

η -pairing ground states in the non-Hermitian Hubbard model

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The introduction of non-Hermiticity has greatly enriched the research field of traditional condensed matter physics, and eventually led to a series of discoveries of exotic phenomena. We investigate the effect of non-Hermitian imaginary hoppings on the attractive Hubbard model. The exact bound-pair solution shows that the electron-electron correlation suppresses the non-Hermiticity, resulting in off-diagonal long-range order (ODLRO) ground state. In a large negative U limit, the ODLRO ground state corresponds to η -spin ferromagnetic states. We also study the system with mixed hopping configuration. The numerical result indicates the existence of the transition from normal to η -pairing ground states by increasing the imaginary hopping strength. Our results provide a promising approach for the non-Hermitian strongly correlated system.

I. INTRODUCTION

Non-Hermitian systems that can only be described by non-Hermitian Hamiltonians are ubiquitous in nature. Many open systems, which are not fully isolated from the rest of world, belong to this class. Comparing to the Hermitian systems, the probability of the non-Hermitian system effectively becomes nonconserving due to the exchange of energy, particles, and information with external degrees of freedom that are out of the Hilbert space. Mainly driven by experimental progress in atomic physics^{1,2}, the last two decades have witnessed remarkable developments in studies of out-of-equilibrium dynamics in isolated quantum many-body systems. It has become possible to study open many-body physics in a highly controlled manner³⁻⁹. Within this burgeoning field, the treasure hunt is sprouting into fascinating new directions ranging from non-Hermitian extensions of Kondo effect^{10,11}, many-body localization¹², to fermionic superfluidity^{13,14}.

Recent advances in quantum simulations of the Hubbard model with ultracold atoms have offered a multifunctional platform to unveil low-temperature properties of the strongly correlated system¹⁵⁻²⁰. A series of cornerstone works have reshaped our understanding of the dissipative strongly correlated system^{3,21-33}. One of the most tantalizing findings is the possible superconductivity in which η -pairing state plays a vital role. This stimulates a plethora of non-equilibrium protocols including photodoping schemes³⁴⁻⁴⁴ and dissipation-induced schemes⁴⁵⁻⁴⁹ to selectively generate such superconducting-like states. However, few people discuss the impact of non-Hermiticity on the low-lying energy spectrum and quantum magnetism of the strongly correlated system from the level of non-Hermitian quantum mechanics⁵⁰.

It is the aim of this paper to investigate the effect of non-Hermiticity on the strongly correlated system in the context of the non-Hermitian quantum mechanics. We show that the non-Hermitian imaginary hopping can indeed induce a robust η -pairing ground state for a wide range of parameters U (particle-particle interaction) and

t (hopping strength), by considering the bipartite non-Hermitian Hubbard system. An exact solution of the bound pair is employed to elucidate the underlying pairing mechanism and pave the way to extend the results to dilute gas. In physics, the particle-particle interaction suppresses the non-Hermiticity leading to the off-diagonal long-range order (ODLRO) ground state with real energy, and the non-Hermitian imaginary hopping, in turn, suppresses the antiferromagnetic correlation, thus ensuring the system has a η -spin ferromagnetic ground state. Numerical results of 1D and 2D systems with corrugation patterns indicate that such property is insensitive to the disorder and the strength of the interaction even though the on-site interaction breaks the $SO(4)$ symmetry, which suggests a promising scheme in a real experiment. We further demonstrate that there can exist a transition from normal to η -pairing ground state associating with the sudden change of the doublon-doublon correlation.

The paper is organized as follows. Sec. II discusses the non-Hermitian Hubbard model, the non-Hermiticity of which originates from the imaginary hopping. Sec. III introduces the exactly two-particle solution providing the mechanism of the formation of the η -pairing ground state. Sec. IV gives the effective magnetic Hamiltonian under the large negative U limit. Sec. V shows the numerical results and the analytical understanding of superconductive η -pairing ground state. Sec. VI demonstrates the transition from a normal to superconductive ground state. Sec. VII concludes this paper. Some details of our calculations are placed in the Appendixes.

II. MODEL

We consider a non-Hermitian Hubbard model on a bipartite lattice

$$H = i \sum_{j,l} \sum_{\sigma=\uparrow,\downarrow} t_{jl} (c_{j,\sigma}^\dagger c_{l,\sigma} + c_{l,\sigma}^\dagger c_{j,\sigma}) + U \sum_j n_{j,\uparrow} n_{j,\downarrow}, \quad (1)$$

with the following notation: the operator $c_{j,\sigma}$ ($c_{j,\sigma}^\dagger$) is the usual annihilation (creation) operator of a fermion with spin $\sigma \in \{\uparrow, \downarrow\}$ at site j , and $n_{j,\sigma} = c_{j,\sigma}^\dagger c_{j,\sigma}$ is the number operator for a particle of spin σ on site j ; the symbol $i = \sqrt{-1}$ represents an imaginary number; U and t_{jl} are required to be real and play the role of interaction and kinetic energy scales, respectively; the system can be divided into two sublattices A and B such that $t_{jl} = 0$ whenever $j \in \{A\}$ and $l \in \{A\}$ or $j \in \{B\}$ and $l \in \{B\}$. The non-Hermiticity of H stems from the imaginary hopping it_{jl} that can be realized by the judicious design of the loss and the magnetic flux^{51,52} which are within the reach of cold atom experiments^{53,54}. Notice that such non-Hermiticity is distinct from the complex particle-particle interaction adopted to describe the inelastic collision of two particles^{50,55}. When the uniform Hermitian hopping is taken, the Hamiltonian can feature a Mott insulating ground state with a strong antiferromagnetic correlation that is generically nonsuperconducting. Evidently, the imaginary hopping inevitably competes with the interaction leading to the unique properties of the considered system. It can be expected that the introducing of such non-Hermiticity will significantly alter the magnetic correlation of the system.

