## Comment on evidence of a transition to the ultimate regime of heat transfer

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Zhu *et al.* [1] carried out DNS of 2D Rayleigh-Bénard convection (RBC) up to Rayleigh number  $Ra = 10^{14}$  and reported evidence of a transition to the 'ultimate regime' of heat transfer predicted by [2] for 3D RBC, with Nusselt number dependence  $Nu \sim Ra^{\gamma}$ , where  $\gamma > 1/3$  for high Ra. Doering *et al.* [3] analysed the results of [1] and concluded that they should rather be interpreted as evidence of absence of a transition. Zhu *et al.* [4] carried out two more simulations at  $Ra > 10^{14}$  and claimed that they had now collected 'overwhelming evidence' of a transition.

The author of this comment would like to point out that none of the simulations at  $Ra > 10^{10}$  presented in [1] reached a statistically stationary state. A sensitive indicator of stationarity is the development of the mean kinetic energy, E. In requesting information from two of the authors of [1] (Detlef Lohse and Xiaojue Zhu), the author was informed that E was still growing in all simulations at  $Ra > 10^{10}$ , when they were ended. For  $Ra \leq 10^{13}$  the simulations were all ended at t = 1000, where time is measured in  $H/u_f$ , H being the height of the domain and  $u_f$  the free fall velocity. Two simulations were carried out at  $10^{13} < Ra < 10^{14}$ , ending at t = 500, and one simulations at  $Ra = 10^{14}$ , ending at t = 250. No information was provided in [4] on how long time the two simulations at  $Ra > 10^{14}$  were run. Lohse & Zhu sent the author a figure depicting the time evolution of the four simulations 7, 8, 9 and 10 listed in the supplementary material of [1]. The simulations had been continued after publication to check the convergence of E. Unfortunately, the figure cannot be shown, because Lohse & Zhu do not grant the author permission to publish it. The figure shows that in the two simulations 7 and 10 ( $Ra = 10^{10}$  and  $Ra = 10^{11}$ ), E reaches approximate stationarity at  $t_s~\approx~1000$  and  $t_s~\approx~3000,$  with stationary values  $E \approx 0.25$  and  $E \approx 0.48 \approx 0.5$ , in each case respectively. The simulation at  $Ra = 10^{11}$  was far from stationarity when it was ended at t = 1000, with  $E \approx 0.38$ . Assuming that E continues to double and t. continues to triple when Ra is increased by a factor of ten, the simulation at  $Ra = 10^{14}$  would reach stationary

first at  $t_s \approx 80000$  with  $E \approx 4$ . Since this simulations was ended at t = 250 with  $E \approx 0.2$ , the Nusselt number was evaluated in a state that was, indeed, very far from stationarity.

A cornerstone of scaling theories of RBC, for example the theory of [5], is the exact expression for the mean kinetic energy dissipation rate in a statistically stationary state,

$$\epsilon = \nu \kappa^2 Ra(Nu - 1)/H^4 \,, \tag{1}$$

where  $\nu$  is the kinematic viscosity and  $\kappa$  the diffusivity. For  $Pr = \nu/\kappa \sim 1$ , a condition for this relation to be satisfied is

$$\frac{\mathrm{d}E}{\mathrm{d}t}|\ll Ra^{-1/2}Nu\,,\tag{2}$$

where the time derivative on the left hand side is nondimensionalized by  $u_f^3/H$ . The high Ra simulations of [1] were far from satisfying this condition in the state where the Nusselt number was evaluated. As pointed out by [6]: 'One can only start to collect statistics when the flow is fully developed and has attained a statistically stationary state.' In conclusion, the issue regarding the scaling of Nu in high Ra 2D RBC is not settled yet.

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