On Sharp Enhancement of Effective Mass of Quasiparticles and Coefficient of T^2 Term of Resistivity around First-Order Metamagnetic Transition Observed in UTe_2

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The mechanism underlying the enhancement of the Sommerfeld coefficient of quasiparticles at the *first-order* metamagnetic transition in UTe_2, reported by Miyake *et al.* in J. Phys. Soc. Jpn. **8**8, 063706 (2019), is discussed theoretically by taking into account the ferromagnetic order-parameter fluctuations on the basis of the Landau theory of phase transition. We find that the enhanced ferromagnetic spin fluctuation gives rise to the enhancement of the effective mass of the quasiparticles or the Sommerfeld coefficient γ, which is consistent with the experimental observations. At the same time, the Kadowaki-Woods type scaling around the metamagnetic transition, reported by Imajo *et al.* in J. Phys. Soc. Jpn. **8**8, 083705 (2019) and Knafo *et al.* in J. Phys. Soc. Jpn. **8**8, 063705 (2019), is also understood semiquantitatively by assuming reasonable values of parameters of Landau-type free energy reproducing key quantities characterizing the metamagnetic transition.

1. Introduction

The discovery of superconductivity in UTe_2 reported in December 2018[1)] gave a strong impact in the heavy fermion community, and it was confirmed soon after by a subsequent experiment,[2)] in which a number of intriguing physical properties other than the superconductivity were reported. In particular, unusual metamagnetic behaviors have attracted considerable attention.[3–5)] Among them, it is a non-trivial phenomenon that the Sommerfeld coefficient and the A_ρ coefficient of the T^2 term in the resistivity exhibit sharp enhancements around the *first-order* metamagnetic transition.[3,4,6)] UTe_2 is considered to be located in the normal phase near the ferromagnetic quantum critical point under the ambient conditions.[1,2,7)] Therefore, it is natural to expect that such anomalous properties arise through the effect of ferromagnetic spin fluctuations of one kind or another, which is enhanced by the *first-order* metamagnetic transition. It is remarked that this first-order metamagnetic transition should be conceptually different from the first-order ferromagnetic transition with the tricritical wings in the $T-P-H$ phase diagram, which was discussed in Ref. 8.

In this paper, the origin of such a non-trivial behavior is clarified on the basis of an extended phenomenological theory of the Landau type and the conventional theory of ferromagnetic spin fluctuations. We are not so ambitious to explain the magnetic field dependence at an arbitrary magnetic field strength and its anisotropic behaviors with respect to the direction of the magnetic field, but to focus on the physical properties only at exactly the metamagnetic field and ambient pressure in a region with sufficiently low temperatures on the basis of a phenomenological uniaxial model for the magnetization, i.e., in the b-direction, as observed in $\mathrm{UTe_2}$. The use of this phenomenological model may be justified by the fact that the direction of the magnetic field (b-direction in $\mathrm{UTe_2}$), in which metamagnetic transition occurs, and that corresponding to the maximum of the magnetic susceptibility (a-direction in $\mathrm{UTe_2}$) are different in general as discussed in Sect. 2.

This paper is organized as follows. In Sect. 2, a theory for the *first-order* metamagnetic transition is formulated on the basis of an extended Landau theory of phase transition. On this basis, in Subsec. 3.1, the structure of ferromagnetic spin fluctuations around the *first-order* metamagnetic transition is presented, which shows that such enhancements in the effective mass and the A_ρ coefficient are caused by effect of the flattening of the curvature of the free energy $F(M)$ around the local minimum corresponding to the lower magnetization at the *first-order* transition. It is shown that such an effect is manifested also in the A_ρ coefficient. On this consideration, in Subsect. 3.2, the observed manner of enhancements of the Sommerfeld coefficient and the A_ρ coefficient is evaluated semiquantitatively. Throughout this paper, we use units of energy, such that $\hbar = 1$, $k_{\mathrm{B}} = 1$, and $\mu_{\mathrm{B}} = 1$, unless explicitly stated. In Sect. 4, the results of this study are summarized, and their relevance to the re-entrant appearance of the superconductivity at $B \gtrsim B_{\mathrm{m}}$ is briefly mentioned.

2. Extended Landau Theory for Metamagnetic Transition

In this section, we discuss the Landau theory for the first-order metamagnetic transition on the basis of a uniaxial model for magnetization. First of all, let us discuss the validity of using the uniaxial model for discussing the metamagnetic transition. If there exists an anisotropy in the magnetic response, we have to introduce three order parameters corresponding to each direction as long as we follow the Landau-type theory. However, a condition for the metamagnetic transition to occur is generally determined independently of its magnetic field direction. This is due to a general concept based on the crystal symmetry. Namely, the crystal structure of $\mathrm{UTe_2}$ is body-centered orthorhombic and centrosymmetric[1)] so that the magnetization is in the b-direction under the magnetic field in the b-direction because there is no coupling term such as M_aM_b or M_cM_b in the Landau-type free energy even when the effect of the spin-orbit interaction is taken into account, in which magnetic space and real space are not independent owing to the effect of the spin-orbit interaction. Therefore, the presence or absence of the metamagnetic transition can be discussed independently of the magnetic field direction, justifying the use of the uniaxial model. Therefore, on the basis of experimental facts reported in Refs. 3–5, we start with the following free energy $F_0(M)$, with M being the magnetization in the b-direction per unit formula of UTe_2 without the magnetic field ($B = 0$):

$$F_0(M) = aM^2 - bM^4 + cM^6 + \cdots \tag{1}$$

where the coefficients a, b, and c are assumed to be positive. Near $B = 0$, the magnetization is given by

$$M \approx \frac{B}{2a} \equiv \chi B, \tag{2}$$

where χ is the magnetic susceptibility. The stationary condition under a general situation is given by

$$0 = \frac{\partial}{\partial M}[F_0(M) - BM] \approx 2aM - 4bM^3 + 6cM^5 - B. \tag{3}$$

When the first-order metamagnetic transition occurs, the free energy has at least two degenerate local minima at $M = M_-$ and $\bar{M}$ as shown schematically in Fig. 1. Therefore, the magnetization $\bar{M}$ at the metamagnetic critical point at $B = B_{\rm m}$ is given by the following two conditions: The stationary condition of the local minimum of $[F_0(M) - B_{\rm m}M]$ for the lower magnetization M_- at $B = B_{\rm m}$ is

$$2aM_- - 4bM_-^3 + 6cM_-^5 - B_{\rm m} \approx 0, \tag{4}$$

and that for the higher magnetization $\bar{M}$ at $B = B_{\rm m}$ is

$$2a\bar{M} - 4b\bar{M}^3 + 6c\bar{M}^5 - B_{\rm m} \approx 0. \tag{5}$$

In addition to these two conditions, the free energy of these two states should be the same:

$$a\bar{M}^2 - b\bar{M}^4 + c\bar{M}^6 - B_{\rm m}\bar{M} \approx aM_-^2 - bM_-^4 + cM_-^6 - B_{\rm m}M_-. \tag{6}$$

Note that the anisotropy of a differential magnetic susceptibility around $B = 0$ and that of the direction in which the metamagnetic transition occurs are generally independent, as noted above. This is because the occurrence of the metamagnetic transition is determined by the combination of the coefficients a, b and c of the extended Landau free energy $F_0(M)$ [Eq. (1)], whereas the anisotropy in the magnetic susceptibility is determined only by the size of coefficient a in each direction of the magnetic field. In this study, we did not touch on the origin of the anisotropy in the magnetic susceptibility, but it is left for a future study.

