# Nodal precession of a hot Jupiter transiting the edge of a late A-type star TOI-1518 

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#### Abstract

TOI-1518b, a hot Jupiter around a late A-type star, is one of the few planetary systems that transit the edge of the stellar surface (the impact parameter $b \sim 0.9$ ) among hot Jupiters around hot stars (Cabot et al. 2021). The high rotation speed of the host star ( $\sim 85 \mathrm{~km} \mathrm{~s}^{-1}$ ) and the nearly polar orbit of the planet ( $\sim 120^{\circ}$ ) may cause a nodal precession. In this study, we report the nodal precession undergone by TOI-1518b. This system is the fourth planetary system in which nodal precession is detected. We investigate the time change in $b$ from the photometric data of TOI-1518 acquired in 2019 and 2022 with TESS and from the spectral transit data of TOI-1518b obtained in 2020 with two high-dispersion spectrographs; CARMENES and EXPRES. We find that the value of $b$ is decreasing with $d b / d t=-0.0116 \pm 0.0036$ year $^{-1}$, indicating that the transit trajectory is moving toward the center of the stellar surface. We also estimate the minimum value of the quadrupole mass moment of TOI-1518 $J_{2, \min }=4.41 \times 10^{-5}$ and the logarithm of the Love number of TOI- $1518 \log k_{2}=-2.17 \pm 0.33$ from the nodal precession.


Key words: planetary systems - planets and satellites: individual (TOI-1518b) — techniques: spectroscopic - techniques: photometric

## 1 Introduction

To date, 20 hot Jupiters have been discovered around hot stars whose effective temperatures are above $7,000 \mathrm{~K}$. These hot stars have a wide range of obliquities, that is, angles between the stellar rotational and orbital axes. The observed spin-orbit misalignment trends of hot Jupiters around hot stars imply that they did not undergo tidal realignment because of their shallow convective envelopes (Albrecht et al. 2012). Hot stars barely sustain stellar winds that lose their spin angular momentum due to mag-
netic braking. Hot stars tend to rotate rapidly as is known for the Kraft break (Kraft 1967). The oblateness of fastrotating stars causes nodal precession of hot Jupiters in misaligned orbits. Nodal precession of three hot Jupiters on nearly polar orbits around rapidly rotating hot stars: Kepler-13Ab (Szabó et al. 2012; Szabó et al. 2014; Herman et al. 2018) and WASP-33b (Johnson et al. 2015; Watanabe et al. 2020; Watanabe et al. 2022; Stephan et al. 2022), and KELT-9b (Stephan et al. 2022), were detected. The nodal precession of a planet enables us to restrict the quadrupole mass moment $J_{2}$ and the Love number $k_{2}$. $J_{2}$ indicates
the oblateness of the host star and its internal mass redistribution due to its rapid rotation. The Love number $k_{2}$ expresses the rigidity of the internal structure, which could be an important clue for understanding the susceptibility to the tidal effects.

Cabot et al. (2021) have discovered a hot Jupiter (planetary radius: $R_{p}=1.875 \pm 0.053 R_{J}$, orbital period: $P_{\text {orb }}=1.902603 \pm 0.000011$ days, scaled semi-major axis: $\left.a / R_{s}=4.291_{-0.061}^{+0.057}\right)$ around a rapidly-rotating late Atype star TOI-1518 with the effective temperature $T_{\text {eff }}=$ $7300 \pm 100 \mathrm{~K}$ and the projected rotational speed $V \sin i_{s}=$ $85.1 \pm 6.3 \mathrm{~km} \mathrm{~s}^{-1}$, where $i_{s}$ is the angle between the stellar spin axis and the line of sight. They have measured the projected spin-orbit obliquity $\lambda=-119^{\circ} 66_{-0.93}^{+0.98} *$ and the impact parameter $b=0.9036_{-0.0053}^{+0.0061}$, indicating that the planet transits the edge of the stellar surface in a near-polar orbit. If the orbit of TOI- 1518 b shifts toward a larger b via nodal precession, then the transit of TOI-1518b will finish in several decades.

Section 2 presents our measurements of the impact parameter of TOI-1518b from the photometric data of The Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2015) and transit spectral data from high-resolution spectrographs. In Section 3, we describe the change of the impact parameter and discuss the nodal precession results. Finally, we present our conclusions in Section 4.

## 2 Observations and Analysises

### 2.1 TESS Photometry

TESS observed TOI-1518 from UT 2019 October 7 to November 27 (Sectors 17 and 18) and from UT 2022 September 30 to November 26 (Sectors 57 and 58). Cabot et al. (2021) utilized the TESS-SPOC HLSP light curves (Caldwell et al. 2020) in 2019. In addition to these datasets, we acquired the datasets of the SPOC light curves (Jenkins et al. 2016) in 2022 for TOI-1518 from the Mikulski Archive for Space Telescopes (MAST). The exposure times are 30 minutes in Sectors 17 and 18, and 2 minutes in Sectors 57 and 58. We used the Presearch Data Conditioning SAP (PDCSAP) light curves, which were included in the SPOC datasets and corrected for systematic trends using other sources on the TESS detector. Subsequently, we excluded the small discontinuities and flux ramps due to the momentum dumps, and the dimming parts due to the secondary eclipse. Figure 1 shows the light curves of TOI-1518 obtained from TESS data.

We created light-curve models of TOI-1518 using

[^0]PyTransit (Parviainen 2015) by a supersampling method to calculate an accurate model of the transit light curve. As the eccentricity is negligible ( $e=0.0031_{-0.0022}^{+0.0047}$; Cabot et al. 2021), which is represented by a circular orbit. Then we used a Matern $3 / 2$ kernel for the Gaussian process to fit the wavelet-shaped features using celerite (ForemanMackey et al. 2017).

