Adaptive Optimal Trajectory Tracking Control Applied to a Large-Scale Ball-on-Plate System

Florian Köpf*, Sean Kille*, Jairo Inga, Sören Hohmann

***Abstract*— While many theoretical works concerning Adaptive Dynamic Programming (ADP) have been proposed, application results are scarce. Therefore, we design an ADP-based optimal trajectory tracking controller and apply it to a large-scale ball-on-plate system. Our proposed method incorporates an approximated reference trajectory instead of using setpoint tracking and allows to automatically compensate for constant offset terms. Due to the off-policy characteristics of the algorithm, the method requires only a small amount of measured data to train the controller. Our experimental results show that this tracking mechanism significantly reduces the control cost compared to setpoint controllers. Furthermore, a comparison with a model-based optimal controller highlights the benefits of our model-free data-based ADP tracking controller, where no system model and manual tuning are required but the controller is tuned automatically using measured data.**

I. Introduction

Model-free Adaptive Dynamic Programming (ADP) is a promising approach to control dynamical systems whenever a system model is unavailable, inaccurate or difficult to achieve [1]–[4]. While many control applications require to track desired reference trajectories, this is non-trivial to incorporate into the ADP formalism adequately [5], [6].

Assuming that the reference trajectory is generated directly by an unknown command system (cf. [7]–[9]) limits the flexibility of the reference trajectory that can be commanded[1]. Alternative approaches extend the system state by the desired state [10]–[12] or the current and next desired state [13]. Shi et al. [13] take into account the desired position of an underwater vehicle model at the current and next time step and train their controller using pseudo-averaged Q-learning in simulation. Although the learned (projected) setpoint controller for an autonomous helicopter [10] and the setpoint controller for a quadrotor [11] have been applied to real systems, in [10] and [11] the training procedure is based on simulations, thus requiring a model of the system to be controlled. Puccetti et al. [12] use model-free ADP for the speed tracking control of a real car, where a velocity setpoint is incorporated into the state-action value function. Nevertheless, these representations of the reference trajectory have limited [10], [13] or no preview capabilities [11], [12], which results in a controller that tends to lag behind.

Therefore, in our previous works, we have incorporated the reference trajectory over a finite horizon into the Q-function

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[1]If the reference trajectory does not result from this unknown command system during training, these methods fail.

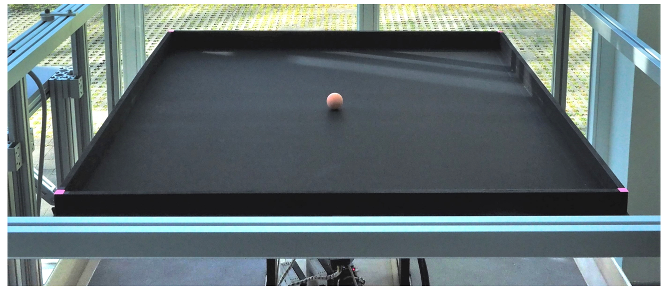

Fig. 1. Large-scale ball-on-plate system for ADP-based trajectory tracking control.

[5] or used an approximated reference trajectory [6]. Instead of assuming an *unknown* underlying command system, the controller approximates an arbitrary reference trajectory in a way that is compatible with ADP allowing flexible reference trajectories. However, [1]–[9], [13] only provide simulation results and no application to a real system—an essential step that is missing in order to validate ADP methods.

In this paper, we propose an ADP tracking controller which incorporates an approximated reference trajectory and apply it to a real large-scale ball-on-plate system (depicted in Fig. 1). The ball-on-plate system is a widely used example for benchmarking controllers. Existing controllers are either fully model-based [14]–[18] or model-based with additional fuzzy supervision [19]. Thus, our work is the first application of a model-free ADP-based controller to a ball-on-plate system. Furthermore, instead of incorporating the reference trajectory, existing controllers either perform no tracking of the ball position at all [14], [15], [18] or simply consider the current deviation from a setpoint causing a trajectory that lags behind [16], [17], [19].

In contrast to existing controllers, our method does not require a model of the ball-on-plate system as we train our optimal tracking controller directly through a policy iteration (PI) mechanism [20] using measured data from a real system. This avoids tedious model design followed by manual tuning. By using an off-policy algorithm, the measured data can be re-used, reducing the effort to record training data[2]. Furthermore, instead of the widely-used setpoint tracking, our ADP controller incorporates information on the course of the reference trajectory which allows predictive rather than reactive behavior and avoids lagging behind. Our automatically tuned controller is also able to learn static offsets to compensate

[2]In contrast, *on-policy* learning would require new data to be collected after each policy improvement step and the estimates would be biased when (indispensable) exploration noise is used [21].

for asymmetries. In summary, our main contributions include an ADP tracking controller which is

- data-efficient as it works off-policy and uses a flexible and compact local approximation of arbitrary reference trajectories that is compatible with ADP
- trained on a real system using measured data, requiring neither system parameters nor manual tuning
- compared to a model-based and a setpoint controller.

