# On gravity unification in SL(2N,C) gauge theories

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**Abstract**

The local $SL(2N,C)$ symmetry is shown to provide, when appropriately constrained, a viable framework for a consistent unification of the known elementary forces, including gravity. Such a covariant constraint implies that an actual gauge field multiplet in the $SL(2N,C)$ theory is ultimately determined by the associated tetrad fields which not only specify the geometric features of spacetime but also govern which local internal symmetries are permissible within it. As a consequence, upon the covariant removal of all "redundant" gauge field components, the entire theory only exhibits the effective $SL(2,C) \times SU(N)$ symmetry, comprising $SL(2,C)$ gauge gravity on one hand and $SU(N)$ grand unified theory on the other. Given that all states involved in the $SL(2N,C)$ theories are additionally classified according to their spin values, many potential $SU(N)$ GUTs, including the conventional $SU(5)$ theory, appear to be irrelevant for standard spin 1/2 quarks and leptons. Meanwhile, applying the $SL(2N,C)$ symmetry to the model of composite quarks and leptons with constituent chiral preons in its fundamental representations reveals, under certain natural conditions, that among all accompanying $SU(N)_L \times SU(N)_R$ chiral symmetries of preons and their composites only the $SU(8)_L \times SU(8)_R$ meets the anomaly matching condition ensuring masslessness of these composites at large distances. This, in turn, identifies $SL(16,C)$ with the effective $SL(2,C) \times SU(8)$ symmetry, accommodating all three families of composite quarks and leptons, as the most likely candidate for hyperunification of the existing elementary forces.

## 1 Introduction

As is well known, there exists a certain similarity between gravity and three other elementary forces when considering gravity within a conventional gauge theory framework [1, 2, 3]. Indeed, the spin-connection fields gauging the local $SL(2,C)$ symmetry group of gravity emerge much like photons and gluons in the Standard Model. It is, therefore, conceivable that these spin-connections could be unified with the ordinary SM gauge bosons in a certain non-compact symmetry group, thereby leading to the hyperunification of all known elementary gauge forces. In the following, we refer to such theories as hyperunified theories (HUTs), and specifically as the $SL(2N,C)$ HUT when speaking about integration of the $SL(2,C)$ gauge gravity with the $SU(N)$ grand unified theory, respectively. We also designate $SU(N)$ as the "hyperflavor" symmetry and fields located in its representations as the "hyperflavored" fields.

In fact, there are many classes of models in the literature where unification of gravity and other interactions goes through a unification of the local Lorentz and internal symmetries in the framework of some non-compact covering symmetry group [4, 5, 6, 7]. Their difficulties are well known and, to varying degrees, they generally appear in the $SL(2N,C)$ HUT as well [8]. Firstly, the vector fields in the total gauge multiplet of this group are always accompanied by the axial-vector fields which must be somehow excluded from the theory as there is no direct indication whatsoever of their existence. Then, while vector fields are proposed to mediate ordinary gauge interactions, tensor fields must provide the minute gravity interactions to align with reality. The crucial point lies in the fact that, whereas in pure gravity case, one can solely consider the action being linear in the curvature ($R$) constructed from the tensor field, the unification with other interactions necessitates the inclusion of quadratic curvature ($R^2$) terms as well. Consequently, tensor fields in these terms will induce interactions comparable to those of the gauge vector fields in the Standard Model. Moreover, the tensor fields, akin to the vector ones, exhibit now the internal $SU(N)$ symmetry features implying the existence of the multiplet of hyperflavored gravitons rather than a single neutral one. Apart from that, such $R+R^2$ Lagrangians for gravity are generally known to contain ghosts and tachyons rendering them essentially unstable. And lastly, but perhaps most importantly, a potential pitfall in hyperunified theories stems from the Coleman-Mandula theorem [9] concerning the impossibility of merging spacetime and internal symmetries. It is worth noting that this theorem initially surfaced precisely in connection with one of the special cases of $SL(2N,C)$, specifically the $SL(6,C)$ symmetry [10], used a long time ago as a possible relativistic version of the global $SU(6)$ symmetry model describing the spin-unitary spin symmetry classification of mesons and baryons [11].

In contrast, we aim to demonstrate here how, in the $SL(2N,C)$ HUT framework, these difficulties can be naturally be overcome in the way as yet unexplored. The key idea is that the extended gauge multiplet in the $SL(2N,C)$ theory – comprising generally the vector, axial-vector and tensor field submultiplets – is suitably constrained by the associated tetrad multiplets which are assumed to not only determine the geometric features of spacetime, but also control which local internal symmetries and associated gauge field interactions are permitted in it. Specifically, we propose that the actual gauge multiplet $I_\mu$ arises as result of the tetrad filtering of some "prototype" nondynamical multiplet $\mathcal{I}_\mu$ which globally transforms similarly to $I_\mu$, but, unlike it, does not itself gauge the corresponding fermion system. These two multiplets are connected in a covariant way using the tetrads $e_\sigma$ and $e^\sigma$ that, instead of being imposed by postulate, can be incorporated into the theory through the Lagrange multiplier type term

$$\mathrm{C}\left(I_\mu - \frac{1}{4}e_\sigma \mathcal{I}_\mu e^\sigma\right)^2 \tag{1}$$

where C is an arbitrary constant. This term, upon variation under the multiplet $\mathcal{I}_\mu$, yields the filtering condition, $I_\mu = e_\sigma \mathcal{I}_\mu e^\sigma/4$, mentioned above. Consequently, the gauge multiplet $I_\mu$ retains only those components of the prototype multiplet $\mathcal{I}_\mu$ which result from the tetrad filtering. In other words, the prototype multiplet $\mathcal{I}_\mu$ becomes partially dynamical to the extent permitted by the tetrads.

Now, in the case of standard or strictly orthonormal tetrads, such filtering excludes the tensor fields in the gauge multiplet $I_\mu$, as we demonstrate below. However, when the tetrad orthonormality condition is slightly broken, the appropriately weakened tensor fields come into play. This occurs in a way that their interaction essentially decouples from other elementary forces and effectively adheres to the Einstein-Cartan type gravity action. The corresponding curvature-squared terms constructed from the filtered tensor fields appear to be vanishingly small and can be disregarded compared to the standard strength-squared terms for vector fields. As a result, the entire theory effectively exhibits a local $SL(2,C)\times SU(N)$ symmetry rather than the unified $SL(2N,C)$ symmetry, which solely provides the structure of total multiplets for gauge and matter fields in the theory. Consequently, this naturally leads to $SL(2,C)$ gauge gravity on one hand and $SU(N)$ grand unified theory (GUT) on the other, thereby effectively bypassing the constraints of the Coleman-Mandula theorem[1].

The paper is organized as follows. In Section 2 we provide a standard presentation of the $SL(2,C)$ gauge gravity which is then discussed using the filtering approach. Section 3 introduces the filtered $SL(2N,C)$ HUT which ultimately lead to the $SL(2,C)$ gauge gravity in conjunction with the $SU(N)$ grand unified theory. Section 4 examines specific HUT models with particular focus on the $SL(16,C)$ theory giving rise to the $SU(8)$ GUT with all three families of composite quarks and leptons that is studied in some detail. Finally, our summary is presented in Section 5.

# 2 $SL(2,C)$ gravity

## 2.1 Standard framework

We first present the $SL(2,C)$ gravity model, partially following the pioneering work [3]. Let there be a local frame at any spacetime point where the global $SL(2,C)$ symmetry

[1]It is worth noting in this connection that, though the Lagrange multiplier term (1) formally breaks gauge invariance of the entire hyperunified $SL(2N,C)$ theory, the local $SL(2,C)\times SU(N)$ symmetry in it remains unbroken.

group acts. According to this symmetry, the basic fermions of the theory transform as

$$\Psi \rightarrow \Omega\Psi, \quad \Omega = \exp\left\{\frac{i}{4}\theta_{ab}\gamma^{ab}\right\} \tag{2}$$

where the matrix $\Omega$ satisfies a pseudounitarity condition, $\Omega^{-1} = \gamma_0\Omega^{+}\gamma_0$ (the transformation parameters $\theta_{ab}$ are assumed to be constant for now). Furthermore, to ensure the invariance of their kinetic terms, $i\overline{\Psi}\gamma^{\mu}\partial_{\mu}\Psi$, one needs to replace $\gamma$-matrices in them with a set of some tetrad matrices $e^{\mu}$ which transform like

$$e^{\mu} \rightarrow \Omega e^{\mu}\Omega^{-1} \tag{3}$$

Generally, the tetrad matrices $e^{\mu}$ , as well as their conjugates $e_{\mu}$, contain the appropriate tetrad fields $e^{\mu}_{a}$ and $e^{a}_{\mu}$, respectively,

$$e_{\mu} = e^{a}_{\mu}\gamma_{a} \ , \ \ e^{\mu} = e^{\mu}_{a}\gamma^{a} \tag{4}$$

which transforms infinitesimally as

$$\delta e^{\mu c} = \frac{1}{2}\theta_{ab}(e^{\mu a}\eta^{bc} - e^{\mu b}\eta^{ac}) \tag{5}$$

They, as usual, satisfy the orthonormality conditions

$$e^{a}_{\mu}e^{\nu}_{a} = \delta^{\nu}_{\mu}, \ \ e^{a}_{\mu}e^{\mu}_{b} = \delta^{a}_{b} \tag{6}$$

and determine the metric tensors in the theory

$$g_{\mu\nu} = \frac{1}{4}Tr(e_{\mu}e_{\nu}) = e^{a}_{\mu}e^{b}_{\nu}\eta_{ab} \ , \ \ g^{\mu\nu} = \frac{1}{4}Tr(e^{\mu}e^{\nu}) = e^{\mu}_{a}e^{\nu}_{b}\eta^{ab} \tag{7}$$

