Dimorphos orbit determination from mutual events photometry

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ABSTRACT

The NASA Double Asteroid Redirection Test (DART) spacecraft successfully impacted the Didymos-Dimorphos binary asteroid system on 2022 September 26 UTC. We provide an update to its pre-impact mutual orbit and estimate the post-impact physical and orbital parameters, derived using ground-based photometric observations taken from July 2022 to February 2023. We found that the total change of the orbital period was -33.240 ± 0.072 min. (all uncertainties are 3σ). We obtained the eccentricity of the post-impact orbit to be 0.028 ± 0.016 and the apsidal precession rate of 7.3 ± 2.0 deg./day from the impact to 2022 December 2. The data taken later in December to February suggest that the eccentricity dropped close to zero or the orbit became chaotic approximately 70 days after the impact. Most of the period change took place immediately after the impact but in a few weeks following the impact it was followed by additional change of -27^{+19}_{-58} seconds or -19 ± 18 seconds (the two values depend on the approach we used to describe the evolution of the orbital period after the impact – an exponentially decreasing angular acceleration or an assumption of a constant orbital period, which changed abruptly some time after the impact, respectively). We estimate the pre-impact Dimorphos-Didymos size ratio was 0.223 ± 0.012 and the post-impact is 0.202 ± 0.018 , which indicates a marginally significant reduction of Dimorphos' volume by $(9 \pm 9)\%$ as the result of the impact.

1. INTRODUCTION

NASA's Double Asteroid Redirection Test (DART) spacecraft intentionally and autonomously collided with Dimorphos, the secondary of the Didymos binary asteroid system, on 2022 September 26 (Daly et al. 2023a). The impact altered the orbit of Dimorphos around Didymos, reducing the orbit period by around 33 minutes, which was determined by two independent approaches using Earth-based photometric and radar observations (Thomas et al. 2023).

DART was NASA's first planetary defense test mission, with the goal of demonstrating the kinetic impactor mitigation technique on an asteroid. The main benefit of targeting a binary asteroid system on a kinetic impactor mission was that it allows the main result of the test – the change in the mutual orbital period – to be measured from Earth via photometric observations, assuming that the binary system exhibits mutual events seen from Earth. Rivkin et al. (2021) discussed the factors that led to the recognition that Didymos was the best candidate for a kinetic impactor test, and its selection as the DART target.

Didymos was discovered in 1996 and photometric observations led to its identification as a binary system (Pravec et al. 2003) and to determination of parameters of the orbit of its secondary (Pravec et al. 2006; Scheirich & Pravec 2009). Radar observations of the Didymos system were also obtained in 2003, confirming that it is a binary system and characterizing its shape and orbit properties (Naidu et al. 2020). The radar observations constrained the diameter of Didymos to about 780 m and the diameter of Dimorphos to about 150 m.

Didymos is classified as an S-type asteroid (Cheng et al. 2018) based on vis-IR spectra obtained by De León et al. (2010), also confirmed by Dunn et al. (2013). Thorough spectral characterization before and after the DART impact was carried also by Lin et al. (2023) and Ieva et al. (2022, 2023).

Following the selection of Didymos as the target of DART, a worldwide observing campaign was organised to observe the system and characterize its pre- and post-impact orbit using ground-based telescopes, with photometric observations being a crucial aspect (Pravec et al. 2022; Moskovitz et al. 2023). These observations resulted in greatly improved knowledge of the orbit of Dimorphos around Didymos and its orbital phase at the time of impact (Scheirich & Pravec 2022; Naidu et al. 2022).

In this paper, we present an update on the pre-impact mutual orbit and results from post-impact mutual orbit modeling using the complete photometry data for mutual events in the Didymos system from 2003 to 2023. An independent derivation of the observed mutual orbit based on an analysis of mutual event timings has been made by Naidu et al. (2023).

For further interpretation of these results and more context on the pre- and post-impact dynamics of the Didymos system, beyond the orbit determinations, see Richardson et al. (2023).

A derivation of the change of the orbital period was made also by Gudebski et al. (2023). We discuss their work in Section 5.

The Didymos system will be investigated by ESA's Hera mission from the beginning of 2027 for about half a year, which will provide a thorough description of the post-impact state of the binary system (Michel et al. 2022).

2. MUTUAL ORBIT MODEL OF DIDYMOS SYSTEM

2.1. Observational data

The photometric data used in our analysis of the pre-impact orbit were obtained during six apparitions of Didymos from 2003 to 2022. Those from apparitions from 2003 to 2021 were published in Pravec et al. (2006, 2022). The data from the pre-impact apparition taken from 2022 July 2 to September 26 (part of which has been published by Thomas et al. 2023) are published in Moskovitz et al. (2023) We briefly summarize all the pre-impact data in Table 1.

The data of the post-impact orbit are published in Moskovitz et al. (2023), and we briefly summarize them in Table 2. In both Tables 1 and 2, we used the term *run* as a time series of photometric data acquired by a single facility during one night.

The data presented in Tables 1 and 2 represent only the data we used in our analysis. Since our criteria for accepting datasets was slightly more strict than those used by Moskovitz et al. (2023), our data are a higher-quality subset, containing 15% less runs, of that reported by Moskovitz et al. (2023).

Time span	No. of runs	Reference
2003-11-20.9 to 2003-12-20.3	17	P06
2015-04-13.3 to 2015-04-14.4	2	P22
2017-02-23.3 to 2017-05-04.3	13	P22
2019-01-31.4 to 2019-03-11.1	5	P22
2020-12-12.6 to 2021-03-06.3	15	P22
2022-07-02.3 to 2022-09-26.4	36	M23

Table 1. Photometric observations of the Didymos system in the pre-impact apparitions 2003 to 2022

References: P06 Pravec et al. (2006), P22 Pravec et al. (2022), M23 Moskovitz et al. (2023).

