

LOOPer: A Learned Automatic Code Optimizer For Polyhedral Compilers

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While polyhedral compilers have shown success in implementing advanced code transformations, they still have challenges in selecting the most profitable transformations that lead to the best speedups. This has motivated the use of machine learning to build cost models to guide the search for polyhedral optimizations. State-of-the-art polyhedral compilers have demonstrated a viable proof-of-concept of this approach. While such a proof-of-concept has shown promise, it still has significant limitations. State-of-the-art polyhedral compilers that use a deep-learning cost model only support a small subset of affine transformations, limiting their ability to apply complex code transformations. They also only support simple programs that have a single loop nest and a rectangular iteration domain, limiting their applicability to many programs. These limitations significantly impact the generality of such compilers and autoschedulers and put into question the whole approach. In this paper, we introduce LOOPer, the first polyhedral autoscheduler that uses a deep-learning based cost model and covers a large set of affine transformations and programs. It supports the exploration of a large set of affine transformations, allowing the application of complex sequences of polyhedral transformations. It also supports the optimization of programs with multiple loop nests and with rectangular and non-rectangular iteration domains, allowing the optimization of an extensive set of programs. We implement and evaluate LOOPer and show that it achieves speedups over the state-of-the-art. On the Polybench benchmark, LOOPer achieves a geometric mean speedup of 1.59× over Tiramisu. LOOPer also achieves competitive speedups with a geometric mean speedup of 1.34× over Pluto, a state-of-the-art polyhedral compiler that does not use a machine-learning based cost model.

Additional Key Words and Phrases: Polyhedral Compiler, Affine Transformations, Automatic Code Optimization, Deep Learning, Machine Learning, Cost Model.

1 INTRODUCTION

In an era where compute-intensive applications are everywhere, writing highly optimized code is becoming highly important. Such code is usually manually optimized by experts. This is a non-trivial task that is time-consuming, laborious, and requires deep knowledge of the underlying hardware.

For these reasons, many efforts in the compiler community have focused on developing automatic code optimization frameworks [1, 2, 5, 6, 8, 10, 13, 15, 24, 29, 34, 37, 38]. Polyhedral compilers have shown success in this area. They can perform complex iteration space transformations [4, 15, 24, 32, 35, 37, 38], data locality optimizations [23, 25], and memory management optimizations [7, 20, 21, 26, 33, 34].

Although polyhedral compilers can apply complex program and data layout transformations, they still face challenges in selecting the right transformations that result in the best performance [11].

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This is mainly because their search techniques rely on less accurate cost models, leading compilers to make suboptimal decisions.

Previous research has tried to address this problem by using machine learning to build a cost model. This cost model is then used to explore code transformations. Tiramisu [11] is a state-of-the-art polyhedral compiler that has adopted this approach [9]. While it avoids the abovementioned problem, the current Tiramisu autoscheduler has significant limitations.

First, supported programs are limited to programs with single loop nests. In addition, those programs can only have rectangular iteration domains. This critical limitation prevents the Tiramisu autoscheduler from optimizing an important class of programs. In the PolyBench benchmark suite [27] for example, 21 out of 30 benchmarks cannot be optimized by the Tiramisu autoscheduler because they are either composed of a sequence of loop nests, or they have non-rectangular iteration domains. Second, the autoscheduler only explores a small subset of affine transformations (polyhedral optimizations). It only explores five transformations (loop interchange, tiling, parallelization, unrolling, and loop fusion of inner loops within a single loop nest). Each of these transformations can only be applied once. It does not cover many important affine transformations. For instance, it does not support loop skewing, which is necessary for parallelizing many stencils. As a result, a stencil, such as a Jacobi 2D with a LARGE input, optimized by the Tiramisu autoscheduler is $10\times$ slower than that optimized by Pluto, which covers a larger space of affine transformations.