Going now to the case when the $SL(2,C)$ transformations (2) become local, $\theta_{ab} \equiv \theta_{ab}(x)$, one have to introduce the spin-connection gauge field multiplet $I_{\mu}$ transforming as usual

$$I_{\mu} \rightarrow \Omega I_{\mu}\Omega^{-1} - \frac{1}{ig}(\partial_{\mu}\Omega)\Omega^{-1} \tag{8}$$

thus providing the fermion field by covariant derivative

$$\partial_{\mu}\Psi \rightarrow D_{\mu}\Psi = \partial_{\mu}\Psi + igI_{\mu}\Psi \tag{9}$$

where $g$ presents the gauge coupling constant extracted for later convenience. The $I_{\mu}$ multiplet gauging the $SL(2,C)$ has by definition the form

$$I_{\mu} = \frac{1}{4}T_{\mu[ab]}\gamma^{ab} \tag{10}$$

where the flat spacetime tensor field components $T_{\mu[ab]}$ transform as

$$\delta T^{[ab]}_{\mu} = \frac{1}{2}\theta_{[cd]}[(T^{[ac]}_{\mu}\eta^{bd} - T^{[ad]}_{\mu}\eta^{bc}) - (T^{[bc]}_{\mu}\eta^{ad} - T^{[bd]}_{\mu}\eta^{ac})] - \frac{1}{g}\partial_{\mu}\theta^{[ab]} \tag{11}$$

The tensor field $T_{\mu[ab]}$ may in principle propagate, while the tetrad $e^{\mu}$ is not considered as a dynamical field. So, the invariant Lagrangian built from its strength

$$
\begin{aligned}
I_{\mu\nu} &= \partial_{[\mu}I_{\nu]} + ig[I_{\mu}, I_{\nu}] = \frac{1}{4}T^{[ab]}_{\mu\nu}\gamma_{ab} \\
T^{[ab]}_{\mu\nu} &= \partial_{[\nu}T^{[ab]}_{\mu]} + g\eta_{cd}T^{[ac]}_{[\mu}T^{[bd]}_{\nu]}
\end{aligned} \tag{12}
$$

can be written in a conventional form

$$
e\mathcal{L}_G = \frac{1}{2\kappa}e^{\mu}_{[a}e^{\nu}_{b]}\ T^{[ab]}_{\mu\nu}\ , \quad e \equiv [-\det Tr(e^{\mu}e^{\nu})/4]^{-1/2} \tag{13}
$$

(where $\kappa$ stands for the modified Newtonian constant $8\pi/M^2_{Pl}$) once the commutator for tetrads and some of standard relations for $\gamma$-matrices have been used[2]. This is, in fact, the simplest pure gravity Lagrangian taken in the Palatini type formulation. Indeed, its variation under the tensor field gives the constraint allowing to express it through the tetrad and its derivative that reduces $e\mathcal{L}_G$ to the standard Einstein Lagrangian. It is written with the scalar factor $e$ which, while irrelevant for $SL(2, C)$ gauge invariance itself, provides an extra invariance of the action under general four-coordinate transformations $GL(4, R)$ as well [3].

Meanwhile, in presence of fermions, the gauge invariant fermion matter coupling given by the covariant derivative (9, 10) implies the extra tensor field interaction with the spin-current density

$$
e\mathcal{L}^{int}_M = -\frac{1}{2}g\epsilon^{abcd}T_{\mu[ab]}e^{\mu}_c\overline{\Psi}\gamma_d\gamma_5\Psi \tag{14}
$$

This is a key feature of the Einstein-Cartan type gravity [2] which eventually results in, apart from the standard GR, the tiny four-fermion (spin current-current) interaction in the matter sector

$$
\kappa\left(\overline{\Psi}\gamma_d\gamma^5\Psi\right)\left(\overline{\Psi}\gamma^d\gamma^5\Psi\right) \tag{15}
$$

On the other hand, one could augment the linear curvature Lagrangian (13) with the curvature squared terms, thereby rendering some tensor field components dynamical. It is known that theories of this type are strongly constrained to a specific form to ensure freedom from ghosts and tachyons [12]. Expressed in terms of tensor field strengths, the acceptable quadratic curvature part in them appears as

$$
e\mathcal{L}^{(2)} = Q(T^{\mu\nu}_{ab}T^{ab}_{\mu\nu} + T^{\mu\nu}_{ab}T^{cd}_{\rho\sigma}e_{\mu c}e_{\nu d}e^{\rho a}e^{\sigma b} - 4T^{\mu\nu}_{ab}T^{ac}_{\rho\nu}e_{\mu c}e^{\rho b}) \tag{16}
$$

[2]We give here some of them used throughout the paper

$$
\begin{aligned}
\gamma^{ab} &= i[\gamma^a, \gamma^b]/2,\ \gamma^a\gamma^b = \gamma^{ab}/i + \eta^{ab}\widehat{1},\ \gamma_c\gamma^{[ab]}\gamma^c = 0 \\
[\gamma^{ab}, \gamma^{a'b'}] &= 2i(\eta^{ab'}\gamma^{ba'} + \eta^{ba'}\gamma^{ab'} - \eta^{aa'}\gamma^{bb'} - \eta^{bb'}\gamma^{aa'}) \\
Tr(\gamma^{ab}\gamma^{a'b'}) &= 4(\eta^{aa'}\eta^{bb'} - \eta^{ab'}\eta^{ba'}),\ \ Tr(\gamma^{ab}\gamma_{cd}) = 4(\delta^a_c\delta^b_d - \delta^b_c\delta^a_d) \\
Tr(\gamma^{ab}\gamma^{a'b'}\gamma^{a''b''}) &= 4i[\eta^{aa'}(\eta^{a''b'}\eta^{bb''} - \eta^{a''b}\eta^{b'b''}) + \eta^{ab'}(\eta^{a''b}\eta^{a'b''} - \eta^{a'a''}\eta^{bb''}) \\
&\quad +\eta^{a'b}(\eta^{aa''}\eta^{b'b''} - \eta^{a''b'}\eta^{ab''}) + \eta^{bb'}(\eta^{a'a''}\eta^{ab''} - \eta^{a'b''}\eta^{aa''})]
\end{aligned}
$$

where $\widehat{1}$ in the above is the $4 \times 4$ unit matrix.

Actually, such a theory contains, apart the graviton, some superheavy scalar excitation $S(0^-)$ with a mass, $m_S^2 \sim M_P^2/Q$, that can hardly be observed unless the numerical parameter $Q$ in (16) is exceedingly large.

### 2.2 Tetrad filtering case

As previously mentioned, the $SL(2,C)$ gauge gravity is implied as a part of the unified set of all elementary forces assembled in the $SL(2N,C)$ symmetry framework. This raises a key issue of how to reconcile the exceptionally weak gravitational force related to the tensor fields with the significant Standard Model forces associated with the vector field submultiplet. The corresponding Lagrangian generally includes terms that are linear and quadratic in the strength of the entire gauge field multiplet (each with independent coupling constants). Interestingly, as is shown later, the linear terms pose no problem, as they only generate gauge gravity, governed by its own coupling constant (related to the Planck mass, as usual). However, the quadratic terms for the tensor fields share the same dimensionless coupling constant as the vector field interactions, which is certainly unacceptable. We propose that the tensor field quadratic terms might become negligible due to the minuscule nature of the filtered tensor fields themselves, while the vector fields remain unaffected by this filtering process. Whereas it would be particularly noteworthy to immediately consider this scenario within the entire framework of $SL(2N,C)$ theory, we nonetheless begin by examining the pure tensor field case to elucidate the proposed tetrad filtering mechanism in greater detail.

In this regard, we propose that the gauge field multiplet $I_\mu$ (10) stems from some prototype nondynamical multiplet $\mathcal{I}_\mu$, which globally transforms akin to $I_\mu$, but does not gauge the matter fermions. They are connected in a covariant way through the tetrads involved

$$I_\mu = \frac{1}{4} e_\sigma \mathcal{I}_\mu e^\sigma, \ \mathcal{I}_\mu = \frac{1}{4} \mathcal{T}_{\mu[ab]} \gamma^{ab} \tag{17}$$

when the Lagrange multiplier term (1) in the gauge gravity Lagrangian is varied under the nondynamical $\mathcal{I}_\mu$ multiplet. However, one can easily find that, as follows from the $\gamma$-matrix algebra[2], and, in particular, from the identity

$$\gamma_c \gamma^{[ab]} \gamma^c = 0 \tag{18}$$

the gauge multiplet $I_\mu$ with the strictly orthonormal tetrads (6) automatically vanishes. This means that the theory only possesses the global $SL(2,C)$ symmetry in this limit and gauge gravity is in fact absent.

The gauge multiplet $I_\mu$ may only appears if the tetrads are no longer orthonormal but their orthonormality conditions (6) include some vanishingly small deviations

$$e_\mu^a e_b^\mu = \delta_b^a + \varepsilon q_b^a, \quad e_\mu^a e_a^\nu = \delta_\mu^\nu + \varepsilon p_\mu^\nu \qquad (\varepsilon \ll 1) \tag{19}$$

given by the infinitesimal tensors $\varepsilon q_b^a$ and $\varepsilon p_\mu^\nu$ (with the tiny constant parameter $\varepsilon$) , respectively. Indeed, now the gauge multiplet $I_\mu$ comes to

$$I_\mu = \frac{1}{4} T_{\mu[ab]} \gamma^{ab} = \frac{\varepsilon}{16} \mathcal{T}_{\mu[ab]} q_d^c (\gamma_c \gamma^{ab} \gamma^d) \tag{20}$$

being solely determined by the tiny tensor $\varepsilon q_b^a$. Multiplying the both sides by $\gamma^{a'b'}$ and taking the traces in them one can readily find using the $\gamma$ matrix algebra[2] the relation between the tensor fields themselves

$$T_{\mu[ab]} = \frac{\varepsilon}{2}(\mathcal{T}_{\mu[bc]}q_a^c - \mathcal{T}_{\mu[ac]}q_b^c + \mathcal{T}_{\mu[ab]}q_c^c/2) = \varepsilon \mathbf{T}_{\mu[ab]} \tag{21a}$$

Thus, the starting prototype, while nondynamical, tensor field multiplet $\mathcal{T}_{\mu[ab]}$ is predominantly extinguished and only its minuscule portion emerges in the gauge multiplet $T_{\mu[ab]}$. Remarkably, the filtering process triggers an important suppression mechanism in the $SL(2,C)$ gauge gravity theory that allows it to be reformulated solely in terms of the weakened tensor field multiplet $\varepsilon \mathbf{T}_{\mu[ab]}$. This multiplet is, in fact, the product of the prototype tensor field $\mathcal{T}_{\mu[ab]}$ with the infinitesimal tensor $\varepsilon q_b^a$ which describes the tiny deviation in the modified orthonormality conditions (19) for tetrads.