The remainder of this paper is structured as follows: In Section II, the system and problem description are given. The theoretical background to our ADP tracking formalism is given in Section III. In Section IV, we present our method. Results are given in Section V, before we conclude the work.

II. System and Problem Description

In the following, the ball-on-plate system that is used as an application example for our ADP tracking method and the problem formulation are given.

A. Ball-on-Plate System

The system used in this work is a custom-built large-scale ball-on-plate system (see Fig. 1). Its centerpiece is a $1\,\mathrm{m}^2$ square plate with a mass of $16.3\,\mathrm{kg}$. The plate can be tilted in two dimensions (denoted by X and Y) that are orthogonal to each other. Each dimension is actuated by its own designated motor. The plate angles $(\alpha^{[X]}, \alpha^{[Y]})$ and angular velocities $(\omega^{[X]}, \omega^{[Y]})$ are measured every $10\,\mathrm{ms}$. A ball with a mass of $0.042\,\mathrm{kg}$ and a radius of $0.02\,\mathrm{m}$ is located on the plate. Its position in plate-fixed coordinates is tracked via a camera, providing an updated ball position $(s^{[X]}, s^{[Y]})$ and ball velocity $(v^{[X]}, v^{[Y]})$ every $\Delta t = 40\,\mathrm{ms}$. For a detailed description of the system architecture and the hardware, see [18][3]. Thus, the resulting system states

$$\boldsymbol{x}_k^{[d]} = \begin{bmatrix} s_k^{[d]} & v_k^{[d]} & \alpha_k^{[d]} & \omega_k^{[d]} \end{bmatrix}^\intercal \tag{1}$$

are defined for both dimensions $d \in \mathcal{D} = \{X, Y\}$. The system input $u_k^{[d]} = I_k^{[d]}$ is the current for the motor driver controller.

As the two dimensions X and Y only slightly depend on each other, they are usually controlled separately (see [14], [15], [17], [18]). Since the controllers for the two dimensions are trained in the same way, the index d is omitted in the following for the sake of readability.

B. Problem Formulation

Consider the discrete-time controllable system dynamics

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}\left(\boldsymbol{x}_k, \boldsymbol{u}_k\right) \tag{2}$$

where $k \in \mathbb{N}_0$ describes the discrete time step, $\boldsymbol{x}_k \in \mathcal{X} \subseteq \mathbb{R}^n$ the system state (1), $\boldsymbol{u}_k \in \mathcal{U} \subseteq \mathbb{R}^m$ the control input $I_k^{[d]}$ and $\boldsymbol{f}$ is *unknown*. From Section II-A, the system order $n = 4$ and number of control inputs $m = 1$ follows for each dimension in $\mathcal{D}$. At each time step k, an approximation of the desired ball position trajectory is denoted by

$$r(\boldsymbol{p}_k, i) = \boldsymbol{p}_k^\intercal \boldsymbol{\rho}(i), \tag{3}$$

[3]Note that we use a heavier plate and a different ball in the present work.

$i \in \mathbb{N}_0$, where $r(\boldsymbol{p}_k, i)$ is the desired ball position at time $k+i$ (i.e. i denotes the time step on the reference from the local perspective at time k), $\boldsymbol{p}_k \in \Theta \subseteq \mathbb{R}^{n_\mathrm{p}}$ a parameter vector and $\boldsymbol{\rho}(i)$ a basis function vector (cf. [6]). The following problem formalizes that the ball position should follow a desired reference trajectory while keeping other system states and the control effort small.

Problem 1. *Assume given basis functions $\boldsymbol{\rho}(i)$ for reference trajectory approximation and measurement tuples $\{\boldsymbol{x}_i, u_i, \boldsymbol{x}_{i+1}\}$, $i = k, \dots, k+N-1$. Let the system dynamics $\boldsymbol{f}(\boldsymbol{x}_k, u_k)$ be unknown. Find the control law $\boldsymbol{\pi}^*(\boldsymbol{x}_k, \boldsymbol{p}_k)$ such that $\forall \boldsymbol{x}_k, \boldsymbol{p}_k$ the control $u_k^* = \boldsymbol{\pi}^*(\boldsymbol{x}_k, \boldsymbol{p}_k)$ minimizes the objective function*

$$J_k = \sum_{i=0}^{\infty} \gamma^i \left(\begin{bmatrix} x_{1,k+i} - r(\boldsymbol{p}_k, i) \\ x_{2,k+i} \\ x_{3,k+i} \\ x_{4,k+i} \end{bmatrix}^\intercal \boldsymbol{Q} \begin{bmatrix} x_{1,k+i} - r(\boldsymbol{p}_k, i) \\ x_{2,k+i} \\ x_{3,k+i} \\ x_{4,k+i} \end{bmatrix} + u_{k+i}^\intercal R u_{k+i} \right) =: \sum_{i=0}^{\infty} \gamma^i c(\boldsymbol{x}_{k+i}, u_{k+i}, r(\boldsymbol{p}_k, i)), \tag{4}$$

where $\gamma \in (0,1)$ denotes a discount factor, $\boldsymbol{Q}$ is assumed to be positive semi-definite and R positive definite.

