The Primordial Black Hole Mass Range

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Abstract

We investigate Primoridal Black Hole (PBH) formation by which we mean black holes produced in the early universe during radiation domination. After discussing the range of PBH mass permitted in the original mechanism of Carr and Hawking, hybrid inflation with parametric resonance is presented as an existence theorem for PBHs of arbitrary mass. As proposed in arXiv:1510.00400, PBHs with many solar masses can provide a solution to the dark matter problem in galaxies. PBHs can also explain dark matter observed in clusters and suggest a primordial origin for supermassive black holes in galactic cores.

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1 Introduction

According to global analyses of the cosmological parameters one quarter, or slightly more, of the energy of the universe is in the form of dark matter whose constituent is the subject of the present paper. Recently it has been proposed [1] that the dark matter constituents are black holes with masses many times the mass of the Sun. In a galaxy like the Milky Way, the proposal is that residing in the galaxy are between ten million and ten billion black holes with masses between one hundred and one hundred thousand solar masses.

Black holes in this range of masses are commonly known as Intermediate Mass Black Hole (IMBHs) since they lie above the masses of stellar-mass black holes and below the masses of the supermassive black holes. It has long been mysterious why there is a mass gap between stellar-mass and supermassive black holes. If the proposed solution of the dark matter problem is correct, it will answer this old question.

There is irrefutable evidence for stellar-mass black holes from observations of X-ray binaries. Such systems were first emphasized in [2] then further studied in [3]. All the known stellar-mass black holes are members of X-ray binaries. The first was discovered over fifty years ago in 1964 in Cygnus X-1 and many stellar-mass black holes have since been discovered from studies of X-ray binaries, with masses in a range between $5M_{\odot}$ and $100M_{\odot}$, where the first-discovered Cygnus X-1 is at about $15M_{\odot}$.

There is irrefutable observational evidence also for supermassive black holes from the observations of fast-moving stars around them and such stars being swallowed or torn apart by the strong gravitational field. The first discovered SMBH was naturally the one, Sag A^* , at the core of the Milky Way which was discovered in 1974 and has mass $M_{SagA*} \sim 4.1 \times 10^6 M_{\odot}$. SMBHs discovered at galactic cores include those for galaxies named M31, NGC4889, among many others. The SMBH at the core of the nearby Andromeda galaxy (M31) has mass $M = 2 \times 10^8 M_{\odot}$, fifty times M_{SagA*} . The most massive core SMBH so far observed is for NGC4889 with $M \sim 2.1 \times 10^9 M_{\odot}$. Some galaxies contain two SMBHs in a binary, believed to be the result of a galaxy merger. Quasars contain black holes with even higher masses up to at least $4 \times 10^{10} M_{\odot}$.

We note historically that dark matter was first discovered by Fritz Zwicky [4,5] in 1933 in the Coma Cluster, and its presence in galaxies was demonstrated convincingly by Vera Rubin in the 1960s and 1970s from the rotation curves of many galaxies [6]. Rubin has more recently made a prescient remark about not liking a universe filled with a new kind of elementary particle and we shall return to this, with the full quote, at the end of our final discussion.

Regarding the PBH mass range, the purpose of the present article is to convince the reader that the possible PBH masses extend upwards to many solar masses and above, far beyond what was was thought possible not many years ago when ignorance about PBHs

with many solar masses probably prevented the MACHO [7] and EROS [8] Collaborations from discovering all the dark matter.

The plan of the present paper is that in Section 2 we review the original implementation \dot{a} la Carr and Hawking of PBH formation. In Section 3 we shall discuss parametric resonance in hybrid inflation which can produce PBHs with arbitrary mass. In Section 4 possible implications are discussed especially for dark matter but also for galactic-core supermassive black holes and unassociated black holes. In Section 5 there is some final discussion.

2 PBHs à la Carr and Hawking

If all black holes were formed by gravitational collapse then black holes with $M_{BH} \ll M_{\odot}$ would be impossible because stars powered by nuclear fusion cannot be far below $M = M_{\odot}$. It was first suggested by Zel'dovich [9, 10] and by Hawking [11] that black holes can be produced in the early stages of the cosmological expansion [12].

Such PBHs are of special interest for several reasons. Firstly, they are the only type of black hole which can be so light, down to $10^{12}kg \sim 10^{-18}M_{\odot}$, that Hawking radiation might conceivably be detected $^{\#2}$. Secondly, PBHs in the intermediate-mass region $100M_{\odot} \leq M_{IMBH} \leq 10^{6}M_{\odot}$ can provide the galactic dark matter. Thirdly, supermassive PBHs with $M_{SMBH} \geq 10^{6}M_{\odot}$ can play a role at galactic centers and provide some of the cluster dark matter.