Analogously, the metric tensor will also include such a deviation which we define from the similar equation

$$e_\mu^a e_{a\nu} = g_{\mu\nu} + r_{\mu\nu}, \ e_a^\mu e^{a\nu} = g^{\mu\nu} + r^{\mu\nu} \tag{22}$$

Multiplying the basic equations (19) by the proper tetrads one can readily find relations between the deviations

$$p_\mu^\nu e_\nu^a = q_b^a e_\mu^b, \quad (pp)_\mu^\nu e_\nu^a = (qq)_b^a e_\mu^b, \quad (p...p)_\mu^\nu e_\nu^a = (q...q)_b^a e_\mu^b \tag{23}$$

so that all deviations can be expressed in terms of the $q$ parameter only. Such $q$-depending deviation appears in the metric tensor in (22) as well provided that one requires general covariance for the metric tensor $g_{\mu\nu}$

$$g_{\mu\nu}e^{\nu b} = e_\mu^b, \ r_{\mu\nu}e^{\nu b} = -e_\mu^b + e_\mu^a e_{a\nu} e^{\nu b} = \varepsilon e_\mu^a q_a^b \tag{24}$$

where we have also used the deviation equations (19). Multiplying then the both sides by $e_{b\rho}$ one finds

$$r_{\mu\nu}(\delta_\rho^\nu + p_\rho^\nu) = e_\mu^a q_a^b e_{b\rho} \tag{25}$$

which after multiplying by the conjugated factor $\delta_\sigma^\rho - p_\sigma^\rho$ finally gives

$$r_{\mu\nu} \simeq [\varepsilon q_a^b - \varepsilon^2 (qq)_a^b] e_\mu^a e_{b\nu} \tag{26}$$

up to the second order terms in $q$. With the same accuracy the metric tensors acquires the form

$$g_{\mu\nu} \simeq e_\mu^a e_\nu^c \eta_{bc}[\delta_a^b - \varepsilon q_a^b + \varepsilon^2 (qq)_a^b], \quad g_{\mu\nu}g^{\nu\rho} = \delta_\mu^\rho + O(\varepsilon^2) \tag{27}$$

and, respectively,

$$\eta_{ab} \simeq g_{\mu\nu}e_a^\mu e_c^\nu [\delta_b^c - \varepsilon q_b^c + \varepsilon^2 (qq)_b^c], \ \eta_{ab}\eta^{bc} = \delta_a^c + O(\varepsilon^2) \tag{28}$$

The exact expressions for them can be symbolically written as

$$g_{\mu\nu} = e_\mu^a e_\nu^c \eta_{bc} \left[\frac{1}{1+\varepsilon q}\right]_a^b, \quad \eta_{ab} = g_{\mu\nu}e_a^\mu e_c^\nu \left[\frac{1}{1+\varepsilon q}\right]_b^c \tag{29}$$

transparently showing their modification compared to the standard case (7).

In the following, we assume for simplicity that the above deviation tensors are traceless ($q_a^a = p_\mu^\mu = 0$) being arisen from some symmetric traceless tensors

$$q_b^a = \eta_{bc} q^{\{ac\}}, \ \ p_\mu^\nu = g_{\mu\sigma} p^{\{\nu\sigma\}} \tag{30}$$

For such a choice, despite the nonzero deviations in (19) the tetrad matrix orthonormality condition remains its form

$$\frac{1}{4} e_\mu e^\mu = \widehat{1} + \varepsilon q_b^a \gamma_a \gamma^b / 4 = \widehat{1} \tag{31}$$

Interestingly, the modified orthonormality conditions (19) may be considered as those that appear due to condensation of the tetrad which can be written in the form

$$e_\mu^a = \delta_\mu^a h + \widehat{e}_\mu^a, \ \ \delta_a^\mu \widehat{e}_\mu^a = 0 \tag{32a}$$

where

$$h = (1 - \widehat{e}_\mu^a \widehat{e}_a^\mu / 4)^{1/2} \tag{33}$$

Such a form provides the vacuum expectation value for the tetrad field $e_\mu$

$$\langle e_\mu \rangle = \gamma_\mu \tag{34}$$

which represents an extremum of the action with $\widehat{e}_\mu^a$ and $h$ being as the effective zero mode and Higgs mode, respectively. This could be considered quite an ordinary example, if tetrads were treated as the dynamical fields in the $SL(2,C)$ gauge theory that is not actually supposed. However, nothing prevents the above conditions from being seen as a would-be spontaneous breakdown of the local frame $SL(2,C)$ symmetry for tetrads, while the theory is still left Poincare-invariant.

Although tetrads are not considered as dynamical fields in the theory, one can take, nonetheless, that the above conditions cause a would-be spontaneous breakdown of the local frame $SL(2N,C)$ symmetry for tetrads.

The basic Lagrangian for the $SL(2,C)$ gravity will now result in the appropriate analogs of the gravity and matter field Lagrangians (13, 14), respectively. Indeed, the minimal gravity Lagrangian (13) remains practically the same form

$$e\mathcal{L}_G = \frac{1}{2\kappa} e_{[a}^\mu e_{b]}^\nu \mathbf{T}_{\mu\nu}^{[ab]} \tag{35}$$

though the tensor field strength $T_{\mu\nu[ab]}$ has been properly modified according to the relations (20, 21a) taken

$$T_{\mu\nu}^{[ab]} = \varepsilon \mathbf{T}_{\mu\nu}^{[ab]}, \ \ \mathbf{T}_{\mu\nu}^{[ab]} = \partial_{[\nu} \mathbf{T}_{\mu]}^{[ab]} + \varepsilon g \eta_{cd} \mathbf{T}_{[\mu}^{[ac]} \mathbf{T}_{\nu]}^{[bd]} \tag{36}$$

Remarkably, once the tiny parameter $\varepsilon$ is absorbed in the gravity constant $\kappa$, the tensor field strength $\mathbf{T}_{\mu\nu}^{[ab]}$ acquires the modified gauge coupling constant $\varepsilon g$ for the new tensor field $\mathbf{T}_\mu^{[ab]}$. Such a modification will also appear for the matter coupling

$$e\mathcal{L}_M^{int} = -\frac{\varepsilon g}{2} \epsilon^{abcd} \mathbf{T}_{\mu[ab]} e_c^\mu \overline{\Psi} \gamma_d \gamma_5 \Psi \tag{37}$$

and for the possible quadratic tensor field strength terms

$$e\mathcal{L}_T = \varepsilon^2 Q_n \mathcal{L}_n^{(2)} \tag{38}$$

where, for generality, the latter is taken to contain all possible combinations of the tensor field strength bilinears with tetrads, each with an arbitrary $Q_n$ constant.

In this context, the propagation of the tensor field $\mathbf{T}_\mu^{[ab]}$ in the hyperunified $SL(2N,C)$ theory seems to be irrelevant. In fact, its kinetic term contained in (38), in sharp contrast to the ordinary kinetic terms of the vector (and axial-vector) fields, scales as $\varepsilon^2$ and therefore can be neglected. As in the standard case considered above, the variation of the total linear Lagrangian in (35, 37) under prototype tensor field $\mathbf{T}_\mu^{[ab]}$ just leads to the constraint equation rather than the normal equation of motion. Meanwhile, the variation of total gravity Lagrangian with respect to the tetrad $e_a^\mu$ leads immediately to the equation of motion of the Einstein-Cartan type gravity.

In conclusion, it is worth noting, that one might believe that all the above modifications are merely fictitious as a simple rescaling of the tensor field $\mathbf{T}_\mu = \mathbf{T}'_\mu/\varepsilon$ seemingly restores the standard case albeit with slightly broken general covariance due to the deviations in tetrads (19). However, this rescaling trick only works for pure gauge gravity with local $SL(2,C)$ symmetry and appears inappropriate in the framework of $SL(2N,C)$ gauge theory. In reality, the weakness of the filtered tensor field is exclusive to the tensor field submultiplet and does not extend to the entire gauge multiplet of $SL(2N,C)$ containing also the vector and axial-vector fields. Consequently, it cannot be circumvented by mere rescaling, as we show in more detail later in Section 3.4.

# 3 Toward $SL(2N,C)$ hyperunification

## 3.1 Basics of $SL(2N,C)$

In general, the $SL(2N,C)$ symmetry group encompasses, among its primary subgroups, the aforementioned $SL(2,C)$ symmetry, which covers the orthochronous Lorentz group, and the internal $U(N)$ symmetry group (including the hyperflavor $SU(N)$ symmetry). Indeed, the $8N^2-2$ generators of $SL(2N,C)$ are formed from the tensor products of the generators of $SL(2,C)$ and generators of $U(N)$ so that the basic transformation applied to the fermions looks as follows

$$\Omega = \exp\left\{\frac{i}{2}\left[\left(\theta^k + i\theta_5^k\gamma_5\right)\lambda^k + \frac{1}{2}\theta_{ab}^K\gamma^{ab}\lambda^K\right]\right\} \quad (K=0,k) \tag{39}$$

Here, among the $\lambda^K$ matrices, $\lambda^k$ ($k = 1, ..., N^2-1$) represent the $SU(N)$ Gell-Mann matrices, while $\lambda^0$ is the unit matrix $\widehat{1}$ corresponding to the $U(1)$ generator (all $\theta$ parameters may be constant or, in general, depend on the spacetime coordinate). Hereafter, we use the uppercase Latin letters $(I,J,K)$ for the $U(N)$ symmetry case, while the lowercase letters

$(i,j,k)$ for the $SU(N)$ symmetry one[3].