Lunation	Time span	Days from impact	No. of runs
L0	2022-09-28.2 to 2022-10-10.3	1.1 to 13.4	47
L1	2022-10-24.4 to 2022-11-02.5	27.4 to 36.5	20
L2	2022-11-17.4 to 2022-12-02.3	51.3 to 66.5	48
L3	2022-12-14.4 to 2022-12-31.3	78.4 to 95.4	25
L4	2023-01-11.2 to 2023-01-30.1	106.1 to 125.3	13
L5	2023-02-11.2 to 2023-02-21.3	137.1 to 147.4	4

Table 2. Photometric observations of the Didymos system in the post-impact apparition 2022-2023

The data were analysed using the standard technique described in Pravec et al. (2006, 2022). Briefly, by fitting a two-period Fourier series to data points taken outside mutual (occultation or eclipse) events, the rotational lightcurves of the primary and the secondary, which are additive in flux units, are separated. Subtracting the rotational lightcurve of the primary from the data, a long-period (orbital) lightcurve component containing the mutual events and the secondary rotation lightcurve is obtained, which is then used for subsequent numerical modeling. We refer the reader to Pravec et al. (2022) for details of the lightcurve decomposition method.

2.2. Numerical model

We constructed a model of the Didymos-Dimorphos system adapting the technique of Scheirich & Pravec (2009) which was further developed by Scheirich et al. (2015, 2021) and used also by Scheirich & Pravec (2022) and Thomas et al. (2023). We outline the basic points of the method below, but we refer the reader to the above references for details of the technique.

For the pre-impact orbit, we applied the method used by Scheirich & Pravec (2022) (hereafter referred to as SP22) on the complete pre-impact dataset (Table 1). For the post-impact orbit, we made a few modifications, which we describe in Section 3.2. The method of SP22 is described in following.

The binary asteroid components were represented with an oblate ellipsoid for the primary and an oblate ellipsoid or a sphere (in the case of the post-impact orbit, see Section 3.2) for the secondary. (The shapes of the bodies were used to calculate the light flux of the system only. The orbital characteristics of the system were treated as free parameters; they were not tied to the dynamics that would come from the non-spherical shapes.) For the pre-impact orbit, we choose a circular orbit for simplicity, as the upper limit on the eccentricity is low (see below). The circular pre-impact orbit is also consistent with the observed post-impact eccentricity of ~ 0.03 (see Section 3.2), as noted by Meyer et al. (2023) (also mentioned in Richardson et al. 2023). The motion was assumed to be Keplerian, but we allowed for a quadratic drift in the mean anomaly, which results from the combined effects of BYORP and tidal dissipation or from differential Yarkovsky force (SP22). The spin axis of the primary was assumed to be normal to the mutual orbital plane of the components. The shapes were approximated with 1016 and 252 triangular facets for the primary and the secondary, respectively. The components were assumed to have the same albedo and their surfaces uniform in albedo and scattering law. The brightness of the system as seen by the observer was computed as a sum of contributions from all visible facets using a ray-tracing code that checks which facets are occulted by or are in shadow from the other body. A combination of Lommel-Seeliger and Lambert scattering laws was used (see, e.g., Kaasalainen et al. 2002).

The quadratic drift in the mean anomaly, ΔM_d , was fitted as an independent parameter on the pre-impact data. It is the coefficient in the second term of the expansion of the time-variable mean anomaly:

$$M(t) = M(t_0) + n(t - t_0) + \Delta M_d (t - t_0)^2,$$
(1)

where

$$\Delta M_d = \frac{1}{2}\dot{n},\tag{2}$$

where n is the mean motion, t is the time, and t_0 is the epoch. ΔM_d was stepped from -10 to $+10 \text{ deg/yr}^2$ with a step of 0.01 deg/yr², and all other parameters were fitted at each step.

Across all pre-impact observations, we found a unique solution for the system parameters, see Table 3. We describe these parameters in Section 3.1.

For the post-impact observations, we developed two alternative modifications of the model in order to describe specific post-impact features seen in the data. For both modifications, we found unique solution for their parameters, both describing the data equally well. We describe and discuss the modifications and their parameters in Section 3.2.

We estimated uncertainties of the fitted parameters using two techniques. The uncertainty of the orbital pole (the pole is strongly determined by the shapes of the mutual events) was estimated using the procedure described in Scheirich & Pravec (2009). The uncertainties of the rest of the parameters, which are determined primarily by the timings of the events, were estimated using the method described in Scheirich et al. (2021), which we outline below.

The residuals of the model fitted to the observational data do not obey the Gaussian statistics because of systematic errors resulting from model simplifications. In particular, the residuals of nearby measurements appear correlated. To eliminate the effect we adopted the following strategy based on the χ^2 test.

We choose a correlation time d and for each data point (i) we calculated how many other data points, K_i , are within $\pm d/2$ from the given point. We then applied a weight of $1/K_i$ to the given data point in the χ^2 sum. We also calculated an effective number of data points as $N_{\text{eff}} = \sum_{i=1}^{N} 1/K_i$, where N is the total number of data points. For normalized χ^2 we then have $\chi^2 = 1/(N_{\text{eff}} - M) \sum_{i=1}^{N} (O - C)_i^2/(\sigma_i^2 K_i)$, where M is the number of fitted parameters of the model and σ_i is a standard deviation of the *i*th point. As the residuals are predominated by model rather than observational uncertainties, we assign each data point the same standard deviation $\sigma_i = \sigma$, where σ is the RMS residual (root mean square of observed magnitudes, O, minus the values calculated from the model, C) of the best fit solution. An illustration of the weights $1/K_i$ determination is shown in Figure 2 of SP22.

The procedure described above is equivalent to reducing the number of data points to one in each time interval with the length d (i.e., to reducing the total number of points to N_{eff}) and assigning $(O - C)^2$ of this point to be a mean of $(O_i - C_i)^2$ of all the points within the interval. However, our approach has the advantage that it does not depend on a particular realization of dividing the observing time span into the intervals of length d.

