Comment on "Topological Pumping in a Floquet-Bloch Band"

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In a recent Letter [1], topological pumping in a Floquet-Bloch band using a plain sinusoidal lattice potential and two-tone driving with relative phase φ, amplitudes K_1 and K_2, and frequencies ω and 2ω, respectively, is experimentally and numerically demonstrated. For a fixed amplitude K_1, the authors obtained numerical estimates of the critical parameters K_{2c}, φ_c for closing the s-p bandgap, thus optimizing directional motion of the center-of-mass position of the atomic cloud of ultracold ^{40}K fermionic atoms, while they found that the efficiency of this topological transport seems to be quantized for sufficiently long driving periods (adiabatic directed transport (DT)). The authors do not provide any theoretical explanation either for these apparently magical values of K_{2c}, φ_c or for the "quantized efficiency" in the adiabatic regime. In this Comment, we argue how the theory of ratchet universality (RU) [2-4] provides a satisfactory explanation of such experimental findings. Although the numerical results discussed in [1] are based on a Floquet two-band model of s, p bands, the authors confirmed that the pump efficiency is the same when evaluated using the full time-dependent Hamiltonian

$$H(\tau) = \frac{p^2}{2} + V(x) - F(\tau)\, x, \tag{1}$$

$$F(\tau) \equiv \frac{\hbar}{a\omega}\left[K_1 \cos(\omega\tau) + 2K_2 \cos(2\omega\tau + \varphi)\right], \tag{2}$$

which characterizes the dynamics of the center-of-mass position of the atomic cloud in the frame co-moving with the driven optical lattice (throughout this Comment, we follow the notations and definitions used in the Letter). Clearly, DT appears due to the de-symmetrization of Floquet eigenstates when the generalized parity symmetry and the generalized time-reversal symmetry are violated, i.e., when $K_1, K_2 \neq 0$ and $\varphi \neq 0, \pm\pi$, respectively [5], while the theory of RU predicts that there exists a universal force waveform that optimally enhances DT by critically breaking such symmetries. This theory has explained previous experimental results concerning DT of fluxons in uniform annular Josephson junctions in one case and of cold atoms in dissipative optical lattices in another, both driven by biharmonic fields [6], and refers to the criticality scenario that emerges when the aforementioned two symmetries are broken, regardless of the nature of the evolution equation in which the breaking of such symmetries results in DT. Specifically, the four equivalent expressions of the biharmonic universal excitation [1] are given by $\cos(\omega\tau) \pm (1/2)\sin(2\omega\tau)$, $\sin(\omega\tau) \pm (1/2)\sin(2\omega\tau)$, i.e., the optimal values predicted from RU for the force $F(\tau)$ [Eq. (2)] to yield optimal enhancement of DT of ultracold ^{40}K fermionic atoms are

$$K_{1opt} = 4K_{2opt}, \varphi_{opt} = \pm\pi/2, \tag{3}$$

where the two signs $\pm$ correspond to DT in opposite directions. For the value $K_1 = 0.8$ considered in [1], Eq. (3) predicts $K_{2opt} = 0.2$ that is very close to the critical value $K_{2c} = 0.22$ stated in [1]. In fact, the authors considered the exact value $K_{2c} = K_{2opt} = 0.2$ for the parametrization of all pumping orbits (cf. Fig. 3 and Supplemental Material in Ref. [1]) and found $\varphi_c = \varphi_{opt} = \pm\pi/2$, as predicted from RU. Additionally, the efficiency of the DT is predicted from RU to reach a constant value that is independent of the pump period when this is the largest timescale of the problem, i.e., when π/ω is sufficiently larger than the inverse of the minimal gap size (adiabatic transport regime), providing thereby an explanation for the "quantized efficiency" indicated by the authors (cf. Figs. 4 and S1 in the Supplemental Material [1]). Otherwise, there is expected from RU a gradual decrease of the efficiency as 2ω is increased from the adiabatic limit, i.e., as the relevant symmetries are gradually restored. This phenomenon of competing timescales leads to the 2ω-force losing ratchet effectiveness, but without deactivating the degree-of-symmetry-breaking mechanism [2,3], thus explaining the gradual decrease of the efficiency when $2\pi/\omega \to 0$ (cf. Fig. 4 in Ref. [1]). It is worth mentioning that the same gradual decrease has been observed previously (cf. Fig. 8 in Ref. [5] and Fig. 5 in Ref. [7]). To conclude, the results of Ref. [1] show a fruitful cooperation between the Berry phase for energy bands and the ratchet universal excitation at optimally inducing charge pumping in driven lattice systems.

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