In this paper, we propose LOOPer (a **L**earned **O**ptimizer for **P**olyhedral compilers), the first deep-learning based polyhedral autoscheduler that covers a large space of programs and code optimizations. It explores sequences of affine transformations and uses a deep-learning based cost model to evaluate them. LOOPer supports programs with multiple loop nests as well as programs with rectangular and non-rectangular iteration domains. In addition, it supports a larger set of transformations, including n -long sequences of affine transformations. To support these new patterns and improve performance, LOOPer's cost model brings a set of improvements to the cost model and data representation presented in [9].

In comparison with other state-of-the-art non-polyhedral autoschedulers that use deep learning (e.g., TVM[17] and Halide[1]), LOOPer is specialized in polyhedral optimizations and therefore can explore complex affine transformations.

The contributions of this paper are as follows:

- We introduce LOOPer, the first deep-learning based polyhedral autoscheduler that covers a large space of programs and code optimizations. It explores a large subset of affine transformations and supports programs with multiple loop nests as well as programs with rectangular and non-rectangular iteration domains.
- We release the dataset we used to train our model, containing 29 million data points.
- We open source the code for LOOPer¹ and its cost model².
- We implement the proposed autoscheduler in Tiramisu and evaluate it on the PolyBench benchmark suite [27], the gold standard benchmark for polyhedral compilers. We show that LOOPer achieves a median speedup of $1.56\times$ and a geometric mean speedup of $1.59\times$ over Tiramisu [9]. It also achieves competitive speedups compared to Pluto [15] with a median speedup of $1.38\times$ and a geometric mean speedup of $1.34\times$.

2 RELATED WORK

This paper proposes an autoscheduler that uses a deep-learning cost model to explore sequences of affine transformations. In comparison with existing work, LOOPer has three distinctions:

¹https://github.com/Tiramisu-Compiler/tiramisu/releases/tag/LOOPer_v1

²https://github.com/Tiramisu-Compiler/cost_model/releases/tag/LOOPer_Cost_Model

Table 1. COMPARISON BETWEEN DIFFERENT AUTOSCHEDULERS.

| Feature | LOOPer | Tiramisu | Pluto | Halide | TVM |
|--|--------|----------|-------|---------|---------|
| Deep-learning based cost model | Yes | Yes | No | Yes | Yes |
| Affine transformations | Yes | Limited | Yes | No | No |
| Rectangular iteration domains | Yes | Yes | Yes | Yes | Yes |
| Non-rectangular iteration domains | Yes | No | Yes | Limited | Limited |
| Multiple loop nests | Yes | No | Yes | Yes | Yes |
| Near-complete modeling of affine transformations | No | No | Yes | No | No |

- It supports polyhedral transformations (in contrast to non-polyhedral compilers such as TVM[17] and Halide[1]).
- It uses a deep-learning based cost model to evaluate transformations (in contrast to classical polyhedral compilers such as Pluto [15]).
- It supports a large space of transformations and programs (in contrast to the Tiramisu autoscheduler[10], which only supports a restricted space of transformations and programs).

In this section, we will first present autoschedulers used in polyhedral compilers (those that use deep learning and those that do not). Then, we will present non-polyhedral compilers that use deep-learning cost models. Finally, we will present other approaches that use deep learning in compilers but have not been designed or demonstrated on the problem of autoscheduling. Table 1 shows a summarized comparison with state-of-the-art compilers that are the closest to our work (Tiramisu[10], Pluto [15], Halide[1] and TVM[17]).