We fitted the light curves in two epochs to the models with the following 12 parameters using Markov Chain Monte Carlo (MCMC): impact parameter $b$ in each year, two hyperparameters of Matern 3/2 kernel, the radius ratio $R_{p} / R_{s}$, mid-transit time $T_{0}$, the orbital period $P_{\text {orb }}$, semimajor axis normalized by stellar radii $a / R_{s}$, two quadratic limb darkening coefficients $u_{1}$ and $u_{2}$, and photometric jitter term $\sigma_{\mathrm{jit}}$ in each year. For this fitting, we set the logarithm of the likelihood, $\ln L_{\text {like }}$ as
$\ln L_{\text {like }}=-\frac{1}{2}\left(\mathbf{r}^{T} \mathbf{K}_{\text {ker }}^{-1} \mathbf{r}+\ln \left|\mathbf{K}_{\text {ker }}\right|\right)$,
where $\mathbf{r}$ is a series of residuals obtained by subtracting the model data from the observation data and $\mathbf{K}_{\text {ker }}$ is the kernel whose element is described as follows:

$$
\begin{align*}
k_{i, j}= & \alpha^{2}\left(1+\frac{\sqrt{3}\left|t_{i}-t_{j}\right|}{l}\right) \exp \left(-\frac{\sqrt{3}\left|t_{i}-t_{j}\right|}{l}\right) \\
& +\left\{\left(\sigma_{i}^{2}+\sigma_{\mathrm{jit}}^{2}\right) \delta_{i, j}\right\} . \tag{2}
\end{align*}
$$

$t_{i}$ and $t_{j}$ are the observation times, $\sigma_{i}$ is the error of data point $i$ and $\delta_{i, j}$ is the Kronecker delta. To calculate these parameter values, we ran 10,000 steps, cut off the first 5,000 steps as burn-ins, and iterated this set 32 times. Table 1 shows the fitted values and priors for the MCMC fitting, and Figure 6 in the Appendix shows the posterior distributions.

### 2.2 Doppler Tomographic Observation

We utilized the reduced transit spectral dataset of TOI-1518b from the 3.5 m telescope with CARMENES (Quirrenbach et al. 2014), a high-resolution spectrograph ( $R \sim 94,600$ ), at the Calar Alto Observatory on UT 2020 October 8. These datasets are reduced using the CARACAL pipeline (Zechmeister et al. 2014; Bauer et al. 2015) automatically at the end of the observation. We used a wavelength range from $5180 \AA$ to $7860 \AA$ except for the wavelength regions around the noticeable telluric lines and bad pixels. The dataset contains 14 spectra obtained with an exposure time of 900 s . The signal-to-noise ratio per pixel for each spectrum is $\sim 100$ at $5500 \AA$. We took the continua of these spectra and shifted them to a barycentric frame with astropy (Astropy Collaboration et al. 2022) to read these fits data. We then adopted the least-squares deconvolution (LSD; Donati et al. 1997, Kochukhov et al.


Fig. 1. The normalized light curve of TOI-1518 in 2019 (top figure) and the one in 2022 (bottom figure) from Presearch Data Conditioning SAP (PDCSAP). The vertical blue lines show the scheduled momentum dumps. The red points are the dimming parts caused by the secondary eclipse and the small discontinuities and flux ramps, which are excluded from our analysis.

Table 1. Parameters of TOI-1518b from the light curve of TESS. In the column of "Prior", we describe uniform priors as $\mathcal{U}$ (lower bound, upper bound) and normal priors as $\mathcal{N}$ (mean, standard deviation).

| Parameter | Fitted Value | Prior for Fitting |
| :--- | :---: | :---: |
| Impact Parameter in 2019 $b_{2019}$ | $0.91497_{-0.00089}^{+0.00093}$ | $\mathcal{U}(0,1)$ |
| Impact Parameter in $2022 b_{2022}$ | $0.8797 \pm 0.0011$ | $\mathcal{U}(0,1)$ |
| Orbital Period (days) $P_{\text {orb }}$ | $1.90261178_{-0.000000018}^{+0.000019}$ | $\mathcal{U}(0,10)$ |
| Mid-transit Time $T_{0}$ (BJD TDB) | $2458766.120581+0.0000098$ | $\mathcal{U}(2458766,2458767)^{\S}$ |
| Radius Ratio $R_{p} / R_{s}$ | $0.10034 \pm 0.00019$ | $\mathcal{U}(0,0.5)$ |
| Scaled Semi-major Axis $a / R_{s}$ | $4.164_{-0.014}^{+0.015}$ | $\mathcal{U}(0,20)$ |
| Limb-darkening Coefficient $u_{1, \text { TESS }}$ | $0.3323_{-0.0015}^{+0.00016}$ | $\mathcal{N}(0.3360,0.0019)^{*}$ |
| Limb-darkening Coefficient $u_{2, \text { TESS }}$ | $0.1625_{-0.0026}^{+0.0025}$ | $\mathcal{N}(0.1587,0.0031)^{*}$ |
| Log of Hyper Parameter of Amplitude Scale ln $\sigma$ | $-8.739_{-0.058}^{+0.058}$ | $\mathcal{U}(-14,14)$ |
| Log of Hyper Parameter of Length Scale ln $\rho(\mathrm{days})$ | $-1.068_{-0.087}^{+0.087}$ | $\mathcal{U}(-7,7)$ |
| Photometric Jitter Term in $2019 \sigma_{\mathrm{jit}, 2019}(\mathrm{ppm})$ | $88.5_{-5}^{+5.6}$ | $\mathcal{U}(0,10000)$ |
| Photometric Jitter Term in $2022 \sigma_{\mathrm{jit}, 2022}(\mathrm{ppm})$ | $147.2_{-7.7}^{+7.3}$ | $\mathcal{U}(0,10000)$ |

[^1]2010) to extract each line profile for each exposure. In this method, we regard a continuum of the observed spectrum as a convolution of a line profile and a series of delta functions, which can be obtained from a list of the absorption lines calculated using the Vienna Atomic Line Database (VALD; Kupka et al. 2000). After creating the line profiles, we made them smoother by averaging the five surrounding values for each data point.