The mechanism of PBH formation involves large fluctuations or inhomogeneities. Carr and Hawking [13] argued that we know there are fluctuations in the universe in order to seed structure formation and there must similarly be fluctuations in the early universe. Provided the radiation is compressed to a high enough density, meaning to a radius as small as its Schwarzschild radius, a PBH will form. Because the density in the early universe is extremely high, it is very likely that PBHs will be created. The two necessities are high density which is guaranteed and large inhomogeneities.

During radiation domination

$$a(t) \propto t^{1/2} \tag{1}$$

and

$$\rho_{\gamma} \propto a(t)^{-4} \propto t^{-2} \tag{2}$$

 $^{^{\#2}}$ We shall, however, confirm at the end of this Section that such detection is impracticable.

Ignoring factors O(1), as we shall do throughout this paper, and bearing in mind that the radius of a black hole is

$$r_{BH} \sim \left(\frac{M_{BH}}{M_{Planck}^2}\right) \tag{3}$$

with

$$M_{Planck} \sim 10^{19} GeV \sim 10^{-8} kg \sim 10^{-38} M_{\odot}$$
 (4)

and using the Planck density ρ_{Planck}

$$\rho_{Planck} \equiv (M_{Planck})^4 \sim (10^{-5}g)(10^{-33}cm)^{-3} = 10^{94}\rho_{H_2O} \tag{5}$$

the density of a general black hole $\rho_{BH}(M_{BH})$ is

$$\rho_{BH}(M_{BH}) \sim \left(\frac{M_{BH}}{r_{BH}^3}\right) = \rho_{Planck} \left(\frac{M_{Planck}}{M_{BH}}\right)^2 \sim 10^{94} \rho_{H_2O} \left(\frac{10^{-38} M_{\odot}}{M_{BH}}\right)^2 \tag{6}$$

which means that for a solar-mass black hole

$$\rho_{BH}(M_{\odot}) \sim 10^{18} \rho_{H_2O} \tag{7}$$

while for a billion solar mass black hole

$$\rho_{BH}(10^9 M_{\odot}) \sim \rho_{H_2O}.$$
(8)

and above this mass the density falls as M_{BH}^{-2} .

The mass of the Carr-Hawking PBH is derived by combining Eqs. (2) and (6). We see from these two equations that M_{PBH} grows linearly with time and using Planckian units or Solar units we find respectively

$$M_{PBH} \sim \left(\frac{t}{10^{-43}sec}\right) M_{Planck} \sim \left(\frac{t}{1sec}\right) 10^5 M_{\odot}$$
 (9)

which implies, if we perversely insisted on PBH formation before the electroweak phase transition, $t < 10^{-12}s$, that

$$M_{PBH} < 10^{-7} M_{\odot} \tag{10}$$

The incorrect upper bound in Eq.(10) explains historically why the MACHO searches around 2000 [7,8], inspired by the 1986 suggestion of Paczynski [14], lacked motivation to pursue searching beyond $100M_{\odot}$ because it was thought incorrectly at that time that PBHs were too light. It was known correctly that the results of gravitational collapse of normal stars, or even large early stars, were below $100M_{\odot}$. Supermassive black holes with $M > 10^6 M_{\odot}$ such as $SagA^*$ in the Milky Way were beginning to be discovered in galactic centers but their origin at that time was mysterious. We shall discuss this again later in the paper. Hawking radiation implies that the lifetime for a black hole evaporating *in vacuo* is given by the cubic formula

$$\tau_{BH} \sim \left(\frac{M_{BH}}{M_{\odot}}\right)^3 \times 10^{64} years \tag{11}$$

so that to survive for the age 10^{10} years of the universe, there is a lower bound on M_{PBH} to augment the upper bound in Eq.(10), giving as the full range of Carr-Hawking PBHs:

$$10^{-18} M_{\odot} < M_{PBH} < 10^{-7} M_{\odot} \tag{12}$$

The lowest mass Carr-Hawking PBH in Eq.(12) has the extraordinary density $\rho \sim 10^{58} \rho_{H_2O}$. It has the radius of a proton and the mass of ten thousand aircraft carriers $^{\#3}$.