We choose the correlation time d to be equal to 1/2 of the mean duration of a decreasing/increasing branch¹ of the secondary mutual event, i.e., the mean time between the first and the second or between the third and the fourth contact between the limbs or terminators of the two bodies in their projection onto the sky. For the observed events in Didymos, it is d = 0.14 h. (We also tested d to be twice as long, i.e., equal to the full mean duration of the secondary event branch, but we found it to be inadequate as the longer correlation time resulted in a substantial loss of information by deweighting the datapoints too much.)

We note that the mutual orbit model fit is sensitive only to data points covering mutual events and their closest neighborhood. Therefore we limited the above analysis only to such data points; points further outside the events were not used, because they do not effectively contribute to the determination of the mutual orbit.

Upon stepping a given parameter on a suitable interval (while the other parameters fitted) and computing the normalized χ^2 for each step, we determined 3- σ uncertainty of the given parameter

¹ We define a *branch* as a part of the mutual event in the lightcurve, where the brightness of the system is rapidly decreasing or increasing, i.e., the time period during which the eclipsed/occulted body is immersing into or emerging from the shadow of (for a mutual eclipse events), or is disappearing behind or reappearing from behind the other body (for a mutual occultation event).

as an interval in which χ^2 is below the p-value of the χ^2 test, corresponding to the probability that the χ^2 exceeds a particular value only by chance equal to 0.27%.

Parameter		Value	Unc.
Secondary:			
Cross-section equiv. diam. ratio	$D_{2,C}/D_{1,C}$	0.220 ± 0.004^{a}	1σ
Volume equivalent diameter ratio	$D_{2,V}/D_{1,V}$	0.223 ± 0.004	1σ
Mutual orbit:			
Sem. axis / primary equat. diam.	$a/(A_1B_1)^{1/2}$	1.49 ± 0.13	3σ
Ecl. longitude of orbital pole	L_P (°)	310.0 ± 15.0^b	3σ
Ecl. latitude of orbital pole	B_P (°)	-80.4 ± 1.9^{b}	3σ
Drift in mean anomaly	$\Delta M_d \; (\mathrm{deg/yr^2})$	0.13 ± 0.10	3σ
Mean motion rate	$\dot{n} \ (\mathrm{rad/s^2})$	$4.56 \pm 3.51 \times 10^{-18}$	3σ
Pre-impact orbital period at $t_{\rm imp}$	$P_{\rm orb}^{\rm pre}$ (h)	11.921493 ± 0.000091	3σ
Eccentricity	e	≤ 0.03	3σ
Epoch of impact	$t_{ m imp}$	JD 2459849.46875 (geod	centric UTC)

Table 3. Parameters of the Didymos-Dimorphos pre-impact model. The indices 1 and 2 refer to Didymos and Dimorphos, respectively.

^{*a*} The ratio is for the average observed aspect of 14.0°. See text for details. ^{*b*} For the actual shape of the uncertainty area, see Figure 3. Semiaxes of the area are $2.0 \times 2.5^{\circ}$.

3. PARAMETERS OF DIDYMOS-DIMORPHOS SYSTEM

3.1. Pre-impact orbit

Using the data taken from 2022 July 2 to September 26, added to the 2003-2021 dataset from SP22, we applied the method of SP22, modeling the system with a circular orbit with the quadratic drift of the mean anomaly. We obtained updated values of the pre-impact mutual orbit parameters, which are given in Table 3 together with their uncertianties. The indices 1 and 2 refer to Didymos and Dimorphos, respectively.

Unlike in SP22, we now used oblate ellipsoids as the approximation of the shapes of the components, with their axes ratios derived from best-fit ellipsoids of shape models obtained from DART data. Barnouin et al. (2023) gives the principal axes of the best-fit ellipsoid of Didymos to be $A_1 =$ 819 ± 14 m, $B_1 = 796 \pm 14$ m and $C_1 = 590 \pm 14$ m. Daly et al. (2023b) gives the principal axes of the best-fit ellipsoid of Dimorphos to be $A_2 = 173 \pm 1$ m, $B_2 = 170 \pm 4$ m and $C_2 = 114 \pm 1$ m. Using those values, we derived $(A_1B_1)^{1/2}/C_1 = 1.369$ as the nominal axes ratio of the oblate primary model and $(A_2B_2)^{1/2}/C_2 = 1.504$ as the nominal axes ratio of the oblate secondary model.

Despite the size ratio of the two components could be computed from the above axes estimates, we fitted it as an independent parameter. This is because the native parameter of our model is the ratio between $D_{1,C}$ and $D_{2,C}$, the mean (rotationally averaged) cross-section equivalent diameters (i.e., the diameter of a sphere with the same cross-section) of the primary and secondary, respectively, at the mean aspect of observed total secondary events (see below). $D_{1,C}$ and $D_{2,C}$ are determined by the actual shapes of the components as well as by the specific geometry of the binary system with respect to the observer and the Sun during the observations.

To quantify the mean aspect we used an asterocentric latitude of the Phase Angle Bisector (PAB), which is the mean direction between the heliocentric and geocentric directions to the asteroid. As discussed in Harris et al. (1984), this is an approximation for the effective viewing direction of an asteroid observed at the non-zero solar phase. The average absolute value of the asterocentric latitude of the PAB for the observed total events across the pre-impact data was 14.0°. (We computed the latitude of the PAB using the nominal pole of the mutual orbit and assuming that the spin poles of both components are the same as the orbit pole.)

