A Simplified Formulation for the Backward/Forward Sweep Power Flow Method

Paulo M. De Oliveira-De Jesus*

November 26, 2021

Abstract

This paper describes a simplified formulation of the Backward/Forward (BW/FW) Sweep Power Flow applied to radial distribution systems with distributed generation under positive sequence modelling. Proposed formulation was applied in an illustrative test system.

Keywords: Backward/forward sweep, load flow, power flow, distribution system analysis

1 Introduction

Several Backward/Forward (BW/FW) sweep algorithms have been discussed in literature. In 1967, Berg presented a paper which can be considered as the source for the all variants of BW/FW sweep methods [1]. Later, a similar approach was presented in [2] based on ladder network theory. The BW/FW Sweep algorithms use the Kirchhoff laws. Different formulations can be found [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 14]. BW/FW sweep methods typically present a slow convergence rate but computationally efficient at each iteration. Using these methods, power flow solution for a distribution network can be obtained without solving any set of simultaneous equations. In this work, the standard BW/FW sweep power flow is reformulated in convenient form. An illustrative four-bus example is solved.

*Electrical & Electronic Engineering Department, School of Engineering, Universidad de los Andes, Colombia, website: https://power.uniandes.edu.co/pdeoliveira, E-mail: pm.deoliveiradejes@uniandes.edu.co

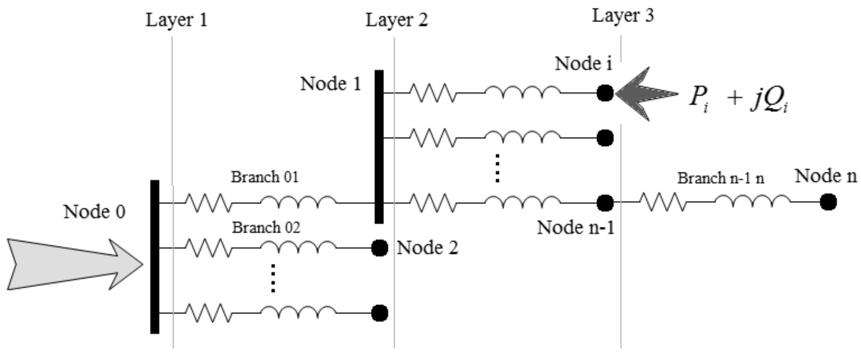

Figure 1: Branch and node numbering of a radial distribution network

2 The Method

The input data of this algorithm is given by node-branch oriented data used by most utilities. Basic data required is: active and reactive powers, nomenclature for sending and receiving nodes, and positive sequence impedance model for all branches.

In the following, the standard BW/FW sweep power flow method is written in matrix notation using complex variables. Branch impedances are stated as a vector $\mathbf{Z}$ corresponding to a distribution line model containing a series positive sequence impedance for line or transformer. Shunt impedances are not considered in this first approach. Fig. 1 shows a radial distribution network with $n+1$ nodes, and n branches and a single voltage source at the root node 0. Branches are organized according to an appropriate numbering scheme (list), which details are provided in [3].

$$\mathbf{Z} = \begin{bmatrix} \overline{Z}_{01} & ... & \overline{Z}_{ij} & ... & \overline{Z}_{mn} \end{bmatrix} \tag{1}$$

where,

$$\overline{Z}_{ij} = R_{ij} + jX_{ij} \quad i,j = 1,...,n \quad i \neq j \tag{2}$$

Bus data is given by

$$\mathbf{S} = \begin{bmatrix} \overline{S}_1 \\ \vdots \\ \overline{S}_i \\ \vdots \\ \overline{S}_n \end{bmatrix} = \begin{bmatrix} P_1 + jQ_1 \\ \vdots \\ P_i + jQ_i \\ \vdots \\ P_n + jQ_n \end{bmatrix} \tag{3}$$

where net nodal active and reactive powers are given by generated and demanded powers:

$$P_i = P_{Gi} - P_{Di} \tag{4}$$

$$Q_i = Q_{Gi} - Q_{Di} \tag{5}$$

The numbering of branches in one layer begins only after all the branches in the previous layer have been numbered. Considering that initial voltages are known: voltage at substation is set $\overline{V}_0 = Vref$ and an initial voltage vector is given by:

$$\mathbf{V}^0 = \left[\begin{array}{ccccc} \overline{V}_1^0 & ... & \overline{V}_i^0 & ... & \overline{V}_n^0 \end{array} \right] \tag{6}$$

The state of the system is reached solving two steps iteratively.

