# Resolving issues of scaling for gramian based input-output pairing methods

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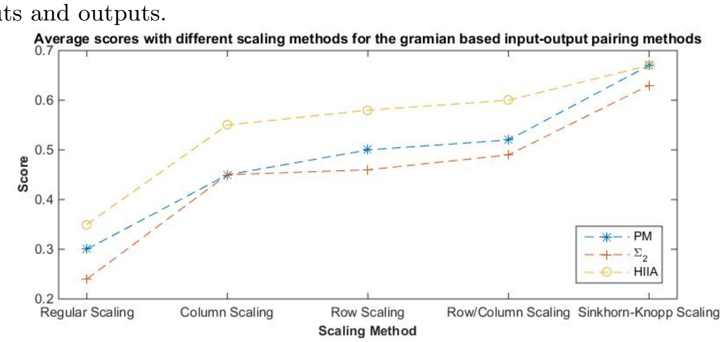
**ABSTRACT**

A key problem in process control is to decide which inputs should control which outputs. There are multiple ways to solve this problem, among them using gramian based measures, which include the Hankel interaction index array, the participation matrix and the $\Sigma_2$ method. The gramian based measures however have issues with input and output scaling. Generally, this is resolved by scaling all inputs and outputs to have equal range. However, we demonstrate how this can result in an incorrect pairing and examine alternative methods of scaling the gramian based measures, using either row or column sums, or by utilizing the Sinkhorn-Knopp algorithm. The benefits of these scaling strategies are first illustrated by applying them to the control structure selection for a heat exchanger network. Then, to more systematically analyze the benefits of the scaling schemes, a multiple input multiple output model generator is used to test the different schemes on a large number of systems. This, along with implementation of automatic controller tuning, allows for a statistical comparison of the scaling methods. This assessment shows considerable benefits to be gained from the alternative scaling of the gramian based measures, especially when using the Sinkhorn-Knopp algorithm. The use of this method also has the advantage that the results are completely independent of the original scaling of the inputs and outputs.

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## 1. Introduction

A common issue in many industrial process control systems is that interaction between different parts of the plant gives rise to a multiple input multiple output (MIMO) system, where the same input may affect multiple outputs, or conversely, the same output is affected by multiple inputs. This is the core of the input-output pairing problem; which control variables should be used to control which process parameters. While one often solves this by matching one input to one output by a decentralized configuration, at times it can be necessary to add additional feed-forward between the inputs, or even implementing MIMO controllers for parts of the system.

There are numerous proposed input-output pairing methods, many of which are discussed by for example, van de Wal and de Jager (2001). The most widely used is probably still the Relative Gain Array (RGA) (Bristol, 1966) and modifications of it, such as the dynamic RGA and the Relative Interaction Array (RIA) (Zhu, 1996). Relatively recently a new group of input-output pairing methods have been introduced, namely the gramian based methods. This group includes the $\Sigma_2$ method (Birk & Medvedev, 2003), the participation matrix (PM) (Conley & Salgado, 2000) and the Hankel interaction index array (HIIA) (Wittenmark & Salgado, 2002). These methods use the controllability and observability gramians to create an interaction matrix which gives a gauge of how much each input affects each output. An attractive property of these interaction matrices is that they can be used to determine both a decentralized controller structure and a sparse structure (a structure which includes feed-forward or MIMO blocks). Moreover, the gramian based measures take into account system dynamics and not only the steady state properties.

The gramian based methods, however, differ from the RGA and its variants in that they suffer from issues of scaling, in the sense that the results of the methods vary depending on input and output scaling. There is a commonly suggested method to solve this problem, presented by for example, Salgado and Conley (2004). We will however demonstrate on a heat exchanger network how this method can be insufficient. Arranz and Birk (2009) have presented a different method to scale the $\Sigma_2$ interaction matrix and here we will examine this method in more detail and also apply it to the PM and the HIIA. Furthermore, we will introduce and examine a new method of scaling, based on the Sinkhorn-Knopp algorithm (Sinkhorn & Knopp, 1967).

To demonstrate the benefit of the new scaling schemes a MIMO model generator will be used. This allows us to generate a large number of systems with predefined statistical properties, which we use for a more general comparison of the different scaling methods. We show that considerable improvements are made with the different scaling schemes, especially when scaling using the method of scaling based on Sinkhorn-Knopp algorithm.

The article is structured as follows: in Section 2 the different scaling methods that will be used are explained. In Section 3 the controller design strategies that will be used are presented, while in Section 4 a example system is used to demonstrate the need for new scaling methods for the gramian based measures. In Section 5 we present an evaluation of the scaling methods. Finally in Section 6 the result is summarised and possible expansions are discussed.

## 2. Gramian based interaction measures, modifications and implementation

### *2.1. Gramian based measures*

The gramian based measures (PM, HIIA and $\Sigma_2$) can be calculated from a system's transfer function matrix (TFM) (Birk & Medvedev, 2003; Conley & Salgado, 2000; Wittenmark & Salgado, 2002). Given a TFM

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & & \\ \vdots & & \ddots & \\ g_{n1}(s) & & & g_{nn}(s) \end{bmatrix}$$

each measure generates an interaction matrix (IM). For the HIIA and $\Sigma_2$ it is generated by

$$[\Gamma]_{ij} = \frac{||g_{ij}(s)||}{\sum_{kl} ||g_{kl}(s)||}$$

using the Hankel norm and 2-norm for the HIIA and $\Sigma_2$, respectively. The PM is derived in a similar fashion, but it uses the squared Hilbert-Schmidt norm, i.e. the IM is generated by

$$[\Gamma]_{ij} = \frac{||g_{ij}(s)||_{HS}^2}{\sum_{kl} ||g_{kl}(s)||_{HS}^2}.$$

Once an IM is generated, a decentralized pairing is generated by choosing the pairing that yields the largest sum of elements from the IM. For efficient implementation in finding which pairing yields the largest sum of elements one can for example use the hungarian algorithm (Fatehi, 2011).

### *2.2. Scaling of the IMs*

An issue with these three methods is that the interaction matrix will be effected by the scaling of the inputs and outputs such that different scalings may yield different results. Generally, this is handled by scaling the input and outputs to range 0 to 1, setting zero to the lowest value they are likely to reach and 1 to the highest value (Salgado & Conley, 2004). However, this scaling is at times insufficient, and we will present a few ways in which the IMs could be rescaled for improved results.