For description of the fermion matter in the theory one needs again to introduce the generalized tetrad multiplet

$$e_\mu = (e_\mu^{aK}\gamma_a + e_{\mu 5}^{aK}\gamma_a\gamma_5)\lambda^K \tag{40}$$

which transforms, as before, according to (3) where the transformation matrix is now given by equation (39). Despite its somewhat cumbersome extension which generally appears in the $SL(2N,C)$ framework, it would be natural for tetrad flat space components in (40) to essentially have the same form as in the pure gravity case. This implies that such an extension might not include the axial-vector part that could be reached through the gauge invariant constraints put on tetrads. In fact, one can introduce for that some special nondynamical $SL(2N,C)$ scalar multiplet in the theory

$$S = \exp\{i[(s^k + ip^k\gamma_5)\lambda^k + t_{ab}^K\gamma^{ab}\lambda^K/2]\} \tag{41}$$

which transforms like as $S \to \Omega S$. With this scalar multiplet one can form a new tetrad in terms of the gauge invariant construction, $S^{-1}eS$. So, choosing appropriately the flat space components in the $S$ field one can turn the tetrad axial part to zero and establish symmetry between Greek and Latin spacetime indices [10].

The issue still lies in the fact that the tetrads in (40), along with a neutral component, also include the $SU(N)$ hyperflavored components, making it impossible to treat them as standard vielbein fields that satisfy the orthonormality conditions (6). As one can see, a strict limitation on the form of tetrads exists for these conditions to be upheld. Indeed, let them generally have the covariant $SL(2N,C)$ form

$$e_\mu = e_\mu^{aK}\gamma_a\lambda^K, \;\; e_\mu^{aK}e_b^{\mu K'} = \Delta_b^{aKK'}, \;\; e_\mu^{aK}e_a^{\nu K'} = \Delta_\mu^{\nu KK'} \tag{42}$$

with some still unspecified constructions $\Delta_b^{aKK'}$ and $\Delta_\mu^{\nu KK'}$ which for a pure gravity case should satisfy the standard arrangement

$$\Delta_b^{a00} = \delta_b^a \;, \;\; \Delta_\mu^{\nu 00} = \delta_\mu^\nu \tag{43}$$

Then multiplying the conditions (42) by the tetrad multiplets $e_\sigma^{bK''}$and $e_a^{\sigma K''}$, respectively, one come after simple calculations

$$e_\mu^{aK} = \Delta_b^{aK0}e_\mu^{b0}, \;\; e_a^{\mu K'} = \Delta_a^{bK'0}e_b^{\mu 0} \tag{44}$$

[3]Some useful relations for the $\lambda$ matrices which will be used below are given here

$$\begin{aligned}[\lambda^k,\lambda^l] &= 2if^{klm}\lambda^m, \; \{\lambda^k,\lambda^l\} = 2(\delta^{kl}\hat{1} + d^{klm}\lambda^m) \\ \lambda^k\lambda^l\lambda^k &= -\lambda^l, \;\; \lambda^K\lambda^l\lambda^K = 0, \;\; Tr(\lambda^k\lambda^l) = N\delta^{kl}\end{aligned}$$

The connections with a standard choice of the $SU(N)$ matrices are given by the links

$$\lambda^K = \sqrt{2N}T^K, \;\; f^{ijk} = \sqrt{N/2}F^{ijk}, \;\; d^{ijk} = \sqrt{N/2}D^{ijk}$$

that finally gives

$$\Delta_b^{aKK'} = \Delta_c^{aK0}\Delta_b^{cK'0} \tag{45}$$

and correspondingly

$$\Delta_\mu^{\nu KK'} = \Delta_\sigma^{\nu K0}\Delta_\mu^{\sigma K'0} \tag{46}$$

As a result, for the constant and multiplicative form of these functions one unavoidably comes to the only possible solution

$$e_\mu^{aK} e_b^{\mu K'} = \Delta_b^{aKK'} = \delta_b^a \delta^{K0}\delta^{K'0} \ , \ \ e_\mu^{aK} e_a^{\nu K'} = \Delta_\mu^{\nu KK'} = \delta_\mu^\nu \delta^{K0}\delta^{K'0} \tag{47}$$

which essentially mirrors the pure gravity case. Consequently, the orthonormality condition for tetrads is consistent with the $SL(2N,C)$ symmetry only if they belong to its $SL(2,C)$ part rather than the entire group

$$e_\mu^{aK} = e_\mu^a \delta^{K0} \tag{48}$$

This may appear as result of the spontaneous violation $SL(2N,C)$ in the tetrad sector, as we will argue later.

Once the $SL(2N,C)$ transformation (39) becomes local one also need, as ever, to introduce the gauge field multiplet $I_\mu$ transforming as usual

$$I_\mu \rightarrow \Omega I_\mu \Omega^{-1} - \frac{1}{ig}(\partial_\mu \Omega)\Omega^{-1} \tag{49}$$

thus providing the fermion multiplet by covariant derivative

$$\partial_\mu \Psi \rightarrow D_\mu \Psi = \partial_\mu \Psi + igI_\mu \Psi \tag{50}$$

with universal gauge coupling constant $g$ of the proposed hyperunification. The $I_\mu$ multiplet includes in general the vector and axial-vector field submultiplets, and also the tensor field submultiplet

$$I_\mu = V_\mu + A_\mu + T_\mu = \frac{1}{2}\left(V_\mu^k + iA_\mu^k \gamma_5\right)\lambda^k + \frac{1}{4}T_{\mu[ab]}^K \gamma^{ab}\lambda^K \quad (K = 0, k) \tag{51}$$

as follows from its decomposition to the flat spacetime component fields. Just the tensor fields provide gravitational interaction in the $SL(2N,C)$ HUTs that, aside from the standard linear curvature Lagrangian for gravity (13), includes the conventional quadratic strength terms for all gauge field submultiplets involved. This, as mentioned, poses the crucial problem how one can selectively suppress tensor field interaction in these terms if the tensor fields are members of the same gauge multiplet $I_\mu$ as vector and axial-vector fields and, therefore, should interact with the same coupling constant $g$. Fortunately, the filtering mechanism described above for the pure gravity case allows for a natural combination of the strong internal symmetry forces related to the vector fields with the tiny quadratic curvature gravity.

### 3.2 Filtering with standard tetrads

We begin by considering the tetrad filtering condition applied directly to the general gauge multiplet itself (51) that could make the nature of this condition clearer. Essentially, we impose the covariant constraint of the form

$$I_\mu = e_\sigma I_\mu e^\sigma / 4 \tag{52}$$

using the "neutral" tetrads (48) which are only permitted in the theory[4]. This yields

$$I_\mu = \frac{1}{4} e_\sigma^{aK} e_b^{\sigma K'} (\gamma_a \lambda^K I_\mu \gamma^b \lambda^{K'}) \tag{53}$$

which, upon employing the orthonormality conditions of the tetrad (47), results in the equality

$$\frac{1}{2}\left(V_\mu^k + iA_\mu^k \gamma_5\right)\lambda^k + \frac{1}{4} T_{\mu[ab]}^K \gamma^{ab} \lambda^K = \frac{1}{2}\left(V_\mu^k - iA_\mu^k \gamma_5\right)\lambda^k$$

This implies that the reduced gauge multiplet (53) comprises solely the vector fields

$$I_\mu = V_\mu^k \lambda^k / 2 \tag{54}$$

while the axial-vector and tensor field submultiplets vanish identically. Remarkably, by imposing the covariant constraint (52) the starting $SL(2N, C)$ symmetry group is effectively reduced to the pure unitary $SU(N)$ symmetry case. In a sense, the constraint acts as a symmetry-breaking mechanism, but unlike typical scenarios, nothing remains of the original $SL(2N, C)$ gauge sector except its $SU(N)$ part.

Now, as claimed, we propose that the gauge field multiplet $I_\mu$ (51) is "originated" from the prototype nondynamical multiplet $\mathcal{I}_\mu$ as per the underlying filtering condition (1). Their connection in the $SL(2N, C)$ symmetry framework acquires the form

$$I_\mu = \frac{1}{4} e_\sigma \mathcal{I}_\mu e^\sigma = \frac{1}{4} e_\sigma^{aK} e_b^{\sigma K'} (\gamma_a \lambda^K \mathcal{I}_\mu \gamma^b \lambda^{K'}) \tag{55}$$

utilizing the "neutral" tetrads (48) permitted in the theory.

The point is, however, that again, as in the pure gravity case (19), the tensor field submultiplet $T_{\mu[ab]}^K$ completely disappear when filtered by tetrads satisfying the standard orthonormality conditions (47). Indeed, using them one immediately comes to the filtering relation

$$I_\mu = \frac{1}{4} e_\sigma \mathcal{I}_\mu e^\sigma = \left(\mathcal{V}_\mu^k - i\mathcal{A}_\mu^k \gamma_5\right)\lambda^k / 2 \tag{56}$$

showing that, while tensor field submultiplet is cancelled, the vector and axial-vector ones practically remain in the gauge multiplet (51) except that the axial-vector fields change

[4]This constraint when properly reiterated converts to an infinite series of constraints

$$I_\mu = e_1 ... e_{\sigma_s} I_\mu e^{\sigma_s} ... e^{\sigma_1} / 4^s \ (s = 1, 2, ...)$$

which all lead to the same outcome though.

the sign. In principle, one could use it to eliminate them from the theory as well. This simply follows when one adds to the condition (55) the double filtering term as well

$$I_\mu = \frac{1}{8}\left(e_\sigma \mathcal{I}_\mu e^\sigma + e_\rho e_\sigma \mathcal{I}_\mu e^\sigma e^\rho/4\right) = \mathcal{V}^l_\mu \lambda^l/2 \tag{57}$$

Comparing this with (54) we can conclude that such filtering is equivalent to the case when the constraint is applied directly to the gauge multiplet itself (52). However, in contrast, the prototype multiplet $\mathcal{I}_\mu$ appears to be free from any constraint. Remarkably, on one hand, for the filtering case (57), one can covariantly transition from the original $SL(2N,C)$ group to the $SU(N)$ symmetry gauged by the vector fields. However, on the other hand, the tensor fields that could induce gravity also disappear from the theory.

