The double doors of the horizon

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In statistical mechanics entropy is a measure of disorder obeying Boltzmann's formula $S = \log \mathcal{N}$, where \mathcal{N} is the accessible phase space volume. In black hole thermodynamics one associates to a black hole an entropy Bekenstein-Hawking S_{BH} . It is well known that S_{BH} is very large for astrophysical black holes, much larger than any collection of material objects that could have given rise to the black hole. If S_{BH} is an entropy the question is thus what is the corresponding \mathcal{N} , and how come this very large phase space volume is only opened up to the universe by a gravitational collapse, which from another perspective looks like a massive loss of possibilities. I advance a hypothesis that the very large increase in entropy can perhaps be understood as an effect of classical gravity, which eventually bottoms out when quantum gravity comes into play. I compare and discuss a selection of the very rich literature around these questions.

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Introduction: The Bekenstein-Hawking entropy S_{BH} of a black hole is a central quantity in modern physics. In its general form S_{BH} equals $\frac{1}{4}\frac{A}{l_{z}^{2}}$ where A is the surface area of the black hole horizon, and $l_p \approx 1, 6 \cdot 10^{-35} \,\mathrm{m}$ is the Planck length. This form of the entropy formula includes extreme cases where the black hole is rotating at high speed and/or is very strongly electrically charged, and where A can be arbitrarily small. In the simplest setting of a non-rotating electrically neutral black hole (Schwarzschild geometry) the area of the horizon is $16 \pi l_p^2 \frac{M^2}{m_p^2}$, where M is the mass of the object and $m_p \approx 2 \cdot 10^{-8} \, \mathrm{kg}$ is the Planck mass. In this case, roughly appropriate for all black holes that have been observed to date, S_{BH} equals $4\pi \frac{M^2}{m_p^2}$ and increases faster with mass than any aggregate of particles and fields that could have formed the black hole [1, 2]. Consequently, for large enough mass, S_{BH} will be larger than than the entropy of any astrophysical object that could have formed the black hole. Well-known estimates say that the entropy of star like the sun would increase about 10^{19} times if it could collapse into a black hole. In the total entropy budget of the universe almost all the entropy is held by super-massive black holes [3].

From the point of view of statistical mechanics entropy is a measure of disorder obeying Boltzmann's formula $S = \log \mathcal{N}$. Disorder means that the system can only be described up to a phase space volume \mathcal{N} . While the formation of an event horizon closes the door on some types of classical evolution, since a classical observer can never get out of a black hole, in some other sense a gravitational collapse must open a door to new possibilities; "Opened are the double doors of the horizon // unlocked are its bolts" [4].

In this note I put forward a hypothesis that the increase of disorder is due to the classical chaotic dynamics of a sufficiently strong gravitational field. The existence of such a mechanism is plausible, and supported by well-

established theories, to be referenced below. The mechanism has nevertheless not previously been put forward as a physical mechanism which can generate S_{BH} .

The relation to Hawking's black hole information paradox: Bekenstein-Hawking entropy is at the center of a very extensive literature motivated by Hawking's black hole information paradox [5–9]. Possible solutions to the information paradox were in [10] classified as (i) fundamental information loss, (ii) remnants or new physics at the horizon, and (iii) information return in Hawking radiation. I am here only concerned with the last scenario, first proposed by Page [11], and more recently dubbed the "central dogma" of black hole quantum physics [12]. Quantum mechanics is thus assumed to hold everywhere in the universe, including in the interior of black holes. Some (unknown) unitary operator evolves an initially pure state of matter to a gravitational collapse and then onto Hawking radiation escaping to infinity. In [13, 14] we considered the kinematics of such a process, where in the final state radiation will be in multi-mode entangled state. That such multi-party entanglement should be present in Hawking radiation is known already from [11], where it was pointed out that when one Hawking particle is emitted with some momentum, the remaining black hole must carry the opposite momentum. Hence later Hawking particles must be entangled with earlier Hawking particles, though only weakly by this recoil ef-

A distinction now has to be made between entropy in the informational-theoretic sense, and entropy in the thermodynamic sense. Entropy of the first kind, usually called "fine-grained entropy" in the black hole literature, does not change under unitary evolution. Entropy of the second kind, usually called "coarse-grained entropy" in the black hole literature, will however tend to a maximum at given external constraints. A part of a resolution of the information paradox of type (iii) must hence be that information-theoretic entropy is preserved in unitary evo-

lution, and that S_{BH} is a thermodynamic entropy, an observation also due to Hawking [15]. Another argument that S_{BH} is a thermodynamic entropy follows from comparing S_{BH} to the entropy of black-body radiation emitted by an evaporating black hole, mode by mode. This can be estimated from the law of Stefan-Boltzmann and its generalizations [16–18], and is somewhat larger than S_{BH} , as appropriate for an irreversible process. The ratio is however a constant independent of mass, meaning that on a relative scale almost all the entropy increase happens from when matter starts collapsing until when the resulting black hole has settled down to a stationary state.

