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de Sitter state as thermal bath

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We consider the local thermodynamics of the de Sitter state. The local temperature, which is the same for all points of the de Sitter space, is $T = H/\pi$, where H is the Hubble parameter. It is twice larger than the Gibbons-Hawking temperature of the cosmological horizon, $T_{\rm GH} = H/2\pi$. The local temperature is not related to the cosmological horizon. It determines the rate of the activation processes, which are possible in the de Sitter environment. The typical example is the process of the ionization of the atom in the de Sitter environment, which rate is determined by temperature $T = H/\pi$. The local temperature determines the local entropy of the de Sitter vacuum state, and this allows to calculate the total entropy inside the cosmological horizon. The result reproduces the Gibbons-Hawking area law, which is related to the cosmological horizon, $S_{\rm hor} = 4\pi KA$, where $K = 1/16\pi G$. We extend the consideration of the local thermodynamics of the de Sitter state using the $f(\mathcal{R})$ gravity. In this thermodynamics, the Ricci scalar curvature $\mathcal{R}$ and the effective gravitational coupling, K, are thermodynamically conjugate variables. The holographic connection between the bulk entropy of the Hubble volume, and the surface entropy of the cosmological horizon remains the same, $S_{\rm hor} = 4\pi KA$, but where the gravitational coupling is $K = df/d\mathcal{R}$. The local temperature of the de Sitter vacuum suggests that it serves as the heat bath for the matter, and thus the de Sitter state is locally unstable towards the creation of matter and its further heating. The decay of the de Sitter vacuum due to such processes determines the quantum breaking time of the space-times with positive cosmological constant. We also consider the thermodynamics of de Sitter in the frame of the multi-metric gravity ensemble, where the heat exchange between different "sub-Universes" in the ensemble leads to the common de Sitter expansion with common temperature $T = H/\pi$. Application of the local thermodynamics to the entropy of the Schwarzschild black hole is also considered.

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Contents

I. INTRODUCTION

We consider the local thermodynamics of the de Sitter state of the expansion of the Universe. The term "local" means that we consider the de Sitter vacuum as the thermal state, which is characterized by the local temperature. This consideration is based on observation, that matter immersed in the de Sitter vacuum feels this vacuum as the heat bath with the local temperature $T = H/\pi$, where H is the Hubble parameter. This temperature is twice larger than the Gibbons-Hawking one, and it has no relation to the cosmological horizon. The existence of the local temperature, suggests the existence of the other local thermodynamic quantities, which participate in the local thermodynamics of the de Sitter state. In addition to the the local entropy density s and local vacuum energy density ϵ, there are also the local thermodynamic variables related to the gravitational degrees of freedom.

Although the local temperature is twice larger than the Gibbons-Hawking temperature assigned to the horizon, $T_{\rm GH} = H/2\pi$, there is the certain connection between the local thermodynamics and the thermodynamics of the event horizon. It appears that the total entropy of the volume V_H bounded by the cosmological horizon coincides with the Gibbons-Hawking entropy, $S_{\rm bulk} = sV_H = A/4G = S_{\rm hor}$. This demonstrates that the local thermodynamics in the $(3+1)$ de Sitter is consistent with the global thermodynamics assigned to the cosmological horizon, although the origin of such bulk-surface correspondence is not very clear.

We tested the local thermodynamics and the bulk-surface correspondence in the extension of general relativity – the $f(\mathcal{R})$ gravity. The $f(\mathcal{R})$ gravity in terms of the Ricci scalar $\mathcal{R}$ is one of the simplest geometrical models, which describes the dark energy and de Sitter expansion of the Universe.[1–7] It was used to construct an inflationary model of the early Universe – the Starobinsky inflation, which is controlled by the $\mathcal{R}^2$ contribution to the effective action. This class of models, $f(\mathcal{R}) \propto \mathcal{R} - \mathcal{R}^2/M^2$, was also reproduced in the so-called q-theory,[8,9] where q is the 4-form field introduced by Hawking[10] for the phenomenological description of the physics of the deep (ultraviolet) vacuum (here the sign convention for $\mathcal{R}$ is opposite to that in Ref.[2]). The Starobinsky model is in good agreement with the observations. However, despite the observational success, the theory of Starobinsky inflation is still phenomenological. Due to a rather small mass scale M compared with the Planck scale it is difficult to embed the model into a UV complete theory.[11–13]

The $f(\mathcal{R})$ theory demonstrates that the effective gravitational coupling K (it is the inverse Newton constant, $K = 1/16\pi G$) and the scalar curvature $\mathcal{R}$ are connected by equation $K = df/d\mathcal{R}$. This suggests that K and $\mathcal{R}$ are the thermodynamically conjugate variables.[14,15] This pair of the gravitational variables is similar to the pair of the electrodynamic variables, electric field $\mathbf{E}$ and electric induction $\mathbf{D}$, which participate in the thermodynamics of dielectrics. Using the local thermodynamics with $T = H/\pi$, we obtained the general result for the total entropy of the Hubble volume, $S_{\rm bulk} = sV_H = 4\pi KA = S_{\rm hor}$, where $K = df/d\mathcal{R}$ is the effective gravitational coupling.

In Sec. II we consider the elements of the local thermodynamics of the de Sitter state. It is based on the observation that the de Sitter vacuum serves as the thermal bath for the external matter – the matter immersed into the de Sitter environment. This is different from the traditional consideration of the vacua of the quantum fields in the de Sitter spacetime, which uses the Euclidean action method. Example of the influence of the de Sitter vacuum to the external matter is provided by an atom in the de Sitter environment (Sec. II A). As distinct from the atom in the flat space, the atom in the de Sitter vacuum has a certain probability of ionization. The rate of ionization is similar to the rate of ionization in the presence of the thermal bath with temperature $T = H/\pi$.[16–20] The same temperature determines the other activation processes, which are energetically forbidden in the Minkowski spacetime, but are allowed in the de Sitter background, see also Refs.[21,22] (Sec. II B). In Sec. II C it is shown that the local temperature also determines the Hawking temperature of radiation from the cosmological horizon without using the Euclidean action.

All that suggests that it is natural to consider the temperature $T = H/\pi$ as the local temperature of the de Sitter vacuum. The local temperature leads to the local entropy of the de Sitter thermal bath (Sec. II H), which integrated over the Hubble volume reproduces the Gibbons-Hawking entropy of the cosmological horizon (Sec.II I). The effect of thermal fluctuations is discussed in Sec. II J.

Sec. III is devoted to the multi-metric gravity, which can be considered as ensemble of the sub-Universes, each described by its own metric $g_{\mu\nu(n)}$ (or tetrads $e^a_{\mu(n)}$), by its own gravitational coupling K_n and cosmological constant Λ_n. The heat exchange between the sub-Universes leads to their thermalization – the formation of the Universe in which the sub-Universes have the common Hubble parameter and thus the common temperature.

In Sec. IV the local entropy of the de Sitter state allows to consider the thermodynamics of the Schwarzschild black hole, assuming that the black hole can be formed by relaxation of the gravastar (Sec. IV C) with the de Sitter core. Here the interior of the gravastar is represented by the contracting de Sitter state (Sec. IV B), which has negative Hubble parameter, and thus the negative temperature $T = H/\pi < 0$ and negative entropy. Since the cosmological and black hole horizons cancels each other, the gravastar has zero entropy. In the process of relaxation of the gravastar to the black hole the de Sitter core with its negative entropy shrinks, which results in the Hawking-Bekenstein entropy of the black hole horizon (Sec. IV D). The entropy of the white hole is considered in Sec. IV E, and the heat exchange between black holes in the multi-metric gravity is in Sec. IV F.