Substituting $B_{\rm m}$ given by Eq. (4) into the r.h.s. of Eq. (6), and $B_{\rm m}$ given by Eq. (5) into the l.h.s. of Eq. (6), we obtain the relation between $\bar{M}$ and M_- as

$$a\bar{M}^2 - 3b\bar{M}^4 + 5c\bar{M}^6 \approx aM_-{}^2 - 3bM_-{}^4 + 5cM_-{}^6. \tag{7}$$

By introducing the parameter δ defined as $\delta \equiv M_-/\bar{M}$, we reduce Eq. (7) to the equation for $\bar{M}$ as

$$a^*\bar{M}^2 - 3b^*\bar{M}^4 + 5c^*\bar{M}^6 \approx 0, \tag{8}$$

where $a^* \equiv a(1-\delta^2)$, $b^* \equiv b(1-\delta^4)$, and $c^* \equiv c(1-\delta^6)$. Solving Eq. (8), we obtain $\bar{M}^2$ for

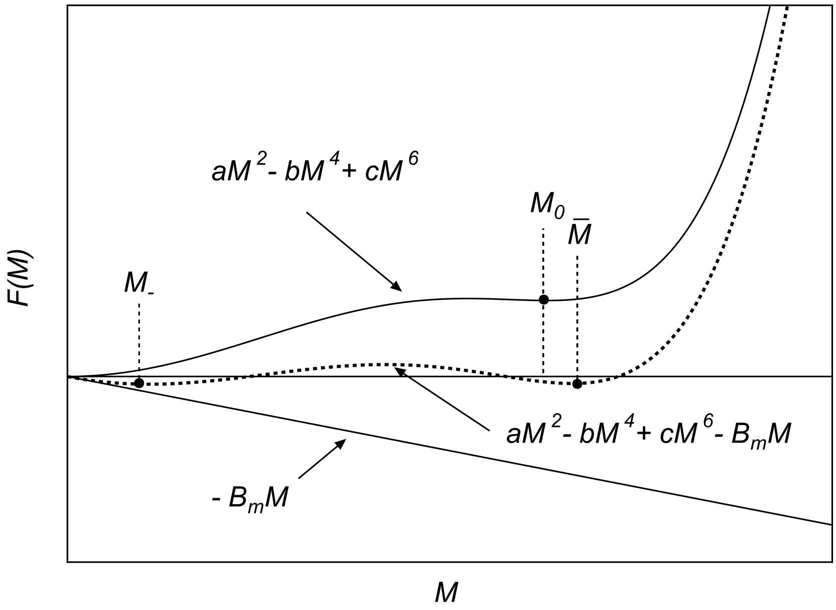

Fig. 1. Schematic behavior of free energies as a function of the uniform magnetization. The solid curve and solid line represent the free energy $F_0(M)$ [Eq. (1)] without the magnetic field B and the Zeeman energy at the metamagnetic field $B_{\rm m}$, respectively. The dotted curve represents the free energy $F_0(M) - B_{\rm m}M$, which has two degenerate minima at $M = M_-$ and $\bar{M}$.

the local minimum of the free energy $[F_0(M) - B_{\rm m}M]$ as

$$\bar{M}^2 \approx \frac{3b^* + \sqrt{9b^{*2} - 20a^*c^*}}{10c^*}. \tag{9}$$

Hereafter, to simplify the analysis, we search the case in which there exists a local minimum of $F_0(M)$ at $M = M_0$, although this is not a necessary condition for the occurrence of the metamagnetic transition but slightly narrows the parameter space explored. Then, there arises a constraint for the coefficient b obtained from the condition $F_0(M_0) > 0$, where M_0 is the magnetization corresponding to the stationary condition $\partial F_0(M_0)/\partial M_0 = 0$. The stationary condition for $M_0 > 0$ is explicitly given as

$$a - 2bM_0^2 + 3cM_0^4 = 0, \tag{10}$$

so that M_0^2 for the local minimum is given by

$$M_0^2 = \frac{b + \sqrt{b^2 - 3ac}}{3c}. \tag{11}$$

Therefore, with the use of the relation Eq. (10), the condition for $F(M_0) > 0$ is given explicitly as

$$F_0(M_0) = aM_0^2 - bM_0^4 + cM_0^6 = \frac{M_0^2}{3}(2a - bM_0^2) > 0. \tag{12}$$

By a straightforward calculation, we find that the inequality $2a > bM_0^2$ gives the following condition:

$$3ac < b^2 < 4ac. \tag{13}$$

3. Effective Mass and Damping Rate of Quasiparticles at Metamagnetic Transition

3.1 Forms of spin fluctuation propagator around two local minima of free energy at metamagnetic transition

Under the assumption that the system is uniform in space at around the first-order metamagnetic transition, the enhancements of effective mass, i.e., the Sommerfeld coefficient γ, and the damping rate of the quasiparticles are given by estimating the effect of ferromagnetic spin fluctuations in the b-direction. Namely, hereafter, we discuss the effect of the longitudinal spin fluctuations in the b-direction, which would be justified by the fact that the enhanced magnetic fluctuations occur along the metamagnetic field around $M = M_{-}$ as shown below. Note that the transverse spin fluctuations in the a- and c-directions are essentially unaltered because the magnetic field in the b-direction does not affect the magnetization perpendicular to the b-direction in the orthorhombic and centrosymmetric crystal systems such as UTe_2 as mentioned in the first paragraph of Sect. 2. To discuss the effect of the magnetic fluctuations, we use the conventional form of the dynamical spin susceptibility of ferromagnetic fluctuations discussed in Refs. 9 and 10 as

$$\chi_{\rm s}(\boldsymbol{q}, i\omega_m) = \frac{qN_{\rm F}^{*}/C}{\omega_{\rm s}(q) + |\omega_m|}, \quad \text{for } q < q_{\rm c} \sim p_{\rm F}, \tag{14}$$

where $N_{\rm F}^{*}$ is the density of states (DOS) of the quasiparticles *per spin* at the Fermi level, which is renormalized only by the *local* correlation effect, and $\omega_{\rm s}(q)$ is defined as

$$\omega_{\rm s}(q) \equiv \frac{q}{C}(\eta + Aq^2), \tag{15}$$

where η parameterizes the closeness to the ferromagnetic criticality.[11)]