#### *2.2.1. Row or Column scaling*

Each column in the IM corresponds to the interactions from one input, while each row corresponds to the interactions affecting one output. If one column contains significantly less interaction than the other columns (as may be the case if one input is relatively poorly suited for control), little importance will be given to the decision of which output should be controlled by this input. This may lead to a poor input-output pairing as will be demonstrated with an example in Section 4. One way to resolve this would be to normalize the columns, that is to divide the elements in each column of the IM by the corresponding column sum. This ensures that, when conducting the pairing algorithm, equal importance is given to each input. In the new IM the scaled elements would become

$$[\Gamma_c]_{ij} = \frac{[\Gamma]_{ij}}{\sum_{k=1}^{N} [\Gamma]_{kj}},$$

where $\Gamma_c$ is an interaction matrix with normalized columns. If we instead wish to ensure that equal importance is given to each output, we can instead normalize the rows, which gives a interaction measure defined by

$$[\Gamma_r]_{ij} = \frac{[\Gamma]_{ij}}{\sum_{k=1}^{N} [\Gamma]_{ik}}.$$

#### *2.2.2. Choosing between row and column scaling*

It may be difficult to determine if it is preferable to scale by rows or columns. We propose an approach to scaling that tries to determine which is the most appropriate for a given IM. In this approach the column sums and row sums were first calculated. If the smallest sum is a row sum, then the rows are scaled, and otherwise the columns are scaled.

#### *2.2.3. Sinkhorn-Knopp algorithm*

By scaling the columns or rows we can guarantee that equal importance is given to either each input or each output when determining pairing. If we, however, wish to have both the columns and rows scaled we can use the Sinkhorn-Knopp algorithm. This algorithm combines row and column scaling by alternating between normalizing the rows and normalizing the columns. In cases where the matrix can be made to have positive elements on the diagonal (as is always the case with gramian based measures) this algorithm is guaranteed to converge to a matrix that will have both rows and columns normalized (Sinkhorn & Knopp, 1967). While the Sinkhorn-Knopp algorithm can be implemented by simply alternating between dividing the elements in each column of the IM by the corresponding column sum and dividing the elements in each row by the corresponding row sum, it can also be implemented as described by Knight (2008), i.e.

$$\begin{aligned}
r_0 &= e \\
c_{k+1} &= \mathcal{D}(\Gamma^T r_k)^{-1} e \\
r_{k+1} &= \mathcal{D}(\Gamma c_{k+1})^{-1} e, \\
\Gamma_{SK} &= \mathcal{D}(r)\Gamma\mathcal{D}(c). \\
\epsilon_k &= ||c_k \circ \mathcal{D}(c_{k+1})^{-1} - e||_1,
\end{aligned}$$

where $\circ$ denotes element-wise multiplication, $e$ is a vector of ones, and $\mathcal{D}(x)$ turns a vector into a diagonal matrix by creating a matrix with the elements of the vector on its diagonal. $\epsilon_k$ is how far the IM ($\Gamma_{SK}$) is from being perfectly scaled (that is having both column and row sums of one), which can be used as a stopping criterion.

Scaling the IMs with the Sinkhorn-Knopp algorithm has the additional benefit of removing the impact of input and output scaling on the IMs. Using the Sinkhorn-Knopp algorithm to scale the system will yield the same IM, regardless of what the original scaling of the system was.

While the Sinkhorn-Knopp algorithm ensures that all inputs and outputs are given equal importance, this is not necessarily what is desired. Some outputs may be particularly important to control well. However, as the Sinkhorn-Knopp algorithm normalizes the entire IM, it can be used to establish a baseline to which further scaling can be done. After scaling using the Sinkhorn-knopp algorithm the user can increase the emphasis on finding a good match for a specific output or input, by multiplying their respective column or row by a factor larger than one.

### *2.3. Niederlinski Index*

The Niederlinski Index (NI) can be used to determine a necessary condition for a decentralized closed loop system to be stable (Grosdidier, Morari, & Holt, 1985). Consider a system described by a TFM $G(s)$ controlled by a decentralized and diagonal controller $C(s)$ with integral action. If $G(s)$ is stable, $G(s)C(s)$ is proper, and all SISO control loops (created by opening the other loops) are stable, a necessary condition for the existence of a stable control scheme with integral action is

$$NI = \frac{\det[G(0)]}{\prod_{i=1}^{n} g_{ii}(0)} \geq 0,$$

where $g_{ii}(0)$ refers to the diagonal elements of $G(0)$. Here, we will use the NI in combination with the gramian based method. That is to say that we will discard solutions which have a negative NI, even if they have the highest total interaction, and instead choose the solution with the highest interaction among those that have a positive NI.

### *2.4. Sparse Controller*

The gramian based IMs can also be used to generate a sparse controller. To do this we first start by deriving the pairing for the decentralized controller, as described previously. Then the system is examined for the possibility to use decoupling feed-forward. To understand how this works, we begin by examining a 3 by 3 system, i.e.

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) & G_{13}(s) \\ G_{21}(s) & G_{22}(s) & G_{23}(s) \\ G_{31}(s) & G_{32}(s) & G_{33}(s) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}.$$

Let us assume that the inputs and outputs have been ordered such that our decentralized controller design decided on a diagonal pairing where $y_i$ is controlled by $u_i$ for $\forall i$. Now, $u_1$ will also affect $y_2$ and $y_3$ by $G_{21}(s)$ and $G_{31}(s)$, respectively. If $u_1$ affects $y_3$ to such an extent that it poses a problem, this can ideally be resolved by using the feed-forward

$$u_3 = u_3^* - \frac{G_{31}(s)}{G_{33}(s)} u_1, \tag{1}$$

where $u_3^*$ is the control signal from the decentralized controller and we assume $\frac{G_{31}(s)}{G_{33}(s)}$ is stable and proper. If we implement this feed-forward loop we will have removed the direct effect of $u_1$ on $y_3$. However, there are other consequences of this implementation since the change of $u_3$ will also affect $y_1$ and $y_2$. If these interactions are significant the feed-forward loop might do more harm than good. Having this in mind, we examine how the IM can be used to determine when feed-forward might be appropriate. Consider an interaction matrix

$$\Gamma = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1N} \\ \vdots & \ddots & \vdots \\ \gamma_{N1} & \cdots & \gamma_{NN} \end{bmatrix}.$$

