Strong Magnetic Interactions and Short Range Magnetic Correlations in $CuTeO_4$

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Abstract

CuTeO₄ has been proposed as a crystallographically distinct, yet electronic structure analog, of superconducting cuprates. Magnetization measurements from T = 2-300 K indicate the presence of antiferromagnetic, low dimensional correlations with J = 148(7) K. Below T \approx 50 K, there is an upturn in magnetization indicative of enhanced ferromagnetic correlations, with neutron powder diffraction data showing the emergence of diffuse magnetic scattering at Q \approx 0.6 Å⁻¹. A Warren line shape analysis yields a characteristic magnetic correlation length of ξ = 4.5(9) Å at T = 40 K, rising to ξ = 10.1(9) Å at T = 10 K. Specific heat measurements reveal a sizable T-linear contribution of γ = 8.3 mJ/mol-f.u/K² at T < 5 K. Together, these results imply a low dimensional, antiferromagnetic spin glass state. Structural modeling of the neutron powder diffraction data reveals the presence of a layered-type of stacking disorder, providing both a rationale for the lack of long range magnetic order, as well as being consistent with the computational predictions of its two-dimensional nature. Unlike the cuprates, we find no evidence of substantive dopability in this wide band-gap, yellow insulator.

I Introduction

High temperature superconductivity in copper oxides is one of the most exciting emergent phenomena to result from strong electron correlations [1][2][3]. Significant progress has been made in the past 30 years in elucidating the key ingredients that underlie this behavior, with a layered, two-dimensional electronic structure, strong Cu-d and O-p orbital hybridization, and proximity to an insulating, magnetically ordered state, all believed to be important.

Recent computational work [4] identified $CuTeO_4$ as a previously unrecognized material containing these ingredients. Figure 1 shows the reported crystal structure of $CuTeO_4$, highlighting the layered nature that is distinct from the cuprates

in not being built of free-standing ${\rm CuO_2}$ planes but rather highly angled, corner sharing ${\rm CuO_4}$ units, arranged as coupled 1D chains, with the Cu-O-Cu bond angle of 126.1°closer to herbertsmithite [5] than to the cuprates. Thus ${\rm CuTeO_4}$ gives a possible new avenue to expand the known cuprate families, and provides an opportunity to test which features are most important in ultimately producing superconductivity.

Despite being known for more than 40 years [6], there is scant experimental data on $CuTeO_4$, particularly with regards to its magnetic and electronic properties. This is likely due to the difficulty in preparation, which required hydrothermal conditions at 650°C and 2000 bar for two months, to produce a multiphase mixture containing small crystals of $CuTeO_4$.

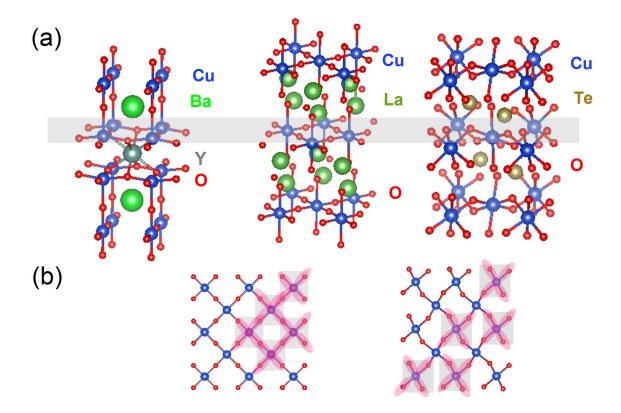


Figure 1: (a) Comparison of the layered structures of known superconductors La_2CuO_4 [7] and $YBa_3Cu_2O_7$ [8] to $CuTeO_4$. (b) While all structures have 2D CuO_2 planes with CuO_4 units (grey squares) $CuTeO_4$ has strong buckling driven by shared oxygens with TeO_6 octahedra. The alternating CuO_4 plaquettes in $CuTeO_4$ suggest a 1D chain like behaviour, mediated through $d_{x^2-y^2}$ orbitals lying in the same plane.