III. ADP Tracking Theory

In this section, we briefly summarize the theoretical background on our ADP tracking formalism related to Problem 1.

Lemma 1. *Define*

$$\boldsymbol{p}_k^{(i)\intercal} = \boldsymbol{p}_k^\intercal \boldsymbol{T}(i), \tag{5}$$

where $\boldsymbol{T}(i)$ is chosen such that

$$r\left(\boldsymbol{p}_k^{(i)}, j\right) = r(\boldsymbol{p}_k, i+j), \quad \forall i, j \in \mathbb{N}_0 \tag{6}$$

holds and

$$\begin{aligned} Q^*(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k) &= c(\boldsymbol{x}_k, u_k, r(\boldsymbol{p}_k, 0)) \\ &\quad + \sum_{i=1}^{\infty} \gamma^i c\Big(\boldsymbol{x}_{k+i}, \pi^*\Big(\boldsymbol{x}_{k+i}, \boldsymbol{p}_k^{(i)}\Big), r(\boldsymbol{p}_k, i)\Big) \\ &= c(\boldsymbol{x}_k, u_k, r(\boldsymbol{p}_k, 0)) \\ &\quad + \gamma Q^*\Big(\boldsymbol{x}_{k+1}, \pi^*\Big(\boldsymbol{x}_{k+1}, \boldsymbol{p}_k^{(1)}\Big), \boldsymbol{p}_k^{(1)}\Big). \end{aligned} \tag{7}$$

Then,

$$u_k^* = \operatorname*{arg\,min}_{u_k} Q^*(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k) \tag{8}$$

is a solution to Problem 1.

Proof. See [6, Lemma 1]. □

Note 1. *$Q^*(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k)$ is the accumulated discounted cost if the system is in state $\boldsymbol{x}_k$, the control u_k is applied at time step k and the optimal control $\pi^*(\cdot)$ thereafter. Using the shifted reference trajectory approximation $\boldsymbol{p}_k^{(i)}$ (cf. (5)) ensures that the Q-function $Q^*(\cdot)$ is compatible with ADP (cf. [6, Note 1]).*

As the optimal Q-function $Q^*(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k)$ is unknown, linear function approximation (FA) (cf. [1]–[9], [12], [20], [22]) is commonly used[4]. Thus, suppose $\hat{Q}(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k) = \hat{\boldsymbol{w}}^\intercal \boldsymbol{\phi}(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k)$, where $\hat{\boldsymbol{w}} \in \mathbb{R}^{n_\mathrm{w}}$ is a weight vector to be adapted and $\boldsymbol{\phi}(\cdot) \in \mathbb{R}^{n_\mathrm{w}}$ a vector of activation functions. A common approach in order to tune $\hat{\boldsymbol{w}}$ is given by a PI (see e.g. [1], [20], [22]). In this iterative procedure, each iteration l consists of two steps. The *policy evaluation* step estimates the Q-function

$$\hat{Q}^{\hat{\pi}_l}(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k) = \hat{\boldsymbol{w}}_l^\intercal \boldsymbol{\phi}(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k) \tag{9}$$

of the current policy $\hat{\pi}_l$, i.e. adapts $\hat{\boldsymbol{w}}_l$ in order to solve

$$\begin{aligned}\hat{Q}^{\hat{\pi}_l}(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k) &= c(\boldsymbol{x}_k, u_k, r(\boldsymbol{p}_k, 0)) \\ &\quad + \gamma \hat{Q}^{\hat{\pi}_l}\left(\boldsymbol{x}_{k+1}, \hat{\pi}_l\left(\boldsymbol{x}_{k+1}, \boldsymbol{p}_k^{(1)}\right), \boldsymbol{p}_k^{(1)}\right).\end{aligned} \tag{10}$$

The *policy improvement* step then greedily updates the policy $\hat{\pi}_{l+1}$ based on $\hat{Q}^{\hat{\pi}_l}$:

$$\hat{\pi}_{l+1}(\boldsymbol{x}_k, \boldsymbol{p}_k) = \underset{u_k}{\arg\min}\, \hat{Q}^{\hat{\pi}_l}(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k). \tag{11}$$

Convergence results of a Q-function-based PI are given in e.g. [20, Theorem 3.1], [8, Theorem 1].

IV. ADP Tracking on the Ball-on-Plate System

The ADP tracking formalism introduced in Section III is applied to the ball-on-plate system described in Section II-A.

A. Quadratic Polynomial Reference Approximation

We choose the reference trajectory to be approximated by means of a quadratic polynomial

$$r(\boldsymbol{p}_k, i) = \boldsymbol{p}_k^\intercal \boldsymbol{\rho}(i) = p_{k,2}(i\Delta t)^2 + p_{k,1} i\Delta t + p_{k,0}, \tag{12}$$

with the basis functions $\boldsymbol{\rho}(i) = \begin{bmatrix}(i\Delta t)^2 & i\Delta t & 1\end{bmatrix}^\intercal$ and the parameter vector $\boldsymbol{p}_k = \begin{bmatrix}p_{k,2} & p_{k,1} & p_{k,0}\end{bmatrix}^\intercal$, where Δt denotes the sampling time.