In order to quantitatively compare the size of Dimorphos before and after the impact (see Section 3.2), we also computed $D_{2,V}/D_{1,V}$, a ratio of volume equivalent diameters (i.e., the diameter of a sphere with the same volume) of the components from $D_{2,C}/D_{1,C}$ and axes ratios of their models. $a/(A_1B_1)^{1/2}$ is the ratio between the semimajor axis of the system and mean equatorial diameter of the primary. L_P, B_P are the ecliptic coordinates of the orbital pole in the equinox J2000, $P_{\text{orb}}^{\text{pre}}$ is the pre-impact orbital period at the nominal time of the DART impact and ΔM_d is the quadratic drift in the mean anomaly. We also give the time derivative of the mean motion \dot{n} , calculated from ΔM_d . e is the orbit eccentricity (only its upper limit was obtained before the impact, see below). Plot of the normalized χ^2 vs ΔM_d is shown in Figure 1. The long-period (orbital) lightcurve component data from 2022 together with the synthetic lightcurve of the best-fit solution are presented in Figure 2. The synthetic curve of the best-fit solution looks indistinguishable from the synthetic curve from SP22 fitted to the data from 2003 – 2021. To save space, we therefore include only plots for the pre-impact 2022 apparition in this paper.

The uncertainty area of the orbital pole is shown in Figure 3. We note that the difference between the orbital pole determined in this work and that derived in SP22 is mainly due to the fact that we now used the ellipsoidal oblate model for Didymos derived from DART data. The axes ratio used in SP22 was systematically off, as the axes ratio we used here (1.369) is close to the 3- σ upper limit of Didymos axes ratio assumed in SP22. The difference in the orbital pole also explains the different value of $D_{2,C}/D_{1,C}$ we obtained with respect to the value from SP22 (0.220±0.004 vs. 0.217±0.004), as the mean aspect is slightly different.

No significant change of the solution was found for the polar flattening of the primary in a range from 1.21 to 1.55 and the polar flattening of the secondary in a range from 1.40 to 1.61. We calculated these limits using the 1- σ uncertainties from (Barnouin et al. 2023) and (Daly et al. 2023b) (see above), which we multiplied by 3 to get 3- σ uncertainties.

Scheirich & Pravec (2009) estimated the upper limit on the eccentricity using the data from the 2003 apparition. SP22 checked that the 3- σ upper limit of 0.03 is consistent with the data taken from 2015 to 2021. In this work, we checked that it is consistent with the data taken from 2022 July 2 to September 26 as well, but those later data do not possess the characteristics necessary to use them for constraining the eccentricity more. Those characteristics include sufficient quality required to resolve offsets in event timings with respect to the solution with circular orbit, and time coverage required to obtain a robust estimate on the apsidal precession. Thus, the upper limit on the eccentricity by Scheirich & Pravec (2009) is still valid.



Figure 1. The normalized χ^2 vs. ΔM_d for solutions of the pre-impact orbit model presented in Section 2.2. The three horizontal lines give the p-values – the probabilities that the χ^2 exceeds a particular value only by chance, corresponding to 1-, 2- and 3σ interval of the χ^2 distribution. The light-gray dots and lines are taken from SP22. The best-fit value has $\Delta M_d = 0.13 \text{ deg/yr}^2$.

3.2. Post-impact orbit

For modeling the post-impact data, we fixed the orbital pole of the mutual orbit on its pre-impact value. We used an oblate ellipsoid as the approximation of the shape of Didymos, with its axes ratios the same as used for the pre-impact orbit modeling (see Section 3.1).

We chose to approximate the post-impact Dimorphos with a sphere for following reasons. As shown by Agrusa et al. (2021) and Meyer et al. (2023), it is possible that Dimorphos entered a tumbling state a few tens of days after the impact due to resonances among system's natural frequencies. When that happened, the orbital eccentricity dropped in just a few days as a result of the exchange of angular momentum between the mutual orbit and the secondary's rotation state. When tumbling occured, the eccentricity also varied chaotically in time. As we do see an indication of the eccentricity drop



Figure 2. The pre-impact orbital lightcurve component of the Didymos system in 2022. The observed data are marked as points. (To avoid confusion between different data sets, different colors are used alternating on each plot.) The solid curve represents the synthetic lightcurve for the best-fit pre-impact solution. The primary and secondary events (the terms refer to which of the two bodies is occulted or eclipsed) are always shown on the left and right sides of the plots, respectively. In some cases, the observations of a secondary event preceded that of a primary event (i.e., their order in the dataset is inverse of that shown on the plot). In order to save space in the plot, we present these events in reverse order to how they were observed. They are separated by the "//" symbol in the plot and one orbital period (0.496 d) is to be subtracted from the x coordinate of data points to the right from this separator.

in the data (see below), we assume that Dimorphos began tumbling in a course of the post-impact apparition. With the chaotically spinning Dimorphos, we choose to approximate Dimorphos with a sphere for simplicity.

An independent fit to the post-impact data (without those from the L0 lunation, which have the apparent depths of mutual events reduced due to strong contamination by the ejecta) gives



Figure 3. Area of admissible poles for the pre-impact mutual orbit of Didymos in ecliptic coordinates (dark grey area). The dot is the nominal solution given in Table 3. This area corresponds to 3σ confidence level. The south pole of the current asteroid's heliocentric orbit is marked with the cross. For comparison, the area of admissible poles from SP22 is shown as light gray area.

 $D_{2,C}/D_{1,C} = 0.209 \pm 0.018$ (3 σ). The average absolute value of the asterocentric latitude of the PAB for the observed total events across the post-impact data was 9.6°.

The cross-section of an oblate ellipsoid is lowest when observed equator-on and highest when observed pole-on. As the mean asterocentric latitude of the PAB in the post-impact data was lower than in the pre-impact data (9.6° vs. 14.0°), the effective post-impact $D_{1,C}$ was lower than it was pre-impact. Also, the mean cross-section of a randomly oriented (tumbling) oblate shape is higher than that of the same shape observed equator-on. Therefore, the effective $D_{2,C}$ in the post-impact data would be higher than it was pre-impact in a case Dimorphos' shape did not change. These two things would therefore independently lead to the post-impact $D_{2,C}/D_{1,C}$ being greater than the preimpact, if the size and shape of Dimorphos had not change. Despite this expectation, the observed