By developing a simpler, less onerous, route to preparing CuTeO₄, utilizing mild hydrothermal conditions at 210°C for 7 days, we are able to overcome these limitations. Here, we report on the low temperature magnetic behavior of CuTeO₄, utilizing a combination of magnetization, specific heat, and neutron powder diffraction measurements. The magnetization measurements show a broad hump at J = 148(7) K, suggesting some low dimensional antiferromagnetic correlations. Elastic neutron scattering reveals diffuse scattering at $Q \approx 0.6 \text{ Å}^{-1}$, which alongside bifurcations at $T \approx 50 \text{ K}$ in zero field cooled (ZFC) and field cooled (FC) susceptibility measurements imply that CuTeO₄ is a low dimensional, antiferromagnetic spin glass compound. Low temperature specific heat measurements for T < 5 K shows a large T-linear contribution of $\gamma = 8.3 \text{ mJ/mol f.u/K}^2$, which is

unexpected for a wide band gap insulator but is consistent with a spin glass state.

II Methods

CuTeO₄ was initially synthesized by Falck et al [6] under high pressure and temperature. We report a novel method of synthesizing yellow, pollycrystalline CuTeO₄ in mild hydrothermal conditions. A 1 : 3 ratio of CuO (2 mmol) and Te(OH)₆ (6 mmol) was used to make the reaction mixture. This was placed in a 23 ml Teflon lined stainless steel autoclave with 10 ml water and 0.03 ml $\rm H_2O_2$. A magnetic stir bar was added to the Teflon Cup and the autoclave was closed and placed on a magnetic hot plate in a sandbath. A temperature of 210°C was mantained for 7 days with the help of a thermocouple placed next to the autoclave in the sand. A difference of a

¹The inclusion of H_2O_2 seems to be crucial for the formation of $CuTeO_4$ in such mild conditions. Repetition of this experiment without H_2O_2 resulted in the formation of Cu_3TeO_6 , the more thermodynamically favourable phase in the formation of $CuTeO_4$. The decomposition of $CuTeO_4$ into Cu_3TeO_6 alongside TeO_2 and O_2 at 510 °C is documented in [9]. H_2O_2 quenches this decomposition by preventing the formation of TeO_2 , oxidizing Te^{4+} to Te^{6+} .

few degrees from the thermocouple sensing and the temperature within the autoclave was expected.

The phase purity of CuTeO₄ was confirmed by Powder X-ray diffraction data (PXRD) collected at room temperature. This was done using Bruker D8 Focus diffractometer with a LynxEye detector and Cu K α radiation (1.54 Å). Rietvield refinements on the PXRD data were done through TOPAS 4.2 [10][11].

The Magnetic Susceptibility was collected using the Vibrating Sample Magnetometer option in the Quantum Design Physical Properties Measurement System(PPMS) at a temperature range of 2 -The susceptibility is defined by the relation $\chi \approx \frac{M}{H}$ where M is the Magnetization of the sample and H is the applied field. The specific heat of the sample was measured using the semiadiabatic pulse method in the PPMS equipped with a Dilution Refrigerator from T = 0.1 - 300 K. Powder Neutron Diffraction data was obtained using the high-resolution neutron powder diffractometer POWGEN, Frame 2, center wavelength 1.5 Å and 6×10^7 total counts at the Oak Ridge National Laboratory. Stacking faults in the material were simulated using DIFFaX v 1.813 [12].

III Results

A Magnetism

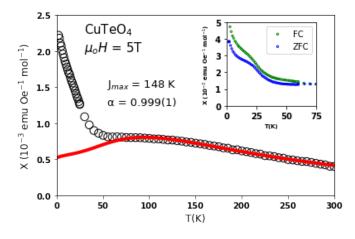


Figure 2: Molar magnetic susceptibility of $CuTeO_4$ fit to a $S=\frac{1}{2}$ Alternating Exchange Heisenberg Chain (red line). Inset shows ZFC/FC splitting indicative of a spin glass state.

The temperature dependent magnetic susceptibility data of $CuTeO_4$ shows no 3D long range ordering as

displayed in Fig. 2. Furthermore, there is no region in the susceptibility data that could be fit with the Curie-Weiss model. However, it seems there may be a paramagnetic regime at $T \ge 150$ K,

Parameters	Fit
$\mathrm{K}\;(\mathrm{erg}\;\mathrm{Oe}^{-2}\;\mathrm{mol}^{-1})$	$7.30(8) \times 10^{-3}$
J_{max} (K)	148(7)
α	0.999(1)
$\chi_o \text{ (emu Oe}^{-1} \text{ mol}^{-1})$	$-2.62(2) \times 10^{-4}$