In the quantum domain the role of informationtheoretic entropy is taken by von Neumann entropy, which conserved for a closed system, but can change for a subsystem. Important progress was in the last less than ten years made in describing a black hole as a unitary quantum system with $\exp\left(\frac{A}{4l_p^2}\right)$ states [19–23]. This approach allowed for the first time a calculation of the von Neumann entropy of Hawking radiation, showing that it first increases, as does the thermodynamic entropy (as above), but then decreases, following the Page curve. This approach however does not deal with the degrees of freedom of the space-time geometry in its entirety, including the full interior of the black hole [24]. The question of how thermodynamic entropy can increase so dramatically in a collapse to a black hole, a process which can be taken to happen before any Hawking particles have been emitted at all, has also not been addressed in this important series of papers.

chaos: I now introduce three assumptions of a physical or plausibility nature, patterned after the "naive model" of [25], see also [26]: (A) after the gravitational collapse has run its course there emerges in the center of the black hole a quantum core state of matter and gravitation; (B) the spatial support of this state is about $r_c \sim l_p \left(\frac{M}{m_p}\right)^{\frac{1}{3}}$ such that the density of the core is about Planck density; (C) outside the core space-time is a meaningful concept, and to a good approximation given by a metric which obeys Einstein's equations. All the mass that went down into the black hole is assumed concentrated in a volume of size r_c , and thoroughly transformed by and entangled with the gravitational field. For a solar mass black hole $r_c \sim 10^{13} \, l_p \approx 10^{-22} \, \mathrm{m}$.

Black holes quantum cores, chaos and space-time

On a general level, during the collapse process the space-time inside a physical black hole can be assumed to be both unique and complex, where in classical theory increasing gravity would eventually lead to a singularity in a way that reflects the black hole's formation [27]. One can try to estimate using classical concepts the increase of disorder when a body falls down in the black hole before it hits the core. First, it is reasonable to take any such body an elementary particle, as macroscopic

bodies are torn apart by tidal forces well before hitting the final singularity [58]. Second, the same diverging tidal forces mean that a small difference between a reference geodesics and a deviation grows without limit as the reference geodesics approaches the singularity. This type of chaos leads to a growth which is algebraic in the distance from the singularity at the final time, see [28] (chap. 32.6) or [29], which in turn translates to a power-law of the ratio $\frac{M}{m_p}$. The number of bits needed to specify an initial condition leading to a given final condition is thus logarithmic in $\frac{M}{m_p}$. Chaos of the geodetic motion of the body falling down the hole can therefore only correspond to a sub-leading fraction of the entropy increase.

Third, disorder can also emerge from the chaotic nonlinear dynamics of the gravitational field itself. This mechanism (BKL scenario) was first proposed for homogeneous cosmology [30–32], and has more recently been conjectured to also describe aspects of the final stages of the collapse of an astrophysical body [33]. For chaotic dynamical systems (coherent in space) the increase in entropy per unit time is measured the Kolmogorov-Sinai entropy. For the homogeneous BKL solution KS-entropy (KS-entropy of the Gauss map) is known, and is positive [31]. This should however again only correspond to a sub-leading fraction of the entropy increase. However, in BKL, every matter particle loses causal contact with every other matter particle, a property known as asymptotic silence [34]. Therefore, one would not expect the gravitational field around different points to be synchronized, and the general BKL-like solution has consequently been postulated to be inhomogeneous (turbulent) with different realizations of the homogeneous solution around different spatial points [35]. If this mechanism of space-time chaos can produce the increase of phase space volume quantified Bekenstein-Hawking entropy is hence the question.

Stability and instability of Schwarzschild and geometry: The exterior of the The Schwarzschild black hole was shown to be linearly stable on the physical level of rigor in the 1950ies and 1960ies [36, 37]; mathematical proofs are quite recent [38, 39]. Very recently a mathematical proof of nonlinear stability was also presented in [40], unfortunately not very accessible to physicists [59]. In another line of investigations from the same community, dynamics in the interior region has also been considered from the rigorous point of view [41, 42] Two further papers have been announced, but have so far not appeared.

This (rigorously established) stability does not exclude that in the interior there can be the kind of instabilities which in plasma physics and hydrodynamics are called convective [43, 44]. Imagine a perturbation of the metric at some distance r from the smeared singularity. Such a perturbation can decay at the point r where it is introduced (absolute stability), but at the same time grow, for a finite or infinite time, at smaller r. In a classical

model this growth can be unlimited by moving to ever smaller r, but by the assumptions made here it would stop at r_c . Unfortunately, the disorder created by such an instability seems quite hard to estimate.