First we choose the elements for the decentralized pairing as described previously and assume, without loss of generality, that the pairing elements are on the diagonal. After this, we look in the interaction matrix for large elements not yet selected for pairing. However, implementing feed-forward on the corresponding inputs needs to be weighed against other potential interactions. For example, assume that $\gamma_{N1}$ is a large value and thus $u_1$ is a potential candidate for feed-forward. However, as described in the example, this will impact $u_N$, which will not only impact $y_N$, but also the other outputs. A gauge of the size of this impact is $\sum_{i=1}^{N-1} \gamma_{iN}$. If these values are very large then the IM indicates that adding the described feed-forward on $u_1$ is unwise. To determine the use of feed-forward in the general case we therefore create a new matrix $IM^*$, whose elements are defined by

$$\gamma_{ij}^* = \gamma_{ij} - \rho \sum_{\substack{k=1 \\ k \neq i}}^{N} \gamma_{ki},$$

where $\rho$ is a tuning parameter. With this new IM, the largest elements where $i \neq j$ are chosen for feed-forward until the sum of elements chosen (both for control and feedforward) is larger than 0.7, a rule of thumb for gramian based measures (Salgado & Conley, 2004). However, as feed-forward increases controller complexity it is only implemented if it seems likely that it will have a positive impact. This is determined by checking if $\gamma_{ij}^* > 0$ in which case feed-forward is considered appropriate, and otherwise it is not implemented. Further precautions also have to be taken to avoid implementing an unstable or non-proper feed-forward block.

## 3. Control schemes

### *3.1. Lambda controller tuning*

For the investigations presented here, lambda tuned PI controllers will be used since both are commonly used in industry. The lambda method (Panagopoulos, Hägglund,

& Åström, 1997) is a two step procedure where the first step is to approximate the transfer function by a first order system with dead time, i.e,

$$G^*(s) = \frac{K}{1 + Ts} e^{-Ls}.$$

Using the PI controller structure

$$C(s) = K_p(1 + \frac{1}{T_i s})$$

the controller parameters are derived from $G^*(s)$ according to

$$K_p = \frac{1}{K} \frac{T}{L + \lambda}$$

$$T_i = T$$

$$\lambda = \eta T,$$

where $\lambda$ is the target time constant of the closed loop system, and $\eta$ is a tuning parameter that will later be used to tune $\lambda$.

### *3.2. IMC controller*

An alternative to lambda tuned controllers, is to use IMC tuning, which uses a model of the system to cancel out as much of the system dynamics as possible. An IMC controller can be implemented in the following way (Rivera, Morari, & Skogestad, 1986). Given a stable transfer function model $\tilde{g}$ of the system, one starts by factorizing the model into two parts:

$$\tilde{g} = \tilde{g}_+ \tilde{g}_-$$

such that $\tilde{g}_+$ contains the delays and the non minimum phase zeros of $\tilde{g}$, while $\tilde{g}_-$ contains the remaining dynamics. This ensures that $\tilde{g}_-^{-1}$ is stable. A controller can then be implemented as

$$C = \frac{f \tilde{g}_-^{-1}}{1 - f \tilde{g}_+}$$

where,

$$f = \frac{1}{(1+\epsilon s)^q}$$

is a user designed filter, $\epsilon$ is a tuning parameter and $q$ is chosen such that the controller is proper. When implementing IMC we will chose

$$\epsilon = \eta Z$$

where $Z$ is the largest time constants of the model's non-minimum phase zeros and $\eta$ is a tuning parameter. For minimum phase systems we will instead chose $\epsilon$ as in lambda tuning, i.e.

$$\epsilon = \eta T,$$

where $T$ is the time constant of the system when approximated by a first order system.

## 4. An illustrative example

To demonstrate some of the issues that can arise with the gramian based measures, we examine a heat exchanger network (HEN), which is a modified version of the configuration designed using pinch technology in Case study 1 by Escobar and Trierweiler (2013). The modifications, implemented to make the HEN interesting from a control configuration selection perspective, amount to removing heaters and coolers and instead adding one more heat exchanger (HE) connected to a new stream (C3). The modified HEN can be seen in Figure 1. The specifications for streams H1-H2 and C1-C2 are the same as in Escobar and Trierweiler (2013), and the new stream C3 has a flow capacity of 11 kW/K. The goal is to control the output temperatures T1 to T4 using bypasses over the heat exchangers U1 to U4. T5 is assumed to be controlled further downstream and is thus not necessary to control here.

### *4.1. Heat exchanger models*

The heat exchangers are modeled as a series of mixing tanks (15 mixing tanks were used to model U4, while 10 were found to be sufficient for the other HEs), as described by the multi-cell model by Mathisen, Skogestad, and Wolff (1991). Moreover, the heat transfer coefficients have been modified from those in Escobar and Trierweiler (2013) to compensate for not using the logarithmic temperature differences, as discussed by Mathisen, Morari, and Skogestad (1994). All heat exchangers are modeled to have a residence time of 10 seconds. Pipe residence time and heat losses are not included in the model.

The actuators are controlled bypasses on the hot side of heat exchangers U1 to U4. The HEs areas were therefore increased to retain the same steady-state temperatures when they bypass 10 percent of the stream, which consequently defines their stationary operating points. The complete system model was implemented in Matlab/Simulink

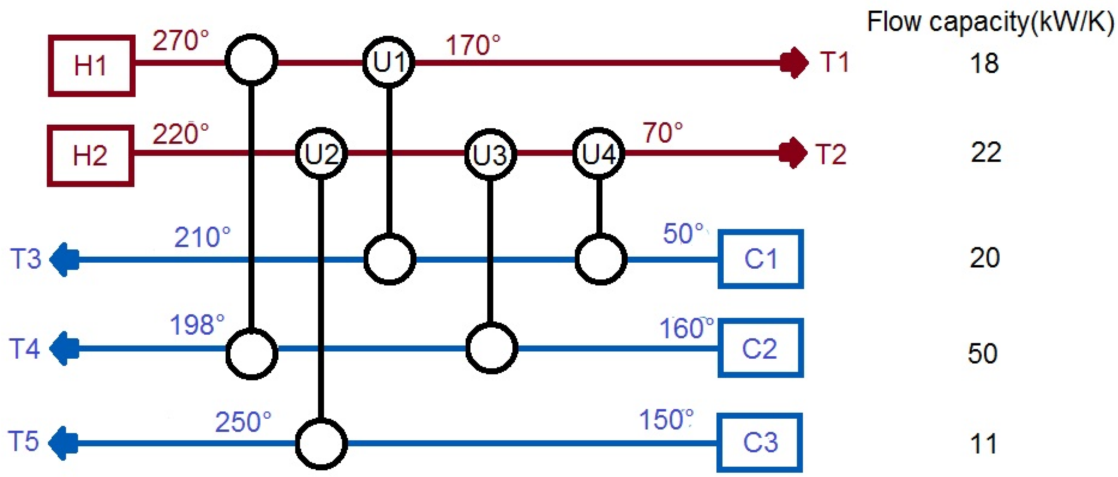


**Figure 1.** The studied heat exchanger network

and linearised with the bypass flow ratio on U1 to U4 as inputs and the temperatures T1 to T4 as outputs.