The Hawking temperature $T_H(M_{BH})$ of a black hole is

$$T_H(M_{BH}) = 6 \times 10^{-8} K\left(\frac{M_{\odot}}{M_{BH}}\right)$$
(13)

which would be above the CMB temperature, and hence there would be outgoing radiation for all of the cases with $M_{BH} < 2 \times 10^{-8} M_{\odot}$. Hypothetically, if the dark matter halo were made entirely of the brightest possible (in terms of Hawking radiation) $10^{-18} M_{\odot}$ PBHs, the expected distance to the nearest PBH would be about 10^7 km. Although the PBH temperature, according to Eq. (13) is $\sim 6 \times 10^{10} K$, the inverse square law renders the intensity of Hawking radiation too small, by many orders of magnitude, to allow its detection by any foreseeable apparatus on Earth.

3 Parametric Resonance in Hybrid Inflation

The original Carr-Hawking mechanism produces PBHs with masses in the range up to $10^{-7}M_{\odot}$. In this Section we shall exhibit formation of PBHs by a different mechanism. As discussed, PBH formation requires very large inhomogeneities. Here we shall merely illustrate how to produce inhomogeneities which are exponentially large.

In a single inflation, no exceptionally large density perturbation is expected. Therefore we use two-stage hybrid inflation with respective fields called [15], inflaton and waterfall. The idea of parametric resonance is that after the first inflation mutual couplings of the inflaton and waterfall fields cause both to oscillate wildly and produce perturbations which

^{#3}The radiation domination ends at $t \sim 47ky \sim 10^{12}sec$ which permits, according to Eq.(20), a PBH with mass $10^{17}M_{\odot}$. This has Schwarzschild radius $\sim 10^4pc$, Hawking temperature, according to Eq.(13), of $\sim 6 \times 10^{-25}K$, and density, according to Eq.(6), of $\sim 10^{-16}g/cm^3$. Such a possible primordial superduper-massive black hole would be a hundred times the mass of the Virgo cluster and one millionth the total mass of the visible universe. Such an object might be unassociated with any galaxy or cluster of galaxies.

grow exponentially. The secondary (waterfall) inflation then stretches further these inhomogeneities, enabling production of PBHs with arbitrarily high mass. The specific model provides an existence theorem to confirm that arbitrary mass PBHs can be produced. The resulting mass function is spiked, but it is possible that other PBH production mechanisms can produce a smoother mass function, as deserves further study.

We follow [16] in using a supergravity framework, defining by S the inflaton superfield and by $\Psi, \bar{\Psi}$ the waterfall superfields. The superpotential is

$$W = S\left(\mu^2 + \frac{(\bar{\Psi}\Psi)^2}{M^2}\right) \tag{14}$$

in which μ is the inflation scale and M is a cut-off.

The Kahler potential is

$$K = |S|^2 + |\Psi|^2 + |\bar{\Psi}|^2 \tag{15}$$

and from Eqs.(14) and (15) the potential is

$$V(\sigma,\psi) \sim \left(1 + \frac{\sigma^4}{8} + \frac{\psi^2}{2}\right) \left(-\mu^2 + \frac{\psi^4}{4M^2}\right)^2 + \frac{\sigma^2\psi^6}{16M^4}$$
(16)

where we have defined $\psi = 2\Re(\Psi)$ and $\sigma = \sqrt{2}\Re(S)$ with $\Re \equiv$ real part.

Stationarizing Eq.(16) gives vacua at $\sigma = 0$ and $\psi = 2\sqrt{\mu M}$. For the case $\sigma > \sqrt{\mu M}/2$ there is a σ -dependent minimum for ψ at

$$\psi_0 \sim \left(\frac{2}{\sqrt{3}}\right) \left(\frac{\mu M}{\sigma}\right).$$
(17)

Because ψ has a large mass, it rolls to ψ_0 and integrating it out results in the potential

$$V(\sigma) = \mu^4 \left(1 + \frac{\sigma^4}{8} - \frac{2}{27} \frac{\mu^2 M^2}{\sigma^4} \right) = \mu^4 + \frac{\mu^4}{8} \left(\sigma^4 - \sigma_0^4 \left(\frac{\sigma_0}{\sigma} \right)^4 \right)$$
(18)

in which $\sigma_0 = \sqrt{2}/3^{\frac{3}{8}} (\mu M)^{\frac{1}{4}}$. So long as the first term in Eq.(18) is largest, the inflaton slow rolls.

After this inflation, the σ and ψ fields oscillate, decaying into their quanta via their self and mutual couplings. Specific modes of σ and ψ are amplified by parametric resonance.

From Eq.(18), we may write the equation of motion for a Fourier mode σ_k as

$$\sigma_k^{''} + 3H\sigma_k^{'} + \left[\frac{k^2}{a^2} + m_\sigma^2 + 3m_\sigma^2 \frac{\tilde{\psi}}{\sqrt{\mu M}} \cos(m_\sigma t)\right] \sigma_k \sim 0 \tag{19}$$

where we defined $m_{\sigma} = \sqrt{8\mu^3/M}$. and $\tilde{\psi}$ is the amplitude of ψ oscillations.