Table 1: Fit parameters for T > 65 K for the magnetic susceptibility data (Figure 2).

as seen in some low-dimensional magnets [13][14][15]. The magnetic behaviour of CuTeO₄ is similar to Na₂Cu₂TeO₆ which displays paramagnetic behaviour at T > 300 K [16]. We find that for T > 65 K, the data for CuTeO₄ is well fit by a two dimensional function for the $S=\frac{1}{2}$ Antiferromagnetic Alternating Exchange Heisenberg chain [17], using the expression,

$$\chi(T) = K\chi^*(\alpha, \frac{T}{J_{max}}) + \chi_0 \tag{1}$$

where K = $\frac{N_A g^2 \mu_B^2}{J_{max} k_B}$ is an overall scaling factor, χ_o is the temperature-independent term and χ^* is the AFM alternating exchange model as a function of reduced temperature $\frac{T}{J_{max}}$. $\chi^*(\alpha, \frac{T}{J_{max}})$ describes a Heisenberg chain with two antiferromagnetic exchange interactions J_1 and J_2 , with $J_2 \leq J_1$ and $\alpha = \frac{J_2}{J_1}$. When $\alpha = 0$, the model recovers the isolated spin dimer model and when $\alpha=1$, the model recovers the uniform AFM Heisenberg The values from the best fit are given in Table 1. $\alpha = 0.999(1)$ indicates highly uniform exchange interactions and suggests that at T > 65 K, CuTeO₄ behaves as system of 1D AFM spin chains. This is perhaps surprising in the context of the prior discussion of twisted CuO₂ planes, but in agreement with a detailed view of the crystal structure, Figure 1(b), which shows how the particular connectivity of CuO₄ plaquettes forms 1D chains within each plane. The value of g = 1.7(3) appears slightly lower than the expected value of 2.2, but the large uncertainty precludes more definitive interpretation. $J_{max} = 148(7) \text{ K},$ is consistent with the experimental data, where a broad maximum is observed around the same temperature range. This value of J is roughly half of that expected from the Cu-O-Cu bond angle [18]; this likely reflects the greater ionicity of the Cu-O bonds in CuTeO₄ due to the large inductive withdrawing effect of neighboring Te⁶⁺ ions [19]. The negative value of χ_o is consistent with the insulating behavior of the compound. The abrupt upturn at T = 50 K is suggestive of a magnetic transition, with some ferromagnetic correlations. To confirm the freezing behaviour of this compound at T = 50 K, field-cooled (FC) and zero field-cooled (ZFC) DC susceptibility data was taken at $\mu_o H = 1$ T for CuTeO₄ (Figure 2).The bifurcation between ZFC and FC data at T ≈ 50 K shows that there is some freezing taking place at that transition temperature.

To unequivocally determine the glassy behvaiour of CuTeO₄, time of flight, elastic neutron scattering was carried out for $CuTeO_4$ at T = 10 K, 40 K, 60 K and 120 K. No Bragg peaks were observed in the data, but instead at $Q \approx 0.6(\text{Å}^{-1})$, there is a broad peak (Figure 3) that emerges, suggestive of short range magnetic order. Further evidence that it is magnetic in nature comes from subtracting the high temperature data from the low temperature data, Figure 4. The excess scattering falls off with angle, which implies a magnetic, rather than nuclear, form factor. The peak center is $Q \approx 0.6 \text{ Å}^{-1}$. This position is same as the one expected for (010) reflection, 0.61 Å^{-1} . As this reflection is disallowed for the nuclear structure due to systematic absences, it is likely this scattering is magnetic in nature and indicates the formation of short range magnetic order, with AFM order along the b axis (perpendicular to the planes). An alternative interpretation that we cannot definitively rule out is that this reflects the appearance of magnetic order with a propagation vector of k = (0.5,0,0) (Q = 0.57 Å⁻¹ for the first peak), but that seems unlikely as the standard crystallographic cell of CuTeO₄ can accommodate in-plane magnetic order without enlarging the unit The breadth of this peak is well-described by a Warren lineshape [20], which is characteristic of short range order along that crystallographic direction. The Warren lineshape function is,

$$P_{2\theta} = C \frac{1 + \cos^2 2\theta}{2(\sin \theta)^{\frac{3}{2}}} \frac{\sqrt{\pi}}{2e^{a^2}}$$
 (2)

where $C = KmF_{hk}^2 \frac{L}{\lambda\sqrt{\pi}}$, $d = \frac{2L\sqrt{\pi}}{\lambda}$ and