### *4.2. The pairing problem*

On the linearised model, different input output pairing algorithms were applied. The three previously mentioned gramian based methods were used to derive decentralized control schemes. For comparison purposes we used the classical RGA (Bristol, 1966) and the more recent ILQIA (Halvarsson, 2010) to derive alternative control schemes . The recommended pairing for each method is shown in Table 1, where we note that all the gramian based methods suggest the same pairing different from the ILQIA and the RGA. To compare the methods decentralized PI control schemes were implemented and tuned using the lambda method applied on the open loop subsystems. Each of the control configurations was simulated on the nonlinear system both with reference steps on the output temperature of streams H1, H2, C1 and C2 and with disturbances on the input temperature and flow rate of streams H1, H2, C1 and C2. The size of the reference steps were two degrees and the size of the disturbances on the input temperatures were negative two degrees. For flow rate the system was tested with a decrease of 5% on the flow rate of H1 and H2, and with an increase of the flow rate of 5% on C1 and C2. These disturbances were chosen to be of a magnitude and direction that is possible for the system to completely compensate for. The simulations ran for 1000 seconds after the reference or disturbance step to fully observe its impact. For assessment, the mean quadratic deviation from the reference was devised as a cost, allowing comparison of the different pairing schemes. This was repeated for controllers designed with different values of $\eta$, and the result of those simulations are presented in Table 2.

A few conclusions can be drawn from these results. We can see that for aggressive control schemes, all controller schemes fail, resulting in an undamped oscillatory system. This is not unexpected as there are obvious limits on the actuators (they cannot bypass more than 100% of the stream or less than 0%), and therefore the controllers

**Table 1.** The results for the pairing suggestions for the HEN

| | RGA | PM | HIIA | $\Sigma_2$ | ILQIA |
|---|---|---|---|---|---|
| T1 | U3 | U1 | U1 | U1 | U1 |
| T2 | U4 | U4 | U4 | U4 | U4 |
| T3 | U1 | U2 | U2 | U2 | U3 |
| T4 | U2 | U3 | U3 | U3 | U2 |

**Table 2.** Costs for different controller tuning $\eta$

| $\eta$ | RGA | Gramian based methods | ILQIA |
|---|---|---|---|
| 1 | 235065 | 342149 | 263431 |
| 2 | 136606 | 210002 | 204696 |
| 3 | 95950 | 126502 | 156832 |
| 3,5 | 61002 | 83096 | 120940 |
| 4 | 28570 | 63220 | 101968 |
| 4,5 | 16528 | 41376 | 85213 |
| 5 | 7595 | 16443 | 69522 |
| 5,5 | **1788** | 4771 | 53765 |
| 6 | 1966 | 3533 | 35830 |
| 6,5 | 2154 | 3282 | 21232 |
| 7 | 2344 | **3235** | 9054 |
| 7,5 | 2537 | 3274 | 1570 |
| 8 | 2732 | 3358 | **990** |
| 10 | 3526 | 3897 | 1064 |
| 15 | 5568 | 5706 | 1687 |

need to be somewhat cautious. However the control configuration suggested by the gramian based methods yields a considerably worse control for the best tuning than the ones recommended by the RGA or ILQIA, with a minimum cost of 3235 as opposed to 1788 or 990 (table entries in bold and blue). To examine why, we need to examine the IMs from the gramian based methods:

$$
\begin{aligned}
PM &= \begin{bmatrix} 0.15 & 0.00046 & 0.056 & 0.13 \\ 0 & 0.000014 & 0.0023 & 0.55 \\ 0.058 & 0.00084 & 0.00052 & 0.017 \\ 0 & 0.0091 & 0.026 & 0 \end{bmatrix} \\
HIIA &= \begin{bmatrix} 0.16 & 0.0084 & 0.097 & 0.15 \\ 0 & 0.0015 & 0.018 & 0.29 \\ 0.1 & 0.011 & 0.009 & 0.054 \\ 0 & 0.032 & 0.063 & 0 \end{bmatrix} \\
\Sigma_2 &= \begin{bmatrix} 0.17 & 0.000035 & 0.0054 & 0.0038 \\ 0 & 0.0000039 & 0.00081 & 0.81 \\ 0.0051 & 0.00003 & 0.0000072 & 0.00022 \\ 0 & 0.00036 & 0.00086 & 0 \end{bmatrix}.
\end{aligned}
$$

As can be seen, all the values in the second columns are small compared to the largest values in the other columns. This means that when choosing elements from the matrix little importance is given to the second column. In other words, little importance is given to which element should be controlled by U2. The reason for this is that U2 is not particularly well suited for control compared to the other inputs, and therefore the values in its column are lower. However, one can still clearly see that the IMs suggest that U2 is much better suited for controlling T4 than controlling T3, but as can be seen in Table 1, none of the gramian based measures recommend this pairing, while both non-gramian based methods do. Similarly, we see that the third and fourth rows contain considerably less interaction than the other rows. Hence, less emphasis is placed on selecting a good actuator for T3 and T4 compared to T1 and T2.