The transformation needed to obtain the propagated version $\boldsymbol{p}_k^{(i)}$ of $\boldsymbol{p}_k$ according to (3) and (6) is given by

$$\begin{aligned} r\left(\boldsymbol{p}_k^{(i)}, j\right) &= \boldsymbol{p}_k^\intercal \boldsymbol{\rho}(i+j) = \boldsymbol{p}_k^\intercal \begin{bmatrix}((i+j)\Delta t)^2 \\ (i+j)\Delta t \\ 1\end{bmatrix} \\ &= \boldsymbol{p}_k^\intercal \underbrace{\begin{bmatrix}1 & 2i\Delta t & (i\Delta t)^2 \\ 0 & 1 & i\Delta t \\ 0 & 0 & 1\end{bmatrix}}_{=:\boldsymbol{T}(i)} \boldsymbol{\rho}(j) = \boldsymbol{p}_k^{(i)\intercal} \boldsymbol{\rho}(j), \end{aligned} \tag{13}$$

$\forall i, j \in \mathbb{N}_0$. For any desired reference trajectory $\bar{r}_k$, a parameter vector $\boldsymbol{p}_k$ is to be found at each time step k, such that $r(\boldsymbol{p}_k, i)$, $i \in \mathbb{N}_0$, is an approximation of $\bar{r}_{k+i}$. The desired reference trajectory is assumed to be known during runtime over a horizon of $h_\mathrm{r} \in \mathbb{N}_{>0}$ timesteps. In each time step, $\boldsymbol{p}_k$ is determined by a weighted least-squares (LS) regression. Therefore, we define

$$\bar{r}_{k:k+h_\mathrm{r}-1} = \begin{bmatrix}\bar{r}_k & \bar{r}_{k+1} & \dots & \bar{r}_{k+h_\mathrm{r}-1}\end{bmatrix}, \tag{14}$$

$$\boldsymbol{W}_\mathrm{p} = \mathrm{diag}(1, \beta, \dots, \beta^{h_\mathrm{r}-1}), \tag{15}$$

$$\boldsymbol{\rho}_{0:h_\mathrm{r}-1} = \begin{bmatrix}\boldsymbol{\rho}(0) & \boldsymbol{\rho}(1) & \dots & \boldsymbol{\rho}(h_\mathrm{r}-1)\end{bmatrix}^\intercal, \tag{16}$$

with $\boldsymbol{W}_\mathrm{p}$ being a weighting matrix with the discount factor $\beta \leq 1$, so that future time steps in the horizon are less important for the fitting process than early time steps. The parameter for the reference trajectory approximation is then calculated with the weighted LS regression according to [6] and given by

$$\boldsymbol{p}_k^\intercal = \bar{r}_{k:k+h_\mathrm{r}-1} \boldsymbol{W}_\mathrm{p} \boldsymbol{\rho}_{0,h_\mathrm{r}-1} \left(\boldsymbol{\rho}_{0,h_\mathrm{r}-1}^\intercal \boldsymbol{W}_\mathrm{p} \boldsymbol{\rho}_{0,h_\mathrm{r}-1}\right)^{-1}. \tag{17}$$

B. Q-Function Approximation

The approximated Q-function (9) is chosen as

$$\begin{aligned}\hat{Q}^{\hat{\pi}_l}(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k) &= \begin{bmatrix}u_k \\ \boldsymbol{x}_k \\ \boldsymbol{p}_k \\ 1\end{bmatrix}^\intercal \begin{bmatrix} h_\mathrm{uu}^{(l)} & \boldsymbol{h}_\mathrm{ux}^{(l)} & \boldsymbol{h}_\mathrm{up}^{(l)} & h_\mathrm{u1}^{(l)} \\ \boldsymbol{h}_\mathrm{xu}^{(l)} & \boldsymbol{h}_\mathrm{xx}^{(l)} & \boldsymbol{h}_\mathrm{xp}^{(l)} & h_\mathrm{x1}^{(l)} \\ \boldsymbol{h}_\mathrm{pu}^{(l)} & \boldsymbol{h}_\mathrm{px}^{(l)} & \boldsymbol{h}_\mathrm{pp}^{(l)} & h_\mathrm{p1}^{(l)} \\ h_\mathrm{1u}^{(l)} & h_\mathrm{1x}^{(l)} & h_\mathrm{1p}^{(l)} & h_\mathrm{11}^{(l)} \end{bmatrix} \begin{bmatrix}u_k \\ \boldsymbol{x}_k \\ \boldsymbol{p}_k \\ 1\end{bmatrix} \\ &= \boldsymbol{z}_k^\intercal \boldsymbol{H}_l \boldsymbol{z}_k = \hat{\boldsymbol{w}}_l^\intercal \boldsymbol{\phi}(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k),\end{aligned} \tag{18}$$

with $\boldsymbol{H}_l = \boldsymbol{H}_l^\intercal$, i.e. $\boldsymbol{\phi}(\boldsymbol{x}_k, u_k, \boldsymbol{p}_k)$ consists of the non-redundant elements of the Kronecker product $\boldsymbol{z}_k \otimes \boldsymbol{z}_k$ and $\hat{\boldsymbol{w}}_l$ corresponds to the non-redundant elements of the unknown matrix $\boldsymbol{H}_l$[5]. This quadratic choice is motivated by the successful control of our system using a model-based linear quadratic (LQ) controller [18] and the fact that the Q-function of LQ optimal control problems is quadratic [6].