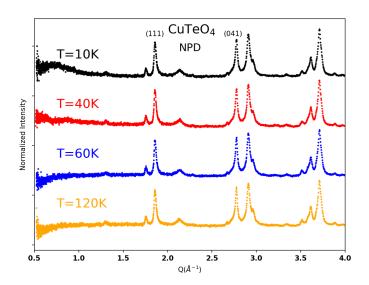


Figure 3: Raw elastic neutron scattering data for T=10 K, 40 K, 60 K and 120 K. No magnetic bragg peaks are visible. The structure peaks like (111) and (041) remain the same at different temperatures. At low Q (\mathring{A}^{-1}) we can see a broad peaks emerging at low temperature indicative of short range magnetic order.

 $a = d(\sin(\theta) - \sin(\theta_0))$. Here K is the overall scale factor, F_{hk} is the structure factor, m is the multiplicity of reflection (hk) centered around $2\theta_0$ and L is the average correlation length. extracted correlation length is $\xi = 4.5(9)$ Å at 40 K, rising to $\xi = 10.1(9)$ Å at 10 K, Table 2. The value of θ_o for T = 10 K is in agreement with the experimental data in Figure 3, where the broad peak appears at $Q \approx 0.6(\text{Å}^{-1})$. The fit for T = 40 K was constrained to this θ_o value for the function in Figure 4. The correlation lengths at T = 10 K and 40 K, are close to the interlayer spacing within CuTeO₄, and suggests little coherence between distinct planes of chains. Thus we conclude that CuTeO₄ exhibits glassy behavior with short correlation lengths between planes.

T(K)	С	θ_o (°)	L (Å)
10	0.732(5)	5.07(1)	10.1(9)
40	0.505(3)	5.07(1)	4.5(9)

Table 2: Fit parameters with the statistical portion of error from the Warren lineshape .

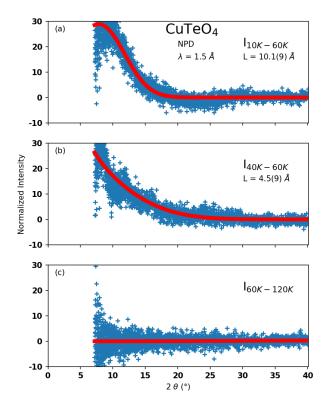


Figure 4: Warren Lineshape analysis characterizing short range order in $\mathrm{CuTeO_4}$. The fit is shown by the solid red line while the subtracted experimental data (a) 10 K - 60 K (b) 40 K - 60 K are the scattered blue markers, (c) 60 K - 120 K is included to show the absence of short range correlations in the spin glass state after the freezing temperature of T \approx 50 K.

B Heat Capacity

To further elucidate the physical properties of ${\rm CuTeO_4}$, temperature-dependent specific heat measurements were carried out. Figure 5 shows the specific heat characterization of ${\rm CuTeO_4}$ from 0.1 - 300 K. No sharp transitions are seen in this temperature range indicating a lack of long range magnetic order as shown by elastic neutron scattering and magnetic susceptibility data discussed previously. The experimental data is well fit at ${\rm T} \geq 4~{\rm K}$ by two Debye modes modelled by the following equation:

$$\frac{C_p}{T} = \frac{D(\theta_{D1}, T)}{T} + \frac{D(\theta_{D2}, T)}{T} \tag{3}$$

The fit was constrained so that $C_{phonon}(T) \leq C(T)$ at higher temperatures so that the phonon contribution is not overestimated. The oscillator strength as well as the two Debye temperatures are shown in Table 3. The first debye mode peaks approximately at $T \approx 50 \text{ K}$, which is around T_g , this is similar to other spin glasses where the maximums are observed after or at T_g [21][22][23]. no sharp maximums are observed in the specific heat data, this may be simply due to a more gradual entropy loss which does not result in any sharp transitions. While, within this temperature range the expected Dulong-Petit value of 3nR is not recovered, this can be explained by the high value of θ_{D2} which means that C(T) will plateau at T $\geq \theta_{D2}$. The low temperature specific heat, $T \le 4$ K is modelled by $\frac{C_p}{T} = \beta T^2 + \gamma$ where we extract $\gamma = 8.30(4)$ mJ mol⁻¹ K⁻² and $\beta = 0.46(2) \text{ mJ mol}^{-1} \text{ K}^{-4}$. This value of β recovers $\theta_D = 293.3$ K, closer to θ_{D1} than θ_{D2} .