**Table 3.** The results for the optimal pairing of HEN using the new scaling

| | RGA | PM column scaling | HIIA column scaling | $\Sigma_2$ column scaling | PM row scaling | HIIA row scaling | $\Sigma_2$ row scaling | ILQIA |
|---|---|---|---|---|---|---|---|---|
| T1 | U3 | U3 | U3 | U1 | U2 | U2 | U1 | U1 |
| T2 | U4 | U4 | U4 | U4 | U4 | U4 | U4 | U4 |
| T3 | U1 | U1 | U1 | U3 | U1 | U1 | U2 | U3 |
| T4 | U2 | U2 | U2 | U2 | U3 | U3 | U3 | U2 |

**Table 4.** Optimal pairing of the HEN using the Sinkhorn-Knopp algorithm

| | PM | HIIA | $\Sigma_2$ |
|---|---|---|---|
| T1 | U3 | U3 | U3 |
| T2 | U4 | U4 | U4 |
| T3 | U1 | U1 | U1 |
| T4 | U2 | U2 | U2 |

It can be argued that this is a matter of scaling. However all the inputs, being bypass percentages, are scaled from 0 to 1 as is the general convention to resolve the issue of input scaling for the gramian based methods (Salgado & Conley, 2004). Moreover, all the outputs are tested with identically sized reference steps and can thus be said to be properly scaled as well. However, in this case this scaling scheme appears to be insufficient. Arranz and Birk (2009) suggest a scaling for the $\Sigma_2$, where each element in the IM is divided by the sum of all the elements in either its column or row. This seems an attractive proposition to resolve this issue as it ensures that either each input or each output is given equal weight. If we scale the PM, $\Sigma_2$ and HIIA interaction measures with this method, we get the configurations shown in Table 3.

As can be seen in Table 3, with column scaling we get the same control configurations as recommended by either the RGA or the ILQIA and consequently a lowered cost according to the assessment. With row scaling we get a new configuration for the HIIA and PM. Testing with this configuration however yields a minimum cost of 3227, so this configuration is not significantly better than the unscaled gramian based configuration.

To conclude, in this case row scaling yields no noteworthy improvement, while column scaling yields a better configuration for all the tested gramian based methods.

### *4.3. Scaling using the Sinkhorn-Knopp algorithm.*

While we could observe here that in this case, normalizing the IMs columns such that each input was given equal weight worked well, while normalizing the rows such that each output was given equal weight yielded poorer results. There is another option, to ensure that the IMs rows and columns all sum up to the same value, and hence all inputs and outputs are given equal weight. This can be achieved by iteratively alternating between scaling the elements by row sum and by column sum, which is the Sinkhorn-Knopp algorithm described in Section 2.2.3. Implementing the Sinkhorn-Knopp algorithm and stopping the algorithm when the error is less than $10^{-3}$ yields the control configurations presented in Table 4.

As can be seen, this resulted in the same configuration as the RGA, which while not being the configuration with the lowest cost, still yielded a considerably better result than the configuration recommended by the unscaled gramian based measures. This is also the same configuration obtained with the PM and HIIA when applying column scaling. However, for the $\Sigma_2$ method column scaling yielded a configuration with a lower cost.

## 5. Large scale assessment of the methods

While the HEN example may have demonstrated potential benefits from rescaling the IMs,a single case study is insufficient to draw general conclusions. To quantitatively assess the methods we have therefore used the MIMO model generator described by Bengtsson and Wik (2017) to generate a large number of linear MIMO-systems, with specifications shown in Table 5. The different scaling methods, i.e. by row sum, column sum, or both using the SK algorithm, were then applied to the generated models and the results were compared to using only the standard scaling. In addition yet another approach was implemented. In this approach one would either scale the rows or the columns, according to the reasoning in Section 2.2.2.

For each control configuration a decentralized control scheme was designed using both internal model control and the lambda method for varying values of $\eta$ (the Matlab function `tfest` was used to approximate the transfer functions as first order systems which were then used for the lambda controller design). The entire feedback system was then tested both by reference step and by load disturbance. For the comparison we define a cost being the squared deviation from the reference for 2000 time units after the reference step or the load disturbance. For each configuration the cost was calculated for values of $\eta$ ranging from 0.1 to 10 and the lowest cost was then saved. Having calculated this cost for each IM, each IM is given a score defined as

$$S = \frac{c_{min}}{c}$$

where $S$ is the score of the IM, $c$ is the IMs cost, and $c_{min}$ is the lowest cost of all IMs for the system. The score was set to zero if the control scheme yielded unstable results. This measure was chosen to normalize the score of each iteration between 0 and 1, to ensure that the results on different systems are comparable.

Three sets of 150 randomly generated systems were assessed having maximum static gains of 10, 100 and 1000 (minimum static gain was always 1). Both decentralized and sparse control schemes (for $\rho = 3$) were implemented. The mean scores are presented in Table 6 and Table 7 for both $\lambda$-tuned controllers and IMC controllers.

The systems generated by the MIMO generator generally contained non-minimum phase transmission zeros. To evaluate how the presence of non-minimum phase dynamics affected the scaling methods, we also tested sets of system without non-minimum phase transmission zeros (still with specifications according to Table 5). The result of this is included in Tables 8 and 9.

The large number of systems investigated allows a statistical evaluation whether the new scalings yield statistically significant improvements or not. Therefore, a t-test for paired samples was performed on the hypothesis that the scaled systems had a higher score than the unscaled system with a 95% confidence interval. This evaluation was carried out on both the systems with feed-forward and without. The statistically significant improvements are highlighted in bold numbers in Tables 6-9.

As can be seen from the tables the scaled IMs fare considerably better than the unscaled IMs. This improvement may be somewhat less pronounced when the gain variation in the TFM is small which is in agreement to what was observed in the HE

**Table 5.** Table showing the MIMO model generatorBengtsson and Wik (2017) settings

| Parameter | Default value |
|---|---|
| **Size** | |
| Number of inputs | 5 |
| Number of outputs | 5 |
| Minimum number of inputs affecting each output | 4 |
| Maximum number of inputs affecting each output | 5 |
| Minimum transfer function order | 1 |
| Maximum transfer function order | 3 |
| Minimum relative degree | 1 |
| Maximum relative degree | 3 |
| **Dynamics** | |
| Maximum static gain | 10-1000 |
| Minimum pole time constant | 1 |
| Maximum pole time constant | 10 |
| Minimum damping for complex poles | 0.1 |
| Distinct time constants | false |
| Basing zeros time constants on poles when possible | true |
| Maximum overshoot percentage | 10 |
| Maximum undershoot percentage | 25 |
| Tolerance when determining overshoot/undershoot | 0.01 |
| Factor used to determine minimum time constant | 100 |
| **Poles and Zeros** | |
| Maximum number of unstable poles | 0 |
| Minimum number of unstable poles | 0 |
| Maximum number of purely imaginary pole pairs | 0 |
| Minimum number of purely imaginary pole pairs | 0 |
| Percentage of unstable poles which are complex | 0 |
| Percentage of stable poles which are complex | 20 |
| Percentage of transfer functions with single integrators | 0 |
| Percentage of transfer functions with double integrators | 0 |
| Percentage of transfer functions with derivatives | 0 |
| Maximum number of non-minimum phase zeros | 4 |
| Minimum number of non-minimum phase zeros | 0 |
| **Delay** | |
| Percentage of transfer functions with delay | 10 |
| Minimum Delay | 0 |
| Maximum Delay | 0.5 |
| Padé approximation order | 2 |