2.1 Step 1 - Backward Sweep

For each iteration k, branch currents are aggregated from loads to origin:

$$\mathbf{J^k} = -\mathbf{T} \cdot \mathbf{I^k} \tag{7}$$

The relationship between nodal currents $\mathbf{I^k}$ and branch currents $\mathbf{J^k}$ is set through an upper triangular matrix $\mathbf{T}$ accomplishing the Kirchhoff Current Laws (KCL). Each element $\overline{I}_i^k$ of $\mathbf{I^k}$ associated to node i is calculated as function of injected powers $\overline{S}_i$ and its voltage profile $\overline{V}_i^k$ as shown below:

$$\overline{I}_i^k = \frac{\overline{S}_i^*}{\overline{V}_i^{k*}} \quad i = 1, ..., n \tag{8}$$

2.2 Step 2 - Forward Sweep

Nodal voltage vector $\mathbf{V}$ is updated from the origin to loads according the Kirchhoff Voltage Laws (KVL), using previously calculated branch currents vector $\mathbf{J}$, branch impedances vector $\mathbf{Z}$:

$$\mathbf{V}^{k+1} = \mathbf{V}_0 - \mathbf{T}^T \cdot \mathbf{D_Z} \cdot \mathbf{J}^k \tag{9}$$

where $\mathbf{V}_0$ is a n-elements vector with all entries set at voltage at origin (swing node) $\overline{V}_0$ and branch impedances $\mathbf{D_Z}$ is the diagonal matrix of vector $\mathbf{Z}$:

Using Eq. 7

$$\mathbf{V}^{k+1} = \mathbf{V}_0 + \mathbf{T}^T \cdot \mathbf{D_Z} \cdot \mathbf{T} \cdot \mathbf{I^k} \tag{10}$$

Updated voltages can be updated using only one equation:

$$\mathbf{V}^{k+1} = \mathbf{V}_0 + \mathbf{TRX} \cdot \mathbf{I^k} \tag{11}$$

where $\mathbf{TRX} = \mathbf{T}^T \cdot \mathbf{D_Z} \cdot \mathbf{T}$

2.3 Convergence

Updated voltages are compared with previous voltages in order to perform convergence check in.

$$\varepsilon \leq |\overline{V}_i^{k+1} - \overline{V}_i^k| \quad i = 1, ..., n \tag{12}$$

3 Illustrative Example: Simply 4-node Network

To illustrate the proposed methodology, it is used the 4-node example shown in Fig. 2. Length of all sections is 1 mile. Load demand at nodes 2 and 3 are 2MW with $\cos\varphi = 1.0$.

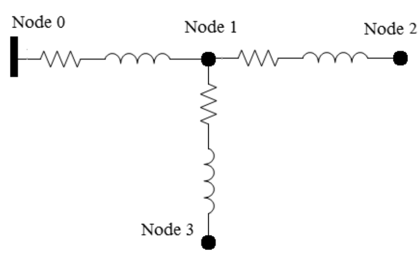

Figure 2: 4-Node Network Topology

Using the following bases S_B =10MW and V_B=12.47kV, data and results are given in per unit. Loads are 0.2 in nodes 2 and 3. Reference voltage at node 0 is $\overline{V}_0 = 1 + j0$ and initial voltages are set $\mathbf{V}^0 = \left[\begin{array}{ccc} 1+0j & 1+0j & 1+0j \end{array}\right]$.

Branches are represented by:

$$\mathbf{Z} = \mathbf{R} + j\mathbf{X} = \begin{bmatrix} .0296 \\ .0296 \\ .0296 \end{bmatrix} + j \begin{bmatrix} .0683 \\ .0683 \\ .0683 \end{bmatrix}$$

Network topology is represented through a 3x3 upper triangular matrix $\mathbf{T}$.

$$\mathbf{T} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, $\mathbf{D_Z}$ is:

$$\mathbf{D_Z} = \begin{bmatrix} .0296 + \jmath.0683 & .0 & 0 \\ 0 & .0296 + \jmath.0683 & 0 \\ 0 & 0 & .0296 + \jmath.0683 \end{bmatrix}$$

Solution reached at iteration 3 for $\varepsilon = 10^{-4}$ and displayed in Table 1. Results are presented in per unit and degrees.

Table 1: 4 Node State of the System - balanced Approach

V_0	θ_0	V_1	θ_1	V_2	θ_2	V_3	θ_3
1.000	0.00	0.987	-1.59	0.981	-2.40	0.981	-2.40

4 Conclusion

This paper describes a convenient formulation of the Backward/Forward (BW/FW) Sweep Power Flow applied to radial distribution systems with distributed generation. Proposed formulation was applied in an illustrative test system.