Eq.(19) is recognized to be of Mathieu type with the required exponentially-growing solutions. Numerical solution shows that the peak wave number k_{peak} is approximately linear in m_{σ} . The resultant PBH mass, the horizon mass when the fluctuations re-enter the horizon, is approximately

$$M_{PBH} \sim 1.4 \times 10^{13} M_{\odot} \left(\frac{k_{peak}}{Mpc^{-1}}\right)^{-2}$$
 (20)

Explicit plots were exhibited in [16] for the cases $M_{PBH} = 10^{-8} M_{\odot}, 10^{-7} M_{\odot}$ and $10^5 M_{\odot}$ but it was checked that the parameters can be chosen to produce arbitrary PBH mass.

In this production mechanism based on hybrid inflation with parametric resonance, the mass function is sharply spiked at a specific mass region. Whether such a mass function is a general feature of PBH formation, or is only a property of this specific mechanism, merits further study.

4 Dark Matter and Supermassive Black Holes

In Section 2 we discussed the method of producing PBHs proposed by Carr and Hawking. Insisting that the production take place before the electroweak phase transition, and bearing in mind the survival to the age of the universe from Hawking radiation, led us to a range of possible PBH masses from $10^{-18} M_{\odot}$ to $10^{-7} M_{\odot}$.

In Section 3, using a different production mechanism based on parametric resonance in hybrid inflation this was augmented to a much bigger mass range

$$10^{-18} M_{\odot} < M_{PBH} < 10^{17} M_{\odot} \tag{21}$$

which adds to Carr-Hawking, *inter alia*, Primordial Intermediate-Mass Black Holes (PIMBHs) in the range

$$10^2 M_{\odot} < M_{PIMBH} < 10^6 M_{\odot}$$
 (22)

and Primordial Supermassive Black Holes (PSMBHs) in the range

$$10^6 M_{\odot} < M_{PSMBH} < 10^{17} M_{\odot}. \tag{23}$$

where we have truncated the upper end at $10^{17} M_{\odot}$ as the heaviest conceivable black hole likely to exist in the Universe.

For dark matter in galaxies, PIMBHs are important, where the upper end may be truncated at $10^5 M_{\odot}$ to stay well away from galactic disk instability [17]. For supermassive black holes in galactic cores, PSMBHs are natural candidates, as they are also for a part of the dark matter in clusters.

4.1 Dark Matter in Galaxies

The dark matter in the Milky Way fills out an approximately spherical halo somewhat larger in radius than the disk occupied by the luminous stars. Numerical simulations of structure formation suggest a profile of the dark matter of the NFW types [18]. The NFW profile is fully independent of the mass of the dark matter constituent.

Our discussion [1] focused on galaxies like the Milky Way and restricted the mass range for the appropriate dark matter to only three orders of magnitude

$$10^2 M_{\odot} < M < 10^5 M_{\odot}$$
 (24)

We shall not repeat the arguments here, just to say that the constituents are Primodial Intermadiate Mass Black Holes, PIMBHs. Given a total dark halo mass of $10^{12} M_{\odot}$, the number N of PIMBHs is between ten million (10⁷) and ten billion (10¹⁰) Assuming the dark halo has radius R of a hundred thousand (10⁵) light years the mean separation \bar{L} of PIMBHs can be estimated by

$$\bar{L} \sim \left(\frac{R}{N}\right) \tag{25}$$

which translates to

$$100ly < \bar{L} < 1000ly \tag{26}$$

which is also an estimate of the distance of the nearest PIMBH to the Earth.

It may be surprising that as many as $10^7 \le N \le 10^{10}$ intermediate-mass black holes in the Milky Way have remained undetected. They could have been detected more than a decade ago had the MACHO Collaboration [7] persisted in its microlensing experiment at Mount Stromlo Observatory in Australia. We shall return to this point in our final discussion.

4.2 Dark Matter in Clusters

The first discovery of dark matter by Zwicky [4,5] was in the Coma cluster which is a large cluster at 99 Mpc containing over a thousand galaxies and with total mass estimated at $6 \times 10^{14} M_{\odot}$ [19]. A nearer cluster at 16.5 Mpc is the Virgo cluster with over two thousand galaxies and whose mass $\sim 10^{15} M_{\odot}$ is also dominated by dark matter, as well as a small amount of X-ray emitting gas [20, 21]. A proof of the existence (if more were needed) of cluster dark matter was provided by the Bullet cluster collision where the distinct behaviors of the X-ray emitting gas which collides, and the dark matter which does not collide, was clearly observable [22].