Sec. V is devoted to thermodynamics of de Sitter in the $f(\mathcal{R})$ gravity. It contains the pair of the thermodynamically conjugate variables, K and $\mathcal{R}$ (Sec. V A). These variables together with the local temperature and local entropy provides the Gibbs-Duhem relation (Sec. V B). The general result for the total entropy of the Hubble volume, $S_{\rm bulk} = sV_H = 4\pi KA = S_{\rm hor}$ is obtained in Sec. V C. This is considered on example of quadratic gravity in Sec. V D.

The Sec. VI is devoted to the de Sitter decay due to the thermalization of matter by the de Sitter heat bath, and finally the conclusion is in Sec. VII.

II. THERMODYNAMICS OF THE DE SITTER STATE

A. Atom in de Sitter environment as thermometer

We consider the de Sitter thermodynamics using the Painlevé-Gullstrand (PG) form,[23,24] where the metric in the de Sitter expansion is

$$ds^2 = -dt^2 + (dr - v(r)dt)^2 + r^2 d\Omega^2 \,. \tag{1}$$

Here the shift velocity is $v(r) = Hr$. This metric is stationary, i.e. does not depend on time, and it does not have the unphysical singularity at the cosmological horizon. That is why it is appropriate for consideration of the local thermodynamics both inside and outside the horizon. It also allows to describe two different states – the expanding de Sitter Universe with $H > 0$ and contracting de Sitter Universe with $H < 0$.

Now let us consider an atom at the origin, $r = 0$. The atom is the external object in the de Sitter spacetime, which is playing the role of the detector (or the role of the static observer) in this spacetime. The electron bounded to an atom may absorb the energy from the gravitational field of the de Sitter background and escape from the electric potential barrier. If the ionization potential is much smaller than the electron mass but is much larger than the Hubble parameter, $H \ll \epsilon_0 \ll m$, one can use the nonrelativistic quantum mechanics to estimate the tunneling rate through the barrier. The corresponding radial trajectory $p_r(r)$ is obtained from the classical equation

$$\frac{p_r^2}{2m} + p_r v(r) = -\epsilon_0 \,, \tag{2}$$

where $p_r(r)v(r)$ is the Doppler shift:

$$p_r(r) = -mv(r) + \sqrt{m^2v^2(r) - 2m\epsilon_0} \,. \tag{3}$$

The integral of $p_r(r)$ over the classically forbidden region, $0 < r < r_0 = \sqrt{2\epsilon_0/mH^2}$, gives the ionization rate

$$w \sim \exp\left(-2\,{\rm Im}\, S\right) = \exp\left(-\frac{\pi\epsilon_0}{H}\right) \,. \tag{4}$$

This is equivalent to the thermal thermal radiation with temperature $T = H/\pi$.

In the interpretation of Ref.[20], the bound state decays by quantum tunnelling from the point $r = 0$ to the point $r = r_0$, at which the energy $-\epsilon_0$ matches the de Sitter gravitational potential $U(r) = -mH^2r^2/2$. The radial trajectory $p_r(r)$ is obtained from the classical equation

$$\frac{\mathbf{p}^2}{2m} - \frac{1}{2}mH^2r^2 = -\epsilon_0 \,. \tag{5}$$

This gives Eq.(4) for the WKB tunneling rate.

B. Decay of composite particles in de Sitter spacetime

The same local temperature $T = H/\pi$ describes the process of the splitting of the composite particle with mass m into two components with $m_1 + m_2 > m$, which is also not allowed in the Minkowski vacuum[17,25–27]. In the limit $m \gg H$, the rate of such decay of the composite particle is

$$w \sim \exp\left(-\frac{\pi(m_1 + m_2 - m)}{H}\right). \tag{6}$$

The similar processes take place in the so-called Cosmological Collider,[21,22] where the new particle created by the Hawking radiation plays the role of the external object which produces the heavy particles. Here there are two different physical processes, which are described by different temperatures. The Hawking radiation from the de Sitter vacuum is determined by the Hawking temperature $T_{\rm GH}$ of the cosmological horizon, while the further process – the splitting of the created particles – is determined by the local temperature $T = 2T_{\rm GH}$.

C. Connection between the local and Hawking temperatures

The local temperature $T = H/\pi$ also determines the process of the Hawking radiation from the cosmological horizon and the Gibbons-Hawking temperature $T_{\rm GH} = T/2 = H/2\pi$. The reason is that in the Hawking process, two particles are coherently created (the analog of cotunneling): one particle is created inside the horizon, while its partner is simultaneously created outside the horizon.[28] The rate of the coherent radiation of two particles, each with energy E, is $w \propto \exp(-\frac{2E}{T})$. However, the observer who uses the Unruh-DeWitt detector can detect only the particle created inside the horizon. For this observer the creation rate $w \propto \exp(-\frac{2E}{T})$ is perceived as

$$w \propto \exp\left(-\frac{E}{T/2}\right) = \exp\left(-\frac{E}{T_{\rm GH}}\right), \tag{7}$$

with the Gibbons-Hawking temperature $T_{\rm GH} = T/2 = H/2\pi$.

On the contrary, in the local process of the decay of the atom, which is not related to the cosmological horizon, only single particle (electron) is radiated from the atom. This process is fully determined by the local temperature, $w \propto \exp(-\frac{\epsilon_0}{T})$.

D. Two detectors: exited atom vs ionized atom

Note the main difference between the temperature measured by the observer using the Unruh-DeWitt detector (see e.g. Ref.[29] and references therein) and the temperature measured by the observer using ionization of an atom. The ionization process is possible, because the radiated electron moves far away from the atom to position $r = r_0$, where its negative gravitational energy compensates the ionization potential. In the Unruh-DeWitt detector, which corresponds to the two-level atom interacting with a quantum field, the electron in the atom is excited but remains in the same position in the same atom. That is why such excitation of electron may only come either by the Hawking radiation, or by radiation of photons, see Sec. II E.

However, in the de Sitter case there is no difference in temperatures measured by two detectors. If in the Unruh-DeWitt detector experiments the Hawking temperature is properly interpreted according to Eq.(7), the measured temperature of the Hawking radiation is also $T = 2T_{\rm GH}$. So, in spite of different physical principles, both detectors in the de Sitter environment show the same physical temperature, $T = 2T_{\rm GH}$. This also demonstrates the uniqueness of the de Sitter state.

E. Radiation of photons by atom in de Sitter environment

Instead of the radiation of electron from the atom, one can consider the radiation of photon by the same atom. In both cases the escape of photon or electron from the atom provides the negative gravitational energy, which allows to consider the processes which are prohibited in the Minkowski state. Let us consider the atom in the excited state with energy $E_0 + \epsilon_0$, where E_0 is the ground state energy. In the de Sitter environment, the radiation of photon is possible even if its energy cp exceeds the energy difference between the excited and the ground state level, i.e. when $cp > \epsilon_0$.

For the relativistic photon the equation (2) takes the form (here we use $c = 1$):

$$\sqrt{p_r^2 + p_\perp^2} + p_r v(r) = -\epsilon_0 \,. \tag{8}$$

This gives the trajectory of photon

$$p_r(r) = \frac{1}{1 - H^2 r^2}\left[\epsilon_0 H r \pm \sqrt{\epsilon_0^2 + p_\perp^2 (H^2 r^2 - 1)}\right] \,. \tag{9}$$

For $cp_\perp > \epsilon_0$, the integration over the classically forbidden region, $r < r_0$, where $H^2 r_0^2 = 1 - \epsilon_0^2/p_\perp^2$, gives the following radiation rate of photon by excited atom:

$$w \sim \exp\left(-2\,\mathrm{Im}\,S\right) = \exp\left(-\frac{\pi(cp_\perp - \epsilon_0)}{H}\right) \,,\ cp_\perp > \epsilon_0 \,. \tag{10}$$

This process is again determined by temperature $T = H/\pi$.