The static susceptibility around $M = 0$ at $B = 0$ is given by

$$\chi_{\rm s}(0,0) = \left[\frac{\partial^2 F_0(M)}{\partial M^2}\Big|_{M=0}\right]^{-1} = \frac{1}{2a} \equiv \frac{N_{\rm F}^{*}}{\eta_0}. \tag{16}$$

This is nothing but the relation Eq. (2). By generalizing this relation around $M = 0$, we obtain the differential spin susceptibility around $M = \bar{M}$ at $B = B_m$ as

$$\chi_{\rm s}(0,0) = \left[\frac{\partial^2 F_0(M)}{\partial M^2}\Big|_{M=\bar{M}}\right]^{-1} \equiv \frac{N_{\rm F}^{*}}{\bar{\eta}}, \tag{17}$$

which is based on the fact that the Zeeman term $(-B_{\rm m}M)$ does not contribute to the curvature of the free energy. Similarly, that around $M = M_{-}$ at $B = B_{\rm m}$ is given by

$$\chi_{\rm s}(0,0) = \left[\frac{\partial^2 F_0(M)}{\partial M^2}\Big|_{M=M_{-}}\right]^{-1} \equiv \frac{N_{\rm F}^{*}}{\eta_{-}}. \tag{18}$$

According to the expression for $F_0(M)$ [Eq. (1)], $\partial^2 F_0(M)/\partial M^2|_{M=\bar{M}}$ is given by

$$\frac{\partial^2 F_0(M)}{\partial M^2}\Big|_{M=\bar{M}} = 2a - 12b\bar{M}^2 + 30c\bar{M}^4. \tag{19}$$

With the use of Eq. (8), this is reduced to

$$\begin{aligned}\frac{\partial^2 F_0(M)}{\partial M^2}\Big|_{M=\bar{M}} &\approx 2a - 12b^*\bar{M}^2 - \frac{6c}{c^*}(a^* - 3b^*\bar{M}^2)\\ &= 2\left(a + \frac{3a^*c}{c^*} - 4a^*\right) + \left(\frac{3c}{c^*} - 2\right)\frac{3}{5}\sqrt{9\kappa^* - 20}\left[\sqrt{9\kappa^* - 20} + \sqrt{\kappa^*}\right]a^*, \end{aligned}\tag{20}$$

where we have used the relation Eq. (9) and $\kappa^* \equiv b^2/(a^*c^*)$ for obtaining the second equality. With the use of the definitions of a^*, b^*, and c^* [see below Eq. (8)],

$$\left(a + \frac{3a^*c}{c^*} - 4a^*\right) = \delta^2 a + \frac{3a\delta^6}{1+\delta^2+\delta^4} \simeq \delta^2 a, \tag{21}$$

$$\left(\frac{3c}{c^*} - 2\right) = 1 + \frac{3\delta^6}{1-\delta^6} \simeq 1, \tag{22}$$

which is based on the fact that the experimental value for UTe_2 is $\delta \simeq 0.4$.[3)] Therefore, $\partial^2 F_0(M)/\partial M^2|_{M=\bar{M}}$ is approximately given by

$$\frac{\partial^2 F_0(M)}{\partial M^2}\Big|_{M=\bar{M}} \simeq 2\left[\delta^2 a + \frac{9}{5}\sqrt{9\kappa^* - 20}\left(\sqrt{9\kappa^* - 20} + \sqrt{\kappa^*}\right)a^*\right]. \tag{23}$$

With the use of the inequality [Eq. (13)], the value of κ^* is restricted in the following region:

$$\frac{3}{(1-\delta^2)(1-\delta^6)} < \kappa^* < \frac{4}{(1-\delta^2)(1-\delta^6)}. \tag{24}$$

Considering the smallness of $\delta^4 \simeq 0.026$ and $\delta^6 \simeq 0.0041$ for UTe_2 because $\delta \simeq 0.4$,[3)] this restriction [Eq. (24)] is technically given by

$$3.6 \lesssim \kappa^* \lesssim 4.8. \tag{25}$$

On the other hand, $\partial^2 F_0(M)/\partial M^2|_{M=M_-}$ is given by

$$\begin{aligned}\frac{\partial^2 F_0(M)}{\partial M^2}\Big|_{M=M_-} &= 2a - 12b{M_-}^2 + 30c{M_-}^4\\ &= 2a - 12b\delta^2\bar{M}^2 + 30c\delta^4\bar{M}^4. \end{aligned}\tag{26}$$

With the use of the relation $\bar{M}^4 \approx (-a^* + 3b^*\bar{M}^2)/5c^*$, given by Eq. (8), and the definitions of a^*, b^*, and c^* [see below Eq. (8)], $\partial^2 F_0(M)/\partial M^2|_{M=M_-}$ is reduced to

$$\frac{\partial^2 F_0(M)}{\partial M^2}\Big|_{M=M_-} \approx 2a\left(1 - 3\delta^4\frac{1-\delta^2}{1-\delta^6}\right) - 6b\left[2\delta^2 - \frac{3\delta^4(1-\delta^4)}{1-\delta^6}\right]\bar{M}^2 \tag{27}$$

$$\simeq 1.87(a - 0.78b\bar{M}^2), \tag{28}$$

where we have used $\delta \simeq 0.4$ to obtain the approximate equality of Eq. (28). Since $\partial^2 F_0(M)/\partial M^2|_{M=M_-}$ should be positive, the coefficients a and b and the upper magnetic field $\bar{M}$ should satisfy the following condition:

$$a - 0.78b\bar{M}^2 \gtrsim 0. \tag{29}$$

Together with the condition [Eq. (24)], this gives the constraint for the parameter set of a, b, and c, and the upper magnetization at $B = B_{\rm m}$.

To conclude this subsection, it is crucial to note that the relation Eq. (28) generally implies that the curvature of the free energy at $M = M_{-}$ is always smaller than $2a$ at around $M = 0$ under the ambient condition $(B = 0)$l, which gives considerably larger ferromagnetic fluctuations leading to the enhancements of the Sommerfeld coefficient γ and the A_ρ coefficient of the T^2 term in the resistivity than those under the ambient condition. The effect arising from the fluctuations around $M = \bar{M}$ at $B = B_{\rm m}$ is smaller than that under the ambient conditions because $\bar{\eta}$ is a few times larger than η_0 under the ambient conditions. For example, if we use the average value for $\kappa^* = 4.2$ over the possible range of the condition [Eq. (25)], $\bar{\eta}$ is given by $\bar{\eta} \simeq 3.8\eta_0$.