Parameter		Value	Unc.		
Secondary:					
Cross-section equiv. diam. ratio	$D_{2,C}/D_{1,C}$	0.209 ± 0.018^a	3σ		
Volume equiv. diam. ratio	$D_{2,V}/D_{1,V}$	0.202 ± 0.018	3σ		
Mutual orbit:					
Sem. axis / primary equat. diam.	$a/(A_1B_1)^{1/2}$	1.46 ± 0.11	3σ		
Ecl. longitude of orbital pole	L_P (°)	310.0 ± 15.0^b	3σ		
Ecl. latitude of orbital pole	B_P (°)	-80.4 ± 1.9^{b}	3σ		
Final post-impact orbital period $(t = \infty)$	$P_{\rm orb}^{\rm post}$ (h)	11.3675 ± 0.0012	3σ		
Total period change	$\Delta P(t = \infty)$ (min.)	-33.240 ± 0.072	3σ		
Eccentricity in L0 to L2	e_{0-2}	0.028 ± 0.016	3σ		
Apsidal precession rate	$\mathrm{d}\varpi/\mathrm{d}t$ (deg./day)	7.3 ± 2.0	3σ		
Solution with the angular acceleration model:					
Angular acceleration	$A \; (\mathrm{deg}/\mathrm{d}^2)$	$(7.3 \pm 5.0)/\tau^2$	3σ		
Angular acceleration timescale	au (d)	14.3 ± 6.5	3σ		
Orbital period at $t = t_{imp}$	$P_{\rm orb}^{\rm imp}$ (h)	$11.3751\substack{+0.0160\\-0.0052}$	3σ		
Period change at $t = t_{imp}$	$\Delta P(t = t_{\rm imp})$ (min.)	$-32.78\substack{+0.96\\-0.31}$	3σ		
Solution with orbital period modeled as step function:					
Mean orbital period before the period drop	$P_{\rm orb}^0$ (h)	11.3729 ± 0.0050	3σ		
Time of the period drop (after the impact)	ΔT (d)	$15.5^{+12.0}_{-7.0}$	3σ		
Period change at $t = t_{imp}$	$\Delta P(t = t_{\rm imp})$ (min.)	-32.92 ± 0.30	3σ		

Table 4. Parameters of the Didymos-Dimorphos post-impact model. The indices 1 and 2 refer to Didymos and Dimorphos, respectively.

^a The ratio is for the average observed aspect of 9.6°. See text for details. ^b Fixed on the pre-impact value.

post-impact value is lower than the pre-impact value at approximately 2- σ level, which indicates that the size of Dimorphos has decreased as a result of the impact. Because $D_{2,C}/D_{1,C}$ is dependent on the observing geometry for the reasons above, we also calculated the ratio of volume equivalent diameters $D_{2,V}/D_{1,V}$. The difference between its post- and pre-impact values $(0.202 \pm 0.006 \text{ vs.} 0.223 \pm 0.004)$ indicates reduction of Dimorphos' volume by $(9 \pm 3)\%$ (1- σ uncertainties).

The spherical approximation of Dimorphos prevent us from fitting the shape-induced rotational lightcurves. The scope of this paper is limited to obtaining the parameters of the mutual orbit. We note that we did see the Dimorphos' rotational lightcurves in the best data that we took in December 2022 and January 2023, suggesting some degree of tumbling, and we will study this topic statistically in another paper (Pravec et al. 2023).

In order to check for the eccentricity and/or its evolution, or for an evolution of the orbital period, we used the following approach. We first fit the post-impact data using a circular orbit with a constant mean orbital period as a reference solution. We then constructed Figure 4 (top panel) showing offsets in mean anomaly with respect to the reference solution for every observed mutual event (here we used only completely covered events, i.e., that have both the decreasing and the increasing branches covered by the data). It was computed as follows. We generated a synthetic lightcurve using the model parameters from the reference solution. Then, for each mutual event, we fitted the mean anomaly of the model in order to obtain the best match between the synthetic lightcurve and the observed data. The values on the vertical axis of Figure 4 are differences between the mean anomaly of the reference model with circular orbit and the fitted one that was shifted in mean anomaly. In other words, the calculated differences correspond to the amount Dimorphos is ahead or behind in time with respect to an ephemeris from the reference solution of the orbit, expressed in degrees of the orbital phase (note that 1 deg. corresponds to $P_{\rm orb}/360 \simeq 1.9$ min.)

For each event, we computed also a standard deviation of the offset using the procedure described in Section 2.2, but with χ^2 computed only from the data points in the vicinity of the given mutual event.

As apparent from Figure 4 (top panel), we see following trends in the offsets:

1. In L0, L1 and L2 lunations, the primary and secondary event offsets are systematically shifted relative to each other, which indicates an eccentric orbit.

- 2. The magnitude of the offsets varies in time and changes a sign periodically with a period of about 50 days, which indicates a presence of apsidal precession with this period.
- 3. There are no systematic shifts of the times of primary vs. secondary events in the data from the L3-L5 lunations, which indicates that the eccentricity dropped to a value close to zero or entered chaotic evolution at a certain time between the L2 and L3 lunation, i.e. about 72 ± 6 days after the impact.
- 4. The gradual trend of the offsets (both of the primary and the secondary events) during L0 lunation suggests a decrease of the orbital period during L0 or between L0 and L1.

In order to describe the above-mentioned features, we augmented the model in following way. The orbit was assumed to be eccentric (with eccentricity e_{0-2} and with apsidal precession with the rate $d\omega/dt$) during the L0, L1 and L2 lunations, while it was assumed to be circular during L3, L4 and L5 lunations.

So that to directly compare the orbital period of the circular orbit with the orbital period of the precessing eccentric orbit, we redefine the orbital period as $P_{\text{orb}}^{\text{post}} = 360^{\circ}/(n + d\varpi/dt)$, which has now the meaning of a mean orbital period in the inertial frame (Meyer et al. 2023, use a term "stroboscopic orbit period").

To describe the apparent change of the mean orbital period during a few weeks after the impact, we used following two approaches.