### 3.3 Filtering with modified tetrads

For incorporation of tensor fields in the gauge sector, it is necessary, as in the pure gravity scenario discussed in Section 2.2, to go to tetrads which are not strictly orthonormal. In such a scenario, the tensor field multiplet will emerge within the filtered gauge multiplet $I_\mu$ once the orthonormality conditions turns out to be slightly shifted

$$e^{aK}_\mu e^{\mu K'}_b = (\delta^a_b + \varepsilon q^a_b)\delta^{K0}\delta^{K'0} \ , \ \ e^{aK}_\mu e^{\nu K'}_a = (\delta^\nu_\mu + \varepsilon p^\nu_\mu)\delta^{K0}\delta^{K'0} \tag{58}$$

being determined again by the tiny tensors $\varepsilon q^a_b$ and $\varepsilon p^\nu_\mu$ ($\varepsilon \ll 1$), respectively. Analogously, the metric tensor deviation in the general $SL(2N,C)$ follows from a similar equation

$$e^{aK}_\mu e^{K'}_{a\nu} = (g_{\mu\nu} + r_{\mu\nu})\delta^{K0}\delta^{K'0} \tag{59}$$

that actually works for neutral components

$$e^{a0}_\mu e^{0}_{a\nu} = g_{\mu\nu} + r_{\mu\nu} \tag{60}$$

and finally gives for the metric tensor

$$g_{\mu\nu} \simeq e^{a0}_\mu e^{0}_{b\nu}[\delta^b_a - \varepsilon q^b_a + \varepsilon^2 (qq)^b_a] \tag{61}$$

being identical to that in the pure gravity case (27).

Actually, tetrads can again be regarded as those which are condensed, thus having the form

$$e^{aK}_\mu = (\delta^a_\mu h + \widehat{e}^{a0}_\mu)\delta^{K0}, \ \ \delta^\mu_a \widehat{e}^{a0}_\mu = 0 \tag{62}$$

where the effective Higgs mode $h$ and zero-modes $\widehat{e}^{a0}_\mu$ are similar those given above in conditions (32a-34). These conditions suggest now a spontaneous-like breakdown of the local $SL(2N,C)$ symmetry for tetrads even though they are not considered as dynamical fields in the theory.

The form (62) remains, in fact, the original pure gravity case form for neutral tetrad component, while the $SU(N)$ flavored tetrad components are absent in the theory.

Note that a right choice of tetrad components is of primary importance since, as in the above pure gravity case, just the tetrad-filtered gauge multiplet is proposed to operate in the extended $SL(2N,C)$ theory. As a consequence of the modified orthonormality conditions for the tetrads taken above (58), one has, after using the identity

$$q_b^a \gamma_a \gamma^b = q^{ab}\gamma_a\gamma_b = q^{ab}\eta_{ab} = 0 \tag{63}$$

for the symmetrical and traceless tensor $q^{ab}$ (30), a unique form for the filtered gauge multiplet $I_\mu$

$$I_\mu = \left(\mathcal{V}_\mu^k - i\mathcal{A}_\mu^k\gamma_5\right)\lambda^k/2 + \frac{\varepsilon}{4}\mathbf{T}^K_{\mu[ab]}\gamma^{ab}\lambda^K \tag{64a}$$

It is noteworthy that, in contrast to the standard tetrad case (56), it also contains the tensor field submultiplet which is given by an expression

$$\mathbf{T}^K_{\mu[ab]} = (\mathcal{T}^K_{\mu[bc]}q_a^c - \mathcal{T}^K_{\mu[ac]}q_b^c)/2 \tag{65a}$$

This actually extends the pure gravity case (21a) in a sense that the hyperflavored tensor field components $\mathbf{T}^k_{\mu[ab]}$ also come into play. However, as we see later they appear insignificant only contributing into the tiny four-fermion (spin current-current) interaction in the fermion matter sector.

As one can readily confirm, the vector and axial-vector submultiplets in the gauge multiplet (51) remain unaffected during the filtering process (unless the special constraint (52) excluding the axial-vector field in the theory is applied). Meanwhile, the tensor field components appear again, as in the pure gravity case, to be completely controlled by the tetrad orthonormality deviations $\varepsilon q_b^a$ and, therefore, are significantly weakened. The product of the deviation tensor $q_b^a$ with the prototype tensor fields $\mathcal{T}^K_{\mu[ab]}$ defines the new tensor field multiplet $\mathbf{T}^K_{\mu[ab]}$ (65a), through which and tiny parameter $\varepsilon$ the theory is ultimately expressed. The hyperunification of the basic elementary forces in this theory does not preclude the tensor field submultiplet from having the vanishingly small quadratic strength terms being scaled as $\varepsilon^2$ and, as a consequence, can be neglected. Therefore, as we confirm below, the final theory tends to the conventional Einstein-Cartan type theory for gravity coupled with the gauge $SU(N)$ theories for other interactions.

### 3.4 Gravity inducing tensor fields

Let us now construct the field strength for the gauge multiplet $I_\mu$ in the $SL(2N,C)$ hyperunified theory

$$I_{\mu\nu} = \partial_{[\mu}I_{\nu]} + ig[I_\mu, I_\nu] = (V+A)_{\mu\nu} + T_{\mu\nu} \tag{66}$$

which includes the terms corresponding the vector, axial-vector and tensor field submultiplets, respectively. Expressed through the prototype $\mathcal{I}_\mu$ multiplet components according to the taken filtered form (64a), this strength tensor comes to

$$\begin{aligned} I_{\mu\nu} &= \frac{1}{2}\partial_{[\mu}\left(\mathcal{V}^k - i\mathcal{A}^k\gamma_5\right)_{\nu]}\lambda^k - \frac{1}{2}f^{ijk}g\left(\mathcal{V}^i - i\mathcal{A}^i\gamma_5\right)_\mu(\mathcal{V}^j - i\mathcal{A}^j\gamma_5)_\nu\lambda^k \\ &+\frac{\varepsilon}{4}\left(\partial_{[\mu}\mathbf{T}^{[ab]K}_{\nu]}\gamma_{ab}\lambda^K + i\frac{\varepsilon g}{4}\mathbf{T}^{[ab]K}_\mu\mathbf{T}^{[a'b']K'}_\nu[\lambda^K\gamma_{ab}, \lambda^{K'}\gamma_{a'b'}]\right) \end{aligned} \tag{67}$$

Similarly, the gauge invariant fermion matter couplings, when given in terms of the $\mathcal{I}_\mu$ submultiplets, take the form

$$e\mathcal{L}_M = -\frac{g}{2}\overline{\Psi}\left\{e^\mu, \left[\frac{1}{2}\left(\mathcal{V}_\mu^k - i\mathcal{A}_\mu^k\gamma_5\right)\lambda^k + \frac{\varepsilon}{16}\mathbf{T}^K_{\mu[ab]}\gamma^{ab}\lambda^K\right]\right\}\Psi \tag{68}$$

As one can readily observe, the vector and axial-vector fields interact everywhere in (67) and (68) with the universal gauge coupling constant $g$ of $SL(2N,C)$. In contrast, all tensor field terms incorporate the aforementioned tiny parameter $\varepsilon$. Consequently, the total Lagrangian will contain the conventional quadratic strength terms for the vector and axial-vector fields, while in the first order in the parameter $\varepsilon$ only the linear strength terms of the tensor fields emerge, alongside the fermion matter couplings. The quadratic strength terms of the tensor field multiplet, which would provide its propagation and an ordinary gauge interaction, scale as $\varepsilon^2$ and are therefore negligible in this regime.

Leaving aside for the moment vector and axial-vector fields, we now focus on the hyperunified gravity Lagrangian taken in the Palatini type form

$$e\mathcal{L}_G \sim Tr\{[e^\mu, e^\nu]I_{\mu\nu}\} \tag{69}$$

In the $SL(2N,C)$ case the strength tensor $I_{\mu\nu}$, apart from tensor submultiplet, comprises the vector and axial-vector submultiplets as well. However, due to the neutral tetrad chosen (48) and their commutator given by

$$[e^\mu, e^\nu] = -2ie_a^{\mu 0}e_b^{\nu 0}\gamma^{ab} \tag{70}$$

one can easily confirm that the they do not contribute to the gravity Lagrangian (69).

Eventually, for the tensor field strength in (67) one has, after taking the necessary traces of products involving $\gamma$ and $\lambda$ matrices, the following gravity Lagrangian

$$e\mathcal{L}_G = \frac{1}{2\kappa}\left(\partial_{[\mu}\mathbf{T}_{\nu]}^{[ab]0} + \varepsilon g\eta_{cd}\mathbf{T}_{[\mu}^{[ac]K}\mathbf{T}_{\nu]}^{[bd]K}\right)e_a^{\mu 0}e_b^{\nu 0} \tag{71}$$

The gravity constant $\kappa$ is assumed to absorb here one power of the tiny parameter $\varepsilon$ (along with the factor $N$ associated with the internal $U(N)$ symmetry). Consequently, the new tensor fields $\mathbf{T}_\mu^{[ab]K}$ appears in the Lagrangian $e\mathcal{L}_G$ with the effective coupling constant $\varepsilon g$ rather than $g$, as the vector and axial-vector fields do in their own quadratic strength Lagrangians. In a similar way, one has for the fermion matter Lagrangian of the tensor submultiplet in (68)

$$e\mathcal{L}_M^{(T)} = -\frac{\varepsilon g}{2}\epsilon^{abcd}\mathbf{T}^K_{\mu[ab]}\overline{\Psi}e_c^{\mu 0}\gamma_d\lambda^K\gamma^5\Psi \tag{72}$$

where couplings of the tensor fields with the neutral and flavored spin density currents also appear with the same effective coupling constant $\varepsilon g$.

Notably, in the Lagrangian (71), there are only kinetic terms for the neutral tensor field component $\mathbf{T}_\mu^{[ab]0}$, while the interaction terms contain the entire $U(N)$ multiplet $\mathbf{T}_{[\mu}^{[ab]K}$. This implies that only the neutral tensor field truly gauges gravity, while the $SU(N)$ flavored ones $\mathbf{T}_\mu^{[ab]k}$ are simply given by the corresponding spin currents $\epsilon^{abcd}\overline{\Psi}e^0_{\mu c}\gamma_d\lambda^k\gamma^5\Psi$.