In this paper, we focus on whether the system can favor the ground state with superconductivity in this non-Hermitian setting. To gain physical insight into this system, we first investigate the symmetry of the considered model. It has two sets of commuting $SU(2)$ symmetries. The first is the spin symmetry characterized by the generators

$$s^+ = (s^-)^\dagger = \sum_j s_j^+, \quad (2)$$

$$s^z = \sum_j s_j^z, \quad (3)$$

where the local operators $s_j^+ = c_{j,\uparrow}^\dagger c_{j,\downarrow}$ and $s_j^z = (n_{j,\uparrow} - n_{j,\downarrow})/2$ obey the Lie algebra, i.e., $[s_j^+, s_j^-] = 2s_j^z$, and $[s_j^z, s_j^\pm] = \pm s_j^\pm$. Large values of the spin quantum number s corresponds to ferromagnetism. The second often referred to as η symmetry has the generators

$$\eta^+ = (\eta^-)^\dagger = \sum_j \eta_j^+, \quad (4)$$

$$\eta^z = \sum_j \eta_j^z, \quad (5)$$

with $\eta_j^+ = \lambda c_{j,\uparrow}^\dagger c_{j,\downarrow}$ and $\eta_j^z = (n_{j,\uparrow} + n_{j,\downarrow} - 1)/2$ satisfying commutation relation, i.e., $[\eta_j^+, \eta_j^-] = 2\eta_j^z$, and $[\eta_j^z, \eta_j^\pm] = \pm \eta_j^\pm$. Here we assume a bipartite lattice and $\lambda = 1$ for $j \in \{A\}$ and -1 for $j \in \{B\}$. Notice that under a particle-hole transformation, $c_{j,\downarrow} \rightarrow \lambda c_{j,\downarrow}^\dagger$, which maps the attractive Hubbard model to a repulsive one in the parent Hermitian Hamiltonian (1), the role of the two sets of $SU(2)$ generators is interchanged. Straight-

forward algebra shows that

$$[H, \eta^\pm] = \pm U \eta^\pm, \quad (6)$$

$$[H, \eta^z] = 0, \quad (7)$$

which indicates that one can construct many exact eigenstates $H(\eta^\pm)^N |\text{Vac}\rangle = NU(\eta^\pm)^N |\text{Vac}\rangle$ with $|\text{Vac}\rangle$ being the vacuum state of fermion $c_{j,\sigma}$. Correspondingly, the large values of the η quantum number are related to a staggered ODLRO and superconductivity^{56,57}.

III. η -PAIRING STATE IN TWO-PARTICLE SUBSPACE

Based on the symmetry of the system, we first elucidate the pairing mechanism through the exact solution within the two-particle subspace. Supposing that the Hamiltonian (1) describes a 1D homogeneous ring system in which $it_{jl} = it$. Owing to the translation symmetry, the basis of such invariant subspace can be constructed as follow

$$|\phi_0^-(K)\rangle = \frac{1}{\sqrt{N}} \sum_j e^{iKj} c_{j,\uparrow}^\dagger c_{j,\downarrow}^\dagger |\text{Vac}\rangle, \quad (8)$$

$$|\phi_r^\pm(K)\rangle = \frac{1}{\sqrt{2N}} e^{iKr/2} \sum_j e^{iKj} (c_{j,\uparrow}^\dagger c_{j+r,\downarrow}^\dagger \pm c_{j,\downarrow}^\dagger c_{j+r,\uparrow}^\dagger) |\text{Vac}\rangle, \quad (9)$$

$$(10)$$

and

$$\frac{s^\pm}{\sqrt{2}} |\phi_r^\pm(K)\rangle = \frac{1}{\sqrt{N}} e^{iKr/2} \sum_j e^{iKj} c_{j,\pm\uparrow}^\dagger c_{j+r,\pm\uparrow}^\dagger |\text{Vac}\rangle, \quad (11)$$

where N is an even number and $K = 2n\pi/N$ is the momentum vector indexing the subspace. r represents the relative distance between the two particles. These bases are eigenvectors of the operators s^2 and s^z , which satisfies

$$s^2 |\phi_r^-(K)\rangle = 0, \quad (12)$$

$$s^z |\phi_r^-(K)\rangle = 0, \quad (13)$$

$$s^2 |\phi_r^+(K)\rangle = 2 |\phi_r^+(K)\rangle, \quad (14)$$

$$s^z |\phi_r^+(K)\rangle = |\phi_r^+(K)\rangle. \quad (15)$$

Evidently, each subspace labeled by K can be further decomposed into four subspaces with $(s, s^z) = (0, 0)$, $(1, 0)$ and $(1, \pm 1)$ in term of spin symmetry. Aiding by the detailed calculation in the Appendix, the bound pair emerges in the $(0, 0)$ subspace with eigen energy being $\epsilon_K = \text{sgn}(U) \sqrt{U^2 + 4\lambda_K^2}$ in which $\lambda_K = 2it \cos(K/2)$. The bound pair state is $|\varphi_K^b\rangle = \sum_r f_K^b(r) |\phi_r^-(K)\rangle$ with