We also analyzed the transit spectral dataset captured by EXPRES, whose resolution is also high ( $R \sim 150,000$ ), mounted on the Lowell Discovery Telescope (Levine et al. 2012) on UT 2020 August 2. This dataset was used to detect its planetary shadow once in Cabot et al. (2021). This dataset includes 41 spectra with an exposure time of 300 s and has already been shifted to a barycentric frame. These spectra have a signal-to-noise ratio per pixel of $\sim$ 30. We then corrected the continuum and telluric lines included in these fits data. The wavelength range adopted in this study is from $3990 \AA$ to $6540 \AA$ except for deep and wide lines, such as the Na D and $\mathrm{H} \beta$ lines. The process of extracting each smooth line profile for each exposure is the same as that used to analyze the CARMENES dataset.

The line profile of the host star TOI-1518 was created by averaging line profiles during out-of-transit. To expose a dark track, called a planetary shadow, we subtracted the stellar line profile from each exposure line profile. We jointly fitted the stellar line profile and observed a planetary shadow to the models that adopted the MCMC using EMCEE. The model of the stellar line profile is composed of an intrinsic stellar line profile and a broadening kernel due to stellar rotation and macro-turbulence (Hirano et al. 2011). Considering the effect of stellar macro-turbulence, we modeled the planetary shadow by combining Equation (10) in Hirano et al. (2011) and the equations in the Appendix of Watanabe et al. (2020).

We derived the values of the following 20 parameters for MCMC fitting: impact parameter in $2020 b_{2020}$ of each instrument, spin-orbit obliquity $\lambda$ of each instrument, the orbital period $P_{\text {orb }}$, mid-transit time $T_{0}$, the radius ratio $R_{p} / R_{s}$, semi-major axis normalized by stellar radii $a / R_{s}$, two quadratic limb darkening coefficients $u_{1}$ and $u_{2}$ of CARMENES and EXPRES, stellar rotational velocity $V \sin i_{s}$, macro-turbulence velocity $v_{\text {mac }}$, FWHM of intrinsic stellar line profile $v_{\text {FWHM }}$, radial velocity of the planetary system $\gamma$, jitter terms for the stellar line profiles $\sigma_{\mathrm{jit}, \text { ste }}$ of CARMENES and EXPRES, and jitter terms for the planetary shadow $\sigma_{\mathrm{jit}, \mathrm{ps}}$ of CARMENES and EXPRES. Here, we estimated the intrinsic stellar line profile as a Gaussian profile. In the MCMC fitting, we set the logarithm of the probability for each dataset of the stellar profile and planetary shadow to

$$
\begin{align*}
\ln L_{\mathrm{prob}}= & -\frac{1}{2} \sum_{i}\left[\frac{\left(O_{i}-C_{i}\right)^{2}}{\sigma_{i}^{2}+\sigma_{\mathrm{jit}}^{2}}+n \ln \left\{2 \pi\left(\sigma_{i}^{2}+\sigma_{\mathrm{jit}}^{2}\right)\right\}\right] \\
& -\frac{1}{2} \sum_{j}\left(\frac{p_{j}-\mu_{j}}{s_{j}}\right)^{2} . \tag{3}
\end{align*}
$$

The first term in Equation (3) shows the logarithm of the likelihood, where $O_{i}$ is the data, $C_{i}$ is the model, and $\delta_{i}$ is the error of the $i$ th data point. The second term represents the Gaussian priors; where $p_{j}$ is the parameter value, $\mu_{j}$ is the center value of the Gaussian prior, and $s_{j}$ is the uncertainty of the Gaussian prior. We ran 20,000 steps, cut off the first 10,000 steps as burn-in, and iterated this set 48 times. Figure 7 in the Appendix 1 shows the posterior distributions.

We also derived these values using a bootstrap analysis for checking systematic errors. In this technique, we first compute the residuals by subtracting the best-fit model from the maximization of Equation 3 for the line profile data and Doppler tomographic data. We then randomly shuffled the residuals with their errors and created new line profile data for the out-of-transit and Doppler tomographic datasets by adding the residuals and the best-fitting model. A total of 200 fake datasets were created. We then executed MCMC fitting by running 4,000 steps, excluding the first 3,000 steps, and iterating 40 times for each mimic dataset. Figure 8 in the Appendix shows the distributions of the optimum values. The systematic errors are negligible because the posterior distributions from the MCMC and the bootstrap methods are comparable. Consequently, we adopted the results from the MCMC method.