When they both, $\mathbf{T}_{[\mu}^{[ab]0}$ and $\mathbf{T}_{\mu}^{[ab]k}$, are independently eliminated from the entire tensor field Lagrangian $e\mathcal{L}_G+e\mathcal{L}_M^{(T)}$, one arrives at the Einstein-Cartan type gravity containing, besides the usual GR, the tiny 4-fermion spin density interaction

$$\varepsilon\kappa\left(\overline{\Psi}\gamma_c\gamma^5\lambda^K\Psi\right)\left(\overline{\Psi}\gamma^c\gamma^5\lambda^K\Psi\right) \tag{73}$$

which in contrast to the standard case [2] includes the flavored four-fermion interaction terms as well, albeit further weakened by the small parameter $\varepsilon$.

We have already mentioned in Section 2.2 that the weakness of the tensor field induced gravity may look fictitious since it might be circumvented by a simple rescaling of the field itself. However, as follows, this rescaling trick only works for pure gauge gravity with local $SL(2,C)$ symmetry. In the framework of $SL(2N,C)$ gauge theory, where the tensor field is proposed to be unified with ordinary vector and axial-vector fields which interact with the $O(1)$ coupling constants, rescaling is not an option. The point is that the weakness of the filtered tensor field is exclusive to the tensor field submultiplet and does not extend to the entire gauge multiplet $I_\mu$ (64a) of $SL(2N,C)$. As a result, it cannot be overcome by mere rescaling. Indeed, rescaling of only the tensor field submultiplet $\mathbf{T}_\mu = \mathbf{T}'_\mu/\varepsilon$ inside $I_\mu$ is not permitted by the $SL(2N,C)$ gauge invariance in the theory, nor by the filtering condition (1) itself. Conversely, rescaling of the entire multiplet $I_\mu = I'_\mu/\varepsilon$, while "normalizing" the tensor field part in total Lagrangian, will "denormalize" the vector and axial-vector field terms,

$$\mathcal{L}(\mathcal{V},\mathcal{A},\varepsilon\mathbf{T}) = \mathcal{L}'(\mathcal{V}'/\varepsilon,\mathcal{A}'/\varepsilon,\mathbf{T}') \tag{74}$$

This necessitates a new rescaling of the entire multiplet $I'_\mu = \varepsilon I''_\mu$ which bring us back to the initial point with the tiny filtered tensor field. Thus, both cases render the rescaling scenarios untenable.

### 3.5 Hyperflavor mediating vector fields

Turn now to the vector and axial-vector fields which are the basic spin-1 carriers of the hyperflavor $SU(N)$ symmetry in the $SL(2N,C)$ theory. Their own sector stemming from the common strength tensor (67) looks as

$$e\mathcal{L}^{(VA)} = -\frac{1}{4}[\partial_{[\mu}\mathcal{V}^k_{\nu]} - gf^{ijk}(\mathcal{V}^i_\mu\mathcal{V}^j_\nu + \mathcal{A}^i_\mu\mathcal{A}^j_\nu)]^2 - \frac{1}{4}[\partial_{[\mu}\mathcal{A}^k_{\nu]}]^2 \tag{75}$$

where, as one can see, the vector fields acquire a conventional gauge theory form, while the axial-vector field couplings break this gauge invariance. At the same time, as follows from the matter sector of the theory (68), the vector fields interact with ordinary matter fermions

$$e\mathcal{L}_M^{(V)} = -\frac{g}{2}\mathcal{V}^i_\mu\overline{\Psi}e^{\mu 0}_a\gamma^a\lambda^i\Psi \tag{76}$$

while axial-vector fields do not, thus being sterile to them.

Generally, one could try to adapt the axial-vector fields to reality though there is no sign of they actually existing. The traditional way would be to make these axial-vector fields superheavy through some enormously extended Higgs sector remaining, at the same time, vector fields gauging the Standard Model massless or enough light. This seems to be quite difficult since the axial-vector fields want to follow the same pattern of the mass formation as the vector fields do. Anyway, despite the gauge $SL(2N, C)$ invariance in the theory, the very presence of the axial-vector fields breaks the gauge $SU(N)$ invariance related to the vector fields, thus only leaving the global $SU(N)$ symmetry in the theory.

In this connection, a rather interesting way could be if these axial-vector fields were condensed at some Planck order scale $\mathcal{M}$, thus providing a true vacuum in the theory, $\langle \mathcal{A}_\mu^i \rangle = \mathfrak{n}_\mu^i \mathcal{M}$, whose direction is given by the unit Lorentz vector $\mathfrak{n}_\mu^i$. Remarkably, in this vacuum, as is directly seen, gauge invariance for the vector fields is completely restored, though a tiny spontaneous breaking of the Lorentz invariance at the scale $\mathcal{M}$ may appear. More details about such a possibility can be found in the recent paper [8].

A more radical approach, as mentioned earlier, would be to impose the specific filtering condition (57) which essentially corresponds to an existence the constraint in the theory being applied directly to the gauge multiplet itself (52). This filtering condition automatically excludes the axial-vector and tensor fields from the gauge multiplet $I_\mu$ when one uses the standard tetrads. However, for the modified tetrads, that satisfy the slightly shifted orthonormality conditions (58), one ultimately obtains, in contrast to (57), the total gauge multiplet $I_\mu$ having the form

$$I_\mu = \mathcal{V}_\mu^l \lambda^l/2 + \frac{\varepsilon}{8}\mathbf{T}_{\mu[ab]}^K \gamma^{ab} \lambda^K + O(\varepsilon^2) \tag{77}$$

This multiplet, in addition to the flavor-mediating vector gauge fields of the local $SU(N)$ symmetry, incorporates the appropriately damped tensor fields that underlie the gravity sector in the hyperunified theory (terms of order $\varepsilon^2$ are disregarded for simplicity).

### 3.6 Symmetry breaking scenario

It is clear that hyperunification of all elementary forces supposes that, while gravity may have an unique linear tensor field strength Lagrangian $L_G$ of the type given in (71), the quadratic strength terms of other components of gauge multiplet $I_\mu$ are naturally unified in their common $SL(2N, C)$ invariant Lagrangian. The tiny quadratic terms for the tensor fields containing the $O(\varepsilon^2)$ and higher order terms appear unessential compared to the strength bilinears for vector fields. So, hyperunification definitely rules out the quadratic curvature terms for tensor fields in the filtered $SL(2N, C)$ gauge theory. The only place where tensor field submultiplet manifests itself is the $SL(2, C)$ gravity gauged by its tiny neutral component $\varepsilon \mathbf{T}_{\mu[ab]}^0$. Accordingly, one eventually has for a gauge sector of the unified Lagrangian

$$eL_U = eL_G - \frac{1}{4} V_{\mu\nu}^k V^{\mu\nu k} \tag{78}$$

containing, apart from the Einstein-Cartan type theory, the standard $SU(N)$ invariant vector field part (provided that the axial-vector fields are properly filtered out of the theory in the way described above).

A conventional breaking scenario of the $SL(2N, C)$ invariance in the theory would depend in general on the proper set of scalar fields which could break this invariance first to the intermediate $SU(N) \times SL(2, C)$ symmetry and then to the Standard Model. In our case, however, one does not need to cause the first stage of symmetry breaking since, as is readily seen in (77), all the gauge submultiplets related to the "nondiagonal" generators of $SL(2N, C)$ are properly weakened (tensor fields) or completely filtered out of the theory (axial-vector fields).

As to the internal $SU(N)$ symmetry violation down to the Standard Model one actually need to have the adjoint scalar multiplets of the type

$$\Phi = (\phi^K + i\phi_5^K\gamma_5 + \phi_{ab}^K\gamma^{ab}/2)\lambda^K \tag{79}$$

which transform under $SL(2N, C)$ as

$$\Phi \to \Omega\Phi\Omega^{-1} \tag{80}$$

It generally contains, apart the scalar components, the pseudoscalar and tensor components as well. However, as in the above gauge multiplet case, one can use again the tetrad projection mechanism to filter away these "superfluous" components, just like as it was done in (57)

$$\Phi = \frac{1}{8}(e_\sigma \mathbf{\Phi} e^\sigma + e_\rho e_\sigma \mathbf{\Phi} e^\sigma e^\rho/4)$$

where $\mathbf{\Phi}$ is some prototype scalar field multiplet. As a result, with tetrads satisfying the orthonormality conditions (58) there is only left the pure scalar components in the $SU(N)$ symmetry breaking multiplet $\Phi$

$$\Phi = \phi^k\lambda^k \qquad (k = 1, ..., N^2 - 1)$$

providing (with other similar scalar multiplets) the breaking of the $SU(N)$ GUT down to the Standard Model. The final symmetry breaking to $SU(3)_c \times U(1)_{em}$ is provided by extra scalar multiplets whose assignment depends on which multiplets are chosen for quarks and leptons.

### 3.7 Final remarks

We have observed that the filtered tensor field only manifests when the orthonormality conditions for tetrads are appropriately shifted, as argued earlier in (58). This, in turn, leads to a slight modification of the metric tensor (61), indicating a tiny departure from general covariance, albeit in a controllable manner determined by the minute parameter $\varepsilon$.

Nevertheless, it is conceivable to manage the filtering mechanism in a way that preserves general covariance. This could only be achieved if there exist two types of tetrads satisfying the orthonormality conditions

$$e_\mu^{aK} e_b^{\mu K'} = \delta_b^a \delta^{K0} \delta^{K'0}, \quad E_\mu^{aK} E_b^{\mu K'} = (\delta_b^a + \varepsilon q_b^a)\delta^{K0}\delta^{K'0} \tag{81}$$

where the primary tetrad $e$ adheres to the standard orthonormality condition, while the auxiliary tetrad $E$ exhibits a slight non-orthonormality deviation $\varepsilon q_b^a$. One can additionally require the entire Lagrangian to remain invariant under the reflection transformation

$$e \to e\ , \quad E \to -E \tag{82}$$

This ensures that the tetrad $E$, in contrast to the primary tetrad $e$ solely parametrizes the spacetime background and does not participate in the matter fermion terms. Consequently, while the tetrad $E$ essentially determines the filtering condition $I_\mu = E_\sigma \mathcal{I}_\mu E^\sigma/4$, a vanishingly small violation of general covariance, caused by its approximate orthonormality condition in (81), appears beyond the scope of the Einstein-Cartan gravity emerged.