$$f_K^-(j) = \begin{cases} 1/\sqrt{2}, & j = 0 \\ e^{-\beta j}, & j \neq 0 \end{cases}, \quad (16)$$

where $\beta = \ln[(-U \pm \sqrt{U^2 + 4\lambda_K^2})/2\lambda_K]$. Here $\pm$ denotes negative and positive U , respectively. We concentrate on

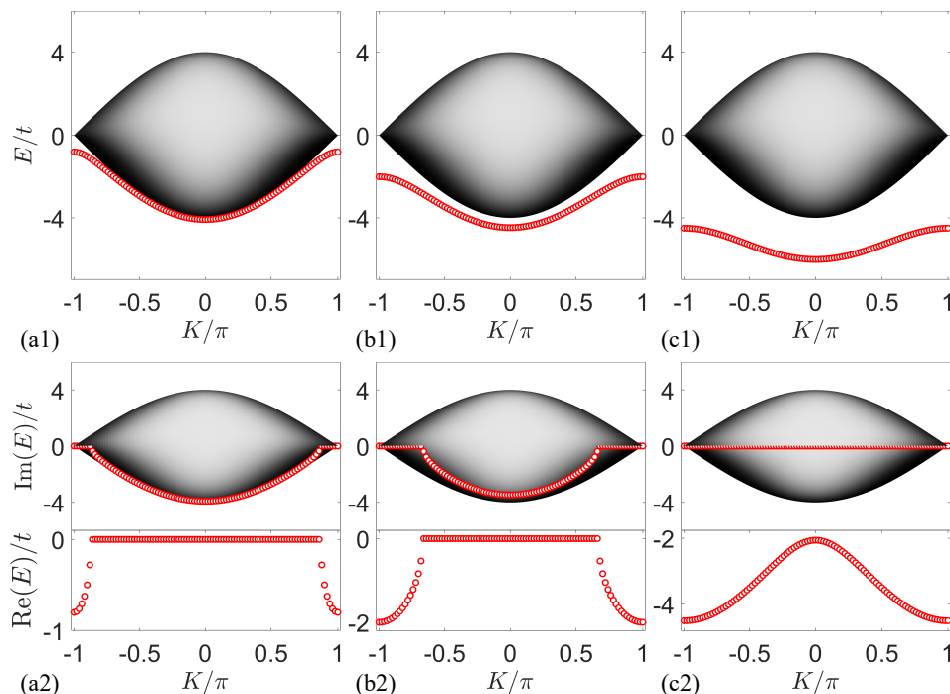


FIG. 1: Comparison of the two-particle spectrum within the subspace $(0,0)$ between the non-Hermitian setting and its parent Hermitian system for (a) $U = -0.8t$, (b) $U = -2t$, and (c) $U = -4.5t$, respectively. The upper and lower panels present the spectrum of the Hermitian and non-Hermitian system, respectively. The red circle and gray shading denote the bound pair and scattering state. The parent Hermitian system can be obtained by assuming $it \rightarrow t$. For the Hermitian system, the bound pair with the lowest energy lies in the $K = 0$ subspace while the ground state of two-particle non-Hermitian setting locates on the subspace indexed by $K = \pi$. It is shown that the presence of the imaginary hopping not only makes all scattering energy bands imaginary but also reverses the whole bound band. In the condition of small U , there can exist an EP characterized by the divergence of $\partial\epsilon_K/\partial K$. Such non-Hermiticity alters significantly the pairing mechanism and hence favors superconductivity.

the negative U in the following unless stated otherwise. In the absence of on-site interaction U , only the scattering eigenstate with imaginary eigenenergy presents and the system does not accommodate the bound pair state. The nonzero interaction U leads to the emergence of the bound pair. When $|U| > |4t|$, the system possesses the full real bound pair spectrum. However, a small U results in the appearance of the imaginary bound pair energy. The corresponding eigenstate is in the form of an oscillation damping wave rather than a monotonic damping wave of the Hermitian parent system. Notice that if $|U| \leq |4t|$, then an exceptional point (EP) $|U| = |2\lambda_{K_c}|$ presents, at which the coalescent eigenstate approaches to a unidirectional plane wave with $\beta = 0$ or π corresponding to $K = 0$ or 2π . In this sense, the non-Hermiticity of the system is suppressed through the pairing mechanism. The emergence of real energy is the consequence of the competition between the on-site interaction and imaginary hopping. Furthermore, the lowest real eigenenergy appears in the $K = \pi$ subspace no matter whether the system possesses the full real spectrum. The corresponding ground state is η -pairing state with the form

$$|\phi_0^-(K)\rangle = (\eta^+)/\sqrt{N}|\text{Vac}\rangle, \quad (17)$$

and thus it favors superconductivity. This is in stark difference from the Hermitian system, i.e., $it \rightarrow t$. In that case, the ground state of the two-particle system locates on the $K = 0$ rather than $K = \pi$ subspace such that the η -pairing state has the highest eigenenergy than the other bound pair state. Fig. 1 shows the typical energy spectrum of subspace $(0,0)$. It demonstrates that the imaginary hopping flips the bound pair spectrum of the parent Hermitian spectrum so that η -pairing state becomes the ground state of the system. It is worthy pointing out that if we consider the dilute Fermi gas formed by many bound pairs in which the pair-pair interaction is neglected, then the mechanism for a single bound pair can be extended to this type of dilute gas.