Parameter	Fit
θ_{D1} (K)	257(1)
s_{D1}	1.78(2)
θ_{D2} (K)	794(13)
s_{D2}	4.08(6)

Table 3: Experimental data is well fitted with two debye modes. The oscillator strength (s_{D1}, s_{D2}) add up to 5.86, just shy of 6 atoms per formula unit found in CuTeO₄.

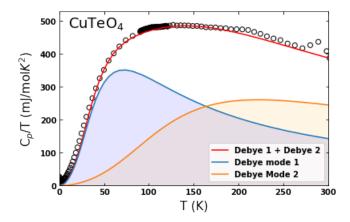


Figure 5: Specific heat measurements modelled by two Debye modes, with θ_{D1} dominant at low temperatures. The small transition at 275 K is due to the vaporization of n-grease.

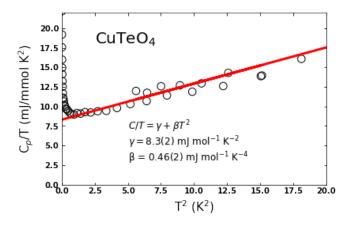


Figure 6: low temperature specific heat measurements of $CuTeO_4$.

This agrees with our Debye heat capacity analysis as at low temperatures the first Debye mode has a larger contribution to the specific heat as shown in Figure 5. The T-linear contribution to heat capacity is shown in Figure 6. At low temperatures, the phonon contribution to the specific heat often scales as the dimensionality of the material. The T³ contribution to the heat capacity therefore suggests that the phonon dispersions of CuTeO₄ are fairly three dimensional. The γ term, despite the material being a large band gap insulator, can be attributed to the large number of available magnetic states at low temperature rather than electronic states [24][25][26]. This is consistent with other reported spin glass compounds, where at $T \leq T_g$, the heat capacity scales as T [21][22]. While we have qualitatively described the magnetic contribution to heat capacity, it should be noted that $C_{magnetic}$ cannot be quantitatively extracted from the specific heat data as the phonon contribution cannot be distinguished from the magnetic contribution due to lack of a non magnetic analog for $CuTeO_4$.

C Crystal Structure

With the magnetic properties of $CuTeO_4$ firmly established, an obvious question arises: why is there short-range, glassy order, with an extremely short correlation length between CuO_2 planes? To answer this, we investigated the crystal structure in greater detail using powder x-ray diffraction. Falck et al. previously characterized the crystal structure in the $P2_1/n$ spacegroup [6]. Rietveld refinement of this model to our powder diffraction data (Figure 7a), shows clear discrepancies, especially in the predicted

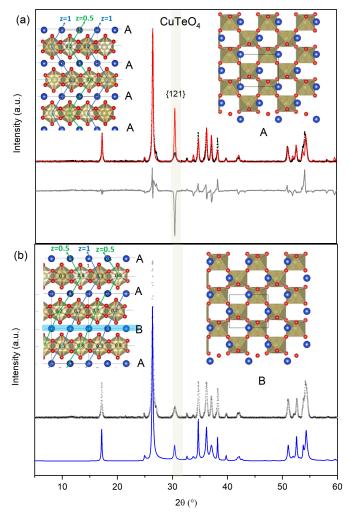


Figure 7: (a) Rietvield refinement(red line) of powder xray-diffraction data (black). The (121) peak is poorly fitted by the P21/n model. Inset in (a) shows a layered view of $CuTeO_4$ in P21/n (left) and a planar view (right) of the 'A' stacking model.(b) The model with 70% original Cu stacking and 30% altered-Cu position layering (blue line) is a good qualitative fit to x-ray data (black), capturing the breadth and height of the (121) peak. Inset in (b)shows the faulted stacking, highlighed in blue (left) and a planar view of the faulted layer 'B' (right).