For the policy evaluation step (10) we utilize least-squares temporal-difference Q-learning (LSTDQ) [20] using the fixed-point objective [20, Section 5.2]. Consequently, N tuples $\left\{\boldsymbol{x}_k, u_k, \boldsymbol{x}_{k+1}, \boldsymbol{p}_k, \boldsymbol{p}_k^{(1)}\right\}$ are used in order to obtain a least-squares solution of $\hat{\boldsymbol{w}}_l$ from (10). Due to its off-policy characteristic, the measured samples can be re-used in each iteration of the PI which renders the method data-efficient. Furthermore, the minimization in (11) requires

$$\frac{\partial \hat{Q}^{\hat{\pi}_l}}{\partial u_k} = 2\left(\boldsymbol{h}_\mathrm{ux}^{(l)} \boldsymbol{x}_k + \boldsymbol{h}_\mathrm{up}^{(l)} \boldsymbol{p}_k + h_\mathrm{u1}^{(l)} + h_\mathrm{uu}^{(l)} u_k\right) \stackrel{!}{=} \boldsymbol{0}. \tag{19}$$

This leads to the explicit[6] policy improvement step (11)

$$\hat{\pi}_{l+1}(\boldsymbol{x}_k, \boldsymbol{p}_k) = -\underbrace{\left(h_\mathrm{uu}^{(l)}\right)^{-1} \begin{bmatrix}\boldsymbol{h}_\mathrm{ux}^{(l)} & \boldsymbol{h}_\mathrm{up}^{(l)} & h_\mathrm{u1}^{(l)}\end{bmatrix}}_{\boldsymbol{L}_l} \begin{bmatrix}\boldsymbol{x}_k \\ \boldsymbol{p}_k \\ 1\end{bmatrix}, \tag{20}$$

which sets a motor current $I_k^{[d]}$ depending on $\boldsymbol{x}_k, \boldsymbol{p}_k$ and a static offset.

Note 2. *The choice of $\hat{Q}(\cdot)$ in (18) extends the approximation used in [6] by an offset term. This allows the controller to learn a static offset compensation, i.e. if the weight of the plate is slightly unbalanced.*

[4]Compared to nonlinear FA, linear FA is easier to handle, usually requires less training data and allows an analytical relation between the Q-function and the optimal controller [22].

[5]Due to the symmetry of $\boldsymbol{H}_l$, the weights corresponding to the off-diagonal elements of $\boldsymbol{H}_l$ are multiplied by 2.

[6]This analytic relation is a result of the quadratic penalty for u in (4).

C. Training Procedure

The offline least-squares policy iteration (LSPI) algorithm [20] utilized in this work iteratively improves a policy by using offline recorded data tuples. These consist to one part of system data extracted through interaction with the system, and to the other part of a generated training reference trajectory.

1) System Data: System data is collected by human interaction with the system. Manual control elements allow to set target plate angles which are controlled with a suboptimal controller. The system states can then be excited by varying the plate angle and data tuples $\{\boldsymbol{x}_k, u_k, \boldsymbol{x}_{k+1}\}$ are collected.

2) Training Reference: The Q-function (18) represents the cost of a chosen control u_k not only referring to the current state $\boldsymbol{x}_k$, but also to a desired target trajectory $\bar{r}_{k:k+h_\mathrm{r}-1}$ which is approximated by $\boldsymbol{p}_k$. Therefore, a training reference trajectory is generated, which consists of a linear combination of multiple sine functions with varying frequencies. A weighted LS approximation (17) is used to approximate the training reference at each time step by means of a quadratic polynomial ($n_\mathrm{p} = 3$) with a discount factor of $\beta = 0.8$ and $h_\mathrm{r} = 10$, resulting in the parameter vector $\boldsymbol{p}_k$. This parameter vector is then propagated according to (13) to find $\boldsymbol{p}_k^{(i)}$.

The collected system data is smoothed (moving average of length 5) and aggregated, together with the training reference parameters, to the tuples $\left\{\boldsymbol{x}_k, u_k, \boldsymbol{x}_{k+1}, \boldsymbol{p}_k, \boldsymbol{p}_k^{(1)}\right\}$. We use $N = 1200$ data tuples for learning, which result with a sampling time of $\Delta t = 40\,\mathrm{ms}$ in $48\,\mathrm{s}$ of excitation data. For numerical stability, we introduce a normalizing factor $V_\mathrm{N} = 10$ which is applied to the state vector and parameter vector ($\bar{\boldsymbol{x}}_k = V_\mathrm{N}\boldsymbol{x}_k$, $\bar{\boldsymbol{p}}_k = V_\mathrm{N}\boldsymbol{p}_k$) such that the values of the system state and control input are in a similar range.