**Table 6.** Average score for the different methods with different gain variation for a reference step for non-minimum phase systems. Bold values indicate statistically significant improvements compared to the system with no scaling.

| Maximum Gain | 10 | 100 | 1000 | 10 | 100 | 1000 |
|---|---|---|---|---|---|---|
| Controller design method | Lambda | | | IMC | | |
| PM | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling | 0.45 | 0.38 | 0.30 | 0.38 | 0.45 | 0.35 |
| Column scaling | **0.57** | **0.54** | **0.45** | **0.48** | **0.58** | **0.51** |
| Row scaling | **0.55** | **0.54** | **0.50** | **0.51** | **0.58** | **0.50** |
| Row/column scaling | **0.60** | **0.59** | **0.52** | **0.51** | **0.64** | **0.51** |
| Sinkhorn-Knopp scaling | **0.61** | **0.64** | **0.67** | **0.58** | **0.71** | **0.67** |
| *Sparse Control* | | | | | | |
| No scaling | 0.46 | 0.39 | 0.31 | 0.38 | 0.43 | 0.40 |
| Column scaling | **0.59** | **0.61** | **0.58** | **0.46** | **0.57** | **0.58** |
| Row scaling | **0.56** | **0.52** | **0.47** | **0.47** | 0.50 | **0.48** |
| Row/column scaling | **0.62** | **0.62** | **0.60** | **0.50** | **0.63** | **0.58** |
| Sinkhorn-Knopp scaling | **0.61** | **0.64** | **0.67** | **0.58** | **0.71** | **0.67** |
| $\boldsymbol{\Sigma_2}$ | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling | 0.42 | 0.37 | 0.24 | 0.41 | 0.40 | 0.33 |
| Column scaling | **0.51** | **0.54** | **0.45** | **0.52** | **0.55** | **0.46** |
| Row scaling | **0.52** | **0.53** | **0.46** | **0.53** | **0.59** | **0.47** |
| Row/column scaling | **0.53** | **0.57** | **0.49** | **0.55** | **0.61** | **0.49** |
| Sinkhorn-Knopp scaling | **0.53** | **0.64** | **0.63** | **0.54** | **0.67** | **0.64** |
| *Sparse Control* | | | | | | |
| No scaling | 0.44 | 0.37 | 0.25 | 0.40 | 0.40 | 0.33 |
| Column scaling | **0.53** | **0.61** | **0.52** | **0.51** | **0.57** | **0.51** |
| Row scaling | **0.50** | **0.51** | **0.47** | **0.52** | **0.53** | **0.41** |
| Row/column scaling | **0.54** | **0.61** | **0.52** | **0.56** | **0.60** | **0.49** |
| Sinkhorn-Knopp scaling | **0.53** | **0.64** | **0.63** | **0.54** | **0.67** | **0.65** |
| **HIIA** | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling | 0.54 | 0.52 | 0.35 | 0.54 | 0.57 | 0.45 |
| Column scaling | **0.60** | **0.61** | **0.55** | 0.58 | **0.66** | **0.58** |
| Row scaling | **0.62** | **0.60** | **0.58** | **0.59** | **0.67** | **0.56** |
| Row/column scaling | **0.65** | **0.63** | **0.60** | **0.59** | **0.69** | **0.59** |
| Sinkhorn-Knopp scaling | **0.64** | **0.66** | **0.67** | **0.62** | **0.72** | **0.67** |
| *Sparse Control* | | | | | | |
| No scaling | 0.53 | 0.54 | 0.37 | 0.54 | 0.56 | 0.45 |
| Column scaling | **0.60** | **0.61** | **0.64** | 0.58 | **0.67** | **0.67** |
| Row scaling | **0.62** | **0.59** | **0.56** | **0.59** | 0.62 | **0.54** |
| Row/column scaling | **0.65** | **0.63** | **0.65** | **0.59** | **0.70** | **0.67** |
| Sinkhorn-Knopp scaling | **0.64** | **0.66** | **0.67** | **0.62** | **0.72** | **0.68** |

**Table 7.** Average score for the different methods with different gain variation for a load disturbance for non-minimum phase systems. Bold values indicate statistically significant improvements compared to the system with no scaling.

| Maximum Gain | 10 | 100 | 1000 | 10 | 100 | 1000 |
|---|---|---|---|---|---|---|
| Controller design method | Lambda | | | IMC | | |
| **PM** | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling | 0.46 | 0.42 | 0.36 | 0.35 | 0.51 | 0.46 |
| Column scaling | **0.58** | **0.61** | **0.56** | **0.45** | **0.65** | **0.64** |
| Row scaling | **0.56** | **0.62** | **0.57** | **0.49** | **0.64** | **0.65** |
| Row/column scaling | **0.61** | **0.66** | **0.61** | **0.49** | **0.70** | **0.65** |
| Sinkhorn-Knopp scaling | **0.61** | **0.71** | **0.78** | **0.56** | **0.80** | **0.82** |
| *Sparse Control* | | | | | | |
| No scaling. | 0.45 | 0.39 | 0.36 | 0.35 | 0.44 | 0.42 |
| Column scaling | **0.56** | **0.59** | **0.57** | **0.43** | **0.57** | **0.57** |
| Row scaling | **0.54** | **0.53** | **0.53** | **0.45** | 0.51 | **0.51** |
| Row/column scaling | **0.60** | **0.63** | **0.62** | **0.47** | **0.61** | **0.56** |
| Sinkhorn-Knopp scaling | **0.61** | **0.71** | **0.78** | **0.56** | **0.80** | **0.82** |
| $\boldsymbol{\Sigma_2}$ | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling. | 0.45 | 0.42 | 0.29 | 0.44 | 0.48 | 0.45 |
| Column scaling | **0.54** | **0.64** | **0.53** | **0.52** | **0.64** | **0.64** |
| Row scaling | **0.56** | **0.61** | **0.55** | **0.56** | **0.68** | **0.62** |
| Row/column scaling | **0.56** | **0.67** | **0.57** | **0.57** | **0.70** | **0.67** |
| Sinkhorn-Knopp scaling | **0.57** | **0.75** | **0.75** | **0.57** | **0.78** | **0.85** |
| *Sparse Control* | | | | | | |
| No scaling. | 0.43 | 0.38 | 0.29 | 0.43 | 0.42 | 0.44 |
| Column scaling | **0.55** | **0.63** | **0.54** | **0.50** | **0.57** | **0.56** |
| Row scaling | **0.51** | **0.52** | **0.51** | **0.54** | **0.52** | 0.48 |
| Row/column scaling | **0.56** | **0.64** | **0.55** | **0.56** | **0.60** | **0.58** |
| Sinkhorn-Knopp scaling | **0.57** | **0.75** | **0.75** | **0.57** | **0.78** | **0.84** |
| **HIIA** | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling. | 0.55 | 0.57 | 0.43 | 0.52 | 0.62 | 0.58 |
| Column scaling | **0.61** | **0.67** | **0.65** | 0.56 | **0.74** | **0.74** |
| Row scaling | **0.63** | **0.67** | **0.67** | **0.58** | **0.75** | **0.72** |
| Row/column scaling | **0.65** | **0.70** | **0.71** | **0.57** | **0.78** | **0.76** |
| Sinkhorn-Knopp scaling | **0.65** | **0.74** | **0.79** | **0.60** | **0.81** | **0.81** |
| *Sparse Control* | | | | | | |
| No scaling. | 0.55 | 0.55 | 0.41 | 0.52 | 0.56 | 0.46 |
| Column scaling | **0.61** | **0.66** | **0.66** | 0.56 | **0.69** | **0.64** |
| Row scaling | **0.63** | **0.64** | **0.60** | **0.58** | **0.66** | **0.60** |
| Row/column scaling | **0.65** | **0.70** | **0.70** | **0.57** | **0.75** | **0.68** |
| Sinkhorn-Knopp scaling | **0.65** | **0.74** | **0.79** | **0.60** | **0.81** | **0.81** |