(a) Angular acceleration

We added an angular acceleration term (see below for its physical reasoning) to the time variable mean anomaly

$$M(t) = M(t_{\rm imp}) + n(t - t_{\rm imp}) + A_c \exp(-(t - t_{\rm imp})/\tau),$$
(3)

assuming that

$$n = \frac{360^{\circ}}{P_{\rm orb}^{\rm post}} - \frac{\mathrm{d}\varpi}{\mathrm{d}t},$$



Figure 4. Offsets in the mean anomaly of the observed mutual events in the post-impact apparition with respect to three reference solutions. Top panel: the offsets are shown with respect to the reference solution that has a constant orbital period and zero eccentricity. The dotted line is an arbitrary sine function with a period of 50 days, plotted there in order to highlight the periodicity of the systematic shifts between the primary and the secondary events during L0, L1 and L2 lunations. Bottom panel: the offsets are shown with respect to the two nominal solutions presented in Table 4, one having the exponentially decreasing angular acceleration (red symbols) and the second having the orbital period described as a step function (green symbols). See text for details. The gray bars at the bottom of each panel indicate spans of the individual lunations (L0 to L5; the full moon occurred in each gap between the bars).

$$\dot{n} = A \exp(-(t - t_{\rm imp})/\tau) \tag{4}$$

and $A_c = A\tau^2$. The epoch t_{imp} is the time of the impact.

We stepped $d\varpi/dt$ and ω on a grid from 0 to 25.0 deg/day and from 0 to 360 deg, respectively, with $M(t_{\rm imp})$, A_c , τ , e_{0-2} , and $P_{\rm orb}^{\rm post}$ varied to get the best fit at each step. Using the best-fit solution obtained with this augmented model, we constructed bottom panel of Figure 4, which is analogous to its top panel, but showing the offsets of the observed events with respect to the new improved solution (red points in the figure).

(b) Orbital period modeled with a step function

Besides the angular acceleration model described above, we also tested a more simple model to quantify the evolution of the mean orbital period following the impact, by modeling the period with step function. We define P_{orb}^0 , a mean orbital period during an interval from the impact to the time $t_{\text{imp}} + \Delta T$, when the orbital period drops from P_{orb}^0 to $P_{\text{orb}}^{\text{post}}$ and remains constant since then.

Instead of Eq. 3, the evolution of the mean anomaly is then described as

If
$$t - t_{\rm imp} < \Delta T : M(t) = M(t_{\rm imp}) + \left(\frac{360^{\circ}}{P_{\rm orb}^0} - \frac{\mathrm{d}\omega}{\mathrm{d}t}\right) (t - t_{\rm imp}).$$
 (5)

If
$$t - t_{\rm imp} \ge \Delta T : M(t) = M(t_{\rm imp}) + \left(\frac{360^{\circ}}{P_{\rm orb}^{\rm post}} - \frac{\mathrm{d}\omega}{\mathrm{d}t}\right) (t - t_{\rm imp}) + \Delta M,$$
 (6)

where $\Delta M = \frac{360^{\circ} \Delta T}{P_{\text{orb}}^0} - \frac{360^{\circ} \Delta T}{P_{\text{orb}}^{\text{post}}}$ secures the continuity of M at $t - t_{\text{imp}} = \Delta T$.

We then fitted P_{orb}^0 , ΔT , $P_{\text{orb}}^{\text{post}}$, and the other parameters of the system described above (except for A_c and τ , this time) to the data. In the same way as above we have calculated the mean anomaly offsets with respect to the best-fit solution obtained with this second approach, and plotted them as green points in the bottom panel of Figure 4.

The parameters of the best-fit solutions using the two approaches are listed in Table 4. We obtained the same values for all the parameters of both solutions, except that they differ in introducing A_c and τ or P_{orb}^0 and ΔT . As the solution with the orbital period modeled as step function gives the same RMS residual as the solution with model of the angular acceleration of the mean anomaly (both give 0.0145 mag.), and both solutions have very similar event timing offsets (see Figure 4) we conclude that the two alternative models for the orbital period change are indistinguishable based on the quality of the fits to the data.

The offsets plotted in the bottom panel of Figure 4 show that with the augmented models we were able to describe the event timings in the L0, L1 and L2 lunations much better than using the simple circular orbit. However, there is still present a systematic shift of the primary vs. secondary events in the data from the L0 lunation. We therefore tried to further improve the fit in the following way. We allowed for a step change of the eccentricity fifteen days after the impact (i.e., during the time interval between the L0 and L1 lunation). The improved model describes the eccentricity as a two-step function with values of e_0 during the L0, $e_{1,2}$ during L1 and L2 and 0 during L3, L4 and L5 (with the former two values fitted as free parameters). We obtained $e_0 = 0.042 \pm 0.021$, describing the event timings in L0 even better, but there still remained small systematic effects during L0 which we cannot explain. Because it is based only on the data from the L0 lunation, the value of e_0 is also poorly constrained. This experiment may indicate, that the dynamics of the orbit during the L0 lunation was more complex than what our simple model is able to describe. However, given the large uncertainties in the L0 lunation offsets, it is also possible that this is just a statistical fluke. Given the above reasons, we do not adopt this model with the additional step in eccentricity as our final solution.

We also computed the 68th percentile of the absolute values of the offsets to quantify the fit in the sense of the event timings. For the L1 and L2 lunations together, which have the smallest offsets, the 68th percentile is 0.7 deg., corresponding to 1.3 minutes in time. For the entire post-impact apparition, the 68th percentile is 1.5 deg., corresponding to 2.8 minutes in time. These values are estimates for 1σ rms residuals of the event times fitted with our model.

Examples of the long-period component data together with the synthetic lightcurve of the best-fit solution are presented in Figures 5 and 6.

Using the solution obtained with the model augmented with the angular acceleration, we also computed $P_{\text{orb}}^{\text{imp}}$ – an orbital period at $t = t_{\text{imp}}$, i.e., at the time immediately after the impact, from the time derivative of the mean anomaly. We give its value in Table 4 as well.