In this context, there are indeed numerous instances in physics where the consideration of a manifold equipped with two distinct vielbein fields becomes necessary. This notably occurs in bimetric theories, where two disparate metrics are defined on the same spacetime manifold, including the case when one of the metrics is nondynamical [13]. Importantly, this concept extends to certain formulations of bigravity theory, which could potentially serve as a base for the massive gravity [14, 15].

Nevertheless, despite the possibility of employing two metrics within the framework of the $SL(2N, C)$ gauge theory, the case with the single metric, even if general covariance is slightly broken, appears to be more economically viable. Consequently, we have proceeded with this version in our hyperunified model.

# 4 From hyperunification to GUTs

## 4.1 $SU(5)$ and its direct extensions

Let us now consider more closely how the $SL(2N, C)$ type model can be applied to some known GUTs starting with a conventional $SU(5)$ [16] which would stem from the $SL(10, C)$ HUT. In this case some of its low-dimensional multiplets of the chiral (lefthanded for certainty) fermions can be given in terms of the $SU(5) \times SL(2, C)$ components as

$$\Psi_L^{i\boldsymbol{a}} \ , \qquad 10 = (\overline{5}, 2) \tag{83}$$

$$\Psi_{L[\boldsymbol{a}i,\ j\boldsymbol{b}]} = \Psi_{L[ij]\{\boldsymbol{ab}\}} + \Psi_{L\{ij\}[\boldsymbol{ab}]}, \ \ 45 = (10,\ 3) + (15,\ 1) \tag{84}$$

where we have used that a common antisymmetry on two or more joint $SL(10, C)$ indices ($i\boldsymbol{a}$, $j\boldsymbol{b}$, $k\boldsymbol{c}$) means antisymmetry in the internal indices ($i, j, k = 1, ..., 5$) and symmetry in the chiral spinor ones ($\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} = 1, 2$), and vice versa (dimension of representations are also indicated). One can see that, while the $SU(5)$ antiquintet can easily be constructed (83), its decuplet is not contained in the pure antisymmetric $SL(10, C)$ representations (84). Moreover, the tensor (84) corresponds in fact to the collection of vector and scalar multiplets rather than the fermion ones.

Note in this connection, that all GUTs where fermions are assigned to the pure antisymmetric representations seem to be also irrelevant since the spin magnitude of appearing states are not in conformity with what we have in reality. The most known example of this kind is the $SU(11)$ GUT [17] with all three quark-lepton families collected in its one-, two-, three-, and four-index antisymmetric representations. No doubt this GUT should also be excluded in the framework of the considered $SL(2N, C)$ theories. Actually, for the right $1/2$ spin value of ordinary quarks and leptons these theories should include more complicated fermion multiplets having in general the upper and lower indices rather than the pure asymmetric ones. The point is, however, that such multiplets appear enormously large and contain in general lots of exotic states which never been detected. This could motivate to seek a possible solution in the composite nature of quarks and leptons for whose constituents – preons – the $SL(2N,C)$ unification might look much simpler.

### 4.2 $SU(8)$ with composite quarks and leptons

Following the recent discussion [18], we introduce $N$ lefthanded and $N$ righthanded preons being the fundamental multiplets $P^{\alpha}_{Li\boldsymbol{a}}$ and $P^{\alpha'}_{Ri\boldsymbol{a}}$ of the vectorlike "metaflavor" $SL(2N,C)$ HUT symmetry ($i=1,...,N$; $\boldsymbol{a}=1,2$) times some local left-right "metacolor" $SO(n)_L \times SO(n)_R$ symmetry ($\alpha = 1,...,n$; $\alpha' = 1,...,n$) binding preons inside quarks and leptons[5]. Both of these symmetries are obviously anomaly-free and the numbers of metaflavors ($N$) and metacolors ($n$) are not yet determined. The metaflavor symmetry describes preons at small distances as well as their composites at large ones. They are produced individually from the lefthanded and righthanded preons due to confining forces of the above metacolor symmetry. Some of these composites, including the observed quarks and leptons, are expected to be much lighter than their composition scale. For that, the accompanying chiral symmetry $SU(N)_L \times SU(N)_R$ of the preons should be preserved at large distances in a way that – when it is considered as the would-be local symmetry group with some spectator gauge fields and fermions – the corresponding triangle anomaly matching conditions [19] are satisfied. Namely, the $SU(N)^3_L$ and $SU(N)^3_R$ anomalies related to $N$ lefthanded and $N$ righthanded preons have to individually match those for lefthanded and righthanded composite fermions being produced by the $SO(n)_L$ and $SO(n)_R$ metacolor forces, respectively.

Moreover, as is turned out, just this condition, when being properly strengthened, can determine the particular metaflavor symmetry $SL(2N,C)$ in the theory. Indeed, we first assume that all composites, both lefthanded and righthanded, have just the three-preon configuration ($n=3$), thus fixing the metacolor symmetry to $SO(3)_L \times SO(3)_R$. And second and most importantly, they belong to a single representation of their chiral symmetries $SU(N)_L$ and $SU(N)_R$, respectively, rather than to some set of representations. Then it turns out that among all their third-rank representations the anomaly matching condition holds individually only for multiplets of the type $\psi^k_{[i\,j]L}$ and $\psi^k_{[i\,j]R}$ ($i,j,k = 1,2,...,N$), that gives the unique solution to the number of preons $N$, both lefthanded and righthanded,

$$N^2/2 - 7N/2 - 1 = 3, \;\; N = 8 \tag{85}$$

This means that among all possible chiral symmetries only the $SU(8)_L \times SU(8)_R$ symmetry can in principle provide masslessness of lefthanded and righthanded fermion composites at large distances. This in turn identifies – among all metaflavor $SL(2N,C)$ symmetries – just $SL(16,C)$ as the most likely candidate for hyperunification. Note that, in contrast to the above global chiral symmetry, in the local $SL(16,C)$ metaflavor theory, being as yet vectorlike, all metaflavor triangle anomalies are automatically cancelled out.

[5]By tradition, we call them the "metaflavor" and "metacolor" symmetry, while still referring to the $SU(N)$ subgroup of $SL(2N,C)$ as the hyperflavor symmetry.

Turning now from the chiral symmetry multiplets $\psi^{k}_{[i\ j]L,R}$ to the corresponding $SL(16,C)$ composite multiplets $\Psi^{k\boldsymbol{c}}_{[i\boldsymbol{a},\ j\boldsymbol{b}]L,R}$ one can write them in terms of the $SU(8)\times SL(2,C)$ components as the collection

$$\Psi^{k\boldsymbol{c}}_{[i\boldsymbol{a},\ j\boldsymbol{b}]} = \Psi^{k\boldsymbol{c}}_{[ij]\{\boldsymbol{ab}\}} + \Psi^{k\boldsymbol{c}}_{\{ij\}[\boldsymbol{ab}]},\ \ 1904 = (216,2) + (216+8,\ 4) + (280+8,\ 2) \qquad (86)$$

which contains some spin 1/2 and 3/2 lefthanded and righthanded composite fermion submultiplets. Meanwhile, as one can easily confirm, among all submultiplets in (86) only the $(216,2)_{L,R}$ ones satisfy individually the anomaly matching condition for the chiral $SU(8)_L$ and $SU(8)_R$ symmetries, respectively. As a result, all the other submultiplets there have then to acquire superheavy masses. This actually means that only the $SU(8)\times SL(2,C)$ subgroup of the $SL(16,C)$ HUT symmetry survives at large distances where the composite fermions emerge. Surprisingly enough, this is consistent with what we had above, albeit from a different perspective. Namely, the filtered $SL(2N,C)$ gauge theory, in which only neutral tensor field and vector field multiplet remain, turns out to be effectively reduced to the $SU(N)\times SL(2,C)$ invariant theory. Now, in the composite model, for the particular case of metaflavored symmetry $SL(16,C)$, this independently follows from the preservation of the accompanying chiral symmetry $SU(8)_L\times SU(8)_R$ at large distances, thus leading to the theory with the residual metaflavor symmetry $SU(8)\times SL(2,C)$.

Remarkably, the above $(216,2)_{L,R}$ submultiplets being decomposed into the standard $SU(5)$ GUT and family symmetry $SU(3)_F$ looks as

$$(216,2)_{L,R} = [(\overline{5}+10,\ \overline{3}) + (45,1) + (5,8+1) + (24,3) + (1,3) + (1,\overline{6})]_{L,R} \qquad (87)$$

where the first term in the squared brackets, when taken for lefthanded states in $216_L$, describes all three quark-lepton families being the family symmetry triplets. However, there are also the similar righthanded states in $216_R$ in our still vectorlike $SL(16,C)$ theory. This means that, while preons are left massless being protected by their own metacolors, the composites (87) being metacolor singlets could in principle pair up and acquire the heavy Dirac masses.

To avoid this for the submultiplet of physical quarks and leptons in (87), $(\overline{5}+10,\ \overline{3})_L$, one may propose, following the scenario developed in [18], some spontaneous breaking of the basic $L$-$R$ symmetry in the theory. This is assumed to follow from the sector of righthanded preons that reduces the chiral symmetry of their composites down to $[SU(5)\times SU(3)]_R$. Actually, such a breaking may readily appear due to a possible condensation of massive composite scalars which unavoidably appear in the theory together with composite fermions. This means that, though the massless righthanded preons still possess the $SU(8)_R$ symmetry, the masslessness of their composites at large distances is now solely controlled by its remained $[SU(5)\times SU(3)]_R$ part. Thus, while nothing really happens with the lefthanded preon composites still completing the total multiplet $(216,2)_L$ in (87), the righthanded preon composites with their residual chiral symmetry no longer include all submultiplets given in $(216,2)_R$. Very remarkably, the corresponding anomaly matching condition "organizes" their composite spectrum in such a way that the submultiplet $(\overline{5}+10,\ \overline{3})_R$ is absent among the righthanded preon composites. As a result, all the lefthanded submultiplets in $(216, 2)_L$, except the $(\overline{5} + 10, \overline{3})_L$, will then pair up, thus becoming heavy and decoupling from laboratory physics [18].