Eq.(10) demonstrates, that radiation takes place even if the atom is not excited, i.e. when $\epsilon_0 = 0$. In this case Eq.(10) describes the process of radiation of photon in the de Sitter environment:

$$w \sim \exp\left(-\frac{cp}{T}\right) \,, \tag{11}$$

The atom here is needed as an external object, which violates the de Sitter symmetry and thus provides the nonzero matrix element for the process of radiation of photon.

F. Accelerating detector: is there connection between the local process and Unruh radiation?

In the same way as in Sections II D and II E, one may consider different independent processes related to the accelerating detector. One of them is the Unruh radiation measured by the Unruh-DeWitt detector, which could be linked to the apparent event horizon in the Rindler spacetime and to the corresponding Unruh temperature $T_U = a/2\pi$, where a is acceleration.[30–32] The other processes, such as the local process of ionization of the accelerating atom,[33] have no relation to the event horizon.

Let us consider the process of ionization of atom. Similar to Eq.(5), the trajectory $p_x(x)$ of the radiated electron is obtained from the classical equation

$$\frac{\mathbf{p}^2}{2m} - max = -\epsilon_0 \,. \tag{12}$$

Here we used the equivalence between the acceleration of the reference frame and the constant gravitational field, $g = a$.

In the WKB approximation and for $a \ll \epsilon_0 \ll m$ this gives the radiation rate:[33]

$$w \sim \exp\left(-\frac{4\sqrt{2}}{3}\frac{\epsilon_0}{a}\left(\frac{\epsilon_0}{m}\right)^{1/2}\right) \,. \tag{13}$$

In principle, one can introduce the effective temperature, which is similar to the effective temperature $T_{\rm eff} \sim El$ of hopping electrons in a strong electric field E, where l is the localization length.[34,35] In our case, the analog of the localization length is $l = 1/\sqrt{m\epsilon_0}$. But otherwise the local process of ionization of the accelerated atom is certainly non-thermal. Since $\epsilon_0 \ll m$, the rate of ionization in Eq.(13) essentially exceeds the ionization rate in any thermal process, which involves acceleration, $w_{\rm thermal} \sim \exp(-\gamma\epsilon_0/a)$. This is very different from the ionization of the atom in the de Sitter environment, which looks thermal.

In principle, with the same approach one can also calculate the Unruh radiation of photons using different arrangements of detectors. The state of the art in theoretical understanding of Unruh radiation can be found in Refs.[36,37] and references therein. Our examples demonstrate that the rate of Unruh radiation depends details of the considered processes and it is not necessarily thermal.

G. de Sitter symmetry and de Sitter heat bath

As distinct from the Unruh effect, different arrangements of detectors in the de Sitter environment show the same temperature. This demonstrates the uniqueness of the de Sitter spacetime in producing the thermal bath with local temperature. The reason for that is that the de Sitter spacetime is homogeneous under the combination of translation and the proper conformal transformations.[38,39] In the PG metric it is the invariance of the de Sitter state under the modified translations, $\mathbf{r} \to \mathbf{r} - e^{Ht}\mathbf{a}$, which for $H \to 0$ corresponds to the conventional invariance under translations $\mathbf{r} \to \mathbf{r} - \mathbf{a}$ in Minkowski spacetime. Due to this combined translational symmetry, all the comoving observers at any point of the de Sitter space observe the same temperature, $T = H/\pi$. That is why one may conclude, that the de Sitter state is the heat bath produced by gravity.

The uniqueness of the de Sitter thermal state lies in the fact that the temperature does not violate the de Sitter symmetry, and thus does not require the preferred reference frame. This is distinct from thermal state of matter, which always has a preferred reference frame where matter is at rest. As a result, thermal matter violates the de Sitter symmetry, which leads to the heat exchange between the two thermal subsystems, gravity and matter.

H. From local temperature to local entropy

The gravity subsystem – de Sitter quantum vacuum – has its own temperature and entropy, see also Ref.[40]. On the other hand the de Sitter spacetime serves as the thermal bath for matter. Then the quasi-equilibrium states of the expanding Universe can be described by two different temperatures: the temperature of the gravitational vacuum (temperature of dark energy) and the temperature of the matter degrees of freedom.[41] In this section we discuss the pure de Sitter vacuum without the excited matter ignoring for the moment the thermal activated creation of matter from the vacuum. The excitation and thermalization of matter by the de Sitter thermal bath will be discussed in Sec. VI.

If the vacuum thermodynamics is determined by the local the activation temperature $T = H/\pi$, then in the Einstein gravity with cosmological constant the vacuum energy density is quadratic in temperature:

$$\epsilon_{\rm vac} = \frac{3}{8\pi G} H^2 = \frac{3\pi}{8G} T^2 \,. \tag{14}$$

This leads to the free energy density of the de Sitter vacuum, $F = \epsilon_{\rm vac} - T d\epsilon_{\rm vac}/dT$, which is also quadratic in T, and thus the entropy density $s_{\rm vac}$ in the de Sitter vacuum is linear in T:

$$s_{\rm vac} = -\frac{\partial F}{\partial T} = \frac{3\pi}{4G} T = 12\pi^2 K T \,. \tag{15}$$

The temperature T and the entropy density $s_{\rm vac}$ are the local quantities which can be measured by the local static observer.

I. Hubble volume entropy vs entropy of the cosmological horizon

Using the entropy density in Eq.(15), one may find the total entropy of the Hubble volume V_H – the volume surrounded by the cosmological horizon with radius $R = 1/H$:

$$S_{\rm bulk} = s_{\rm vac} V_H = \frac{4\pi R^3}{3} s_{\rm vac} = \frac{\pi}{G H^2} = 4\pi K A = S_{\rm hor} \,, \tag{16}$$

where A is the horizon area. This Hubble-volume entropy coincides with the Gibbons-Hawking entropy of the cosmological horizon. However, here it is the thermodynamic entropy coming from the local entropy of the de Sitter quantum vacuum, rather than the entropy of the horizon degrees of freedom.

Anyway, the relation between the bulk and surface entropies in the local vacuum thermodynamics suggests some holographic origin. However, such bulk-surface correspondence is valid only in the $(3+1)$-dimension. In the general $d+1$ dimension of spacetime, the same approach gives the factor $(d-1)/2$ in the relation between the entropy of the Hubble volume and the Gibbons-Hawking entropy of the cosmological horizon, $S_{\rm bulk} = \frac{d-1}{2} S_{\rm hor}$.

This may add to the peculiarities of the $d = 3$ space dimension,[42] where in particular the mass dimension of the gravitational coupling, $[K] = d - 1$, coincides with the mass dimension of curvature, $[\mathcal{R}] = 2$. The same concerns such pair of thermodynamically conjugate variables as electric field with the mass dimension $[E] = 2$ and the electric induction with mass dimension $[D] = d - 1$. Their dimensions also coincide only for $d = 3$. Discussion of the natural dimensions of physical quantities in $d = 3$ can be found in Ref.[43].

J. Hubble volume vs the volume of Universe, and thermal fluctuations of de Sitter state

It is not excluded that our Universe is finite. Its volume V might be comparatively small, not much larger than the currently observed Hubble volume V_H.[44]

If the Universe is finite and if the de Sitter state represents the excited thermal state of the quantum vacuum, the thermal fluctuations of the deep quantum vacuum may become important. According to Landau-Lifshitz,[45] the thermal fluctuations are determined by the compressibility of the system and by its volume. In case of the Universe with the volume V, the fluctuations of the vacuum energy density are given by:[46]

$$\left\langle (\Delta\epsilon_{\rm vac})^2 \right\rangle = \left\langle (\Delta P_{\rm vac})^2 \right\rangle = \frac{T}{V\chi_{\rm vac}} \,. \tag{17}$$

Here $\chi_{\rm vac}$ is the vacuum compressibility[47] – the compressibility of the fully equilibrium Minkowski vacuum with $\epsilon_{\rm vac} = -P_{\rm vac} = 0$.