Tiramisu. The Tiramisu autoscheduler [10] is an automatic code optimization module included in the Tiramisu compiler [12]. It uses tree-based search techniques to explore code transformations that include loop interchange, parallelization, tiling, unrolling, and loop fusion within a single loop nest (i.e., fusing inner loops within a loop nest). It relies on a deep-learning based cost model for steering the exploration toward finding interesting transformations. The cost model is an LSTM-based neural network that takes as input a set of simple features representing the unoptimized code and a set of boolean tags representing code optimizations. This model works by recursively embedding a program depending on its AST (Abstract Syntax Tree) structure and then, from the final embedding, predicts the performance of the given transformations. The current Tiramisu autoscheduler has many limitations, though. First, it only supports five transformations (parallelization, unrolling, tiling, interchange, and loop fusion of inner loops within a loop nest). Each of these transformations can only be applied once. It also does not support reversal, skewing and shifting, three common affine transformations. Second, all code transformations are represented as boolean tags (a boolean tag for each loop level indicating whether the loop is parallelized, unrolled, tiled, or interchanged). While such a representation is enough for certain transformations, such as parallelization, it is not expressive enough to represent sequences of affine transformations and does not capture the order of transformations. Third, the Tiramisu autoscheduler only supports programs with a single loop nest, limiting its applicability to many programs. Fourth, it only supports programs with rectangular iteration domains, limiting its applicability even further. Due to these limitations, the current Tiramisu autoscheduler only supports one-third of the PolyBench benchmark suite and does not apply many important optimizations, leading to significant slowdowns in the generated code. LOOPer solves all these problems and adds many other improvements, such as an improved expression representation and support for different data types.

Polyhedral autoschedulers. Polyhedral compilers such as Pluto [15], Pluto+ [14], Polly [24], Tensor Comprehensions [37], PENCIL [3, 5], and PolyMage [29] are fully automatic. Some of them are designed for specific domains (such as Tensor Comprehensions and PolyMage), while Pluto, PENCIL, and Polly are more general. Pluto is the most well-established among these compilers, and many of them use its algorithm internally [5, 15, 24, 37]. LOOPer’s deep-learning based cost model is complementary to these compilers as it provides a data-driven model that would allow them to replace their heuristics (decisions about parallelism, loop fusion, tiling, and unrolling). One of the main differences between our approach and that of Pluto is that we use a tree-search algorithm and a deep-learning based cost model to guide the search for all the optimizations that we consider in our space, while Pluto uses an ILP solver along with a linear objective function to find most of its affine transformations and then uses heuristics to decide about the remaining transformations (parallelism, loop fusion, tiling, and unrolling). On one hand, an important limitation of Pluto’s linear objective function is that it does not consider important factors in performance modelization such as the data sizes, the complexity of control in the generated code and the characteristics of the hardware being targeted. A data-driven model automatically avoids this limitation and learns these factors from data. On the other hand, due to the simplicity of the objective function, Pluto can cover a larger space of transformations (near-complete modeling of affine transformations in the case of Pluto+ [14]), which is harder for a deep-learning based autoscheduler since the space of affine transformations is infinite and building a deep-learning model that is accurate on all of that space is hard (due to the amount of data needed). Therefore the core tradeoff between the two approaches is that LOOPer considers a smaller set of affine transformations yet uses a more precise, data-driven cost model, while Pluto considers a larger space of affine transformations but uses a simpler linear cost function. We believe that bridging the gap between the two approaches is an interesting research direction for the future.

PolyGym. Brauckmann et al [16] propose PolyGym, a reinforcement learning environment for finding affine schedules. They propose a formulation for the space of legal transformations in the polyhedral model as a Markov Decision Process (MDP). While PolyGym and Tiramisu both search for polyhedral transformations, one of the main differences between the two approaches is that PolyGym does not use a deep-learning based cost model to evaluate optimizations. It compiles and executes programs instead. Compiling and executing programs takes much more time than running a deep-learning cost model, as we do, and, therefore, the search for optimizations is slower in PolyGym compared to Tiramisu. The other difference is that PolyGym uses reinforcement learning, which we do not cover in this paper.

Iterative Optimization in the Polyhedral Model. Pouchet et al [30, 31] propose a framework for iterative optimization in the polyhedral model. The authors use an algorithm to find the space of legal polyhedral transformations, then, they select candidate transformations from this space of legal transformation. Multiple methods to select candidate transformations have been explored, including random selection and genetic algorithms. Once a schedule is selected, it is applied to the code. The code is then compiled and run, and the speedup is obtained. The search continues until a satisfactory schedule is found or there is a time-out. Our work is complementary to this work. LOOPer’s cost model can be used to evaluate transformations instead of running them, accelerating the search. LOOPer would also benefit from Pouchet’s algorithm for finding legal transformations a priori.