Our goal in this work is to track the position of the ball. Additionally, we want the plate to preferably stay in a horizontal position. Therefore, we set $\boldsymbol{Q} = \mathrm{diag}(800, 0, 400, 0)$ to strongly penalize the deviation of the ball position (i.e. $\boldsymbol{x}_1$) from the parametrized reference as well as a deviation of the plate angle (i.e. $\boldsymbol{x}_3$) from its horizontal position $\alpha = 0$ (cf. (4)). We set the discount factor to $\gamma = 0.9$. For the initial iteration, we set all weights $\hat{\boldsymbol{w}}_0$ to 1.

Using the LSPI algorithm, where the policy evaluation is done using a least-squares fixed-point approximation [20, Section 5.2], we obtain updated weights $\hat{\boldsymbol{w}}_l$ in each iteration[7] l. The algorithm converges towards a fixed-point and is stopped when the stopping criterion

$$||\hat{\boldsymbol{w}}_l - \hat{\boldsymbol{w}}_{l-1}||_2 \leq \epsilon = 1 \times 10^{-6} \tag{21}$$

is fulfilled. The final policy improvement step yields the control matrix $\boldsymbol{L}$ (cf. (20)), i.e. the final control policy

$$\hat{\boldsymbol{\pi}}(\boldsymbol{x}_k, \boldsymbol{p}_k) = -\begin{bmatrix}\boldsymbol{L}_\mathrm{x} & \boldsymbol{L}_\mathrm{ref} & L_\mathrm{off}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_k^\intercal & \boldsymbol{p}_k^\intercal & 1\end{bmatrix}^\intercal \tag{22}$$

after being re-normalized. All steps are summarized in Fig. 2.

[7]The complexity of each iteration is dominated by the policy evaluation step with $\mathcal{O}(n_\mathrm{w}^3 + n_\mathrm{w}^2 N)$.

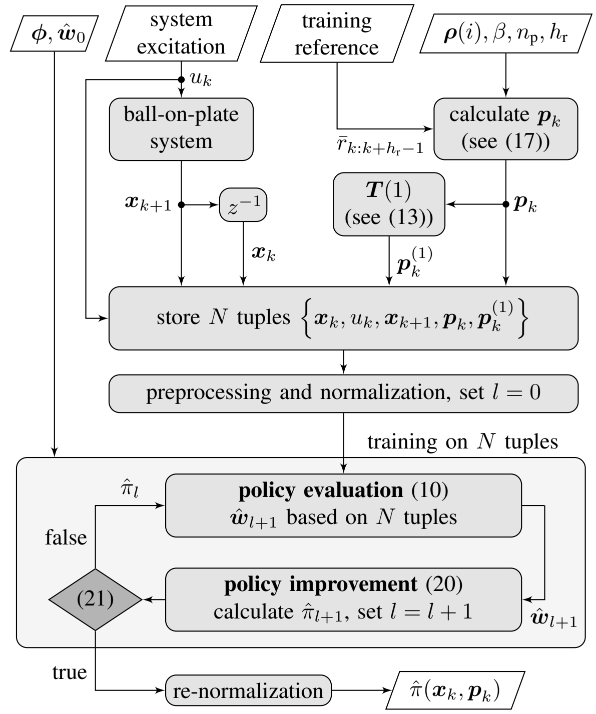

Fig. 2. Training procedure to obtain the approximate optimal tracking controller $\hat{\pi}(\boldsymbol{x}_k, \boldsymbol{p}_k)$ for the ball-on-plate system.

V. Results

To validate the learned ADP controller, we compare a learned controller $\boldsymbol{L}_\mathrm{ADP}$ with a model-based controller $\boldsymbol{L}_\mathrm{model}$. Since both dimensions are learned using the same approach, we firstly focus on a comparison in one dimension. The controllers are compared using a sine-like step function as well as a composite validation trajectory. In the second half of this section, we present the ability of two simultaneous controllers to follow a 2-dimensional trajectory.

We train a controller as described in Section IV-C. The convergence of $\hat{\boldsymbol{w}}_l$ is depicted in Fig. 3. The resulting learned control matrix is

$$\boldsymbol{L}_\mathrm{ADP}^{[Y]} = [\underbrace{64.8\ 32.3\ 145.3\ 16.2}_{\boldsymbol{L}_\mathrm{x}}\ \underbrace{-27.9\ -36.9\ -60.7}_{\boldsymbol{L}_\mathrm{ref}}\ \underbrace{-0.1}_{L_\mathrm{Off}}]. \tag{23}$$

The model-based solution is calculated according to [6, Theorem 2] which solves the optimization problem described