From $P_{\text{orb}}^{\text{post}}$ and the pre-impact orbital period $P_{\text{orb}}^{\text{pre}}$, we obtain that total Dimorphos' period change was -33.240 ± 0.072 min. The higher values of $P_{\text{orb}}^{\text{imp}}$ and P_{orb}^{0} than $P_{\text{orb}}^{\text{post}}$ indicate that the period change did not remain at a constant value, but it was lower (in absolute values) shortly after the impact and it took a few weeks ($\Delta T = 15.5^{+12.0}_{-7.0}$ days, or $\tau = 14.3 \pm 6.5$ days) for the orbital period to stabilize. From the solution with the exponentially decreasing angular acceleration we obtain that the period change (caused by the DART impact) immediately after the impact was

JD₀ (Date)



Figure 5. Selected post-impact events from the orbital lightcurve component of the Didymos system. The observed data are marked as points. (To avoid confusion between different data sets, different colors are used alternating on each plot.) The solid curve represents the synthetic lightcurve for the best-fit solution with the angular acceleration. The synthetic lightcurve for the best-fit solution with orbital period modeled as step function is identical within the thickness of the line used, and is therefore not shown in the figure. For comparison, the dashed curve is the model with the circular orbit and constant mean orbital period. The primary and secondary events (the terms refer to which of the two bodies is occulted or eclipsed) are always shown on the left and right sides of the plots, respectively. In some cases, the observations of a secondary event preceded that of a primary event (i.e., their order in the dataset is inverse of that shown on the plot). In order to save space in the plot, we present these events in reverse order to how they were observed. They are separated by the "//" symbol in the plot and one orbital period (0.496 d) is to be subtracted from the x coordinate of data points to the right from this separator.



Figure 6. Continuation of Figure 5.

 $P_{\rm orb}^{\rm imp} - P_{\rm orb}^{\rm pre} = -32.78_{-0.31}^{+0.96}$ min. and the later period drop (caused by the following evolution of the system, see below) was $P_{\rm orb}^{\rm post} - P_{\rm orb}^{\rm imp} = -27_{-58}^{+19}$ seconds. The solution with the period modeled as step function gives the period change $P_{\rm orb}^0 - P_{\rm orb}^{\rm pre} = -32.92 \pm 0.30$ min. and the period drop $P_{\rm orb}^{\rm post} - P_{\rm orb}^0 = -19 \pm 18$ seconds (all uncertainties are 3σ). We note that $P_{\rm orb}^0$ represents only the lower limit of the orbital period just after the impact (it is the mean value over the time interval from $t_{\rm imp}$ to $t_{\rm imp} + \Delta T$).

JD₀ (Date)

One possible explanation for the small drop in the orbit period during the first few weeks following the DART impact is the idea of binary hardening due to the gravitational interaction of the ejecta cloud with Didymos and Dimorphos (Richardson et al. 2023). The DART impact released a significant amount of ejecta from Dimorphos, from micrometer-sized dust to meter-sized boulders (e.g., Moreno et al. 2023, Roth et al. 2023, Jewitt et al. 2023). This observed ejecta is already outside Didymos's Hill sphere on escape trajectories. However, a significant quantity of ejecta was likely initially bound to the system, where it would have made repeated close passes by Didymos and/or Dimorphos. This material would have then been reaccreted or scattered out of the system during close approaches. The net effect of this process could lead to an overall reduction in the orbit period, as Dimorphos would lose angular momentum by ejecting the bound debris. As discussed by Richardson et al. (2023), this orbital hardening will decay exponentially as the debris is cleared out of the system. In the approach involving the angular acceleration (see above), we included this effect as the exponentially decaying drag-like term in Equation 3.

However, this process will depend strongly on the mass-velocity distribution of ejecta at low speeds (6 cm/s < v < 25 cm/s), which is not well constrained. Further investigation of this apparent period drop is needed.

Bottom panel of Figure 4 reveals that while most of the offsets of the observed events with respect to the two solutions are close to zero in L1 and L2 lunations, indicating that the model explains the data well, in lunations L3 to L5 (i.e., starting approximately 70 days after the DART impact) there occur larger random-like deviations of up to about 5 deg. (corresponding to ~ 10 minutes in time). Since these deviations seem to have the same sign for both primary and secondary events (where both types of events occur close in time to each other), we cannot attribute them to eccentricity. However, the deviations of mutual event timings may indicate variations in the orbital period. Meyer et al. (2023) (also Meyer et al. 2021) showed that when Dimorphos enters a tumbling state, the variations of the orbital period, driven by an exchange of angular momentum between the secondary's rotation and its orbit, become chaotic, varying on a time scale of twelve hours and can reach up to several minutes. Together with the suggested decrease in orbital eccentricity, these observed variations also suggest an onset of the tumbling state about 70 days after the impact.

4. MUTUAL EVENTS EPHEMERIS FOR 2022–2025

In order to facilitate planning ground-based observations in the next years, or to analyze the observations made in the pre- or post-impact apparitions, we computed times of mutual events that occurred and will occur from July 2022 to the end of 2025. It is included here as a supplemental data file. [supplementary_data.tar]

The prediction for pre- and post-impact apparition is made using the nominal solutions presented in Sections 3.1 and 3.2, respectively. Due to the random-like deviations in the mutual events timings with respect to the nominal solution, which we see since approximately 70 days after the DART impact (see bottom panel of Figure 4), the uncertainties may be up to ± 10 minutes larger for dates after 2022-12-02 than indicated in the link above. A future evolution of the mutual orbit may increase the uncertainties further.

5. DISCUSSION AND CONCLUSIONS

We have presented the results of modeling the mutual orbit of Didymos-Dimorphos system using the mutual events data that were obtained within the photometric campaign carried out in support of the DART mission. We updated the parameters of the pre-impact orbit and obtained the solution for the new, post-impact orbit.