## 3 Result and Discussion

Figure 2 shows the phase-folded TESS light curves and the best-fitting light curve models. Figure 3 displays the line profile residuals and the best-fitted models. Figure 4 shows the averaged line profiles during out-of-transit and best-fitted line-profile models. Tables 1 and 2 list the best values and priors for the MCMC fitting from the transit photometric and spectral observations, respectively. Our values of $P_{\text {orb }}, T_{0}$ and $R_{p} / R_{s}$ are consistent with those of Cabot et al. (2021) within $\sim 1 \sigma\left(P_{\text {orb }}=1.902603 \pm 0.000011\right.$ days, $T_{0}=2458787.049255 \pm 0.000094 \mathrm{BJD}_{\mathrm{TDB}}{ }^{\dagger}, R_{p} / R_{s}=$ $0.0988_{-0.0012}^{+0.0015}$ in Cabot et al. 2021). However, our values of $\lambda$ from EXPRES, $b$ in 2019 and $a / R_{s}$ and differ from those in Cabot et al. (2021) by $\sim 2 \sigma\left(\lambda_{E}=-119^{\circ} 66_{-0.93}^{+0.98}\right.$, $b_{2019}=0.9036_{-0.0053}^{+0.0061}, a / R_{s}=4.291_{-0.061}^{+0.057}$ in Cabot et al.

[^2]Table 2. Measured parameters of TOI-1518b from CARMENES and EXPRES. In the column of "Priorfor Fitting", we describe uniform priors as $\mathcal{U}$ (lower bound, upper bound) and normal priors as $\mathcal{N}$ (mean, standard deviation).

| Parameter | MCMC (Adopted) | Bootstrap | Prior for Fitting |
| :---: | :---: | :---: | :---: |
| Impact Parameter in 2020 from CARMENES $b_{2020, \mathrm{C}}$ | $0.8953{ }_{-0.0048}^{+0.0050}$ | $0.8954_{-0.0047}^{+0.0051}$ | $\mathcal{U}(0,1)$ |
| Spin-orbit Obliquity from CARMENES $\lambda_{\mathrm{C}}$ (deg) | $-118.96_{-0.65}^{+0.68}$ | $-118.93_{-0.65}^{+0.66}$ | $\mathcal{U}(-180,180)$ |
| Impact Parameter in 2020 from EXPRES $b_{2020, \mathrm{E}}$ | $0.8574{ }_{-0.0043}^{+0.0051}$ | $0.8572_{-0.0043}^{+0.0049}$ | $\mathcal{U}(0,1)$ |
| Spin-orbit Obliquity from EXPRES $\lambda_{\mathrm{E}}$ (deg) | $-117.24_{-0.69}^{+0.64}$ | $-117.19_{-0.70}^{+0.62}$ | $\mathcal{U}(-180,180)$ |
| Orbital Period $P_{\text {orb }}$ (days) | $1.902611788_{-0.00000019}^{+0.0000018}$ | $1.90261178 \pm 0.00000019$ | $\mathcal{N}(1.90261178,0.00000019)^{\ddagger}$ |
| Mid-transit Time $T_{0}$ ( $\mathrm{BJD}_{\text {TDB }}$ ) | $2459064.83065 \pm 0.00010$ | $2459064.83066 \pm 0.00010$ | $\mathcal{N}(2459064.83063,0.00010)^{\ddagger \S}$ |
| Radius Ratio $R_{p} / R_{s}$ | $0.10043 \pm 0.00019$ | $0.10043 \pm 0.00019$ | $\mathcal{N}(0.10037,0.00020)^{\ddagger}$ |
| Scaled Semi-major Axis $a / R_{s}$ | $4.171_{-0.014}^{+0.015}$ | $4.171 \pm 0.015$ | $\mathcal{N}(4.167,0.014)^{\ddagger}$ |
| Limb-darkening Coefficient $u_{1}$ for CARMENES $u_{1, \mathrm{C}}$ | $0.4216_{-0.0022}^{+0.0021}$ | $0.4215 \pm 0.0021$ | $\mathcal{N}(0.4226,0.0021)^{*}$ |
| Limb-darkening Coefficient $u_{2}$ for CARMENES $u_{2, \mathrm{C}}$ | $0.1617_{-0.0034}^{+0.0031}$ | $0.1617 \pm 0.0032$ | $\mathcal{N}(0.1628,0.0032)^{*}$ |
| Limb-darkening Coefficient $u_{1}$ for EXPRES $u_{1, \mathrm{E}}$ | $0.5321_{-0.0028}^{+0.0027}$ | $0.5320_{-0.0027}^{+0.0028}$ | $\mathcal{N}(0.5314,0.0028)^{*}$ |
| Limb-darkening Coefficient $u_{2}$ for EXPRES $u_{2, \mathrm{E}}$ | $0.1672 \pm 0.0038$ | $0.1670 \pm 0.0038$ | $\mathcal{N}(0.1665,0.0038)^{*}$ |
| Apparent Stellar Rotational Velocity $V \sin i_{s}\left(\mathrm{~km} \mathrm{~s}^{-1}\right)$ | $76.624_{-0.052}^{+0.051}$ | $76.621 \pm 0.051$ | $\mathcal{U}(0,200)$ |
| Macro-turbulence Velocity $v_{\text {mac }}\left(\mathrm{km} \mathrm{s}^{-1}\right)$ | $13.90_{-0.50}^{+0.46}$ | $13.911_{-0.50}^{+0.47}$ | $\mathcal{U}(0,50)$ |
| FWHM of Gaussian Line Profile $v_{\text {FWHM }}\left(\mathrm{km} \mathrm{s}^{-1}\right)$ | $7.12_{-0.35}^{+0.36}$ | $7.12_{-0.36}^{+0.37}$ | $\mathcal{N}(3.5,1)^{\dagger}$ |
| Radial Velocity of System $\gamma\left(\mathrm{km} \mathrm{s}^{-1}\right)$ | $-12.671_{-0.039}^{+0.038}$ | $-12.668_{-0.038}^{+0.039}$ | $\mathcal{U}(-30,30)$ |
| Jitter for Stellar Line Profile of CARMENES $\sigma_{\mathrm{jit}, \text {,ste, } \mathrm{C}}$ | $0.00651_{-0.00047}^{+0.00052}$ | $0.00651_{-0.00047}^{+0.00051}$ | $\mathcal{U}(0,1)$ |
| Jitter for Planetary Shadow of CARMENES $\sigma_{\mathrm{jit}, \mathrm{ps}, \mathrm{C}}$ | $<0.0011$ (36) | $<0.0011$ (36) | $\mathcal{U}(0,1)$ |
| Jitter for Stellar Line Profile of EXPRES $\sigma_{\text {jiit,ste, }}$ | $0.00483_{-0.00041}^{+0.00042}$ | $0.00484_{-0.00041}^{+0.00044}$ | $\mathcal{U}(0,1)$ |
| Jitter for Planetary Shadow of EXPRES $\sigma_{\mathrm{jit}, \mathrm{ps}, \mathrm{E}}$ | $<0.0021$ (3 $\sigma$ ) | $<0.0021$ (3 $\sigma$ ) | $\mathcal{U}(0,1)$ |