In a quantum theory of gravity the information would instead be stored in the state at the smeared singularity. The ensemble describing all such space-times, assuming they will in the end be thoroughly mixed and indistinguishable from the outside, must be able to encode all quantum states of all systems that could have given rise to the black hole of given mass, angular momentum and electric charge. By one of Bekenstein's original estimates such an ensemble has an entropy of the order of S_{BH} [1]. More refined estimates in the same direction were more recently given by Mukhanov [45]. Hence, one can at least say that the classical chaos of the gravitational field should not lead to an entropy increase larger than S_{BH} .

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- [58] On a body of size l and mass m at a distance r from a black hole of mass M ($l \ll r$ the tidal forces are about $GmMl/r^3$ Imagine a system of size l and binding energy V held together by a force about V/l; it is then torn apart at a distance from the singularity of about $r_* \sim r_s \left(\frac{l^2}{r_s^2} \frac{mc^2}{V}\right)^{\frac{1}{3}}$. For a hydrogen atom in a solar mass black hole r_* is about one millimeter $(10^{-6}r_s)$, as follows from $l \sim 10^{-10} \mathrm{m}$, $r_s \sim 10^3 \mathrm{m}$, $mc^2 \sim 10^9 \mathrm{eV}$ and $V \sim 13 \mathrm{eV}$. A proton would similarly be torn apart at a distance of one nanometer from the singularity $(10^{-12}r_s)$, as follows from $l \sim 10^{-15} \mathrm{m}$ and $V \sim mc^2$, though one may imagine that pulling apart the quarks could also lead to particle production and a transition to another phase. In any case, both distances are small compared to r_s but very large compared to r_c .
- [59] This so far unpublished monograph is 519 pages in length

Remarks on classical and quantum entropy increase and thermalization:

The mechanism proposed above is based on the distinction between information-theoretical entropy and thermodynamic entropy. Thermodynamic entropy can in turn be interpreted as lack of information (ignorance, or coarse-graining) [46–49], or as a property of an ensemble describing the outcome of possible (sufficiently simple) experiments after a process of thermalization [50, 51].

If and how a closed quantum system thermalizes (or does not) is an important topic, often referred to as the Eigenvalue Thermalization Hypothesis (ETH) [52, 53]. In quantum chaos growth of disorder can often be described by classical dynamics which lead to generation of smaller and smaller structures in phase space, until quantum effects eventually take over [54]. The classical picture proposed here is therefore not necessarily in contradiction with the idea that the quantum state in the center of a black hole is quantum chaotic and in itself scrambles information in a fast way [55]. A precise mechanism in this direction, however assuming the holographic principle, was discussed in [56].

Remarks on Bekenstein's 2001 critique of his 1973 interpretation of BH entropy

Above I evoked Bekenstein's argument in his pioneering 1973 paper that S_{BH} is the number of quantum states of matter than could have formed a black hole [1]. I thus have to address the counter-argument advanced by Bekenstein in a 2001 review [57]. Bekenstein there considered a black hole emitting Hawking radiation and at the same time being fed by a stream of matter so that its mass, angular momentum and charge stay constant. Bekenstein formulated his objection (page 8 in [57]) as follows:

[The black hole then] does not change in time, and neither does its entropy. But surely the inflowing matter is bringing into the black hole fresh quantum states; yet this is not reflected in a growth of S_{BH} ! [...] If we continue thinking of the Hawking radiation as originating outside the horizon, this does not sound possible.

A part of a counter-argument runs as what Bekenstein outlines is not a state of thermal equilibrium but a non-equilibrium stationary state (NESS). An entropy flow can be defined as the expected loss of entropy of the stream of infalling matter, or as the expected gain of entropy of the outgoing Hawking radiation: in a stationary state these

should match. A mechanism supporting such an entropy flow was discussed by the author and Michał Eckstein and Paweł Horodecki in [13].

Another part of the counter-argument is that the entropy brought into the black hole by fresh quantum matter is anyway only an extremely small fraction of all the entropy in play in the process. The increase of Bekenstein-Hawking entropy of the black hole when its mass increases by Δm is $8\pi \frac{M}{m_p^2} \delta m$. This is the same as

if an impacting body of mass Δm would carry an entropy per unit mass of c^2/k_BT_H where T_H is the Hawking temperature of the black hole. For a solar mass black hole this is about $10^{47} \cdot \text{kg}^{-1}$. By comparison, the standard molar entropy of water at room temperature is 75.33J/(mol K) which means about $3 \cdot 10^{26} \cdot \text{kg}^{-1}$ in the same units. To emphasize this discrepancy in entropy per mass of more than twenty orders of magnitude was the main motivation of this note.