Note the main difference between the thermal fluctuations and quantum fluctuations. The contribution of the quantum fluctuations of the relativistic quantum fields to the vacuum energy density is typically on the order of $M^4_{\rm Pl}$, where $M_{\rm Pl}$ is the Planck mass. But in the fully equilibrium vacuum state this contribution is cancelled by the ultraviolet trans-Planckian degrees of freedom due to the thermodynamic Gibbs-Duhem relation.[46,47] This cancellation is universal, being valid both for the relativistic vacuum states and for the non-relativistic grounds states of the condensed matter systems. The contribution of thermal fluctuations to the vacuum energy is in the range of the infrared physics, which is determined by temperature T.

The vacuum compressibility is not constrained by the thermodynamic laws and is determined by the ultraviolet physics[47] with its Planck energy scale, $\chi^{-1}_{\rm vac} \sim M^4_{\rm Pl}$. In this relation it is similar to the gravitational coupling, which is also determined by the Planck scale, $K \sim M^2_{\rm Pl}$. On the other hand, the temperature corrections to $\chi^{-1}_{\rm vac}$ and K as well as the Casimir corrections are within the range of the infrared physics, see Ref.[48] for the universal temperature correction to the gravitational coupling K. The contribution to the compressibility from the infrared range was discussed in Refs.[49–51]. The negative contributions to compressibility obtained in these papers do not violate the stability of the quantum vacuum, since these contributions represent the corrections, which are small compared to the main value of the vacuum compressibility, $\chi^{-1}_{\rm vac} \sim M^4_{\rm Pl}$.

In the excited vacuum – the de Sitter state with the temperature $T = H/\pi$ and the energy density $< \epsilon_{\rm vac} > \sim M^2_{\rm Pl} H^2$ – the relative magnitude of thermal fluctuations is determined by the ratio of the Hubble volume to the volume of the Universe:

$$\frac{\left\langle (\Delta\epsilon_{\rm vac})^2 \right\rangle}{\left\langle \epsilon_{\rm vac} \right\rangle^2} \sim \frac{V_H}{V} \,. \tag{18}$$

The volume of the present Universe exceeds the Hubble volume, $V > V_H$, and thus the thermal fluctuations of the vacuum energy density are still relatively small.

III. THERMODYNAMICS FROM THE HEAT TRANSFER IN THE MULTI-METRIC GRAVITY ENSEMBLE

A. Multi-metric gravity

Since the heat exchange between the bodies or between the systems is the main source of the emergent thermodynamics,[52] we can consider the de Sitter thermodynamics from the point of view of the heat transfer between different cosmological objects.

Such heat exchange can be discussed in the frame of the so-called multi-metric gravity, see Ref.[53] and references therein. The corresponding model action of the whole system can be written as the sum of actions of the sub-systems in the same coordinate spacetime:

$$S = -\int d^4x \sum_{n=1}^{N} \mathcal{L}_n \;,\; \mathcal{L}_n = \sqrt{-g_{(n)}} \left(K_n \mathcal{R}\{g_{\mu\nu(n)}\} + \Lambda_n \right) \,. \tag{19}$$

Then the Universe can be seen as the system of N sub-Universes, each with its own gravitational coupling K_n, cosmological constant Λ_n and metric $g_{\mu\nu(n)}$.

Following Frogatt and Nielsen[54] one can introduce N independent tetrad fields $e_\mu^{a(n)}$ for N fermionic species. In this multi-tetrad gravity one has:

$$\mathcal{L}_n = e_{abcd} K_n \, R^{ab(n)} \wedge e^{c(n)} \wedge e^{d(n)} + \\ + e_{abcd} \Lambda_n \, e^{a(n)} \wedge e^{b(n)} \wedge e^{c(n)} \wedge e^{d(n)} \,, \tag{20}$$

with the corresponding action for the fermionic species:

$$S_M = e_{abcd} \int \sum_n \Theta^{a(n)} \wedge e^{b(n)} \wedge e^{c(n)} \wedge e^{d(n)} \,, \tag{21}$$

$$\Theta^{a(n)} = \frac{i}{2} \left[\bar{\Psi}^{(n)} \gamma^a D_\mu \Psi^{(n)} - D_\mu \bar{\Psi}^{(n)} \gamma^a \Psi^{(n)} \right] dx^\mu \,. \tag{22}$$

This can be extended to the multi-fünfbein gravity, where instead of the tetrad fields the Dirac fermions are described by the rectangular vielbein (fünfbein).[55]

On the other hand, gravity with multiple tetrad fields may come from the Akama-Diakonov-Wetterich theory,[56–64] where the tetrads are formed as composite objects – the bilinear combinations of the fundamental fermionic fields:

$$e_\mu^{a(n)} = \left\langle \Theta^{a(n)} \right\rangle \,. \tag{23}$$

In this approach, the metric is the quartet of fermions. In principle, the so called vestigial gravity can be realized, in which the bilinear combination of fermions in Eq.(23) is zero, $e_\mu^{a(n)} = 0$, while the metric – the quartet of fermions – is nonzero:[65]

$$g_{\mu\nu(n)} = \eta_{ab} \left\langle \Theta_\mu^{a(n)} \Theta_\nu^{b(n)} \right\rangle \,. \tag{24}$$

B. Heat exchange in multi-metric gravity

The heat exchange between the sub-Universes leads to their equilibration with formation of the common expansion rate and thus the common temperature. We consider first the system of two Universes, assuming that in each of them the entropy of horizon obeys the area law, and show that the maximum entropy corresponds to the state in which both states have the same expansion rate.

In the de Sitter state, which is determined by the cosmological constant, the equation of state for the vacuum energy is $\epsilon_{\rm vac} = -P_{\rm vac}$. The total vacuum energy is proportional to the volume V of the system, if we assume that the volume V is much larger than the Hubble volume, $V \gg V_H$, so that the boundary terms are not important. Then we have:

$$E_V = \epsilon_{\rm vac} V = 6KH^2 V \,. \tag{25}$$

Let us assume that the bulk-surface correspondence is valid, i.e. the entropy of the Hubble volume V_H is equal to the Gibbons-Hawking entropy of cosmological horizon, $S_{V_H} = S_{\rm hor} = 4\pi K A$. Then the total entropy$S_V$ in the volume $V \gg V_H$ can be obtained from the entropy of the Hubble volume V_H:

$$S_V = S_{\rm hor} \frac{V}{V_H} = 12\pi K H V \,. \tag{26}$$

Let us now consider two de Sitter sub-states with different values of the gravitational coupling, K_1 and K_2, and different values of the Hubble parameter, H_1 and H_2:

$$S = -\int d^4x \sqrt{-g_{(1)}} \left(K_1 \mathcal{R}\{g_{\mu\nu(1)}\} + \Lambda_1 \right) - \\ - \int d^4x \sqrt{-g_{(2)}} \left(K_2 \mathcal{R}\{g_{\mu\nu(2)}\} + \Lambda_2 \right) \,. \tag{27}$$

This corresponds to the higher dimensional analog of the bilayer graphene,[66] where two 2+1 dimensional Universes are in the neighbouring layers of the 3+1 spacetime. In this interpretation we have two 3+1 dimensional Universes in the neighbouring layers in the 4+1 space.