Halide and TVM. Halide [1] also uses a deep-learning based cost model to find efficient code transformations. It combines beam search with a feedforward neural network that predicts the

execution time of programs from a set of manually-engineered features. In a similar way, AutoTVM [18] uses a deep learning model to search for code transformations. Both of these compilers are not polyhedral and therefore do not cover the same space of transformations and programs as Tiramisu.

Deep-learning Models in Compilers. Previous research has used deep learning to build models that allow compilers to make certain decisions. For example, DeepTune [19] uses LSTMs to predict whether an OpenCL kernel should be mapped to CPU or GPU. It also proposes another model to predict whether thread coarsening (a code transformation for GPUs) should be used. MILEPOST GCC [22] uses a 1-nearest-neighbor model with a set of features representing code and predicts the best combinations of compiler flags for GCC (GNU Compiler Collection). Ithemal [28] uses an LSTM-based model to predict the throughput of basic blocks of the control-flow graph of assembly code.

3 OVERVIEW OF THE PROPOSED APPROACH

In this paper, we propose LOOPer, a data-driven polyhedral code optimizer. LOOPer explores a large space of code transformations and uses a deep learning cost model to guide the search. The design of LOOPer is made of two main components: the search space exploration module and the evaluation module. These two modules cooperate to optimize the input program.

The role of the search space exploration module is to iteratively build a sequence of code transformations that optimizes a given program. This module can be further dissected into two parts: a candidate generation algorithm and a search method. The candidate generation algorithm is responsible for suggesting new transformation candidates based on the input program and the current search state. The search method is responsible for defining the space traversal strategy by choosing which candidates to explore next.

The evaluation module is in charge of assessing the quality of candidates that are encountered during the exploration. This module consists of a deep learning model that is trained to predict the potential speedup that sequences of transformations would yield if they were to be applied to the input program.

Scope

Space of supported programs. We are interested in optimizing programs composed of a sequence of rectangular and non-rectangular loop nests that have static affine control. The loop sizes have to be known at compile time (have to be literal constants).

Space of supported transformations. We explore a subset of affine transformations. These transformations are composed of sequences of the following primitive transformations: loop shifting, fusion, skewing, interchange, reversal, parallelization, tiling, and unrolling. Skewing, interchange, and reversal can be applied multiple times and in any order. The maximal length of the transformation sequence is a user-defined hyperparameter.

Target hardware. LOOPer is designed to target multi-core CPU architectures. In this paper, we implemented and deployed LOOPer for an Intel Xeon E5-2695 v2 processor.

4 SEARCH SPACE EXPLORATION

We structure LOOPer’s transformation search space as a tree where each node represents a primitive transformation and a branch is an ordered sequence of transformations. The root of the tree is the original untransformed program. Two main components enable exploring the space of supported transformations. First, a candidate generation algorithm that decides for each node in the tree