[^3]

Fig. 2. Phase-folded light curves of TOI-1518b in 2019 (left panel) and 2022 (right panel). These light curves are subtracted using the Gaussian process. The gray and red points show the observed data and the 10 -minute binned data, respectively. The cyan lines show the best-fit light curve models. The bottom panels show the residuals between the observed data and model data.



Fig. 3. Doppler tomographic datasets of TOI-1518b from CARMENES (left) and EXPRES (right). Top panel: Observed residuals of line profile series from CARMENES. Middle panel: Model of planetary shadows using best-fit values via MCMC. Bottom panel: Difference between the top and the middle panels.
2021). In Table 2, the values and uncertainties from the bootstrap are comparable to those from MCMC fitting. Here, we adopted the values obtained via the MCMC as the measured values.

The values of $b$ in 2020 from CARMENES and EXPRES differ by $5 \sigma$ in both analyses. Additionally, Figure 5 shows that the measured $b$ from CARMENES agrees with that from the precession model, whereas that from EXPRES disagrees with the model. Concerning the dominant stellar rotational angular moment and orbital angular moment in this system, the impact parameter should not change immediately over several months. Observed planetary shadow from EXPRES in Figure 3 is less continual than that from CARMENES because the signal-to-noise of EXPRES is low. This would be the cause of the discrepancies. This may imply that the precession of TOI-1518b
has a short-term variation due to other factors; a resonant normal mode of the host star driven by tidal excitation is one of the possible causes of the variation (Alexander 2023).

From the values calculated in Section 2, we derived the change in the impact parameter of TOI-1518b using weighted least squares. The value of $b$ is decreasing with $d b / d t=-0.0116 \pm 0.0036 \mathrm{yr}^{-1}$. Figure 5 shows the changes in $b$ for TOI-1518b. If $d b / d t$ of TOI-1518b is constant while its transit trajectory is on the stellar disk, this transit would have begun at $2003_{-7}^{+4} \mathrm{CE}$ and be going to end at $2194_{-39}^{+70} \mathrm{CE}$.

We derived an equation to estimate the nodal precession speed from $d b / d t$. Given the short orbital period of a planet and the rapid rotation of its host star, we can consider the stellar rotational vector as a steady vector. In


Fig. 4. Averaged line profiles of TOI-1518 during out-of-transit from CARMENES (left, averaged 5 exposures) and EXPRES (right, averaged 15 exposures). The black dots are the observed data, and the red line shows the line profile model. The bottom panels show the residuals between the observed and model data. The different number of exposures should be a cause for their equivalent error bars although their signal-to-noises differ.
this case, $b$ and $\lambda$ are expressed as follows.

$$
\begin{equation*}
b(t)=\frac{a}{R_{s}}\left(\cos \psi \cos i_{s}+\sin \psi \sin i_{s} \cos \theta(t)\right) \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
\tan \lambda(t)=\frac{\sin \psi \sin \theta(t)}{\sin \psi \cos i_{s} \cos \theta(t)-\cos \psi \sin i_{s}}, \tag{5}
\end{equation*}
$$

where $\psi$ and $\theta(t)$ are the real spin-orbit obliquity and the nodal angle, respectively (Watanabe et al. 2022). The derivation of Equations (4) and (5) is written in the Appendix 2. From Iorio (2011), $\psi$ can be described as

$$
\begin{equation*}
\cos \psi=\frac{b(t) R_{s} \cos i_{s}}{a}+\sin i_{s} \cos \lambda(t) \sqrt{1-\left(\frac{b(t) R_{s}}{a}\right)^{2}} . \tag{6}
\end{equation*}
$$

Using Equations (4), (5), and (6), the nodal precession speed $d \theta / d t$ can be expressed as

$$
\begin{equation*}
\frac{d \theta}{d t}=\frac{1}{\sin i_{s} \sin \lambda \sqrt{\left(\frac{a}{R_{s}}\right)^{2}-b^{2}}} \frac{d b}{d t} \tag{7}
\end{equation*}
$$

where we assume that $\psi$ and $i_{s}$ are constants and $\theta$ is a time-variable. Barnes et al. (2013) also give the expression of $d \theta / d t$ for a planet in a circular orbit as

$$
\begin{equation*}
\frac{d \theta}{d t}=-\frac{3 \pi J_{2} R_{s}^{2} \cos \psi}{P_{\text {orb }} a^{2}} \tag{8}
\end{equation*}
$$