Since there is not the same disk stability limit as for galaxies, the constituents of the cluster dark matter can involve also PSMBHs up to much higher masses. In the Universe, we may speculate here that there may be unassociated PBHs with any mass up to $10^{17} M_{\odot}$ drifting outside of any galaxy or cluster of galaxies.

4.3 Supermassive Black Holes at Galactic Centres

As mentioned in the Introduction, in the Milky Way there is SMBH, SagA*, with mass $M_{SagA*} \sim 4 \times 10^6 M_{\odot}$. Other galaxies have SMBHs with masses ranging up to $2.1 \times 10^9 M_{\odot}$ (for the galaxy NGC4889). Only a tiny fraction of galaxies have been studied, so the range of galaxies' core SMBHs is likely broader.

A black hole with the mass of $SagA^*$ would disrupt the disk dynamics [17] were it out in the spiral arms but at, or near to, the center of mass it is more stable. $SagA^*$ is far too massive to have been the result of a gravitational collapse, and if we take the view that all black holes either are the result of gravitational collapse or are primordial then the galaxies' core SMBHs must be primordial. This offers a new explanation of their origin.

4.4 Galaxy formation

If our discussion is correct, it provides a clear time-ordering for galaxy formation that the dark matter precedes star formation by half a billion years. Let us consider the history of the Milky Way.

The constituents of the Milky Way's dark matter halo, PIMBHs, were produced in the era of radiation domination which ended at time $t \sim 47ky$ (red shift $Z \sim 4760$). Only much later, after 560 million years ($Z \sim 8$), did star formation begin in the Milky Way.

In this version of cosmic history, much of the large-scale structure formation including of galaxies such as the Milky Way progresses during the half billion years represented by the red shifts 4760 > Z > 8. This stage importantly involves *only* dark matter. Baryonic astrophysical objects like the Solar System appear only when Z < 8 and are demoted to an afterthought with respect to the Milky Way's formation.

5 Discussion

Such a bold solution of the dark matter problem cries out for experimental verification. Three methods have been discussed: wide binaries, distortion of the CMB, and microlensing. Of these, microlensing seems the most direct and the most promising.

Microlensing experiments were carried out by the MACHO [7] and EROS [8] Collaborations several years ago. At that time, it was believed that PBH masses were below $10^{-7}M_{\odot}$ by virtue of the Carr-Hawking mechanism. Heavier black holes could, it was then believed, arise only from gravitational collapse of normal stars, or heavier early stars, and would have mass below $100M_{\odot}$. For this reason, there was no motivation to suspect that there might be MACHOs which led to higher-longevity microlensing events. The longevity, \hat{t} , of an event is

$$\hat{t} = 0.2yrs \left(\frac{M_{PBH}}{M_{\odot}}\right)^{\frac{1}{2}}$$
(27)

which assumes a transit velocity 200 km/s. Subsituting our extended PBH masses, one finds approximately $\hat{t} \sim 6, 20, 60$ years for $M_{PBH} \sim 10^3, 10^4, 10^5 M_{\odot}$ respectively, and searching for light curves with these higher values of \hat{t} could be very rewarding.

Our understanding is that the original telescope used by the MACHO Collaboration [7] at the Mount Stromlo Observatory in Australia was accidentally destroyed by fire, and that some other appropriate telescopes are presently being used to search for extasolar planets, of which two thousand are already known.

It is seriously hoped that MACHO searches will resume and focus on greater longevity microlensing events. Some encouragement can be derived from this, written this month by a member of the original MACHO Collaboration :

There is no known problem with searching for events of greater longevity than those discovered in 2000; only the longevity of the people!

That being written, convincing observations showing only a fraction of the light curves could suffice? If so, only a fraction of the *e.g.* six years, corresponding to PIMBHs with one thousand solar masses, could well be enough to confirm the theory.

Finally, going back to the 2010 Vera Rubin quote mentioned in the Introduction, it is

"If I could have my pick, I would like to learn that Newton's laws must be modified in order to correctly describe gravitational interactions at large distances. That's more appealing than a universe filled with a new kind of sub-nuclear particle."

If our solution for the dark matter problem is correct, Rubin's preference for no new elementary particle filling the Universe would be vindicated, because for dark matter microscopic particles become irrelevant. Regarding Newton's law of gravity, it would not need modification beyond general relativity theory which is needed for the black holes. In this sense, Rubin did not need to pick either alternative to explain dark matter.

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