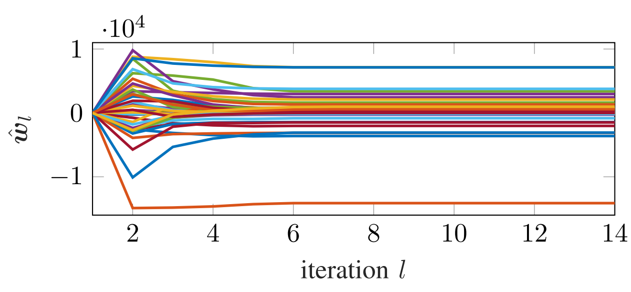

Fig. 3. Estimated weights over all iterations of the LSPI algorithm.

in Problem 1 but uses a system model established specifically for our system (cf. [18]). The resulting model-based control matrix is given by

$$\boldsymbol{L}_{\text{model}}^{[Y]} = [\underbrace{53.4\ 41.0\ 167.8\ 28.0}_{\boldsymbol{L}_{\text{x}}}\ \underbrace{-33.7\ -41.0\ -52.9}_{L_{\text{ref}}}]. \tag{24}$$

A. Setpoint Control: Step

In order to compare the model-free learned controller with the model-based calculated controller, both controllers are to follow a sine-like step function $\bar{r}$. Fig. 4a depicts the average ball position when using a learned (blue) and a model-based (yellow) setpoint controller with $n_{\text{p}} = 1$, over 11 repetitions. The standard deviation is shown shaded. Both controllers lag behind as they only have information about the current setpoint. The learned controller shows a slightly faster step response, which is reflected by lower accumulated one-step costs $\sum_{\kappa=0}^{k} c\left(\boldsymbol{x}_\kappa, u_\kappa, r(\boldsymbol{p}_\kappa, 0)\right)$ (see Fig. 4a).

B. Trajectory Control: Step

A comparison between a learned trajectory controller (red) and a model-based trajectory controller (green), both with $n_{\text{p}} = 3$, is depicted in Fig. 4b. Both trajectory controllers allow a significantly better tracking of the reference trajectory compared to the learned setpoint controller (blue), as they receive information about the future course of the trajectory. This leads to significantly lower accumulated one-step costs, as seen in Fig. 4b. Similarly to the setpoint controllers, the learned trajectory controller shows lower accumulated costs compared to the model-based trajectory controller.

C. Trajectory Control: Validation Trajectory

Fig. 4c compares a learned trajectory controller ($n_{\text{p}} = 3$) with the learned setpoint controller ($n_{\text{p}} = 1$) on a validation reference trajectory, which is composed of overlaid sines, step functions and ramps. Again, an evidently better tracking of the trajectory is possible with the trajectory controller than with the setpoint controller, which leads to significantly lower accumulated one-step costs.

D. 2D Trajectory Control

In order to use the ball-on-plate system to its full extent, we apply two separately learned controllers, one for each plate dimension respectively. Learning with the same parameters as for the Y-dimension, but with system data tuples for the X-dimension, we receive the learned control law:

$$\boldsymbol{L}_{\text{ADP}}^{[X]} = [\underbrace{65.3\ 37.0\ 135.1\ 18.6}_{\boldsymbol{L}_{\text{x}}}\ \underbrace{-28.8\ -38.2\ -61.2}_{\boldsymbol{L}_{\text{ref}}}\ \underbrace{2.2}_{\boldsymbol{L}_{\text{Off}}}]. \tag{25}$$

$L_{\text{off}}^{[X]} = 2.2$ leads to a static offset current of $-2.2\,\text{A}$, since the plate exhibits a mass-imbalance which needs to be compensated. For a model-based solution, this current would have to be determined heuristically, as the mass-imbalance is not described by the system model. Not using a static offset current leads to an asymmetric behavior of the ball position, as depicted in Fig 5. In comparison, a learned controller that allows the learning of an offset current leads to a symmetric