IV. η -PAIRING STATE IN THE LARGE U LIMIT

Now we turn to investigate the situation with arbitrary filling but in the large U limit. system. Following the standard step of quantum mechanics, the system can be divided into the kinetic part H' and interaction part H_0 ,

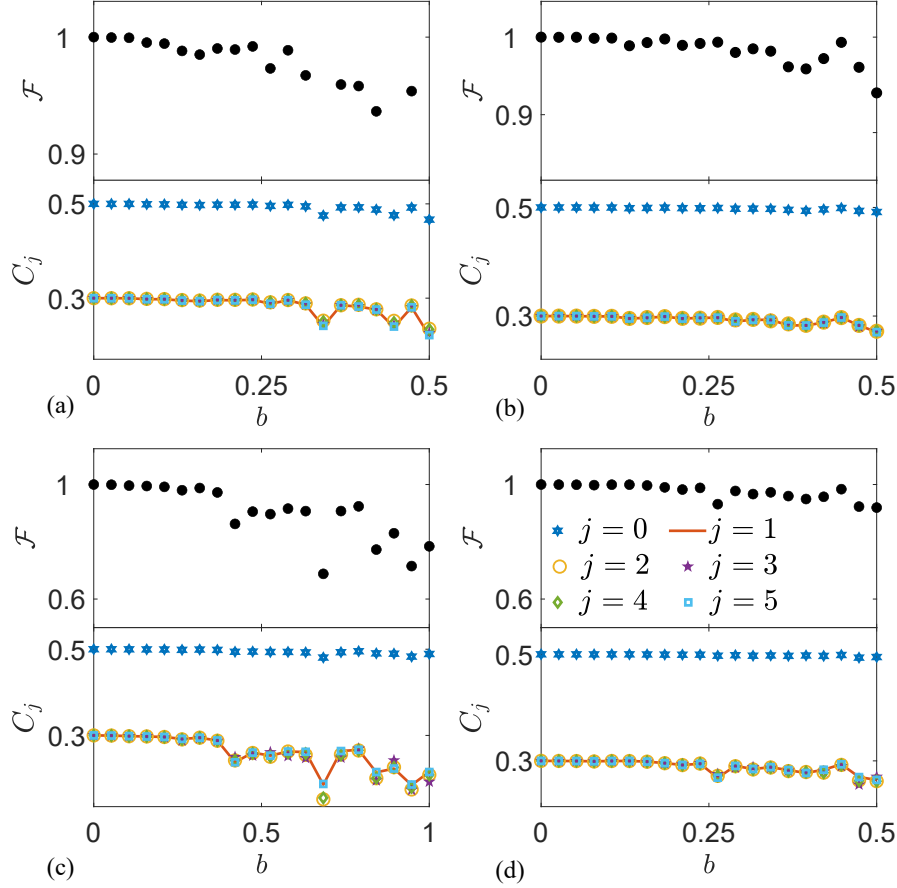


FIG. 2: Plots of the overlap $\mathcal{F}$ and correlator C_j as a function of the strength of interaction disorder b for (a) $t = 1$, $a = 0.1t$, $U = -0.5t$ (b) $t = 1$, $a = 0.3t$, $U = -1.5t$ (c) $t = 1$, $a = 0$, $U = -4t$, and (d) $t = 1$, $a = 0.2t$, $U = -4t$. The numerical simulation is performed for the 6 site 1D Hubbard model at half filling and $s^z = 0$. Here the strength of the hopping disorder a is set to be constant for each subfigure and the correlator C_j is averaged over all sites separated by a distance j . When $b = 0$, no matter what value a takes, as long as U is non-zero, one can always get a perfect η -pairing ground state. The variation of C_j indicates that the increase of b will not result in the significant change of the η -pairing ground state; the presence of the hopping disorder can suppress the fluctuation of C_j compared to the disorder free case, which can be seen from (c)-(d). Therefore, the value of the correlator is the consequence of the interplay between two such disorders, which provides a scheme to prepare η -pairing ground state in the experiment.