where $J_{2}$ is the stellar quadrupole moment.
We impose a-priori constraints on $i_{s}$ (Iorio 2011),
$\sin i_{s}>V_{s} \sqrt{\frac{R_{s}}{G M_{s}}}$,
where $V_{s}\left(=V \sin i_{s} / \sin i_{s}\right)$ denotes the stellar rotation
speed. This equation was derived based on the condition that the gravitational acceleration at the stellar surface should be greater than the centrifugal acceleration at the stellar equator. The value of $i_{s}$ for TOI-1518 is estimated to be $9.71 \leq i_{s} \leq 170 .{ }^{\circ} 29$ using our value of $V \sin i_{s}$ for $V_{s}$ and the values of $R_{s}$ and $M_{s}$ from Cabot et al. (2021). Then, we set the range of $\psi$ to $81.84 \leq$ $\psi \leq 121.98$ from Equation (6) using our values of $b, a / R_{s}$, and $\lambda$ within an $1 \sigma$-confidence level. However, considering $d b / d t<0, \sin \lambda<0$ and $\sin i_{s}>0, d \theta / d t$ must be positive from Equation (7), and $\psi$ must be greater than $90^{\circ}$ from Equation (8). Thus, the possible ranges of $\psi$ and $i_{s}$ should be $90^{\circ}<\psi \leq 121.98$ and $22^{\circ} 86 \leq i_{s} \leq 170^{\circ} 28$, respectively. Therefore, we can set the limit of the nodal precession speed as $0 .{ }^{\circ} 13 \mathrm{yr}^{-1} \leq d \theta / d t \leq 1 .{ }^{\circ} 43 \mathrm{yr}^{-1}$ from Equation (7), and the lower limit of the stellar quadrupole moment of TOI-1518 as $J_{2, \text { min }}=4.41 \times 10^{-5}$ from equation (5). The minimum value of $J_{2}$ for TOI-1518 indicates that the shape of TOI-1518 is more oblate than that of the $\operatorname{Sun}\left(J_{2, \odot} \sim 2 \times 10^{-7}\right.$; Roxburgh 2001). TOI-1518 has a flattened shape like other rapidly-rotating hot stars such as Kepler-13A $\left(J_{2}=(6.1 \pm 0.3) \times 10^{-5}\right.$; Masuda 2015 $)$, WASP-33 $\left(J_{2}=\left(1.36_{-0.12}^{+0.15}\right) \times 10^{-4}\right.$; Watanabe et al. 2022 $)$, and KELT-9 $\left(J_{2}=\left(3.26_{-0.80}^{+0.93}\right) \times 10^{-4}\right.$; Stephan et al. 2022 $)$. These values of $J_{2}$ also indicate that hot stars are more likely to redistribute their internal mass than the Sun.

We calculated the Love number $k_{2}$ of TOI-1518, which is an index of the stellar rigidity. From Equations (2) and (3) in Ragozzine \& Wolf (2009), $J_{2}$ can be expressed with as $k_{2}$


Fig. 5. Change in $b$ of TOI-1518b for short term (top) and long term (bottom). The red circles are values from TESS, the magenta inverted triangle is a value from CARMENES, the blue triangle is a value from EXPRES, and the black solid lines show the best-fit model of the nodal precession. The two blue-dashed lines show the edges of the stellar disc of TOI-1518b. The dark gray and light gray lines represent model likelihoods within the $1 \sigma$ and $3 \sigma$ confidence, respectively.
$J_{2}=\frac{k_{2} R_{s}^{3}}{3 a^{3}}\left(\frac{P_{\mathrm{orb}}^{2}}{P_{\mathrm{spin}}^{2}}+\frac{3 M_{p}}{2 M_{s}}\right)$,
where $M_{p}$ is the planetary mass and $P_{\text {spin }}\left(=2 \pi R_{s} / V_{s}\right)$ is the stellar rotation period. $\quad P_{\text {spin }}$ varies 0.21 days $<$ $P_{\text {spin }}<1.31$ days within the range of $i_{s}$. Thus, the range of $P_{\text {spin }} / P_{\text {orb }}$ is $0.11<P_{\text {spin }} / P_{\text {orb }}<0.69$. The upper limit of the mass of TOI- 1518 b is $M_{p}<2.3 M_{J}$ (Cabot et al. 2021), which gives $M_{p} / M_{s}<0.0014$. The second term in Equation (10) should be negligible for this system.

We made 10,000 samples by randomly selecting values of $P_{\text {orb }}, a / R_{s}, \psi, i_{s}$ and $d \theta / d t$ within the $1 \sigma$ ranges for $P_{\text {orb }}$ and $a / R_{s}$, and within the certain ranges for $\psi$, $i_{s}$ and $d \theta / d t\left(90^{\circ}<\psi<121^{\circ} .98,22 .^{\circ} 86<i_{s} \leq 170^{\circ} 28\right.$, $0 .{ }^{\circ} 13 \mathrm{yr}^{-1}<d \theta / d t<1^{\circ} .43 \mathrm{yr}^{-1}$ ). Using equations (8) and (10), we determine the distribution of $k_{2}$ from 10,000 sam-
ple systems. We obtained $\log k_{2}=-2.17 \pm 0.33$ for the Love number of TOI-1518, which is smaller than that of a sun-like star $\left(\log k_{2} \sim-1.52\right.$; Claret \& Gimenez 1995). This result is consistent with the model of the relationship between $M_{s}$ and $k_{2}$ in Claret \& Gimenez (1995). For reference, we calculated the $k_{2}$ values of other hot stars from Equation (10). These calculated values are listed in Table 3. The Love number of TOI-1518 is similar to that of the other hot stars. This implies that the interior of a hot star is stiffer and less susceptible to tidal deformation than that of a sun-like star.