We obtained that the total Dimorphos' period change, i.e., the difference between the post-impact and the pre-impact orbital period, was -33.240 ± 0.072 min. (all uncertainties are 3σ , unless otherwise stated). The results show that while the majority of the period change (~ -33 min.) occurred immediately after the impact, it was slightly smaller (in absolute values) shortly after the impact and it took a few weeks for the orbital period to stabilize, i.e., it was followed by an additional change of a few tens of seconds. By introducing two alternative models to describe the evolution of the orbital period in the days and weeks following the impact, we found that the orbital period immediately after the impact was longer by 27^{+58}_{-19} seconds (with the model of exponentially decreasing angular acceleration) or by 19 ± 18 seconds (with the model of the orbital period as step function) than its final value.

As the primary and secondary events were phase-shifted with respect to each other in the lunations L0 to L2 after the impact, and such shifts were not apparent in the lunations L3 to L5 (i.e., from about 70 days after the impact), we modeled the orbit as eccentric (with apsidal precession) during the L0, L1 and L2 lunations, while we modeled it as circular during the L3, L4 and L5. We obtain the eccentricity in the L0 to L2 lunations was 0.028 ± 0.016 and the apsidal precession rate of $7.3 \pm 2.0 \text{ deg/day}$. We also tried to fit the whole post-impact dataset with constant eccentricity, but we obtained an unsatisfactory fit.

The observed value of the apsidal precession rate is consistent with Meyer et al. (2021), who predicted the precession period to be from 22 to 62 days for Dimorphos' axis ratio A_2/B_2 from 1.4 to 1.3, respectively. As the pre-impact value of A_2/B_2 was close to 1, see Daly et al. (2023b), the observed apsidal precession rate suggests some reshaping of Dimorphos, as shown also by Richardson et al. (2023).

A gravitational oblateness parameter of Didymos, J_2 , is estimated by Naidu et al. (2023) to be 0.090 ± 0.008 , which is consistent with the oblate spheroidal shape of Didymos, assuming its uniform density. Classical perturbation theory (Murray & Dermott 2000) which assumes the secondary to be a point mass, gives an apsidal precession rate of around 13 deg/day, i.e., about twice our estimated rate. However, as shown by Ćuk & Nesvorný (2010) (see also Meyer et al. 2023), the apsidal precession might be dominated by the perturbations from the prolate and librating secondary, rather than the oblate primary.

Since approximately 70 days after the DART impact, we see random-like deviations of up to ~ 10 minutes in the mutual events timings with respect to the nominal solution, which we cannot attribute to the eccentricity. Those deviations may indicate short-term variations in the orbital period. The observed variations together with the suspected decrease of the orbital eccentricity at about the same time suggest that Dimorphos entered a tumbling state around 70 days after the impact. Due to the conservation of angular momentum, the binary eccentricity can only change as a result of

a change in orbital angular momentum. If the post-impact Didymos binary is considered a closed system, then the only sources of angular momentum for the mutual orbit are in the primary and secondary rotations. The primary's spin/shape has not changed by a measurable amount (Ďurech & Pravec 2023), which suggests that the secondary would have had to undergo some change in angular momentum to account for the eccentricity change. A natural way to do this is if the secondary enters an excited rotation state (i.e., tumbling) where angular momentum can be transferred from the mutual orbit to the secondary's rotation. We consider this as the most plausible explanation for the eccentricity change.

From the depths of the mutual events, we estimated the pre-impact Dimorphos-Didymos size ratios was $D_{2,V}/D_{1,V} = 0.223 \pm 0.004$ and the post-impact is 0.202 ± 0.006 (1 σ uncertainties). The difference between the two values indicates the reduction of Dimorphos' volume by $(9 \pm 3)\%$ (1 σ) as the result of the impact. This reduction is about an order of magnitude larger than the estimate for the mass of the ejecta from observations and modeling (the largest estimate of 5.5×10^7 kg is by Ferrari et al. 2023; see Richardson et al. 2023 for a list of published estimates) and therefore it deserves further discussion. First, we point out the relatively large uncertainty of our estimate, so the inferred reduction of Dimorphos's volume is only a marginally significant detection. As the DART impact caused global deformation and resurfacing of Dimorphos (see Raducan et al. 2023), the reduction of the Dimorphos volume might suggest an impact-induced change in its macroporosity. Further, the Dimorphos-Didymos size ratios are based on depths of total secondary events and an assumption that albedos of both components were the same, both before and after the impact. Albedo differences can introduce a systematic error into the estimates.

The change of the orbital period was also estimated by Gudebski et al. (2023), who compared the mutual events prediction from SP22 with events observed on 2022 October 30 and 31, and obtained $\Delta P = -34.2 \pm 0.1$ min. To compare their results with ours, we constructed Figure 7, which contains the bottom part of their Figure 1, to which we had added the data used in our analysis and the synthetic curves of our nominal solutions shown in Figure 5. The figure shows that the lightcurve decomposition and the times of the mutual events derived by Gudebski et al. are fairly consistent

with our nominal solution (although they did not identify the second part of the primary event from October 31 in their data). However, the description of the method they used to derive the new orbital period from the data (especially, the first paragraph of their Section 3) is quite vague and a description of how they estimated its uncertainty is also missing. They also did not take into account the uncertainty of the event ephemeris from SP22, which was ± 20 minutes (3σ) at the end of October 2022. Therefore, we are unable to compare their result with the conclusions of this paper.

In this work, we used a simplified model, that did not include a full two-body interaction and describes the orbit parameters as dynamically independent variables. The advantage of this approach is that it does not involve any a priori dynamical assumptions and therefore serves as an independent check of more complex models. An independent derivation of the post-impact mutual orbit based on an analysis of mutual event timings by integrating Dimorphos' orbit using a 2-dimensional model has been made by Naidu et al. (2023) Their estimated parameters are in agreement with our values within the uncertainties.

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Figure 7. Figure 1 from Gudebski et al. (2023) with the orbital lightcurve component (cyan circles), the synthetic lightcurve for the best-fit solution with the angular acceleration (solid cyan curve) and the model with the circular orbit and constant mean orbital period (dashed cyan curve) from this work overplotted.

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