# Comment on evidence of a transition to the ultimate regime of heat transfer 

Erik Lindborg ${ }^{1}$<br>${ }^{1}$ Department of Engineering Mechanics, KTH, Osquars backe 18, SE-100 44, Stockholm, Sweden

(Dated: November 7, 2023)

PACS numbers:

Zhu et al. 1] carried out DNS of 2D Rayleigh-Bénard convection (RBC) up to Rayleigh number $R a=10^{14}$ and reported evidence of a transition to the 'ultimate regime' of heat transfer predicted by [2] for 3D RBC, with Nusselt number dependence $N u \sim R a^{\gamma}$, where $\gamma>1 / 3$ for high Ra. Doering et al. [3] analysed the results of [1] and concluded that they should rather be interpreted as evidence of absence of a transition. Zhu et al. [4] carried out two more simulations at $R a>10^{14}$ and claimed that they had now collected 'overwhelming evidence' of a transition.

The author of this comment would like to point out that none of the simulations at $R a>10^{10}$ presented in [1] reached a statistically stationary state. A sensitive indicator of stationarity is the development of the mean kinetic energy, $E$. In requesting information from two of the authors of [1] (Detlef Lohse and Xiaojue Zhu), the author was informed that $E$ was still growing in all simulations at $R a>10^{10}$, when they were ended. For $R a \leq 10^{13}$ the simulations were all ended at $t=1000$, where time is measured in $H / u_{f}, H$ being the height of the domain and $u_{f}$ the free fall velocity. Two simulations were carried out at $10^{13}<R a<10^{14}$, ending at $t=500$, and one simulations at $R a=10^{14}$, ending at $t=250$. No information was provided in 4] on how long time the two simulations at $R a>10^{14}$ were run. Lohse \& Zhu sent the author a figure depicting the time evolution of the four simulations $7,8,9$ and 10 listed in the supplementary material of [1]. The simulations had been continued after publication to check the convergence of $E$. Unfortunately, the figure cannot be shown, because Lohse \& Zhu do not grant the author permission to publish it. The figure shows that in the two simulations 7 and $10\left(R a=10^{10}\right.$ and $\left.R a=10^{11}\right), E$ reaches approximate stationarity at $t_{s} \approx 1000$ and $t_{s} \approx 3000$, with stationary values $E \approx 0.25$ and $E \approx 0.48 \approx 0.5$, in each case respectively. The simulation at $R a=10^{11}$ was far from stationarity when it was ended at $t=1000$, with $E \approx 0.38$. Assuming that $E$ continues to double and $t_{s}$ continues to triple when $R a$ is increased by a factor of ten, the simulation at $R a=10^{14}$ would reach stationary
first at $t_{s} \approx 80000$ with $E \approx 4$. Since this simulations was ended at $t=250$ with $E \approx 0.2$, the Nusselt number was evaluated in a state that was, indeed, very far from stationarity.

A cornerstone of scaling theories of RBC, for example the theory of [5], is the exact expression for the mean kinetic energy dissipation rate in a statistically stationary state,

$$
\begin{equation*}
\epsilon=\nu \kappa^{2} R a(N u-1) / H^{4} \tag{1}
\end{equation*}
$$

where $\nu$ is the kinematic viscosity and $\kappa$ the diffusivity. For $\operatorname{Pr}=\nu / \kappa \sim 1$, a condition for this relation to be satisfied is

$$
\begin{equation*}
\left|\frac{\mathrm{d} E}{\mathrm{~d} t}\right| \ll R a^{-1 / 2} N u \tag{2}
\end{equation*}
$$

where the time derivative on the left hand side is nondimensionalized by $u_{f}^{3} / H$. The high $R a$ simulations of [1] were far from satisfying this condition in the state where the Nusselt number was evaluated. As pointed out by [6]: 'One can only start to collect statistics when the flow is fully developed and has attained a statistically stationary state.' In conclusion, the issue regarding the scaling of $N u$ in high $R a$ 2D RBC is not settled yet.
[1] X. Zhu. V. Mathai, R.J.A.M. Stevens, R. Verzicco, and D. Lohse, Phys. Rev. Lett. 120, 144502 (2018).
[2] R.H. Kraichnan, Phys. Fluids, 5, 1374 (1962).
[3] C.H. Doering, S. Toppoladoddi, and J.S. Wettlaufer, Phys. Rev. Lett. 123, 259401, (2019).
[4] X. Zhu. V. Mathai, R.J.A.M. Stevens, R. Verzicco, and D. Lohse, Phys. Rev. Lett. 123, 259402 (2019).
[5] S. Grossmann, and D. Lohse, J. Fluid. Mech. 407, 27 (2000)
[6] Ahlers, G., Bodenschatz, E., Hartmann, R., He, X., Lohse, D., Reiter, P., Stevens, R., Verzicco, R., Wedi, M., Weiss, S., Zhang. X., Zwirner, L. \& Shishkina, O. Phys. Rev. Lett. 128, 084501, Supplementary material (2022).