The total energy and total entropy of two layers are (if the interaction between the layers is neglected):

$$E_V = E_1 + E_2 = 6(K_1 H_1^2 + K_2 H_2^2)V \,, \tag{28}$$

$$S_V = S_1 + S_2 = 12\pi(K_1 H_1 + K_2 H_2)V \,. \tag{29}$$

Let us now allow for the energy exchange (the heat exchange) between these two Universes (analogs of the two layers of graphene). This exchange can be realized by the matter field, which interacts with both metrics. It leads to the variations of the Hubble parameters H_1 and H_2 at fixed E_V. If we ignore the thermalization of matter by de Sitter environment, the heat exchange will finally produce the equilibrium state with the maximum entropy S, in which the Hubble parameters become equal:

$$H_1^2 = H_2^2 = \frac{E_V}{6(K_1 + K_2)V} \equiv H^2 \,. \tag{30}$$

The equilibration of the Hubble parameters demonstrates that the Hubble parameter (with some numerical factor) plays the role of the temperature of the de Sitter Universe.

The temperature of the de Sitter Universe can be obtained by variation over the Hubble parameter:

$$\begin{aligned} \frac{1}{T_1} &= \frac{dS_1}{dE_1} = \frac{dS_1/dH_1}{dE_1/dH_1} = \frac{\pi}{H_1} \,, \\ \frac{1}{T_2} &= \frac{dS_2}{dE_2} = \frac{dS_2/dH_2}{dE_2/dH_2} = \frac{\pi}{H_2} \,, \end{aligned} \tag{31}$$

with $T_1 = T_2 = H/\pi$ in equilibrium.

In case of the arbitrary number N of the sub-Universes, the heat exchange between them leads to the state of the Universe in which all the sub-Universes coherently expand with the same rate H, i.e., with the same de Sitter metric in all the subsystems. The whole Universe has the gravitational coupling K equal to the sum of the individual couplings in the sub-Universes and the vacuum energy density equal to the sum of energy densities of subsystems:

$$E_V = 6KH^2V \ , \ K = \sum_n K_n \ , \ \Lambda = \sum_n \Lambda_n \,, \tag{32}$$

with $T_n = T = H/\pi$.

C. Thermodynamics from the multi-metric ensemble

Let us remind that in the above approach we used the bulk-horizon correspondence $S_{V_H} = S_{\rm hor} = 4\pi KA$, which finally leads to the equilibrium Universe with temperature $T = H/\pi$. Let us now consider the thermodynamics of the whole de Sitter system without assumption about the entropy of the cosmological horizon. For that we consider the statistical ensemble of N de Sitter sub-Universes with random Hubble parameters H_n. This is the extension of the multi-metric gravity to the statistical ensemble with the randomly distributed parameters K_n and Λ_n.

For large N, the random distribution of the parameters results in the exponential behaviour of the distribution functions, $w_n \propto \exp(-E_n/T)$, with the same parameter T for all subsystems. As in the statistical ensemble of atoms, where the temperature of the system is determined by the physical processes, the temperature of the ensemble of the sub-Universes factor is also determined by the physical processes. In our case it is the of the behaviour of matter (atom) in the de Sitter environment, which gives $T = H/\pi$. This connection between T and H is rather natural. Both, the parameter T, which plays the role of temperature, and the Hubble parameter H are the quantities which in equilibrium become common for all the subsystems in the ensemble, and they have the same dimension of inverse time, $[T] = [H] = [1/t]$.

The physical temperature in turn gives rise to the total entropy and to the local entropy, $S_V = \sum_n S_n = 12\pi KHV = s_{\rm loc}V$. So, in this scenario the de Sitter entropy comes from a set of many randomly distributed subsystems with the expansion rates H_n. Due to the heat exchange at fixed total energy E_V these states are organized in the equilibrium thermal state, which corresponds to the coherent de Sitter expansion of the whole system. The coherence due to thermalization may explain the horizon problem, i.e. why the causally-disconnected regions of the CMB are in thermal equilibrium.

D. Regularization vs thermalization

The multi-metric ensemble may include the ensemble of N species of Weyl or Dirac fermions. At large N, all tetrads in the random ensemble approach the same value, $e_\mu^{a(n)} \to e_\mu^a$, and thus in the equilibrium state all fermionic species experience the same geometry. In Refs.[54,67] the formation of the common Lorentz invariance for different fermionic species is also considered. But this is achieved by the renormalization group effect in the infrared limit, instead of thermalization. This suggests the possible connection between renormalization and thermalization.

One may expect that the heat exchange between subsystems leads not only to the coherence of the de Sitter states, but to the general coherence of the metric fields, when the metric fields $g_{\mu\nu(n)}$ of the subsystems become equal, thus forming the common metric $g_{\mu\nu}$. If this is true, this could be some kind of the thermodynamic gravity, but without the holographic principle.

E. Coherence vs thermalization

At first glance this formation of the time dependent coherent state looks similar to the formation of the coherent spin precession (magnon BEC) from the incoherent precessions about the local magnetic fields with random frequencies ω_n.[68] The latter corresponds to random H_n. The coherence develops due to the spin currents between the regions of local precessions – the analog of the heat exchange. The common frequency ω of the coherent precession corresponds to the common Hubble parameter H. They have the same dimension, $[H] = [\omega] = [1/t]$, while the dimensionless magnon number (or the spin projection S_z) corresponds to the entropy which is also dimensionless. However, the source of the coherence is the exchange of spin instead of exchange of the energies, and also the formation of coherent state is due to minimization of the total energy E at fixed projection S_z of the total spin on magnetic field. This leads to the common frequency, which plays the role of the chemical potential μ, which emerges due to the exchange of particles between the regions. That is why this process is quite opposite to the formation of temperature and entropy, where energy is fixed, while entropy reaches maximum value. This process corresponds to the minimization of energy at fixed total entropy. Anyway, in all the cases the parameters, which are the same in all subsystems in equilibrium – common frequency ω, common chemical potential μ, common temperature T and common Hubble parameter H – have the same dimension, $[T] = [H] = [\mu] = [\omega] = [1/t]$.

IV. CONTRACTING DE SITTER AND BLACK HOLE ENTROPY

A. de Sitter vs black hole

The thermodynamics of de Sitter is very different from the black hole thermodynamics. Black hole is the compact object, and the temperature of the Hawking radiation T_H is well determined (which is also supported by the analogs). On the other hand, the origin of the black hole entropy is still not clear, although it can be determined from the equation $dM = T_H dS$, assuming that the laws of thermodynamics are applicable to this compact object.

On the contrary, the de Sitter state is not the compact object. It is the homogeneous vacuum state without boundaries, which has the homogeneous energy density as the local quantity. The local energy density allows us to introduce also the local temperature T. This temperature can be measured for example by an atom, which is stationary with respect of the shift velocity. It determines the rate of the ionization of the atom, $\exp(-E/T)$.

B. Entropy of expanding, contracting and static de Sitter

The static de Sitter vacuum with singularity at the cosmological horizon is the intermediate state between the two opposite states – the expanding and contracting states. The entropy of the static state is zero, and correspondingly the temperature must be infinite, see also Ref.[69]. In the contracting de Sitter vacuum with $H < 0$ the local temperature is negative, $T = H/\pi < 0$, and the local entropy is also negative, $s_{\rm contracting} = 3H/4G = -3/(4Gr_h) < 0$. The total entropy in the Hubble volume of contracting de Sitter is

$$S_{\rm contracting} = s_{\rm contracting} V_h = -\frac{A}{4G}\,, \tag{33}$$

where A is the area of the cosmological horizon.

C. Gravastar – black hole with de Sitter core

The black hole can be formed from the gravastar. This is the object, which contains the de Sitter state inside the black hole horizon.[70–72] We consider the gravastar in which the black hole horizon coincides with the de Sitter horizon, $r_{\rm bh} = r_h$ (the metric in the state with the critical value of the mass parameter $m = 2MG|H| = 1$ at which two horizons merge is illustrated in Fig. 1 of Ref.[73]). Such gravastar is stationary, it has no Hawking radiation and its entropy is zero. In the Painlevé-Gullstrand form the metric is given by Eq.(1) with the following shift velocity:

$$v(r) = -\sqrt{\frac{r_h}{r}}\ ,\ r > r_h\ , \tag{34}$$

$$v(r) = -\frac{r}{r_h}\ ,\ r < r_h\ . \tag{35}$$

Here $r_h = 1/|H| = r_{\rm bh}$, where $r_{\rm bh} = 2MG$ and $M = 1/(2G|H|)$ is the mass of the black hole, which is formed by the de Sitter core.