3.2 Expressions for enhancements of effective mass and damping rate near first-order metamagnetic transition

On the basis of the the theoretical framework discussed in the previous subsection and the formulae given in Appendices A and B, we show that the enhancements of the Sommerfeld coefficient of the quasiparticles and the coefficient A of T^2 term in the resistivity, reported in Refs. 3 and 6, can be evaluated semiquantitatively by taking a reasonable set of parameters, a part of which is fixed using the physical quantities experimentally observed.

First of all, the magnetization ratio $\delta = M_{-}/\bar{M}$ at the metamagnetic transition is fixed as $\delta \simeq 0.4$,[3] which has already been used in the previous subsection. The upper magnetization at the metamagnetic transition is also determined as $\bar{M} \simeq 1.0$.[3] Therefore, the condition [Eq. 29] is reduced to

$$a - 0.78b \gtrsim 0. \tag{30}$$

With the use of Eqs. (17) and (23), $\bar{\eta}$ is given by

$$\bar{\eta} = N_{\rm F}^* \frac{\partial^2 F_0(M)}{\partial M^2}\Big|_{M=\bar{M}} \simeq \left[2\delta^2 a + \frac{9}{5}\sqrt{9\kappa^* - 20}\left(\sqrt{9\kappa^* - 20} + \sqrt{\kappa^*}\right) a^*\right] N_{\rm F}^*. \tag{31}$$

Similarly, with the use of Eqs. (18) and (28), η_{-} is given by

$$\eta_{-} = N_{\rm F}^* \frac{\partial^2 F_0(M)}{\partial M^2}\Big|_{M=M_{-}} \simeq 1.87(a - 0.78b\bar{M}^2) N_{\rm F}^*. \tag{32}$$

Note that, according to Eq. (16), the parameter η_0 for the ambient condition $(B = 0)$ is given by

$$\eta_0 = 2aN_{\rm F}^*. \tag{33}$$

According to Eqs. (A·9) and (B·4), the coefficient A_ρ of the T^2 term in the resistivity is given as

$$A_\rho \simeq A_{\rho 0} + \frac{m^*}{Ne^2}\frac{VN_{\rm F}^* g^2 C}{4\pi^2 N v_{\rm F}^* k_{\rm F} A}\left(\frac{1}{\eta} - \frac{1}{\eta + Aq_{\rm c}^2}\right), \tag{34}$$

where $A_{\rho 0}$ is the coefficient without ferromagnetic spin fluctuations and is given by

$$A_{\rho 0} \approx \frac{m^*}{Ne^2}\frac{2}{\epsilon_{\rm F}^*} = \frac{m^*}{Ne^2}\frac{8}{3}N_{\rm F}^*, \tag{35}$$

where the damping rate $1/2\tau^*$ of quasiparticles at the Fermi level is assumed to be given by $1/2\tau^* \approx sT^2/\epsilon_{\rm F}^* = (4/3)sN_{\rm F}^* T^2$, with $\epsilon_{\rm F}^*$ being the Fermi energy of quasiparticles and s being a constant of $\mathcal{O}(1)$. In deriving the equality in Eq. (35), the relation $N_{\rm F}^* = (3/4)\epsilon_{\rm F}^*$ has been used. Note that the second term in Eq. (34) is not given by $\Sigma''_{k_{\rm F}}(0;T)$ [Eq. (A·8)], but is given by $\Sigma''_{{\rm tr},k_{\rm F}}(0;T)$ [Eq. (A·9)]. Note also that the mass m^* in the Fermi energy $\epsilon_{\rm F}^*$ appearing in Eqs. (34) and (35), and formulae hereafter, is the effective mass renormalized both by local correlations among 4f electrons at the U site and the effect of spin fluctuations both longitudinal (parallel to b-axis) and transverse (perpendicular to b-axis) under the ambient condition (at $B = 0$).

Similarly, with the use of Eq. (A·12), the increase in the Sommerfeld coefficient due to the ferromagnetic spin fluctuations, $\partial\Sigma_{\rm s}^{\rm R}(p_{\rm F},\epsilon)/\partial\epsilon|_{\epsilon=0}$, leads to

$$\gamma \simeq \gamma_0\left\{1 + \frac{VN_{\rm F}^* g^2}{8\pi^2 N v_{\rm F}^* A}\left[\log\frac{Aq_{\rm c}^2+\eta}{\eta} - \frac{1}{2}\log\frac{Aq_{\rm c}^2 + (Cv_{\rm F}^*)^2}{\eta^2 + (Cv_{\rm F}^*)^2}\right]\right\}, \tag{36}$$

where $\gamma_0 \equiv (2\pi^2/3)N_{\rm F}^*$ is the Sommerfeld coefficient without the effect of ferromagnetic spin fluctuations or renormalized only by *local* correlations.

The coefficients of $VN_{\rm F}^* g^2 C/4\pi^2 N v_{\rm F}^* k_{\rm F} A$ in Eq. (34) and $Vg^2 N_{\rm F}^*/8\pi^2 N v_{\rm F}^* A$ in Eq. (36) are estimated as

$$\frac{VN_{\rm F}^* g^2 C}{4\pi^2 N v_{\rm F}^* k_{\rm F} A} \approx \frac{3N_{\rm F}^*(Cv_{\rm F}^*)g^2}{16(Ak_{\rm F}^2)\epsilon_{\rm F}^{*2}}, \tag{37}$$

and

$$\frac{VN_{\rm F}^* g^2}{8\pi^2 N v_{\rm F}^* A} \approx \frac{3N_{\rm F}^* g^2}{16(Ak_{\rm F}^2)\epsilon_{\rm F}^*}, \tag{38}$$

respectively. In deriving Eqs. (37) and (38), we have assumed that the dispersion of quasiparticles is given by the free dispersion, i.e., $\epsilon_{\boldsymbol{k}} = k^2/2m^*$, so that $v_{\rm F}^* k_{\rm F} = 2\epsilon_{\rm F}^*$ and $k_{\rm F}^3 = 3\pi^2 N/V$.