where

$$H' = i \sum_{j,l} \sum_{\sigma=\uparrow,\downarrow} t_{jl} (c_{j,\sigma}^\dagger c_{l,\sigma} + c_{l,\sigma}^\dagger c_{j,\sigma}), \quad (18)$$

$$H_0 = U \sum_j n_{j,\uparrow} n_{j,\downarrow}. \quad (19)$$

Here the imaginary hopping is assumed to be homogeneous $it_{jl} = it$. In the strongly correlated regime $|U| \gg t$, the kinetic term H' can be treated as a perturbation and one can derive an effective Hamiltonian for the degenerate space. To second order in perturbation theory, the effective Hamiltonian is given by

$$H_{\text{eff}} = P_0 H_0 P_0 + P_0 H' P_1 \frac{1}{E_0 - H_0} P_1 H' P_0 + O\left(\frac{t^3}{U^2}\right), \quad (20)$$

where P_0 is a projector onto the Hilbert subspace in which there are M lattice sites occupied by two particles with opposite spin orientation, and $P_1 = 1 - P_0$ is the complementary projection. Here the energy E_0 of the unperturbed state is set to $E_0 = MU$ where M denotes the number of doublons. Since H' acting on states in P_0 annihilates only one double occupied site, all states in $P_1 H' P_0$ have exactly $N - 1$ doubly occupied sites. Therefore, the effective Hamiltonian regarding doublon-hole creation and recombination process is given as

$$H_{\text{eff}} = MU + \frac{4t^2}{U} \sum_j \left(\boldsymbol{\eta}_j \cdot \boldsymbol{\eta}_{j+1} - \frac{1}{4} \right), \quad (21)$$

where $\boldsymbol{\eta}_j = (\eta_j^x, \eta_j^y, \eta_j^z)$. In the Appendix, a simple two-site case is provided to elucidate this mechanism. This indicates that the non-Hermitian virtual exchange

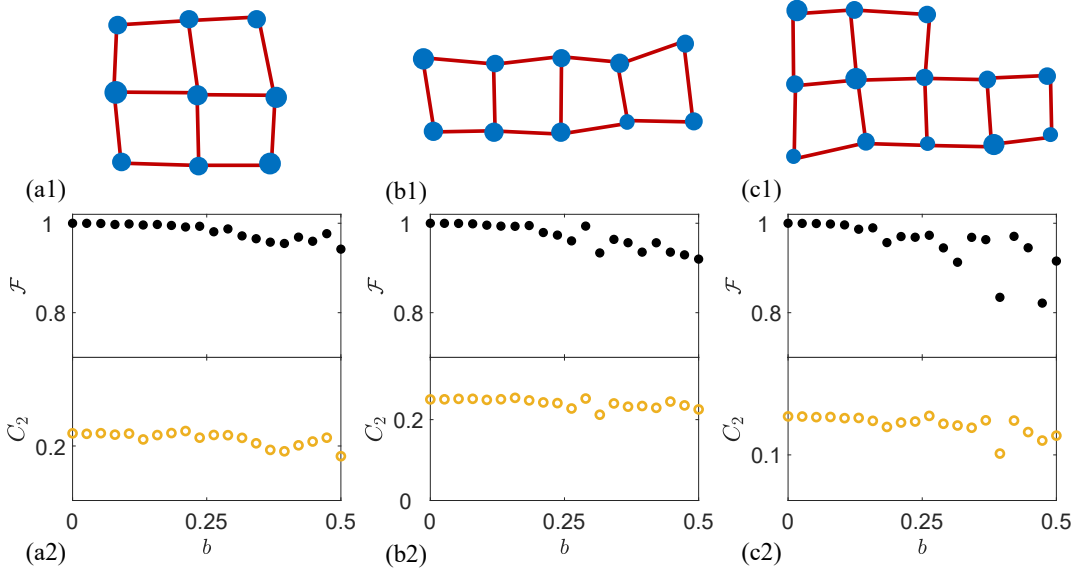


FIG. 3: Numerical simulation of 2D corrugation pattern for (a1) 8 site Hubbard model with 4 filled particles, (b1) 9 site Hubbard model with 6 filled particles, and (c1) 13 site Hubbard model with 2 filled particles. The different sizes of solid circles and different lengths of red edges denote the disorder of the interaction strength U_j and hopping strength t_j of Eq. (23). (a2)-(c2) Plots of $\mathcal{F}$ and C_1 as function of disorder strength b . The system parameters are (a) $t = 1$, $a = 0$, $U = -0.8t$ (b) $t = 1$, $a = 0.2t$, $U = -0.8t$ and (c) $t = 1$, $a = 0.2t$, $U = -0.5t$. Similar with 1D system, the induced fluctuation of C_1 is minor in comparison with disorder free case ($b = 0$) indicating that the existing result of 1D can be extended to 2D or higher dimensional system.

mediates an interaction between the pseudo spins. It is similar to the Heisenberg interaction in the Hermitian Hubbard model in the way that the doubly occupied and vacuum sites correspond to spin up and spin down states, respectively. Owing to the fact that $4t^2/U < 0$, the effective Hamiltonian is ferromagnetic Heisenberg model of pseudo spins. The eigenstate of the lowest eigenenergy within each doublon subspace is the η -pairing state with different pair number. As such the ground state of H_{eff} is $(\eta^+)^N |\text{Vac}\rangle$ dubbed as η -spin ferromagnetic state. Note in passing that for the case of repulsive Hubbard model ($U > 0$) at half filling, the extra minus sign induced by the non-Hermitian virtual exchange leads to an effective ferromagnetic rather than an antiferromagnetic Heisenberg Hamiltonian

$$H_{\text{eff}}^1 = -\frac{4t^2}{U} \sum_j \left(\mathbf{s}_j \cdot \mathbf{s}_{j+1} - \frac{1}{4} \right) \quad (22)$$

describing the behavior of the ground state and low energy excitations. The eigenenergy of the ground state is zero. Notably, the interplay between the imaginary hopping and particle-particle interaction fundamentally alters the magnetism of the Hubbard model leading to sign reversal of magnetic correlation.