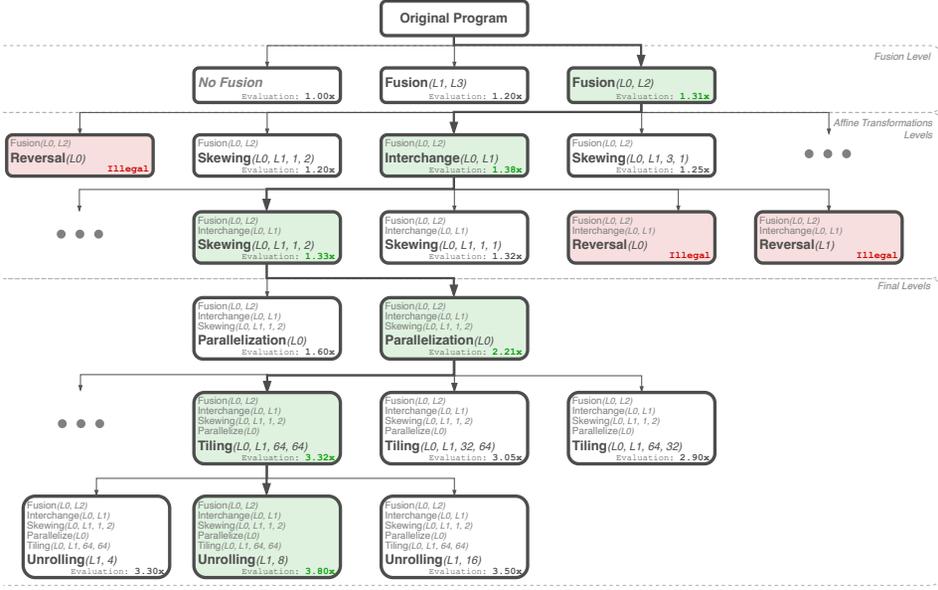


Fig. 1. Example of the search space exploration of a program with two statements showing the different exploration levels.

what transformations can be applied and added to this branch. Second, a search method to choose between the candidates and decide which candidates to explore further.

4.1 Search Method

First, let us look at the search method regardless of which transformations are contained in the nodes, i.e., regardless of the candidate generation algorithm, which we will present in the next section, and regardless of the evaluation technique, which we will present in section 5.

In this work, we use beam search to explore the tree. The beam search algorithm is simple: given an evaluation function that quantifies the quality of each branch in the tree, we pick K elements for each level. K is a search parameter that is referred to as the beam size. At the first level, where we generate the first set of candidates, we evaluate each candidate using our evaluation function and then pick the best K candidates to be explored further in the tree. For each one of these picked candidates, we repeat the same process: we generate new candidates and evaluate them. All of the newly generated candidates are put in the same pool, and only the k -best elements are picked to be explored further. This process continues until a stopping condition is met. In our autoscheduler, the stopping condition is when no new candidates can be generated. Throughout the exploration, illegal transformations (i.e., schedules that violate data dependencies for the explored program) are detected and pruned (we use classical polyhedral dependence analysis and legality check to check the legality of transformations [21, 36]). At each level, we make sure that the option of no transformation being applied is explored. This is useful if all the generated candidates have bad performance. We also check each generated transformation to make sure no sequence of transformations with the same parameters is explored twice.

4.2 Candidate Generation

The candidate generation algorithm generates possible transformations that could be added to a specific branch in the tree. Because we need to generate new transformations depending on what was explored previously on each branch, we structure our candidate generation algorithm depending on the level we are at in the tree:

4.2.1 Fusion Level. At the first level of the tree (where the root is the original, unmodified program), we generate possible loop fusion candidates for the input program AST.

4.2.2 Affine Transformations Levels. After exploring the possible loop fusions and picking the best candidates to explore further using the previously presented search method, we explore affine transformations. To be more specific, at this level, we are interested in exploring affine unimodular transformations. This would exclude affine transformations like tiling. Concretely, we want to explore an n -long sequence of interchange, skewing, reversal, and their parameters. This allows us to reach a large space of affine unimodular transformations while keeping the space restricted to transformations that are more likely to be beneficial. To do this, we would need to generate an arbitrary n -long sequence of interchanges, reversals, and skewings. To make sure we are exploring all the combinations of these three transformations, we do the following: Given n , the maximum size of the sequence of affine transformations, we add n levels where, at each level, we explore all possible interchange candidates, reversal candidates, and a subset of skewing parameters that optimize for locality and inner/outer parallelism. We only explore a subset of skewings because the space of possible skewing parameters is infinite. We use a Pluto-like algorithm to pick the skewing parameters (the main difference is that this algorithm is only applied on two or three loop levels that will be skewed instead of all the loop levels as in Pluto).

Design choices on exploring affine transformations. We have explored other methods for generating affine transformations, including a random generation of the schedule coefficients (in affine schedules). Although this allows the exploration of a wider space of affine transformations, there are two main limitations: First, since the space of all affine transformations is infinite, most of the transformations chosen by the random generation method were not beneficial and often illegal. Second, it is harder to build a deep-learning based evaluation technique that can predict the performance of all the possible transformations. This arises from the difficulty of having a large enough dataset to cover all affine transformations. To avoid these limitations, our autoscheduler explores sequences of common affine transformations. In practice, we find that this is sufficient.