In an analysis below, we estimate the coupling constant as $g = 4\epsilon_{\rm F}^*$ borrowing the theoretical result for effective interaction $U^* = 4T_{\rm K}$ in the single-impurity Anderson model in the Kondo limit.[12,13] For simplicity, we assume $Ak_{\rm F}^2 \approx Aq_{\rm c}^2 \approx 1$ and $Cv_{\rm F}^* \approx 1$. These approximations affect to some extent the numerical estimate of the increases in the Sommerfeld coefficient γ and the A_ρ coefficient. However, such an uncertainty will be absorbed in an ambiguity of taking many other parameters characterizing the system. On this approximation scheme, the coefficients in Eqs. (37) and (38) are reduced to

$$\frac{VN_{\mathrm{F}}^{*}g^{2}C}{4\pi^{2}Nv_{\mathrm{F}}^{*}k_{\mathrm{F}}A}\approx 3N_{\mathrm{F}}^{*}, \tag{39}$$

and

$$\frac{VN_{\mathrm{F}}^{*}g^{2}}{8\pi^{2}Nv_{\mathrm{F}}^{*}A}\approx\frac{9}{4}, \tag{40}$$

respectively. It is crucial to note that the factor N_{F}^{*} arises from the density of states per quasiparticle (per spin) contained in the expression of the dynamical spin susceptibility [Eqs. (14) and (A·1)].

One might wonder whether the magnetic field dependence of N_{F}^{*} (or m^{*}) cannot be neglected because, at first sight, the conventional relation $N_{\mathrm{F}}^{*}=3/4\epsilon_{\mathrm{F}}^{*}$ implies that N_{F}^{*} is proportional to the effective mass of the quasiparticles, which is affected by the magnetic field of $B_{\mathrm{m}}\sim 35\,\mathrm{T}$.[3] However the size of the Zeeman energy (per formula unit) is estimated as $\mu_{\mathrm{eff}}B_{\mathrm{m}}/k_{\mathrm{B}}\simeq 9.2\,\mathrm{K}$, with the effective magnetic moment $\mu_{\mathrm{eff}}\simeq 0.4\mu_{\mathrm{B}}$ at the metamagnetic field $B=B_{\mathrm{m}}$, which is only about 1/5 of the so-called Kondo temperature T_{K} of $\sim 47\,\mathrm{K}$ that is estimated from the Sommerfeld coefficient under the ambient conditions, i.e., $\gamma_0\simeq 1.2\times 10^{2}\,\mathrm{mJ/K^{2}mole}$[6] by comparing with $\gamma\simeq 1.6\times 10^{3}\,\mathrm{mJ/K^{2}mole}$ in $CeCu_6$ whose $T_{\mathrm{K}}\simeq 3.5\,\mathrm{K}$.[14]

Furthermore, there is a chance that the density of states N_{F}^{*} related to the magnetic susceptibility and the Sommerfeld coefficient is technically robust against the magnetic field if the *local* spin fluctuations, which are the origin of mass enhancement under the ambient conditions, originate mainly from the Van Vleck process through the renormalized c-f hybridization in the system with the nonmagnetic (singlet) crystalline-electric-field (CEF) ground state in the f^2 configuration. That is, the Zeeman energy of the quasiparticles is essentially given by $\sim -\mu_{\mathrm{B}}^{2}(N_{\mathrm{F}})_{\mathrm{cond}}B^{2}$ because the magnetic susceptibility of the quasiparticles, χ_{quasi}, is given by that of conduction electrons, χ_{cond}. Indeed, such a property was observed in the NMR Knight shift measurements of UPt_3,[15] and supported by theoretical discussions in Ref. 16–18. Namely, N_{F}^{*} appearing in Eqs. (35) and (39), and γ_0 in Eq (36) can be relatively robust against the applied magnetic field. Although the CEF ground state of UTe_2 has not been observed, we assume that this case is realized as a working hypothesis.

Then, finally, the relations Eqs. (34) and (36) are respectively reduced to compact forms as

$$A_{\rho}\approx\frac{m^{*}}{Ne^{2}}N_{\mathrm{F}}^{*}\left[\frac{8}{3}+3\left(\frac{1}{\eta}-\frac{1}{\eta+Aq_{\mathrm{c}}^{2}}\right)\right], \tag{41}$$

and

$$\gamma\approx\frac{2\pi^{2}}{3}N_{\mathrm{F}}^{*}\left\{1+\frac{9}{4}\left[\log\frac{Aq_{\mathrm{c}}^{2}+\eta}{\eta}-\frac{1}{2}\log\frac{Aq_{\mathrm{c}}^{2}+(Cv_{\mathrm{F}}^{*})^{2}}{\eta^{2}+(Cv_{\mathrm{F}}^{*})^{2}}\right]\right\}. \tag{42}$$

3.3 Analysis of experiment of UTe_2

On the parameterization discussed in the previous subsection, we focus on understanding the enhancements of γ and A_ρ at the *first-order* metamagnetic transition reported in Refs. 6 and 4. The ratios of the Sommerfeld coefficient and the A_ρ coefficient at the metamagnetic transition to those at the ambient ($B = 0$) state are $\gamma(B = B_{\rm m})/\gamma(B = 0) \simeq 2.2$,[6] and $[A_\rho(B = B_{\rm m})/A_\rho(B = 0)]^{1/2} = 2.3 \pm 0.1$,[4] respectively. We have these two experimental data to explain and two adjustable theoretical parameters, η_0 [Eq. (16)] and η_- [Eq. (32)], that essentially affect the enhancements of γ and A_ρ, other than fundamental parameters, such as $Aq_{\rm c}^2$ and $Cv_{\rm F}^*$ in Eq. (36), at the ambient states, and experimentally fixed δ and $\bar{M}$ as mentioned above. On the other hand, $\bar{\eta}$ is restricted to a relatively narrow region. Namely, the range of κ^* is restricted by Eq. (25) between 3.6 and 4.8 so that the value of $\bar{\eta}$ is restricted in the range

$$2.85\,\eta_0 \lesssim \bar{\eta} \lesssim 4.78\,\eta_0, \tag{43}$$

where we have used Eq. (31) and the relation $2aN_{\rm F}^* = \eta_0$ [Eq. (33)]. This relation implies that the ferromagnetic fluctuations around $M = \bar{M}$ are less important than those around $M = 0$ at the ambient conditions. To find one of the possible sets of parameters that reproduce the observed values of enhancements in $\gamma(B = B_{\rm m})/\gamma(B = 0)$ and $[A_\rho(B = B_{\rm m})/A_\rho(B = 0)]^{1/2}$ mentioned above, let us fix $\kappa^* = 4.2$, which is the average in the possible range [Eq. (25)]. Then, $\bar{\eta}/\eta_0$ is fixed as $\bar{\eta}/\eta_0 \simeq 3.85$, which is nearly equal to the average over its possible range given by Eq. (43).

On the other hand, the ferromagnetic fluctuations around $M = M_-$ give the most dominant contribution to the enhancements of γ and $A_{\rho 0}$, because η_- [Eq. (32)], with $\bar{M} \simeq 1.0$, can be considerably smaller than $\eta_0 = 2aN_{\rm F}^*$ [Eq. (33)].