The shift velocity $v(r)$ must be continuous across the horizon, i.e. there is no jump in the shift velocity, while the gradient of the shift velocity dv/dr experiences jump at the horizon. The shift velocity $v(r)$ is negative both outside and inside the horizon. Since the shift velocity is negative in the de Sitter core, this means that the de Sitter spacetime in this gravastar is contracting, $v(r) = Hr = -r/r_h < 0$, i.e. the Hubble parameter is negative, $H = -1/r_h < 0$.

D. Entropy of black hole from negative entropy of contracting de Sitter

According to Eq.(33) the region of the contracting de Sitter state has negative entropy, $S_{\rm contracting} = -A/4G$. That is why the gravastar is unstable towards the shrinking of the de Sitter region, since this leads to the increase of the entropy of the whole system due to decrease of the negative entropy of the contracting de Sitter state. Due to the energy conservation the singularity is formed ar $r = 0$. In the final state – the black hole – the de Sitter region with negative entropy fully disappears by shrinking to the singularity with mass M. The resulting black hole with mass M acquires the positive entropy, $A/4G$:

$$S_{\rm BH} = S_{\rm gstar} - S_{\rm contracting} = 0 - s_{\rm contracting} V_h = \frac{A}{4G}\,. \tag{36}$$

In terms of the entropies of horizons, this means that in the initial gravastar state the entropy of the contracting de Sitter horizon fully compensates the entropy of the black hole horizon:

$$S_{\rm gstar} = S_{\rm contracting} + S_{\rm BH} = -\frac{A}{4G} + \frac{A}{4G} = 0\,. \tag{37}$$

E. White hole and anti-gravastar

In the same way the anti-gravastar can be obtained as the white hole with the de Sitter core. This object has the following shift velocities:

$$v(r) = \sqrt{\frac{r_h}{r}}\ ,\ r > r_h\ , \tag{38}$$

$$v(r) = \frac{r}{r_h}\ ,\ r < r_h\ . \tag{39}$$

It is obtained from the pure white hole, which has the negative horizon entropy $S_{\rm WH} = -A/4G$,[74,75] by growing the de Sitter state in its core with positive entropy. In the anti-gravastar state the entropies of two horizons cancel each other:

$$S_{\rm antigstar} = S_{\rm expanding} + S_{\rm WH} = \frac{A}{4G} - \frac{A}{4G} = 0\,. \tag{40}$$

F. Heat exchange between black holes in the multi-metric ensemble

Let us first consider the two-metric gravity with $K_1 = K_2$. In this case, the heat exchange between the black holes with masses M_1 and M_2 at fixed $M = M_1 + M_2$ leads to formation of the black holes with masses $M_1 = M_2 = M/2$ in each of the two sub-Universes. For sub-Universes with $K_1 \neq K_2$ one obtains $M_1 = MK_1/(K_1 + K_2)$ and $M_2 = MK_2/(K_1 + K_2)$.

For the general ensemble of the multi-metric gravities, the heat exchange between the substates gives rise to the black hole with mass M and gravitational coupling K:

$$M = \sum_n M_n \ , \ K = \sum_n K_n \ , \ M_n = M \frac{K_n}{K} \,. \tag{41}$$

The Hawking temperature of the black hole is the common temperature in all the substates:

$$\frac{1}{T_n} = \frac{dS_n}{dM_n} = \frac{dS}{dM} = \frac{1}{T_H} \,. \tag{42}$$

V. THERMODYNAMICS OF DE SITTER STATE AND $f(R)$ GRAVITY

A. Thermodynamic variables in $f(R)$ gravity

The T^2 dependence of the de Sitter vacuum energy on temperature suggests the modification of the thermodynamic Gibbs-Duhem relation for the quantum vacuum and to the reformulation of the vacuum pressure. The conventional vacuum pressure $P_{\rm vac}$ obeys the equation of state $w = -1$ and enters the energy momentum tensor of the vacuum medium in the form:

$$T^{\mu\nu} = \Lambda g^{\mu\nu} = {\rm diag}(\epsilon_{\rm vac}, P_{\rm vac}, P_{\rm vac}, P_{\rm vac}) \ , \ P_{\rm vac} = -\epsilon_{\rm vac} \,. \tag{43}$$

In the de Sitter state the vacuum pressure is negative, $P_{\rm vac} = -\epsilon_{\rm vac} < 0$.

This pressure $P_{\rm vac}$ does not satisfy the standard thermodynamic Gibbs-Duhem relation, $Ts_{\rm vac} = \epsilon_{\rm vac} + P_{\rm vac}$, because the right hand side of this equation is zero. The reason for that is that in this equation we did not take into account the gravitational degrees of freedom of quantum vacuum. Earlier it was shown, that gravity contributes with the pair of the thermodynamically conjugate variables: the gravitational coupling $K = \frac{1}{16\pi G}$ and the scalar Riemann curvature $\mathcal{R}$, see Refs.[9,74,75]. The contribution of the term $K\mathcal{R}$ to thermodynamics is similar to the work density.[76–79]

The quantities K and $\mathcal{R}$ can be considered as the local thermodynamic variables, which are similar to temperature, pressure, chemical potential, number density, spin density, etc., in condensed matter physics. Indeed, since the de Sitter spacetime is maximally symmetric, its local structure is characterized by the scalar curvature alone, while all the other components of the Riemann curvature tensor are expressed via $\mathcal{R}$:

$$R_{\mu\nu\alpha\beta} = \frac{1}{12} \left(g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}\right) \mathcal{R} \,. \tag{44}$$

That is why the scalar Riemann curvature as the covariant quantity naturally serves as one of the thermodynamical characteristics of the macroscopic matter.[80,81] Another argument is related to the so-called Larkin-Pikin effect.[82] This is the jump in the number of degrees of freedom, when the fully homogeneous state is considered. One has the extra parameters, which are space independent, but participate in thermodynamics.[47,83,84] The same concerns the constant electric and magnetic fields *in vacuo*, which add three more degrees of freedom. These constant fields are mutually independent, in contrast to the spacetime-dependent fields connected by the Maxwell equations.[47] The scalar curvature $\mathcal{R}$ in the de Sitter vacuum, which is constant in space-time, also serves as such thermodynamic parameter. Then the gravitational coupling $K = df/d\mathcal{R}$ serves as the analog of the chemical potential, which is constant in the full equilibrium.