## 4 Conclusion

We investigated the nodal precession of TOI-1518b using transit photometric datasets from TESS and Doppler tomographic datasets from CARMENES and EXPRES and measured the change in its impact parameter $d b / d t=$ $-0.0116 \pm 0.0036 \mathrm{yr}^{-1}$. TOI- 1518 b is the fourth planetary system in which the nodal precession is detected. We estimate that the transit has started in $2003_{-8}^{+4} \mathrm{CE}$ and will cease in $2194_{-39}^{+70} \mathrm{CE}$ if $b$ changes linearly while the transit trajectory is on the stellar surface. This result suggests that despite the nodal precession of TOI-1518 b, its transit is observable with the next-generation telescopes such as Ariel (Tinetti et al. 2018) and the Thirty Meter Telescope (TMT). We calculated the minimum value of $J_{2}$ for TOI$1518 J_{2, \min }=4.41 \times 10^{-5}$, which indicates that TOI-1518 is more oblate and more prone to their internal mass redistribution than the sun. We also derived the logarithm of $k_{2}$ for TOI- $1518 \log k_{2}=-2.17 \pm 0.33$. TOI- 1518 may have a stiffer interior and may be less susceptible to tidal effects than the sun-like star.

The TESS mission has discovered hot Jupiters around the B- and A-type stars. The real spin-orbit obliquity $\psi$ of these systems is key to revealing the orbital evolution of a hot Jupiter. Measuring $k_{2}$ of hot stars and $\psi$ through nodal precession observations every few years (Watanabe et al. 2022) would provide insights into the tidal evolution and orbital migration of a planetary system with a hot Jupiter around a hot star.

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Table 3. Calculated $\log k_{2}$ of hot stars and referred parameters for equations (8) and (10).

| Star | TOI-1518 | WASP-33 | KELT-9 | Kepler-13A |
| :---: | :---: | :---: | :---: | :---: |
| $\log k_{2}$ | $-2.17 \pm 0.33$ | $-2.14 \pm 0.11$ | $-2.16_{-0.30}^{+0.26}$ | $\overline{-2.17_{-0.20}^{+0.22}}$ |
| $J_{2}$ | $>4.41 \times 10^{-5(\mathrm{i})}$ | $\left(1.36_{-0.12}^{+0.15}\right) \times 10^{-4(\text { iii })}$ | $\left(3.26_{-0.80}^{+0.93}\right) \times 10^{-4(v i i)}$ | $(6.1 \pm 0.3) \times 10^{-5(x)}$ |
| $\psi$ (deg) | $90<\psi<121.98{ }^{\left({ }^{\text {i }}\right.}$ | $108.09_{-0.97}^{+0.95}$ (iii) | $87_{-11}^{+10}$ (viii) | $60 \pm 2^{(\mathrm{x})}$ |
| $d \theta / d t\left(\operatorname{deg} \mathrm{yr}^{-1}\right)$ | $0.13<d \theta / d t<1.43{ }^{(\mathrm{i})}$ | $0.507_{-0.022}^{+0.025}$ (iii) | $0.404_{-0.074}^{+0.076}(\mathrm{vii}) *$ | $0.240_{-0.028}^{+0.037}(\mathrm{x}) *$ |
| $i_{s}$ (deg) | $22.86 \leq i_{s}<170.28^{(\mathrm{i})}$ | $58.3_{-4.2}^{+4.6}$ (iii) | $139 \pm 7^{\text {(vii) }}$ | $81.8 \pm 0.2^{(\mathrm{x})}$ |
| $V \sin i_{s}\left(\mathrm{~km} \mathrm{~s}^{-1}\right)$ | $76.624_{-0.052}^{+0.051}{ }^{\text {(i) }}$ | $86.63_{-0.32}^{+0.37}(\mathrm{iv})$ | $111.4 \pm 1.3^{(\mathrm{ix})}$ | $78 \pm 15^{(x)}$ |
| $R_{s}\left(R_{\odot}\right)$ | $1.950 \pm 0.048^{(i i)}$ | $1.444 \pm 0.034^{(\mathrm{v})}$ | $2.362_{-0.063}^{+0.075}(\mathrm{ix})$ | $1.74 \pm 0.04^{(\mathrm{xi})}$ |
| $a / R_{s}$ | $4.171_{-0.014}^{+0.015}(\mathrm{i})$ | $3.69 \pm 0.01{ }^{(\mathrm{v})}$ | $3.153 \pm 0.011^{(\text {(ix) }}$ | $4.5007_{-0.0040}^{+0.0039(x i)}$ |
| $M_{s}\left(M_{\odot}\right)$ | $1.79 \pm 0.26^{(i i)}$ | $1.495 \pm 0.031^{(\mathrm{v})}$ | $2.52_{-0.20}^{+0.25(i x)}$ | $1.72 \pm 0.10^{(\mathrm{xi})}$ |
| $M_{p}\left(M_{J}\right)$ | $<2.3{ }^{\text {(ii) }}$ | $2.81 \pm 0.53{ }^{(\mathrm{vi})}$ | $2.88 \pm 0.84{ }^{\text {(ix) }}$ | $9.28 \pm 0.16^{(\mathrm{xi})}$ |
| $P_{\text {orb }}$ (days) | $1.90261178_{-0.00000019}^{+0.0000018(i)}$ | $1.2198675 \pm 0.0000011{ }^{(\mathrm{vi})}$ | $1.4811235 \pm 0.0000011{ }^{\text {(ix) }}$ | $1.763588 \pm 0.000001^{(\mathrm{xi})}$ |

${ }^{(i)}$ This work, ${ }^{(i i)}$ Watanabe et al. (2022), ${ }^{(\text {iii })}$ Cabot et al. (2021), (iv) Johnson et al. (2015), (v) Collier Cameron et al. (2010), (vi) von Essen et al. (2014), ${ }^{(\text {vii })}$ Stephan et al. (2022), ${ }^{(\text {viii) }}$ Ahlers et al. (2020), ${ }^{(\mathrm{ix})}$ Gaudi et al. (2017), ${ }^{(\mathrm{x})}$ Masuda (2015), (xi) Esteves et al. (2015)

* These values are derived from the nodal precession period presented in the earlier studies.