Although the numbers of enhancements of γ and A_ρ given by Eqs. (41) and (42) depend on those of $Aq_{\rm c}^2$ and $Cv_{\rm F}^*$ other than the important parameter η characterizing the strength of ferromagnetic fluctuations, we are interested in the ratio of those values under the metamagnetic field and the ambient condition. Therefore, we adopt the set $Aq_{\rm c}^2 = 1$ and $Cv_{\rm F}^* = 1$, considering that such uncertainties are absorbed in some ambiguities for taking η_0 and η_-. Furthermore, we adopt an approximation that a combination $[8/3 - 3/(\eta + Aq_{\rm c}^2)]$ in the bracket of Eq. (41) can be safely neglected compared with $3/\eta$, which is far larger than 1 near the ferromagnetic critical point as expected in UTe_2.

Then, it is shown by straightforward arithmetic that the experimental values, $\gamma(B = B_{\rm m})/\gamma(B = 0) \simeq 2.2$[6] and $[A_\rho(B = B_{\rm m})/A_\rho(B = 0)]^{1/2} = 2.3 \pm 0.1$,[4] are reproduced by taking, for example, $\eta_0 = 1/10$ and $\eta_- = 1/50$, which leads to

$$\frac{\gamma(B = B_{\rm m})}{\gamma(B = 0)} = \frac{\bar{\gamma} + \gamma_-}{\gamma_0} \simeq 2.19, \tag{44}$$

and

$$\left[\frac{A_\rho(B=B_\mathrm{m})}{A_\rho(B=0)}\right]=\left[\frac{\bar{A}_\rho+A_\rho^-}{A_\rho^0}\right]\simeq 2.29, \tag{45}$$

respectively. Here, γ_0 and A_ρ^0 are those under the ambient conditions, γ_- and A_ρ^- are those arising from fluctuations with η_- around $M=M_-$, and $\bar{\gamma}$ and $\bar{A}_\rho$ are those from fluctuations with $\bar{\eta}$ around $M=\bar{M}$. These values [Eqs. (44) and (45)] reproduce the observed values.

Of course, there are almost infinite sets of parameters η_0, η_-, and $\bar{\eta}$ [or κ^* through Eqs. (17) and (20)] to reproduce the observed values of $\gamma(B=B_\mathrm{m})/\gamma(B=0)$ and $[A_\rho(B=B_\mathrm{m})/A_\rho(B=0)]^{1/2}$ other than those we have adopted above. Furthermore, the ambiguity in the above adopted coupling constant $g=4\epsilon_\mathrm{F}^*$ between the quasiparticles and the ferromagnetic spin fluctuations may affect the choice of the set of η_0, η_- and $\bar{\eta}$. Nevertheless, it does not considerably affect the result because the quantities in question are the ratios of values between those at $B=B_\mathrm{m}$ and $B=0$.

4. Conclusion and Perspective

Motivated by the experimental observation of the sharp enhancements of the Sommerfeld coefficient γ and the coefficient A_ρ of the T^2 term in the resistivity at the *first-order* metamagnetic transition in UTe_2, we developed the extended Landau theory of the phase transition on the basis of the *uniaxial* model. As a result, the enhanced ferromagnetic spin fluctuations, caused by the flattening of the curvature of the free energy around the shifted local minimum, give such enhancements of γ and A_ρ around the metamagnetic field B_m. However, the anisotropy in the magnetic response has not been discussed in this paper, although there exists the pronounced anisotropy in UTe_2. The effect of the anisotropy in the magnetic space is left for a future study.

Another interesting aspect of the present theory is that superconductivity is expected to be induced at around $B \lesssim B_\mathrm{m}$ owing to the enhancement of ferromagnetic spin fluctuations there, which was the origin of the enhancements of γ and A_ρ around $B=B_\mathrm{m}$. This mechanism may have some relevance to the appearance of the re-entrant superconductivity around the metamagnetic field,[5)] along with the discussions in the case of URhGe given in Refs. 19 and 20.

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Appendix A: Estimation of Self-energy due to Ferromagnetic Fluctuations

In this appendix, the self-energy of the quasiparticles due to the ferromagnetic fluctuations is discussed. For this purpose, we adopt an exponentially decaying phenomenological form for the spin fluctuation propagator (dynamical spin susceptibility) $\chi_{\rm s}(\boldsymbol{q}, i\omega_m)$ in the Matsubara frequency representation as

$$\chi_{\rm s}(\boldsymbol{q}, i\omega_m) = \frac{qN_{\rm F}^*/C}{\omega_{\rm s}(q) + |\omega_m|}, \quad \text{for } q < q_{\rm c} \sim p_{\rm F}, \tag{A·1}$$

where $N_{\rm F}^*$ is the DOS of the quasiparticles per quasiparticle, and $\omega_{\rm s}(q)$ is defined as

$$\omega_{\rm s}(q) \equiv \frac{q}{C}(\eta + Aq^2), \tag{A·2}$$

where η parameterizes the closeness to the ferromagnetic criticality.[9,10]

The retarded self-energy $\Sigma_{\rm s}^{\rm R}(p, \epsilon + {\rm i}\delta)$ gives a measure of the quasiparticle effective mass and lifetime in its real and imaginary parts, respectively. It can be calculated using a simple one-fluctuation mode exchange process (see Fig. A·1) and is given as

$$\begin{aligned}
{\rm Re}\Sigma_{\rm s}^{\rm R}(p, \epsilon) &= -\frac{N_{\rm F}^*}{2\pi CN}\sum_{\boldsymbol{q}} q\, g_q^2 \int_{-\infty}^{+\infty} dx \frac{x}{\omega_{\rm s}(q)^2 + x^2} \\
&\qquad \times \frac{\coth\frac{x}{2T} + \tanh\frac{\xi^*_{\boldsymbol{p}-\boldsymbol{q}}}{2T}}{-\epsilon + \xi^*_{\boldsymbol{p}-\boldsymbol{q}} + x},
\end{aligned} \tag{A·3}$$

$$\begin{aligned}
{\rm Im}\Sigma_{\rm s}^{\rm R}(p, \epsilon) &= -\frac{N_{\rm F}^*}{2CN}\sum_{\boldsymbol{q}} q\, g_q^2 \frac{\epsilon - \xi^*_{\boldsymbol{p}-\boldsymbol{q}}}{\omega_{\rm s}(q)^2 + (\epsilon - \xi^*_{\boldsymbol{p}-\boldsymbol{q}})^2} \\
&\qquad \times \left(\coth\frac{\epsilon - \xi^*_{\boldsymbol{p}-\boldsymbol{q}}}{2T} + \tanh\frac{\xi^*_{\boldsymbol{p}-\boldsymbol{q}}}{2T}\right),
\end{aligned} \tag{A·4}$$

where N is the number of U sites, g_q is the coupling between quasiparticles and spin fluctuation modes, and ξ_p^* is the dispersion of the quasiparticle measured from the chemical potential. Hereafter, for simplicity, g_q is assumed to be constant without wavenumber dependence. because it can be essentially approximated as a constant.