intensity of the (121) peak, indicating this model is an incomplete description of the structure. The (121) peak is an allowed reflection specifically due to the Cu atoms on $P2_1/n$ Wyckoff positions 2a and 2c, which prompted an investigation into Cu disorder. The potential of possible stacking disorder is further supported by (1) the inconsistent broadening of peaks in our experimental diffraction data, (2) the higher symmetry which is suggested

by $\gamma = 90$ ° in the monoclinic P2₁/n model but which could not be solved in a spacegroup, and (3) Falck et al.'s report [6] that their P2₁/n model could also be described as the ${\rm TeO_4}$ layers belonging to spacegroup Pnma along with the lower-symmetry Cu atoms possessing only n-glide symmetry. We posit based on the structure model that a plane of Cu atoms might occur on $P2_1/n$ Wyckoff positions 2b and 2d instead of 2a and 2c (Inset Fig. 7a,7b). While the two are similar, Cu atoms in the faulted plane have a slightly greater overlap with the Te-O coordination octahedra of the underlying layer. As both original and faulted Cu planes have the same intralayer arrangement, the subsequent TeO₄ layer positions identically with respect to the Cu. Using DIFFaX, we simulated models with 0-100% alternate Cu planes and found a probability of 70% original and 30% faulted Cu planes (Fig. 7b) is in good agreement with the experimental data. We note that because the fault is extended over an entire plane, post-synthetic measures such as annealing would be dynamically insufficient to resolve the structure to the $P2_1/n$ "ideal."

IV Conclusion

 $CuTeO_4$ was proposed as a high T_c candidate amongst other copper tellurates due to its crystal structure and antiferromagnetic, insulating ground state which is similar to other superconducting cuprates. While CuTeO₄ was predicted to have a 2D antiferromagnetic ground state [4], our experiments have shown CuTeO₄ to display 1D AFM chain behavior at T > 65 K, due to the CuO_4 plaquettes highlighted in Figure 1. First principles calculations in [4] showed CuTeO₄ to have a small bandgap of 0.13 eV, increasing to 1 eV upon inclusion of a Hubbard U, making it a suitable candidate for superconductivity upon doping. the yellow colour of the polycrystalline powder qualitiatively suggests a much larger band gap than the one proposed. While the GGA functional used for the calculations is known to underestimate the band gap, upon closer inspection of the band structures it seems that the origin of the underestimation is not solely due to the exchange correlation functional used. We propose that the large difference in experimental and predicted band gap may be due to the failure of DFT to consider the effect of hypervalent cations like Te⁶⁺ which have a strong inductive withdrawal effect on O^{2-} . This also explains the apparent undopability of the parent structure as attempts to dope CuTeO₄ with Sb were unsuccessful due to the covalent character of the TeO bond. Thus, we have shown CuTeO₄ to be an undopable, yellow, wide band-gap insulator. CuTeO₄ harbors a spin glass state with T_g occurring at T = 50K as shown by (i) bifurcation in ZFC and FC magnetic susceptibility measurements at T = 50 K(ii) Low temperature specific heat measurements which reveal a sizable T-linear contribution of $\gamma = 8.3(1) \text{ mJ/mol f.u/K}^2$, consistent with other reported spin glasses. (iii) Elastic neutron scattering showing diffuse peaks at $Q \approx 0.6 \text{ Å}^{-1}$ at T = 10 Kand T = 40 K, which flatten out at T = 60 Kand T = 120 K, indicating a freezing transition in between the two temperature ranges. Warren line shape analysis reveals short range correlations, with average magnetic correlation length of $\xi = 10.1(9)$ Å at T = 10 K, decreasing to $\xi = 4.5(9)$ Å at T = 40 K. The spin glass behavior of $CuTeO_4$ can be explained by it's disordered layering as shown by the x-ray powder diffraction data. Simulating the disordered layering reveals a qualitative fit to disorder in copper positions which disturbs the stacking pattern.

While it is unclear, if the stacking faults are the thermodynamically stable state for this compound, lack of layering faults would not change its undopable, wide band gap insulating nature which is mainly due to the hypervalency of ${\rm Te}^{6+}$ ions. Therefore, our characterization of ${\rm CuTeO}_4$ as an undopable, wide band gap insulator, rules out high temperature superconductivity in copper tellurates where Te is found in the ${\rm Te}^{6+}$ state. Furthermore, it presents new opportunities to explore high temperature superconductivity in copper tellurates where there is a reduced inductive withdrawal effect by Te, such as those compounds where Te is present in the 4+ state, thus making them better suited candidates for doping.

V Acknowledgements

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