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## Appendix 1 MCMC Results of Photometric and Doppler Tomographic Measurements

In this appendix, we display the corner plots of posteriors after calculating by MCMC in Figures 6 and 7. We also show the corner plots of posteriors via bootstrap analysis in Figure 8.

## Appendix 2 Derivation of Changes of Impact Parameter and Projected Spin-orbit Obliquity

We define the angles $i_{s}, \theta(t), \lambda, \psi$ and $i_{p}$ (the planetary inclination: $\left.i_{p}=\arccos \left(b R_{s} / a\right)\right)$ in Figure 9. Considering the coordinate system with $z$ axis and stellar rotation axis aligned is rotated by $90^{\circ}-i_{s}$ around $x$ axis, we can write the unit vector of the planetary orbital axis $\overrightarrow{k_{p}}$ as

$$
\begin{align*}
\overrightarrow{k_{p}} & =\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \sin i_{s} & \cos i_{s} \\
0 & -\cos i_{s} & \sin i_{s}
\end{array}\right)\left(\begin{array}{c}
-\sin \psi \sin \theta(t) \\
\sin \psi \cos \theta(t) \\
\cos \psi
\end{array}\right) \\
& =\left(\begin{array}{c}
-\sin \psi \sin \theta(t) \\
\cos \psi \cos i_{s}+\sin \psi \sin i_{s} \cos \theta(t) \\
\cos \psi \sin i_{s}-\sin \psi \cos i_{s} \cos \theta(t)
\end{array}\right) \tag{A1}
\end{align*}
$$

$\overrightarrow{k_{p}}$ also can be expressed as
$\overrightarrow{k_{p}}=\left(\begin{array}{c}\sin i_{p} \sin \lambda \\ \cos i_{p} \\ \sin i_{p} \cos \lambda\end{array}\right)$,
which is also described in Iorio (2016) as Equations (21), (22) and (23). From $y$ components of Equations (A1) and


Fig. 6. Corner plots for the free parameters from the photometric datasets of TESS. We created these plots with corner.py (Foreman-Mackey 2016).


Fig. 7. Corner plots for the free parameters from the spectral datasets of CARMENES and EXPRES via MCMC method. We created these plots in the same way as Figure 6.


Fig. 8. Corner plots for the free parameters from the spectral datasets of CARMENES and EXPRES via bootstrap method. We created these plots in the same way as Figure 6.
(A2), we can derive the change in $b$ as

$$
\begin{align*}
b(t) & =\frac{a}{R_{s}} \cos i_{p} \\
& =\frac{a}{R_{s}}\left(\cos \psi \cos i_{s}+\sin \psi \sin i_{s} \cos \theta(t)\right) . \tag{A3}
\end{align*}
$$

On the other hand, dividing $x$ component of $\overrightarrow{k_{p}}$ by its $z$ component, we can obtain the change in $\lambda$ as
$\tan \lambda(t)=\frac{\sin \psi \sin \theta(t)}{\sin \psi \cos i_{s} \cos \theta(t)-\cos \psi \sin i_{s}}$.


Fig. 9. Cartoon of planetary system. We set $y$ axis as the line of sight and $x z$ plane as the plane of sky. The solid red vector and the solid blue vector show the stellar rotational axis and the planetary orbital axis, respectively. We define $\theta(t)=0$ when the planetary orbital axis is on $y z$ plane.

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[^0]:    * They defined the range of $\lambda$ as $0^{\circ}<\lambda<360^{\circ}$, so they measured $\lambda=$ $240^{\circ} 34_{-0.98}^{+0.93}$. In this study, we define this range as $-180^{\circ}<\lambda<180^{\circ}$. Thus, we write $\lambda=-119^{\circ} 66_{-0.93}^{+0.98}$.

[^1]:    * Median and standard deviation of these coefficients are calculated by PyLDTk (Husser et al. 2013; Parviainen \&

    Aigrain 2015) using the values of $T_{\text {eff }}, \log g$ and $[\mathrm{Fe} / \mathrm{H}]$ from Cabot et al. (2021).
    $\S$ This mid-transit time is during the first transit in the TESS observation.

[^2]:    ${ }^{\dagger}$ From their values of $P_{\text {orb }}$ and $T_{0}$, forecasted $T_{0}$ during the first transit in the TESS observation and the transit observation by EXPRES are $T_{0}=$ $2458766.12062 \pm 0.00015$ BJD $_{\text {TDB }}$ and $T_{0}=2459064.8293 \pm 0.0016$ $\mathrm{BJD}_{\mathrm{TDB}}$, respectively.

[^3]:    * Median and standard deviation of these coefficients are calculated by PyLDTk (Husser et al. 2013; Parviainen \& Aigrain 2015) using the values of $T_{\text {eff }}, \log g$ and $[\mathrm{Fe} / \mathrm{H}]$ from Cabot et al. (2021).
    $\dagger$ To set the median and standard deviation of this parameter, we referred this value from the typical range of Gaussian dispersion of spectral lines from Table 1 in Hirano et al. (2011).
    $\ddagger$ Median and standard deviation of these parameters are from the best values and the uncertainties of those derived in Section 2.1 .
    $\S$ This mid-transit time is during the transit observation by EXPRES.