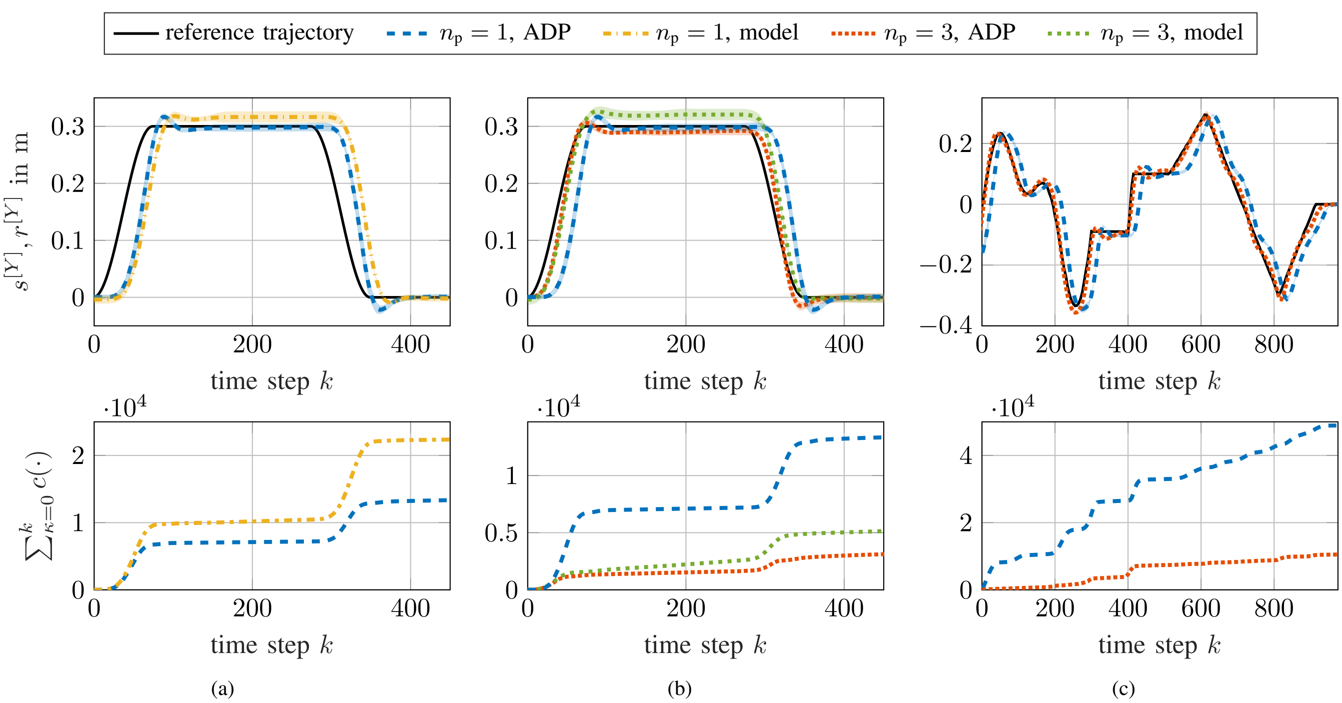

Fig. 4. **(a)** Setpoint controllers, learned in blue, model-based in yellow, compared on a sine-step function. *Top:* Average ball position and standard deviation over 11 repetitions. *Bottom:* Average accumulated one-step cost; **(b)** Trajectory controllers, learned (red), model-based (green), compared to a setpoint controller (blue) on a sine-step function. *Top:* Average ball position and standard deviation over 18, 13 and 11, repetitions. *Bottom:* Average accumulated one-step cost; **(c)** Learned trajectory controller ($n_{\text{p}} = 3$, red) and learned setpoint controller ($n_{\text{p}} = 1$, blue) on a validation trajectory. *Top:* Average ball position and standard deviation over 4 repetitions. *Bottom:* Average accumulated one-step cost.

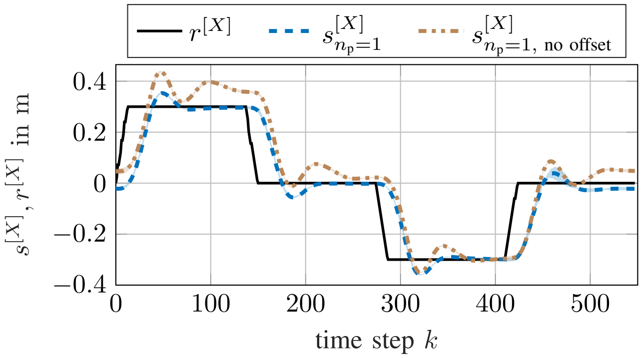

Fig. 5. Comparison of a learned setpoint controller with base functions that allow the learning of a static offset current (blue) versus a controller with base functions that do not allow the learning of a static offset current (brown).

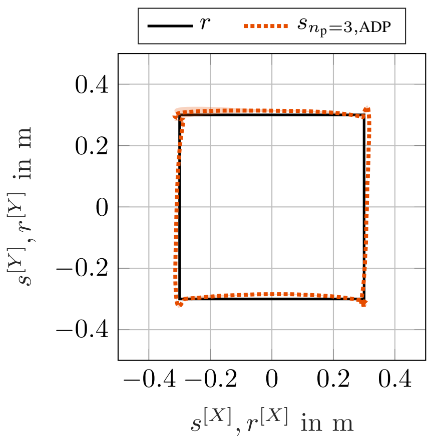

Fig. 6. Trajectory control of a rectangle. Average ball position and standard deviation over 4 repetitions.

behavior of the ball position. Fig. 6 displays the tracking of a 2-dimensional reference trajectory.

VI. Conclusion

In this paper, we presented the application of an ADP-based learned trajectory tracking controller on a large-scale ball-on-plate system. With less than one minute of measured real data, our model-free ADP-based method successfully learned an optimal tracking controller which allows the tracking of 2-dimensional reference trajectories and outperforms its model-based counterpart. In addition, the implemented reference trajectory approximation led to a faster accelerated ball, a smaller static error and therefore to overall reduced accumulated costs compared to setpoint controllers. In summary, the experimental results show that our ADP method is suitable for real systems. It includes the autonomous learning of an offset correction and avoids tedious modeling and manual tuning. The resulting control law was proved to be more cost-effective in a real scenario, benefiting from being trained with real measured data. Finally, due to the flexibility of function approximation, other basis functions can be studied in the future in order to allow for even more complex control tasks.

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