B. Gibbs-Duhem relation in $f(R)$ gravity

The new thermodynamic variables, K and $\mathcal{R}$, which come from the gravity, and Eq. (15) for the entropy density allow us to introduce the corresponding Gibbs-Duhem relation for de Sitter vacuum, which has the conventional form:

$$Ts_{\rm vac} = \epsilon_{\rm vac} + P_{\rm vac} - K\mathcal{R} \,. \tag{45}$$

This equation is obeyed, since $\epsilon_{\rm vac}+P_{\rm vac}=0$; $\mathcal{R}=-12H^2$; and $Ts_{\rm vac}=12\pi^2KT^2=12KH^2$, which supports the earlier proposal that K and $\mathcal{R}$ can be considered as the thermodynamically conjugate variables.[74,75]

The Eq.(45) can be also written using the effective vacuum pressure, which absorbs the gravitational degrees of freedom:

$$P=P_{\rm vac}-K\mathcal{R}\,. \tag{46}$$

Then the conventional Gibbs-Duhem relation is satisfied:

$$Ts_{\rm vac}=\epsilon_{\rm vac}+P\,. \tag{47}$$

The equation (47) is just another form of writing the Gibbs-Duhem relation (45). But it allows to make different interpretation of the de Sitter vacuum state. The introduced effective de Sitter pressure P is positive, $P=\epsilon_{\rm vac}>0$, and satisfies equation of state $w=1$, which is similar to matter with the same equation of state. As a result, due to the gravitational degrees of freedom, the de Sitter state has many common properties with the non-relativistic Fermi liquid, where the thermal energy is proportional to T^2, and also with the relativistic stiff matter with $w=1$ introduced by Zel'dovich.[85]

C. Entropy of cosmological horizon in terms of effective gravitational coupling

Let us show that equation $S_{\rm hor}=4\pi KA$ remains valid also in the $f(\mathcal{R})$ gravity, but with the gravitational coupling determined as the thermodynamic conjugate to the curvature. In the $f(\mathcal{R})$ gravity the action is:

$$S=-\int d^4x\sqrt{-g}f(\mathcal{R})\,. \tag{48}$$

The generalization of the modified Gibbs-Duhem relation for the de Sitter states (i.e. for the states with constant four-dimensional curvature) in the $f(\mathcal{R})$ gravity is:

$$Ts_{\rm vac}=\epsilon_{\rm vac}+P_{\rm vac}-K\mathcal{R}=-K\mathcal{R}\,, \tag{49}$$

$$\epsilon_{\rm vac}=f(\mathcal{R})-K\mathcal{R}\ ,\ K=\frac{df}{d\mathcal{R}}\,. \tag{50}$$

Here K is the natural definition of the variable, which is thermodynamically conjugate to the curvature $\mathcal{R}$, while $\epsilon_{\rm vac}$ serves as the corresponding thermodynamic potential. In the equilibrium de Sitter state the curvature is determined by equation:

$$2f(\mathcal{R})=\mathcal{R}\frac{df}{d\mathcal{R}}\,. \tag{51}$$

The value of the local entropy of the de Sitter state $s_{\rm vac}$ follows from Eq.(49), assuming that the local temperature of the equilibrium dS states is $T=H/\pi$. Then the total entropy of the Hubble volume V_H is given by the same Eq.(16):

$$S_{\rm bulk}=s_{\rm vac}V_H=4\pi KA=S_{\rm hor}\,. \tag{52}$$

But now K is the effective gravitational coupling in Eq.(50). This generalization of the Gibbons-Hawking entropy was discussed in Refs.[9,86–88]. But here it was obtained using the local thermodynamics of the de Sitter vacuum. This demonstrates that the local thermodynamics of the de Sitter vacuum is valid also for the $f(\mathcal{R})$ gravity. The effective gravitational coupling K serves as one of the thermodynamic variable of the local thermodynamics. This quantity plays the role of the chemical potential, which is thermodynamically conjugate to the curvature $\mathcal{R}$, and it is constant in the thermodynamic equilibrium state of de Sitter spacetime.

D. Example of quadratic gravity

For illustration, we consider an example of the modification of the gravitational coupling K in the de Sitter environment. In the conventional Einstein gravity, where $f(\mathcal{R})=K_0\mathcal{R}+\Lambda$, the de Sitter state has the equilibrium value of the curvature, $\mathcal{R}_0=-2\Lambda/K_0=-12H^2$. Let us add the quadratic term to the Einstein action:[9,86]

$$f(\mathcal{R})=K_0\mathcal{R}-p\mathcal{R}^2+\Lambda\,. \tag{53}$$

Then one obtains the following equations for the equilibrium value of the curvature $\mathcal{R}_0$, the entropy of the Hubble volume $S_{\rm hor}$ and the equilibrium value of the effective coupling K:

$$\mathcal{R}_0 = -2\frac{\Lambda}{K_0} = -12H^2\,, \tag{54}$$

$$S_{\rm hor} = s_{\rm vac}V_H = 4\pi K A\,, \tag{55}$$

$$K = \frac{df}{d\mathcal{R}}\Big|_{\mathcal{R}=\mathcal{R}_0} = K_0 + p\frac{\Lambda}{K_0}\,. \tag{56}$$

The equilibrium curvature in the de Sitter space $\mathcal{R}_0$ is obtained from Eq.(51). It is the same as in Einstein gravity, because the quadratic terms in Eq.(51) are cancelled. The local entropy $s_{\rm vac}$, which follows from Eq.(49), is determined by the modified gravitational coupling K. As a result, the entropy of the Hubble volume in Eq.(55), which we identify with the entropy of the horizon $S_{\rm hor}$, is also determined by the modified coupling K. The latter is given by Eq.(56).

The local entropy $s_{\rm vac}$ changes sign for $K < 0$, while the cosmological expansion is still described by the de Sitter metric. However, the negative K requires the negative parameter $p < 0$, which marks the instability of such de Sitter vacuum.[86]

VI. FROM DE SITTER THERMODYNAMICS TO DE SITTER DECAY

A. de Sitter state as thermal bath for matter

The extension of the thermodynamics to the $f(\mathcal{R})$ gravity supports the idea that the de Sitter vacuum is the thermal state with the local temperature $T = H/\pi$. It is interesting that the gravitational temperature, which is twice the Hawking temperature, has been also obtained in the de Sitter limit in Ref.[89], see the footnote 2 on page 4.

The nonzero local temperature of the gravitational vacuum suggests that the de Sitter vacuum is locally unstable towards the creation of thermal matter from the vacuum by thermal activation. This is distinct from the mechanism of creation of the pairs of particles by Hawking radiation from the cosmological horizon, which may or may not lead to the decay of the vacuum energy. There are still controversies concerning the stability of the de Sitter vacuum caused by Hawking radiation, see e.g.[84,90,91] and references therein.

B. de Sitter decay due to thermalization of matter by de Sitter heat bath

To describe the decay of the vacuum due to activation and thermalization of matter, the extension of the Starobinsky analysis of the vacuum decay[92–95] is needed. Especially the revolutionary stochastic inflation approach pioneered by Starobinsky[96,97] is extremely useful, although it requires some modifications.[98–101] This also includes the so-called separate universe approach, which is somewhat similar to the multi-metric gravity discussed in Sec. III. Here we consider the simple phenomenological scenario based on the energy exchange between the thermal de Sitter and the thermal matter.

The thermal exchange between the de Sitter heat bath and the excited matter generates the thermal relativistic gas. The temperature of relativistic gas tends to approach the temperature $T = H/\pi$ of the de Sitter background. Then the energy density of this matter ϵ_M tends to approach the value $\epsilon_M \sim T^4$. In terms of the Hubble parameter, $\epsilon_M \to bH^4$, were the dimensionless parameter b depends on the number of massless relativistic fields. For example, $b = 7N_F/120\pi^2$ for N_F species of massless Weyl fermions.

The energy exchange between the vacuum heat bath and matter can be described by the following dynamical modification of the Friedmann equations,[102] where the dissipative Hubble friction equation $\partial_t\epsilon_M = -4H\epsilon_M$ is extended to

$$\partial_t\epsilon_M = -4H(\epsilon_M - bH^4)\,. \tag{57}$$

This equation describes the tendency of matter to approach the local temperature of the vacuum, $T = H/\pi$. The extra gain of the matter energy, $4bH^5$, must be compensated by the corresponding loss of the vacuum energy:

$$\partial_t\epsilon_{\rm vac} = -4bH^5\,, \tag{58}$$

Here we use for simplicity the conventional general relativity with $\epsilon_{\rm vac} = \Lambda = -\frac{1}{2}K_0\mathcal{R}$. This phenomenological description of the energy exchange between the de Sitter vacuum and matter does not depend on the details of the microscopic (UV) theory, and requires only the slow-roll condition, i.e. the slow variation of the Hubble parameter, $|\dot{H}| \ll H^2$, or which is the same $|\dot{T}| \ll T^2$.