**Table 8.** Average score for the different methods with different gain variation for a reference step for minimum phase systems. Bold values indicate statistically significant improvements compared to the system with no scaling.

| Maximum Gain | 10 | 100 | 1000 | 10 | 100 | 1000 |
|---|---|---|---|---|---|---|
| Controller design method | Lambda | | | IMC | | |
| **PM** | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling | 0.24 | 0.27 | 0.22 | 0.26 | 0.28 | 0.26 |
| Column scaling | **0.35** | **0.44** | **0.38** | **0.31** | **0.39** | **0.38** |
| Row scaling | **0.36** | **0.41** | **0.32** | **0.35** | **0.41** | **0.39** |
| Row/column scaling | **0.35** | **0.47** | **0.38** | **0.33** | **0.45** | **0.40** |
| Sinkhorn-Knopp scaling | **0.39** | **0.52** | **0.54** | **0.37** | **0.48** | **0.50** |
| *Sparse Control* | | | | | | |
| No scaling. | 0.26 | 0.27 | 0.23 | 0.27 | 0.34 | 0.27 |
| Column scaling | **0.35** | **0.51** | **0.49** | 0.31 | **0.50** | **0.47** |
| Row scaling | **0.36** | **0.44** | **0.39** | **0.35** | **0.44** | **0.41** |
| Row/column scaling | **0.34** | **0.52** | **0.47** | **0.34** | **0.54** | **0.46** |
| Sinkhorn-Knopp scaling | **0.39** | **0.52** | **0.55** | **0.37** | **0.48** | **0.50** |
| $\boldsymbol{\Sigma_2}$ | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling. | 0.66 | 0.46 | 0.34 | 0.64 | 0.50 | 0.40 |
| Column scaling | 0.69 | **0.60** | **0.50** | **0.70** | **0.59** | **0.50** |
| Row scaling | **0.71** | **0.58** | 0.42 | **0.71** | **0.60** | **0.52** |
| Row/column scaling | **0.71** | **0.61** | **0.47** | **0.72** | **0.62** | **0.55** |
| Sinkhorn-Knopp scaling | 0.71 | **0.69** | **0.62** | **0.75** | **0.66** | **0.66** |
| *Sparse Control* | | | | | | |
| No scaling. | 0.70 | 0.50 | 0.39 | 0.70 | 0.55 | 0.43 |
| Column scaling | 0.74 | **0.73** | **0.63** | 0.73 | **0.76** | **0.69** |
| Row scaling | **0.77** | **0.66** | 0.51 | **0.77** | **0.66** | **0.54** |
| Row/column scaling | 0.74 | **0.73** | **0.58** | **0.76** | **0.76** | **0.63** |
| Sinkhorn-Knopp scaling | 0.71 | **0.69** | **0.62** | 0.75 | **0.66** | **0.66** |
| **HIIA** | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling. | 0.36 | 0.41 | 0.29 | 0.34 | 0.38 | 0.30 |
| Column scaling | **0.41** | **0.51** | **0.43** | 0.36 | **0.46** | **0.44** |
| Row scaling | **0.46** | **0.47** | **0.39** | **0.39** | **0.44** | **0.40** |
| Row/column scaling | **0.42** | **0.51** | **0.45** | 0.37 | **0.47** | **0.47** |
| Sinkhorn-Knopp scaling | **0.45** | **0.55** | **0.51** | 0.38 | **0.50** | **0.51** |
| *Sparse Control* | | | | | | |
| No scaling. | 0.36 | 0.43 | 0.44 | 0.34 | 0.41 | 0.35 |
| Column scaling | **0.41** | **0.54** | **0.58** | 0.36 | **0.51** | **0.55** |
| Row scaling | **0.46** | 0.48 | 0.51 | **0.39** | 0.45 | **0.44** |
| Row/column scaling | **0.42** | **0.52** | **0.58** | 0.37 | **0.49** | **0.55** |
| Sinkhorn-Knopp scaling | **0.45** | **0.55** | 0.52 | 0.38 | **0.50** | **0.51** |

**Table 9.** Average score for the different methods with different gain variation for a load disturbance for minimum phase systems. Bold values indicate statistically significant improvements compared to the system with no scaling.