V. η -PAIRING STATE IN SYSTEM WITH MIXED HOPPINGS

In the aforementioned sections, we have demonstrated that the η -pairing state can be either the ground state of the system under the large U limit, or the ground state of the two-particle subspace with non-zero U . Then a natural question arises: (i) For any nonzero U , is the η -pairing state still the ground state of the system in the subspace of other particle numbers? (ii) If yes, can the existing 1D results be extended to 2D or higher dimensional system? (iii) If the disorder is introduced, does the property of ground state be changed? To answer these questions, we first investigate the 1D non-Hermitian system with disordered imaginary hoppings and interaction since the system parameter does not hold the uniform in real experiments. The corresponding disordered Hamiltonian can be obtained by taking two sets of random numbers $\{t_j\}$ and $\{U_j\}$ around t and U in Eq. (1). The random number parameter can be taken as

$$t_j = t + \text{rand}(-a, a), U_j = U + \text{rand}(-b, b), \quad (23)$$

where $\text{rand}(-a, a)$ denotes a uniform random number within $(-a, a)$. It is too cumbersome to obtain an analytical result. Hence, we perform the numerical simulation to check the fidelity between the ground state and target η -pairing state, which can be given as

$$\mathcal{F} = |\langle \psi_g(m) | \psi_c(m) \rangle|, \quad (24)$$

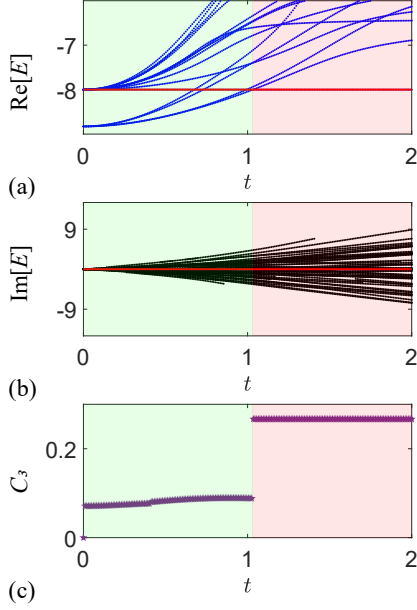


FIG. 4: Transition of the ground state driven by the imaginary hopping. (a)-(b) depict the variation of the energy spectrum with respect to hopping strength t , and (c) presents the switch of the correlator C_3 . The numerical simulation is performed for the 6 site Hubbard model with 4 filled particles. The other system parameters are $t_0 = 1$, and $U = -1.5t_0$. The hopping strength t is in units of t_0 . The red line denotes the η -pairing state. C_3 experiences an evident jump at the critical point indicating a dramatic change of the ground state. Such a first-order transition may associate with quantum phase transition to some extent.

with m being an even number representing filled particle number of the system. Here $|\psi_g(m)\rangle$ is the ground state of such subspace and the target η -pairing state is

$$|\psi_c(m)\rangle = \Omega^{-1} (\eta^+)^{m/2} |\text{Vac}\rangle, \quad (25)$$

with renormalization coefficient $\Omega = \sqrt{C_N^{m/2}}$. Fig. 2 shows that if U is homogeneous ($b = 0$) then the ground state is the η -pairing state in the subspace with particle numbers $m = 2, 4$, and 6 . The corresponding energies are $U, 2U$, and $3U$ respectively. Such a result indicates that the formation of the η -pairing ground state does not depend on the values of U and is immune to the hopping disorder. However, the introduction of disordered U will cause the ground state of the system to deviate from the η -pairing state. The underlying mechanism is clear, that is, the system fulfills the η symmetry even in the presence of the hopping disorder, but if one introduce disorder into the interaction, this symmetry will be destroyed leading to such deviation. One may think that the disorder of interaction scrambles the background spin configuration and disturb the spin correlation. Then a question arises: to what extent does the ground state maintain the superconductivity? To capture supercon-

ductivity, the doublon-doublon correlator

$$C_j = \sum_i \langle \eta_i^+ \eta_{i+j}^- \rangle / N \quad (26)$$

is introduced. It is averaged over all sites separated by a distance j . The nonzero value of such quantity implies both the Meissner effect and flux quantization and hence provides a possible definition of superconductivity.^{56,58} For the target state $|\psi_c(m)\rangle$, the expectation value can be given as

$$\langle \psi_c(m) | \eta_i^+ \eta_{i+j}^- | \psi_c(m) \rangle = \begin{cases} \frac{M(N-M)}{N(N-1)}, & \text{for } j \neq 0 \\ \frac{M}{N}, & \text{for } j = 0 \end{cases}, \quad (27)$$

where $M = m/2$. Notice that it is irrelative to the distance j and hence the correlator C_j obeys the same law such that $C_j = M(N-M)/[N(N-1)]$ for $j \neq 0$ or $C_j = M/N$ for $j = 0$. Fig. 2 shows that the value of correlator and the overlap between the ground state and target state. It indicates that $\mathcal{F}$ is around 0.9 and the correlator C_j stays at a non-zero value ensuring the ground state of the system possesses the superconductivity even though the strong inhomogeneity of interaction presents.