Since the vacuum energy density is $\epsilon_{\rm vac} \propto KH^2$, one obtains from Eq.(58) the following time dependence of the Hubble parameter and of energy densities:

$$H \sim M_{\rm Pl} \left(\frac{t_{\rm Pl}}{t+t_0}\right)^{1/3} , \tag{59}$$

$$\epsilon_{\rm vac} \sim M_{\rm Pl}^4 \left(\frac{t_{\rm Pl}}{t+t_0}\right)^{2/3} , \tag{60}$$

$$\epsilon_M = bH^4 \sim M_{\rm Pl}^4 \left(\frac{t_{\rm Pl}}{t+t_0}\right)^{4/3} . \tag{61}$$

Here $M_{\rm Pl}$ is the Planck mass, $M_{\rm Pl}^2 = K$, and $t_{\rm Pl} = 1/M_{\rm Pl}$ is Planck time. We assume that $t_0 \gg t_{\rm Pl}$, and thus $\dot{H} \ll H^2$.

Thus the thermal character of the de Sitter state determines the process of its decay. The obtained power law decay of H in Eq.(59) was also found in Refs.[103–108], although using different approaches. In the Padmanabhan model[103,104] the de Sitter horizon is considered as the photosphere with the Gibbons-Hawking temperature and with the radiative luminosity $dE/dt \propto T^4 A_H$, where $A_H = 4\pi/H^2$ is the area of horizon. Since the energy of the Hubble volume is $E \sim M_{\rm Pl}^2/H$, and $T^4 A_H \sim H^2$, this leads to Eq.(60) for the vacuum energy density. As was mentioned by Padmanabhan,[104] in his model the late time cosmological constant is independent of the initial value, see Eq.(60) at $t \gg t_0$.

In Refs.[105–107] the Starobinsky stochastic inflation approach has been used. The parameter b therein is proportional to the number N of conformal fields and the parameter t_0 is related to the initial value of the Hubble parameter at the beginning of inflation at $t = 0$:

$$H(t=0) \sim M_{\rm Pl} \left(\frac{t_{\rm Pl}}{t_0}\right)^{1/3} \ll M_{\rm Pl} . \tag{62}$$

This $H(t = 0)$ corresponds to the scaleron mass M in Starobinsky inflation. The time $t_0 \sim E_{\rm Pl}^2/H_{t=0}^3$ is called the quantum breaking time of space-times with positive cosmological constant.[109,110]

So, the scenario of thermalization of matter by the de Sitter heat bath gives Eqs. (57) and (58). They produce the inflation in terms of two phenomenological parameters, b and t_0, which determine the decay of the vacuum energy density:

$$\epsilon_{\rm vac}(t) = 6M_{\rm Pl}^4 \left(\frac{t_{\rm Pl}}{b(t+t_0)}\right)^{2/3} . \tag{63}$$

C. de Sitter decay and Zel'dovich stiff matter

As was mentioned by Padmanabhan,[104] his model leads to a late time cosmological constant in Eq.(63), which is independent of the initial value, but its value is still far too large. Can we fix this? In Sec. V B we obtained that the thermodynamics of de Sitter thermal bath corresponds to the Zel'dovich stiff matter with $w = 1$. Let us try the stiff matter scenario using the phenomenological approach.

For the matter with $w = 1$, the dissipative Hubble friction equation is $\partial_t \epsilon_{\rm stiff} = -6H\epsilon_{\rm stiff}$. In our case, the stiff matter analog is thermal, with linear in T entropy density, and its temperature tends to approach the heat bath temperature $T = H/\pi$. Then instead of Eqs.(57) and (58) one obtains

$$\partial_t \epsilon_{\rm stiff} = -6H(\epsilon_{\rm stiff} - \tilde{b}H^2) , \tag{64}$$

and

$$\partial_t \epsilon_{\rm vac} = -6\tilde{b}H^3 . \tag{65}$$

This gives the following power law decay of the vacuum energy density:

$$\epsilon_{\rm vac} \sim M_{\rm Pl}^4 \left(\frac{t_{\rm Pl}}{t+t_0}\right)^{2} , \tag{66}$$

with the reasonable value of the vacuum energy density in the present time:

$$\epsilon_{\rm vac}(t = t_{\rm present}) \sim \frac{M_{\rm Pl}^2}{t_{\rm present}^2} \,. \tag{67}$$

The same behaviour of the vacuum energy was obtained in Ref.[9] (see also Ref.[46]). This was also obtained in Ref.[111], where the Polyakov scenario[90] of the infrared instability of the de Sitter space was discussed.

Note that in this approach, the analog of the stiff matter comes from the gravitational degrees of freedom. In this sense it has relations to Refs.[112–114] and references therein, where the role of the gravitational degrees of freedom is discussed. Since in this approach the stiff matter is on the order of vacuum energy density, $\epsilon_{\rm stiff} \sim \epsilon_{\rm vac}$, these gravitational degrees of freedom can be responsible for dark matter.[115]

So, the phenomenological approach to the vacuum energy decay may produce different power-law decays: in Eq.(66) and in Eq.(63). But it is not excluded that these two asymptotic laws correspond to different epochs.

VII. CONCLUSION

The local thermodynamics of the de Sitter state in the Einstein gravity gives rise to the Gibbons-Hawking area law for the total entropy inside the cosmological horizon. Ee extended the consideration of the local thermodynamics to the $f(\mathcal{R})$ gravity and obtained the same area law, but with the modified gravitational coupling $K = df/d\mathcal{R}$. The agreement with the traditional global thermodynamics of de Sitter supports the suggestion that the de Sitter vacuum is the thermal state with the local temperature $T = H/\pi$, and that the local thermodynamics is based on the thermodynamically conjugate gravitational variables K and $\mathcal{R}$. The variable K plays the role of the chemical potential, which is constant in the thermal equilibrium. The gravitational variables modify the thermodynamic Gibbs-Duhem relation, due to which the de Sitter thermal state becomes similar to the Zel'dovich stiff matter.

The local temperature $T = H/\pi$ has the definite physical meaning. It is temperature, which is experienced by the external object in the de Sitter environment. In particular, this temperature determines the local activation processes, such as the process of ionization of an atom in the de Sitter environment. The nonzero local temperature of the de Sitter state suggests the thermal instability of this state due to the thermalization of matter by the de Sitter heat bath. The process of thermalization of matter with the corresponding decay of the vacuum energy density determines the so-called quantum breaking time of the space-times with positive cosmological constant. The thermalization of matter by the de Sitter heat bath leads to the power law decay of the vacuum energy density, which is described by two phenomenological parameters. This behaviour is rather universal, and in particular it reproduces the result of the Padmanabhan model.[104] On the other hand, the analogy with the Zel'dovich stiff matter suggests another universal behaviour, which leads to the reasonable value of the vacuum energy density in the present time.

The obtained connection between the bulk entropy of the Hubble volume, and the surface entropy of the cosmological horizon suggests a kind of the bulk-surface correspondence, which may have the holographic origin.[116–118] It would be interesting to check this correspondence using the more general extensions of the Einstein gravity and also different types of the generalized entropy.[79,119–122]

We also discussed the thermodynamics of de Sitter in the frame of the statistical ensemble of the multi-metric gravities. The heat exchange between different "sub-Universes" in the ensemble leads to the common de Sitter expansion with the common temperature $T = H/\pi$. Application of the local thermodynamics to the entropy of the Schwarzschild black hole was also considered. We obtained the Bekenstein-Hawking entropy of black hole from the negative entropy of the contracting de Sitter core of the gravastar object.

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