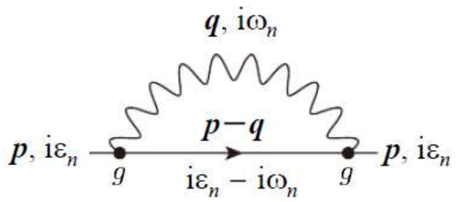

Fig. A·1. Feynman diagram for the self-energy given by Eqs. (A·3) and (A·4). The solid line with an arrow represents the Green function $\bar{G}_{\rm f}$ of the f electron renormalized only by local correlations, the wavy line represents the spin fluctuation propagator $\chi_{\rm s}$, and g is the coupling constant between the localized 4f electron and the spin fluctuation mode.

In typical limiting cases, (A·4) can be straightforwardly calculated on the approximation, $\xi^*_{\boldsymbol{p}-\boldsymbol{q}} \simeq -v^*_{\rm F} qx$, where $x \equiv \cos\theta$ with θ being the angle between $\boldsymbol{p}$ and $\boldsymbol{q}$ and $v^*_{\rm F}$ being the velocity of the quasiparticles at the Fermi level.

In the case $T = 0$, $\epsilon \neq 0$,

$$\mathrm{Im}\Sigma^{\rm R}_{\rm s}(p_{\rm F}, \epsilon) \simeq -\frac{Vg^2 N^*_{\rm F} C}{32\pi N v^*_{\rm F} \sqrt{A}} \frac{1}{\eta^{3/2}} \epsilon^2, \tag{A·5}$$

where V is the system volume, and we have used the fact that the factor $\{\coth[(\epsilon - \xi^*_{\boldsymbol{p}-\boldsymbol{q}})/2T] + \tanh(\xi^*_{\boldsymbol{p}-\boldsymbol{q}}/2T)\}$ is nonvanishing only in the region $0 < \xi^*_{\boldsymbol{p}-\boldsymbol{q}} < \epsilon$, assuming the case $\epsilon > 0$, and performed integrations with respect to $q \equiv |\boldsymbol{q}|$ and $x \equiv (\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}})$. Here, we have retained the most divergent term in $1/\eta$ with $\eta \to 0$ and assumed $\eta \ll Aq^2_{\rm c}$.

It is shown by a straightforward calculation that the coefficient of the ϵ^2 term for the resistivity, $\mathrm{Im}[\Sigma^{\rm R}_{\rm s}(p_{\rm F}, \epsilon)]_{\rm tr}$, is less singular owing to the extra factor $(q/k_{\rm F})[1 - (\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}})] = (q/k_{\rm F})(1 - x)$, which is necessary for taking into account the effect of the momentum change $\boldsymbol{p} \to \boldsymbol{p} + \boldsymbol{q}$ contributing to the resistivity. Namely,

$$\begin{aligned}\mathrm{Im}[\Sigma^{\rm R}_{\rm s}(p_{\rm F}, \epsilon)]_{\rm tr} \simeq & -\frac{Vg^2 N^*_{\rm F} C}{16\pi^2 N v^*_{\rm F} k_{\rm F} A} \frac{\epsilon^2}{\eta} + \frac{Vg^2 N^*_{\rm F} C}{12\pi^2 N v^{*2}_{\rm F}} \frac{\epsilon^3}{(\eta + Aq^2_{\rm c})^2} \log \frac{q_{\rm c}\eta}{C\epsilon} \\ & - \frac{Vg^2 N^*_{\rm F} C}{32\pi N v^{*2}_{\rm F} k_{\rm F} \sqrt{A}} \frac{\epsilon^3}{\eta^{3/2}}.\end{aligned} \tag{A·6}$$

In the case $\epsilon = 0$, $0 < T \ll \epsilon^*_{\rm F}$,

$$\begin{aligned}\mathrm{Im}\Sigma^{\rm R}_{\rm s}(p_{\rm F}, 0) \quad \simeq \quad & -\frac{Vg^2 N^*_{\rm F}}{8\pi^2 N v^*_{\rm F} C} \int_0^{q_{\rm c}} dq\, q^2 \int_{-vq/T}^{vq/T} dy \times \\ & \frac{y}{[\omega_{\rm s}(q)/T)^2 + y^2]} \left(\coth \frac{y}{2} - \tanh \frac{y}{2} \right),\end{aligned} \tag{A·7}$$

where $y = v^*_{\rm F} qx/2T$. The integration with respect to y can be approximately performed, leading to

$$\mathrm{Im}\Sigma^{\rm R}_{\rm s}(p_{\rm F}, 0) \simeq -\frac{Vg^2 N^*_{\rm F} C}{8\pi N v^*_{\rm F} \sqrt{A}} \frac{1}{\eta^{3/2}} T^2, \tag{A·8}$$

where we have made the approximation that the range of integration is technically restricted as $-1 < y < 1$, in which the last factor in (A·7) is approximated as $2/y$. This is because the factor $(\coth \frac{y}{2} - \tanh \frac{y}{2})$ in Eq. (A·7) decreases rapidly in proportion to e^{-y} in the region $|y| > 1$, and $v^*_{\rm F} q \gg T$ holds in the dominant region of q-space. Similarly to the case $T = 0$, $\epsilon \neq 0$ above, the imaginary part of the self-energy for the resistivity, $\mathrm{Im}[\Sigma^{\rm R}_{\rm s}(p_{\rm F}, 0)]_{\rm tr}$, is given as

$$\mathrm{Im}[\Sigma^{\rm R}_{\rm s}(p_{\rm F}, 0)]_{\rm tr} \simeq -\frac{Vg^2 N^*_{\rm F} C}{4\pi^2 N v^*_{\rm F} k_{\rm F} A} \left(\frac{1}{\eta} - \frac{1}{\eta + Aq^2_{\rm c}} \right) T^2. \tag{A·9}$$

Note that x in the extra factor $(q/k_{\rm F})(1 - x)$ gives no extra contribution because the integrand with respect to $y = v^*_{\rm F} qx/2T$ in Eq. (A·7) is an even function of y.