Accordingly, once the $L$-$R$ symmetry is violated in the theory, the vectorlike metaflavor symmetry $SU(8) \times SL(2, C)$, while still working for preons, will also break down to its subgroup $[SU(5) \times SU(3)_F] \times SL(2, C)$ for their large-distance composites. So, one eventually comes to the conventional $SU(5)$ GUT [16] together with the extra local $SU(3)_F$ family symmetry [20] describing just three standard families of composite quarks and leptons. Both types of the triangle anomalies, $SU(5)^3$ and $SU(3)_F^3$, emerging at this stage are properly cancelled out in the theory.

The further symmetry violation is related, as was mentioned above, to the adjoint scalar field multiplet $\Phi$ (79) which in the present context breaks the $SU(5)$ to the Standard Model. As to the final breaking of the SM and accompanied family symmetry $SU(3)_F$, it appears through the extra multiplets $H^{[i\boldsymbol{a},j\boldsymbol{b},k\boldsymbol{c},l\boldsymbol{d}]}$, and $\chi_{[i\boldsymbol{a},j\boldsymbol{b}]}$ and $\chi_{\{i\boldsymbol{a},j\boldsymbol{b}\}}$ of $SL(16, C)$, respectively. These multiplets contain, among others, the true scalar components which develop the corresponding VEVs and give masses to the weak bosons, as well as the flavor bosons of the $SU(3)_F$. They also generate masses to quarks and leptons located in the lefthanded fermion multiplet (86, 87) through the $SL(16, C)$ invariant Yukawa couplings

$$\frac{1}{\mathcal{M}}\left[\Psi^{i\boldsymbol{c}}_{[j\boldsymbol{a},\ k\boldsymbol{b}]L} C \Psi^{l\boldsymbol{d}}_{[m\boldsymbol{e},\ n\boldsymbol{f}]L}\right] H^{\{[j\boldsymbol{a},k\boldsymbol{b}],[m\boldsymbol{e},n\boldsymbol{f}]\}} (a_u \chi_{[i\boldsymbol{c},l\boldsymbol{d}]} + b_u \chi_{\{i\boldsymbol{c},l\boldsymbol{d}\}})$$
$$\frac{1}{\mathcal{M}}\left[\Psi^{i\boldsymbol{c}}_{[j\boldsymbol{a},\ k\boldsymbol{b}]L} C \Psi^{l\boldsymbol{d}}_{[i\boldsymbol{c},\ m\boldsymbol{e}]L}\right] H^{\{[j\boldsymbol{a},k\boldsymbol{b}],[m\boldsymbol{e},n\boldsymbol{f}]\}} (a_d \chi_{[l\boldsymbol{d},n\boldsymbol{f}]} + b_d \chi_{\{l\boldsymbol{d},n\boldsymbol{f}\}}) \qquad (88)$$

with different index contraction for the up quarks, and down quarks and leptons, respectively ($i, j, k, l, m, n = 1, ..., 8$; $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f} = 1, 2$). The mass $\mathcal{M}$ stands for some effective scale in the theory that in the composite model of quarks and leptons can be related to their compositeness scale, while $a_{u,d}$ and $b_{u,d}$ are some dimensionless constants of the order of 1. Actually, these couplings contain two types of scalar multiplets with the following $SU(8) \times SL(2, C)$ components – the $H$ multiplet $H^{\{[j\boldsymbol{a},k\boldsymbol{b}],[m\boldsymbol{e},n\boldsymbol{f}]\}}$ containing the true scalar components

$$H^{[jkmn]\{[\boldsymbol{ab}],[\boldsymbol{ef}]\}}(70, 1) \qquad (89)$$

and symmetric and antisymmetric $\chi$ multiplets, $\chi_{\{i\boldsymbol{c},l\boldsymbol{d}\}}$ and $\chi_{[i\boldsymbol{c},l\boldsymbol{d}]}$, whose scalar components look as

$$\chi_{[il][\boldsymbol{cd}]}(28, 1), \quad \chi_{[\boldsymbol{cd}]\{il\}}(36, 1) \qquad (90)$$

Decomposing them into the components of the final $SU(5) \times SU(3)_F$ symmetry one finds the full set of scalars

$$\begin{aligned} 70 &= (5, 1) + (\overline{5}, 1) + (10, \overline{3}) + (\overline{10}, 3) \\ 28 &= (5, 3) + (10, 1) + (1, \overline{3}) \\ 36 &= (5, 3) + (15, 1) + (1, 6) \end{aligned} \qquad (91)$$

containing the $SU(5)$ quintets $(5, 1)$ and $(\overline{5}, 1)$ to break the Standard Model at the electroweak scale $M_{SM}$ and the the $SU(3)_F$ triplet and sextet, $(1, \overline{3})$ and $(1, 6)$, to properly break the family symmetry at some large scale $M_F$. One may refer to the scalars (89) and

(90) as the "vertical" and "horizontal" ones, respectively, which are actually the simplest choice to form the above Yukawa couplings. Working in pairs in them, they presumably determine masses and mixings of all quarks and leptons. And the last but not the least, they may be indeed composed, in the model considered, from the same preons as quarks and leptons [18].

# 5 Conclusion

We have investigated the potential of the local $SL(2N, C)$ symmetry to unify all fundamental forces, including gravity. The key idea is that this symmetry can be "trimmed down" in a controllable covariant way by the accompanied tetrad fields. These tetrads are assumed not only determine the spacetime geometry but also function as a kind of discerning filter, dictating which local symmetries can operate within spacetime and how the related elementary forces interact via gauge fields. As a result of this filtering, the theory ends up with a simpler and more effective symmetry, $SL(2, C) \times SU(N)$, that translates to two separate but interacting parts: the local $SL(2, C)$ symmetry describes gravity as a gauge force, while the local $SU(N)$ symmetry represents a grand unified theory for other forces.

For the gravitational part, its unification with other interactions necessitates the inclusion of quadratic curvature terms in the gravitational sector. However, this would be experimentally unacceptable since the coupling constant in these terms may typically be comparable to that of vector fields. Fortunately, despite both vector and tensor fields are located in the same $SL(2N, C)$ gauge multiplet, the tensor field submultiplet appears naturally suppressed through the tetrad filtering that enables the neglect of quadratic curvature terms. An essential problem related to this type of HUT models is also a possible presence of ghosts being related to the tensor rather than vector submultiplet of the $SL(2N, C)$ gauge multiplet. But, again, as the quadratic strength terms of tensor fields appears to be significantly diminished in the theory this problem is proving to be quite surmountable. As a result, one eventually comes to the conventional Einstein-Cartan type gravity with an extra four-fermion (spin current-current) interaction (73) properly suppressed in the theory.

For the grand unification, in turn, since all states involved in $SL(2N, C)$ theories are additionally classified by spin magnitude, the $SU(N)$ GUTs with purely antisymmetric matter multiplets, including the usual $SU(5)$ theory, turn out to be irrelevant for the standard 1/2-spin quarks and leptons. Meanwhile, the $SU(8)$ grand unification with the certain mixed representation for all three families of composite quarks and leptons, arising from the $SL(16, C)$ theory, appears to be particularly interesting and was studied in some detail. Indeed, starting from $N$ lefthanded and $N$ righthanded preons being fundamental multiplets of the "metaflavor" $SL(2N, C)$ symmetry one can show, under some natural conditions, that among all possible chiral symmetries only the $SU(8)_L \times SU(8)_R$ symmetry meets the anomaly matching condition and can in principle ensure masslessness of lefthanded and righthanded fermion composites at large distances. This, in turn, identifies – among all metaflavor $SL(2N, C)$ symmetries – just the $SL(16, C)$ one as the most likely candidate for hyperunification of all elementary forces.

At the same time, it is important to clarify that $SL(2N, C)$ hyperunification does not imply a single universal coupling constant for gravity and other interactions, as is usually assumed in unified theories. Instead, it suggests that all these forces are provided by vector and tensor fields being the members of the same $SL(2N, C)$ gauge multiplet. A universal constant is indeed necessary for the standard quadratic strength terms of vector and tensor fields. However, the pure gravitational interaction has a fundamentally different coupling, linear in the tensor field strength (71). This unique coupling arises solely due to the presence of tetrads, which are essential ingredients for an $SL(2N, C)$ invariant theory. It comes with its own independent coupling constant $(1/2\kappa)$, conventionally related to the Planck mass. Significantly, the vector (and axial-vector) fields cannot have these linear strength terms alongside the standard quadratic ones.

Finally, we should emphasize the special role of tetrads and their orthogonality condition (58) within the entire $SL(2N, C)$ theory framework. The key point is that, irrespective of the filtering process, tetrads should be truly neutral, devoid of any $SU(N)$ hyperflavored components. Otherwise, they cannot be treated as standard vielbein fields satisfying the invertibility conditions. Remarkably, only such tetrads perform the filtering process that significantly weakens the tensor field submultiplet, while leaving the vector and axial-vector fields unaffected. The axial-vector fields may pose some problem in the theory, which, as argued earlier, could be resolved through their condensation [8] or a double filtering mechanism that completely expels them from the theory. However, there exists an essentially different approach that could generically solve this problem just for the tetrads considered. Notably, unlike the vector and tensor fields, axial-vector fields have no direct coupling with fermion matter, as was demonstrated above. This suggests a new scenario for hyperunification, wherein all gauge fields of the $SL(2N, C)$ symmetry appear as the composite bosons formed by fermion pairs, rather than being elementary fields. This approach, well-known for decades as a viable alternative to conventional quantum electrodynamics [21], gravity [22], and Yang-Mills theories [23, 24, 25], has never been applied to noncompact unified symmetries. In such a scenario, where only the global $SL(2N, C)$ symmetry is initially postulated for the pure fermionic Lagrangian with appropriately filtered fermion currents, one could expect that only composite vector and tensor fields emerge in an effective gauge theory, while axial-vector fields are never formed.

Another avenue for further study concerns the phenomenological aspects of the theory. The spontaneous breaking of the $SL(2N, C)$ HUT through the filtered $SL(2, C) \times SU(N)$ symmetry down to the Standard Model and below will lead to many new processes caused by generalization of both gravity and SM sectors due to new particles and new couplings, as partially discussed above.

We may return to these important issues elsewhere.

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