Now we switch gears to the cases of the 2D system. In Fig. 3, the disordered 2D system is sketched. For simplicity, we fix the strength of the hopping disorder a and examine two quantities $\mathcal{F}$ and C_2 . It is shown that the system still possesses the η -pairing state even though the small homogeneous U and the disordered imaginary hopping present, which is similar to that of the 1D system. Although the disorder U affects the correlation of ground state, the correlator C_2 has a small fluctuation around the value of uniform case supporting the superconductivity of the ground state. Therefore, one can conclude that all the results of 1D can be extended to 2D lattice system. It can be expected that this conclusion is still valid for the higher dimensional bipartite system.

VI. TRANSITION FROM NORMAL TO η -PAIRING GROUND STATES

In this section, we focus on how does the ground state transits from normal to superconductive state. To observe such a transition, we consider a 1D Hubbard system with only two nearest neighbour (NN) sites coupled through Hermitian hopping t . The corresponding Hamiltonian can be given as

$$H = - \sum_{j,\sigma=\uparrow,\downarrow} t_j (c_{j,\sigma}^\dagger c_{j+1,\sigma} + c_{j+1,\sigma}^\dagger c_{j,\sigma}) + U \sum_j n_{j,\uparrow} n_{j,\downarrow}, \quad (28)$$

where homogeneous U is supposed and $t_1 = t_0$ otherwise $t_j = 0$. The Hamiltonian still possesses the η symmetry and hence supports the η -pairing eigenstate. However, such a state is not the ground state of the system.

Now we switch on the other coupling of the NN sites, which are the imaginary hoppings rather than Hermitian hoppings, that is $t_j = it$ for $j \neq 1$. Fig. 4 shows the variation of the low-lying energy spectrum with respect to it . The η -pairing state is denoted by the red line, which is suppressed to the ground state by the increase of imaginary hopping. There exists a transition window in which the ground state is transformed from a normal state to a superconducting state. We perform the numerical simulation to demonstrate this process through the correlator C_3 . Evidently, C_3 undergoes a leap around the critical point which leads to the divergence of the first order of derivative $\partial C_3 / \partial t$. It witnesses the formation of the η -pairing ground state. Notice that all the conclusions can be extended to a higher dimension. Before ending this section, we want to point out that the imaginary hopping plays the key to achieve the superconducting ground state, however, it does not mean that the system must have the η -pairing ground state as long as the imaginary hopping is applied. The transition of the ground state always requires a process such that there is a threshold beyond which the system favors the superconductivity. Such property is reminiscent of quantum phase transition, that is, the ground state will experience a dramatic change when the system crosses the quantum phase transition point. This findings paves the way to understand the η -spin ferromagnetic state of the non-Hermitian strongly correlated system.

VII. SUMMARY

In summary, we have systematically studied the effect of the non-Hermitian imaginary hopping on the low-lying energy spectrum of the Hubbard model. The analytical solution within the two-particle subspace shows that the introduction of the imaginary hopping results in a full imaginary scattering spectrum and a flip of the bound pair spectrum comparing to its Hermitian parent model. It indicates that the particle-particle correlation suppresses the non-Hermiticity making the ground state to be η -pairing state with ODLRO. The η symmetry plays the vital role in this mechanism. In the large negative U limit, the magnetism of the Hubbard model is altered fundamentally due to the interplay between the particle-particle interaction and non-Hermitian imaginary hopping. The ground state experiences a transition from normal to η -spin ferromagnetic states. Such a transition holds for any pair filled, that is, the ground state in each invariant subspace is $(\eta^+)^M |\text{Vac}\rangle$ with M being the pairs of particles. Through numerical simulation of 1D and 2D non-Hermitian Hubbard system, we demonstrate that the η -pairing ground state can still survival albeit a small negative U presents. This evidence is robust against disorder even if the system does not fulfill the $SO(4)$ symmetry. Our results open a new avenue toward populating a η -pairing ground state and suppressing antiferromagnetic correlation of η spins in the attractive

Hubbard model.

Acknowledgments

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Appendix A: Two-particle solutions

In this section, we show the detailed calculation for the two-particle solution in each invariant subspace. For the simplicity, we only focus on the solutions in subspaces $(0, 0)$ and $(1, 0)$, since the solution in subspace $(1, \pm 1)$ can be obtained directly from that in subspace $(1, 0)$ by operator $s^\pm$. A two-particle state can be given as

$$|\varphi_K^\pm\rangle = \sum_r f_{K,k}^\pm(r) |\phi_r^\pm(K)\rangle, \quad (f_K^+(0) = f_{K,k}^+(-1) = 0), \quad (\text{A1})$$

where r denotes the relative distance between the two particles and the wave function $f_{K,k}^\pm(r)$ obeys the Schrödinger equations

$$Q_r^K f_{K,k}^+(r+1) + Q_{r-1}^K f_{K,k}^+(r-1) + [(-1)^n Q_r^K \delta_{r,N_0} - \varepsilon_K] f_{K,k}^+(r) = 0, \quad (\text{A2})$$