The real part of the self-energy, (A·3), can be calculated easily at $T = 0$ and $\epsilon \sim 0$, leading to

$$\begin{aligned}&\mathrm{Re}\left[\Sigma_{\mathrm{s}}^{\mathrm{R}}(p_{\mathrm{F}},\epsilon)-\Sigma_{\mathrm{s}}^{\mathrm{R}}(p_{\mathrm{F}},0)\right]\\&\quad\simeq -\epsilon\,\frac{Vg^2N_{\mathrm{F}}^*}{4\pi^2Nv_{\mathrm{F}}^*}\int_0^{q_{\mathrm{c}}}dq\,q\frac{1}{\eta+Aq^2}\left[1-\frac{(\eta+Aq^2)^2}{(Cv_{\mathrm{F}}^*)^2+(\eta+Aq^2)^2}\right],\end{aligned}\tag{A·10}$$

where we have put the external momentum on the Fermi surface, and we also used the following approximate relations $\xi^*_{\boldsymbol{p}-\boldsymbol{q}}\simeq -v_{\mathrm{F}}^*qx$ and

$$\begin{aligned}&\sum_{\boldsymbol{q}} q\int_{-\infty}^{\infty}dw\frac{w}{w^2+[\omega_{\mathrm{s}}(q)]^2}[\mathrm{sign}(w)+\mathrm{sign}(\xi^*_{\boldsymbol{p}-\boldsymbol{q}})]\left(\frac{1}{-\epsilon+\xi^*_{\boldsymbol{p}-\boldsymbol{q}}+w}-\frac{1}{\xi^*_{\boldsymbol{p}-\boldsymbol{q}}+w}\right)\\&\quad\approx\frac{\epsilon V}{2\pi}\int_0^{q_{\mathrm{c}}}dqq^3\frac{1}{v_{\mathrm{F}}^*q\omega_{\mathrm{s}}(q)}\left\{1-\frac{[\omega_{\mathrm{s}}(q)]^2}{(v_{\mathrm{F}}^*q)^2+[\omega_{\mathrm{s}}(q)]^2}\right\}+\mathcal{O}(\epsilon^2).\end{aligned}\tag{A·11}$$

Performing the $\boldsymbol{q}$-integration, we obtain

$$\begin{aligned}&\mathrm{Re}\left[\Sigma_{\mathrm{s}}^{\mathrm{R}}(p_{\mathrm{F}},\epsilon)-\Sigma_{\mathrm{s}}^{\mathrm{R}}(p_{\mathrm{F}},0)\right]\\&\quad\approx -\epsilon\,\frac{Vg^2N_{\mathrm{F}}^*}{8\pi^2Nv_{\mathrm{F}}^*A}\left[\log\frac{Aq_{\mathrm{c}}^2+\eta}{\eta}-\frac{1}{2}\log\frac{Aq_{\mathrm{c}}^2+(Cv_{\mathrm{F}}^*)^2}{\eta^2+(Cv_{\mathrm{F}}^*)^2}\right]+\mathcal{O}(\epsilon^2).\end{aligned}\tag{A·12}$$

Note that the less singular terms in η in the logarithm have been retained.

Appendix B: Structure of Green Function of Quasiparticles

In this appendix, we briefly recapitulate the discussion on the relationship between the resistivity and the self-energy of quasiparticles, which are strongly renormalized by the self-energy. Let us start with the general expression of the retarded Green function $G^{\mathrm{R}}(\boldsymbol{k},\epsilon)$:

$$[G^{\mathrm{R}}(\boldsymbol{k},\epsilon)]^{-1}=\epsilon-\xi^*_{\boldsymbol{k}}-\Sigma'_{\boldsymbol{k}}(\epsilon)-i\Sigma''_{\boldsymbol{k}}(\epsilon),\tag{B·1}$$

where $\Sigma'_{\boldsymbol{k}}(\epsilon)$ and $\Sigma''_{\boldsymbol{k}}(\epsilon)$ are the real and imaginary parts of the self-energy, respectively. In the region $\epsilon\sim 0$, $[G^{\mathrm{R}}(\boldsymbol{k},\epsilon)]^{-1}$ is approximated as

$$\begin{aligned}&[G^{\mathrm{R}}(\boldsymbol{k},\epsilon)]^{-1}\simeq\left[1-\frac{\partial\Sigma'_{\boldsymbol{k}}(\epsilon)}{\partial\epsilon}\right]_{\epsilon=0}\times\\&\qquad\left\{\epsilon-\left[1-\frac{\partial\Sigma'_{\boldsymbol{k}}(\epsilon)}{\partial\epsilon}\right]^{-1}_{\epsilon=0}\xi^*_{\boldsymbol{k}}-i\left[1-\frac{\partial\Sigma'_{\boldsymbol{k}}\partial(\epsilon)}{\partial\epsilon}\right]^{-1}_{\epsilon=0}\Sigma''_{\boldsymbol{k}}(\epsilon)\right\}.\end{aligned}\tag{B·2}$$

Therefore, the effective mass $\tilde{m}^*$ near the Fermi level and the damping rate $1/\tilde{\tau}^*_{\mathrm{tr}}(\epsilon;T)$ with energy ϵ are renormalized as

$$\tilde{m}^*\simeq\left[1-\frac{\partial\Sigma'_{\boldsymbol{k}}(\epsilon)}{\partial\epsilon}\right]_{\epsilon=0}m^*,\tag{B·3}$$

and

$$\frac{1}{\tau^*_{\mathrm{tr}}(\epsilon;T)}\simeq -\left[1-\frac{\partial\Sigma'_{\boldsymbol{k}}(\epsilon)}{\partial\epsilon}\right]^{-1}_{\epsilon=0}\Sigma''_{\mathrm{tr},\boldsymbol{k}}(\epsilon;T),\tag{B·4}$$

respectively. Here, $\Sigma''_{\mathrm{tr},\boldsymbol{k}}(\epsilon;T)$ is the imaginary part of the self-energy in which the effect of the momentum change is taken into account as mentioned in the discussion leading to Eq.

(A·6). Then, it is immediately found that the approximate Drude formula for the resistivity $\rho \simeq (\tilde{m}^*/Ne^2)[1/\tilde{\tau}^*_{\rm tr}(0;T)]$ is not affected by the renormalization factor $[1-\partial\Sigma'_{\boldsymbol{k}}(\epsilon)/\partial\epsilon]^{-1}_{\epsilon=0}$.[21,22] Here, we have made the approximation that $\langle 1/\tilde{\tau}^*_{\rm tr}(\epsilon;T)\rangle$, where $\langle\cdots\rangle$ is the average over ϵ with the weight of the ϵ-derivative of the Fermi distribution function $[-\partial f(\epsilon)/\partial\epsilon]$, is replaced by $1/\tilde{\tau}^*_{\rm tr}(0;T)$, considering that ϵ and T dependences appear through a combination of $[\epsilon^2+(\pi T)^2]$ as in the Fermi liquid state.[23]

Namely, the resistivity ρ is given approximately as

$$\rho \simeq \frac{\tilde{m}^*}{Ne^2}\frac{1}{\tilde{\tau}^*_{k_{\rm F},{\rm tr}}(0;T)} \approx \frac{m^*}{Ne^2}\left[-\Sigma''_{{\rm tr},k_{\rm F}}(0;T)\right]. \tag{B·5}$$

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