| Maximum Gain | 10 | 100 | 1000 | 10 | 100 | 1000 |
|---|---|---|---|---|---|---|
| Controller design method | Lambda | | | IMC | | |
| PM | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling | 0.22 | 0.32 | 0.32 | 0.29 | 0.39 | 0.35 |
| Column scaling | **0.35** | **0.52** | **0.59** | 0.32 | **0.53** | **0.53** |
| Row scaling | **0.34** | **0.47** | **0.52** | **0.37** | **0.55** | **0.54** |
| Row/column scaling | **0.35** | **0.55** | **0.60** | 0.34 | **0.60** | **0.57** |
| Sinkhorn-Knopp scaling | **0.38** | **0.58** | **0.78** | **0.38** | **0.63** | **0.73** |
| *Sparse Control* | | | | | | |
| No scaling | 0.23 | 0.30 | 0.33 | 0.29 | 0.37 | 0.34 |
| Column scaling | **0.33** | **0.49** | **0.53** | 0.31 | **0.49** | **0.49** |
| Row scaling | **0.32** | **0.44** | **0.47** | **0.36** | **0.49** | **0.51** |
| Row/column scaling | **0.33** | **0.50** | **0.56** | 0.33 | **0.54** | **0.54** |
| Sinkhorn-Knopp scaling | **0.38** | **0.58** | **0.78** | **0.38** | **0.63** | **0.73** |
| $\boldsymbol{\Sigma_2}$ | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling | 0.72 | 0.56 | 0.50 | 0.79 | 0.69 | 0.58 |
| Column scaling | 0.77 | **0.76** | **0.75** | 0.82 | **0.82** | **0.75** |
| Row scaling | 0.77 | **0.72** | **0.68** | 0.83 | **0.78** | **0.75** |
| Row/column scaling | 0.78 | **0.76** | **0.73** | 0.84 | **0.82** | **0.81** |
| Sinkhorn-Knopp scaling | 0.78 | **0.86** | **0.89** | 0.83 | **0.86** | **0.92** |
| *Sparse Control* | | | | | | |
| No scaling | 0.69 | 0.55 | 0.48 | 0.65 | 0.63 | 0.56 |
| Column scaling | **0.78** | **0.73** | **0.69** | **0.76** | **0.76** | **0.66** |
| Row scaling | 0.72 | **0.67** | **0.62** | 0.70 | 0.68 | **0.66** |
| Row/column scaling | **0.78** | **0.72** | **0.69** | **0.75** | **0.74** | **0.71** |
| Sinkhorn-Knopp scaling | **0.78** | **0.86** | **0.89** | **0.83** | **0.86** | **0.92** |
| **HIIA** | | | | | | |
| *Decentralized Control* | | | | | | |
| No scaling | 0.34 | 0.48 | 0.45 | 0.35 | 0.49 | 0.42 |
| Column scaling | **0.40** | **0.60** | **0.66** | 0.38 | **0.60** | **0.63** |
| Row scaling | **0.44** | **0.56** | **0.62** | **0.41** | **0.61** | **0.58** |
| Row/column scaling | **0.41** | **0.59** | **0.68** | 0.38 | **0.62** | **0.68** |
| Sinkhorn-Knopp scaling | **0.45** | **0.62** | **0.75** | 0.40 | **0.65** | **0.75** |
| *Sparse Control* | | | | | | |
| No scaling | 0.34 | 0.46 | 0.41 | 0.35 | 0.45 | 0.37 |
| Column scaling | **0.40** | **0.59** | **0.60** | 0.38 | **0.58** | **0.60** |
| Row scaling | **0.44** | **0.55** | **0.59** | **0.41** | **0.56** | **0.52** |
| Row/column scaling | **0.41** | **0.59** | **0.66** | 0.38 | **0.60** | **0.64** |
| Sinkhorn-Knopp scaling | **0.45** | **0.62** | **0.74** | 0.40 | **0.65** | **0.75** |

example as high gain variation increases the likelihood that a row or column in the IM will have considerably less interaction than the other rows or columns. However, even in the case of low gain variation there are statistically significant improvements for many of the scaling schemes.

One can also clearly see that the scaling method which yielded the best results was the one based on the Sinkhorn-Knopp algorithm, as it consistently yielded the highest score. The benefits of using the Sinkhorn-Knopp algorithm were the most apparent with a high gain variation. With a high gain variation one can see a very pronounced improvement for the PM and $\Sigma_2$ method when using the Sinkhorn-Knopp algorithm compared to the other scaling methods. When using the HIIA, the improvement was somewhat smaller but still considerable.

The scaled systems also fared better both for decentralized and sparse control configurations. This indicates that the scaling methods can be used for both. However the algorithm for implementing feedforward was somewhat cautious, especially for Sinkhorn-Knopp scaling, as can be seen in that there was often little difference between the cost for the sparse and decentralized controllers. This also led to Sinkhorn-Knopp algorithm yielding comparably poor results in the cases where implementing feedforward had a very positive effect, namely reference following for minimum phase systems. A more thorough and individual feedforward implementation for the different scaling methods might have alleviated this issue.

Generally the results are similar regardless whether the controllers are tuned using IMC or the lambda method; in both cases the Sinkhorn-Knopp algorithm generally outperformed the other scaling methods.

Another observation is that without scaling it appears as if HIIA, on average, gave better results than PM and $\Sigma_2$ for non-minimum phase systems, while the $\Sigma_2$ method was superior for minimuim phase systems. However, SK-scaling has an effect on the performance in many cases exceeding the difference between the different methods.

Some caveats are however necessary. Only two methods for automatic controller design was tested; it is possible that other control schemes might yield different results. Furthermore, other than changing the gain and the properties of the transmission zeros, only one set of model generator settings was used, modifications to other system properties may possibly also yield different results.

## 6. Conclusions and further work

The gramian based interaction measures (IMs) are well known to be affected by scaling and the standard is therefore to scale all inputs and outputs to an equal range. A heat exchanger network control problem illustrated a case where the conventional scaling scheme failed. It was shown that by adapting the scaling of the IMs, this issue could be resolved. A few possible scaling methods were tested on a large number of systems using a MIMO model generator. It was found that all the tested scaling methods statistically improved the performance of the pairing methods consistently when designing a decentralized, as well as a sparse controller. The scaling method that yielded the best results was the one based on the Sinkhorn-Knopp algorithm, in particular so when there are large variations in the static gains. In addition, this scaling method has the advantage of yielding results that are identical no matter what the original scaling of the inputs and outputs is.

There is however some room for further research in this field. The automatic implementation of sparse controllers is something that can be explored further, especially when using Sinkhorn-Knopp scaling.

Morover there are other possible control strategies and controller tuning on which the scaling methods can be explored. When testing the scaling methods here we specifically examined the impact of non-minimum phase transmission zeros and differences in static gain. There are of course other system propeties which can be evaluated as well using the same techniques.

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