and

$$Q_r^K f_{K,k}^-(r+1) + Q_{r-1}^K f_{K,k}^-(r-1) + [U\delta_{r,0} + (-1)^n Q_r^K \delta_{r,N_0} - \varepsilon_K] f_{K,k}^-(r) = 0, \quad (\text{A3})$$

with $N_0 = (N-1)/2$ and the eigen energy ε_K in the invariant subspace indexed by K . Here factor $Q_r^K = -2\sqrt{2}it \cos(K/2)$ for $r = 0$ and $-2it \cos(K/2)$ for $r \neq 0$, respectively. U appears in the $(0,0)$ subspace and therefore admits the bound pair solution. In the large N limit, we can neglect the effect of on-site potential $(-1)^{n+1} 2it \cos(K/2)$ at N_0 th site. The solution of (A3) is equivalent to that of the single-particle semi-infinite tight-binding chain system with nearest-neighbour (NN) hopping amplitude Q_j^K , and on-site potentials U at 0th site, respectively. Moreover the solution of (A2) corresponds to the same chain with infinite U . In this scenario, the bound state solution $|\varphi_K^\pm\rangle = \sum_r f_{K,k}^\pm(r) |\phi_r^\pm(K)\rangle$ can be determined by substituting the ansatz

$$f_K^-(j) = \begin{cases} 1/\sqrt{2}, & j = 0 \\ e^{-\beta j}, & j \neq 0 \end{cases} \quad (\text{A4})$$

into the following equivalent Hamiltonian

$$H_{\text{eq}}^K = U |0\rangle \langle 0| + \sum_{i=0}^{\infty} (Q_i^K |i\rangle \langle i+1| + \text{H.c.}). \quad (\text{A5})$$

Straightforward algebra shows that $\beta = \ln[(-U \pm \sqrt{U^2 + 4\lambda_K^2})/2\lambda_K]$ where $\lambda_K = 2it \cos(K/2)$ and $\pm$ denotes negative and positive U , respectively. Correspondingly, the energy of the bound pair is

$$\epsilon_K = \text{sgn}(U) \sqrt{U^2 - 16t^2 \cos^2(K/2)}. \quad (\text{A6})$$

For the case of negative U , the lowest energy of bound pair is $\epsilon_\pi = -U$ locating on the subspace with $K = \pi$. As such the corresponding eigenstate is $|\phi_0^-(K)\rangle$ that represents a η -pairing state in the coordinate space with the form of $(\eta^+)/\sqrt{N}|\text{Vac}\rangle$.

Appendix B: Simple example of two-site case for the effective Hamiltonian H_{eff}

In this subsection, we provide a detailed calculation of the two-site case for the effective Hamiltonian H_{eff} which may shed light to obtain the effective Hamiltonian (21). In the simplest two-site case, $P_0 = \sum_{\alpha \in \text{d.o.}} |\alpha\rangle\langle\alpha|$ is the projection operator to the doublon subspace spanned by the configuration $\{|x0\rangle, |0x\rangle\}$, and $P_1 = 1 - P_0 = \sum_{a \notin \text{d.o.}} |a\rangle\langle a|$ is the complementary projection. Here the abbreviation d.o. means the doubly occupied subspace and $|x0\rangle = c_{1,\uparrow}^\dagger c_{1,\downarrow}^\dagger |\text{Vac}\rangle$, $|0x\rangle = c_{2,\uparrow}^\dagger c_{2,\downarrow}^\dagger |\text{Vac}\rangle$. The first term of Eq. (21) clear gives $P_0 H_0 P_0 = U$. The second term can be simplified by noting: (i) the unperturbed energy E_0 is U ; (ii) $P_1 H' P_0$ annihilates the doubly occupied

site. Then H_{eff} can be written as

$$\begin{aligned} H_{\text{eff}} &= U + \sum_{\alpha, \beta \in \text{d.o.}} \sum_{a, b \notin \text{d.o.}} |\alpha\rangle\langle\alpha| H' |a\rangle\langle a| \\ &\quad \times \frac{1}{U - H_0} |b\rangle\langle b| H' |\beta\rangle\langle\beta| \\ &= U + \frac{1}{U} \sum_{\alpha, \beta \in \text{d.o.}} \langle\alpha| (H')^2 |\beta\rangle |\alpha\rangle\langle\beta|. \end{aligned} \quad (\text{B1})$$

The second term describes the virtual exchange of the fermions. The non-Hermitian imaginary hopping brings about an additional sign to this process yielding that

$$H_{\text{eff}} = U - \frac{2t^2}{U} (|x0\rangle\langle 0x| + |0x\rangle\langle x0| + |x0\rangle\langle x0| + |0x\rangle\langle 0x|). \quad (\text{B2})$$

Combining the cases in the subspaces of $|xx\rangle$ and $|\text{Vac}\rangle$, the pseudo spin Hamiltonian can be given by the non-Hermitian Heisenberg-like model

$$H_{\text{eff}} = MU + \frac{4t^2}{U} \left(\boldsymbol{\eta}_1 \cdot \boldsymbol{\eta}_2 - \frac{1}{4} \right), \quad (\text{B3})$$

where M can be 0, 1, and 2 denoting the number of pairs of the doublon subspace. Evidently, the ground state of H_{eff} is the η -spin ferromagnetic state with the form of $(\eta^+)^2 |\text{Vac}\rangle$.

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