5 Nomencalture

List of Symbols

$\mathbf{D_Z}$	Diagonal matrix of branch impedance vector $\mathbf{Z}$
$\mathbf{R}$	Diagonal matrix of branch resistance vector $\Re e\mathbf{Z}$
$\mathbf{X}$	Diagonal matrix of branch reactance vector $Im\mathbf{Z}$
ε	Convergence criteria
$\mathbf{I}$	Current vector
$\mathbf{J}$	Branch Current vector $\mathbf{J}$
n	Number of nodes, excluding origin

$\mathbf{P}$	Active Power Injected vector
$\mathbf{Q}$	Reactive Power Injected vector
P_j	Active Power Injected at node j
Q_j	Reactive Power Injected at node j
P_{Dj}	Active Power Demanded at node j
Q_{Dj}	Reactive Power Demanded at node j
P_{Gj}	Active Power Generated at node j
Q_{Gj}	Active Power Generated at node j
R_{ij}	Resistance between node i and node j
S_{Dj}	Apparent Power Demanded at node j
S_{Gj}	Apparent Power Generated at node j
$\mathbf{T}$	Triangular matrix
$\mathbf{V}$	Voltage vector
X_{ij}	Reactance between node i and node j
$\mathbf{Z}$	Branch Impedance vector $\mathbf{Z}$
$\overline{Z}^{ij}$	Branch Impedance between node i and node j
$\mathbf{Z}^{ij}$	Impedance matrix between node i and node j

Operators

T	Transpose Matrix
D	Diagonal Matrix
$*$	Conjugate of a complex number

Sub-Indexes

i	Associated to node i
j	Associated to node j
k	Associated to iteration k

References

[1] R. Berg, E.S. Hawkins, and W.W. Pleines, "Mechanized Calculation of Unbalanced Load Flow on Radial Distribution Circuits," *IEEE Transactions on Power Apparatus and Systems*, Volume PAS-86, No. 4, pp.415-421, Apr 1967

[2] W. H. Kersting and D. L. Mendive, "An application of ladder network theory to the solution of three phase radial load flow problem," *in Proc. IEEE PES Winter Meeting*, 1976, New York, paper A76044-8 (IEEE, New York, 1976).

[3] D. Shirmohammadi, H.W. Hong, A. Semlyen, and G.X. Luo, "A compensation-based power flow method for weakly meshed distribution and transmission networks," *IEEE Transactions on Power Delivery*, Vol. 3, No. 2, pp.753-762, 1988.

[4] S. Ghosh and D. Das, "Method for load-flow solution of radial distribution networks," *IEE Proc. Generat. Transm. Distrib.*, Vol. 146, No. 6, pp. 641-648, 1996.

[5] Y Fukuyama, Y Nakanishi, H-D Chiang "Fast distribution power flow using multi-processors," *Electrical Power & Energy Systems*, Vol. 18, No. 5, pp. 331-337, 1996

[6] D. Thukaram, H.M.W.Banda, and J. Jerome, "A robust three-phase power flow algorithm for radial distribution systems," *Electric Power Systems Research*, Vol. 50, No. 3, pp. 227-236, 1999.

[7] S. Jovanovic and F. Milicevic, "Triangular distribution load flow," *IEEE Power Engineering Review*, pp. 60-62, 2000.

[8] D. Rajicic, R. Taleski, "Two novel methods for radial and weakly meshed network analysis," *Electric Power Systems Research*, Vol. 48, No.2, pp. 79-87, Dec. 1998

[9] Aravindhababuba, P., Ganapathy, S., and Nayar, K. R., "A novel technique for the analysis of radial distribution systems," *Electric Power Systems Research*, Vol. 23, No. 3, pp. 167-171, 2001.

[10] Y. Zhu and K. Tomsovic, "Adaptive power flow method for distribution systems with dispersed generation," *IEEE Transactions on Power Delivery*, Vol. 17, No. 3, pp. 822-827, 2002.

[11] J. Liu, M.M.A. Salama and R.R. Mansour, "An efficient power flow algorithm for distribution systems with polynomial load," *Int. J. Elect. Eng. Educat.*, Vol. 39, No. 4, pp. 371-386, 2002.

[12] M. Afsari, S. P. Singh G. S. Raju G. K. Rao "A Fast Power Flow Solution of Radial Distribution Networks," *Electric Power Components and Systems*, 30:1065-1074, 2002

[13] J.H. Teng,"A direct approach for distribution system load flow solutions," *IEEE Transactions on Power Delivery*, Vol. 18, No. 3, pp. 882-887, 2003.

[14] K, Prasad, N.C. Sahoo, A. Chaturvedi and R. Ranjan. "A simple approach for branch current computation in load flow analysis of radial distribution systems," *Int. J. Elect. Eng. Educat.* Vol. 44 No. 1 pp. 49-63, Jan. 2007

[15] G.W. Chang, S.Y. Chu and H.L. Wang "An Improved Backward/Forward Sweep Load Flow Algorithm for Radial Distribution Systems," *IEEE Transactions on Power Systems*, Vol. 22, No. 2, pp.882-884, May 2007