4.2.3 Final Exploration Levels. The best candidates from the previous levels (fusion and affine transformations) are set to explore the following transformations in this order: parallelization, tiling, and unrolling. This means we have a level for each transformation in the exploration tree. We generate a candidate for each possible parallelization on the tree. For tiling, we tile each branch of perfectly nested loops in the program AST. Currently, we only explore unrolling for the innermost nodes in the tree. For the parameters, we explore the following set of parameters for tiling and unrolling, respectively: $\{32, 64, 128\}$ and $\{4, 8, 16\}$.

4.3 Exploration Example

Below is an example program composed of two loop nests and Figure 1 illustrates a hypothetical exploration tree for this program.

```

1 for L0 in 0..128
2   for L1 in 1..128
3     Statement00
4   for L2 in 0..128
5     for L3 in 1..128
6       Statement01

```

In this example, for the sake of simplification, we set the beam size to 1. This means we only explore the single best candidate at each level. We also set the depth of the affine transformation levels to be 2.

At the first level, we have two fusion candidates. That is either fusing the two loop nests at the outermost level (fusing $L0$ and $L2$) or at the innermost one (fusing $L1$ and $L3$). Using the evaluation module, we see that the best candidate is fusing at the innermost level. This transforms the program into a single loop nest with two statements. The evaluation module posits that it improves the execution time by $1.31\times$ times. We move on to exploring this branch further. In the first level from the affine transformation levels, we generate two reversals for each loop, which are illegal and directly discarded from the candidate pool. We generate the possibility of interchanging two loops from the fused loop nest. This candidate has the best evaluation so it will be explored further. At the second and final level of affine transformation levels (since we set n to 2 for this example), we again generate the same set of affine transformations except for the same interchange selected in the current branch. Applying the same interchange twice would lead both interchanges to cancel each other and the initial loop order to be restored. At this level, the skewing of the two loops with parameters 1 and 2 has the best evaluation. So far, the best sequence of transformations for the first two levels is the following: $Fusion(L0, L2)Interchange(L0, L1)Skewing(L0, L1, 1, 2)$.

At the final levels, we start by exploring all the parallelization candidates. We only have two loops in this example, so two parallelization candidates are generated, one for each loop. Applying the previous transformations ($Fusion(L0, L2)Interchange(L0, L1)Skewing(L0, L1, 1, 2)$) and then parallelizing the outermost loop ($L0$) has the best evaluation by being more than twice as fast as the original program. We emphasize that the evaluations are for the whole branch of explored candidates and not for a single transformation each time. After parallelization, tiling and unrolling are explored. The best sequence of transformations for this input program is: $Fusion(L0, L2)Interchange(L0, L1)Skewing(L0, L1, 1, 2)Parallelization(L0)Tiling(L0, L1, 64, 64)Unrolling(L1, 4)$ which has a speedup of $3.8\times$ in comparison with the unoptimized program.

5 COST MODEL

Due to the large search space size, we need a fast and accurate way to evaluate the transformation candidates encountered during exploration. To do so, we use a deep-learning based cost model for evaluating the quality of candidate code transformations. We use this model instead of executing and measuring the speedup.

The cost model used for LOOPER is inspired by the Tiramisu cost model presented by Baghdadi et al. [9]. LOOPER's cost model takes as input an unoptimized Tiramisu program along with a sequence of code transformations and predicts the speedup this sequence of optimizations would yield if applied on the program. This model is then used to explore a transformation search space consisting of combinations of loop shiftings, fusions, interchanges, skewings, reversals, tilings, parallelizations, and unrollings. Our contribution with regard to the original Tiramisu cost model lies in adding the support of a more complex and extensive space of programs and transformations.

In the following sections, we describe LOOPER's cost model. These sections will briefly describe the original Tiramisu cost model while highlighting its limitations to contrast our contributions better.

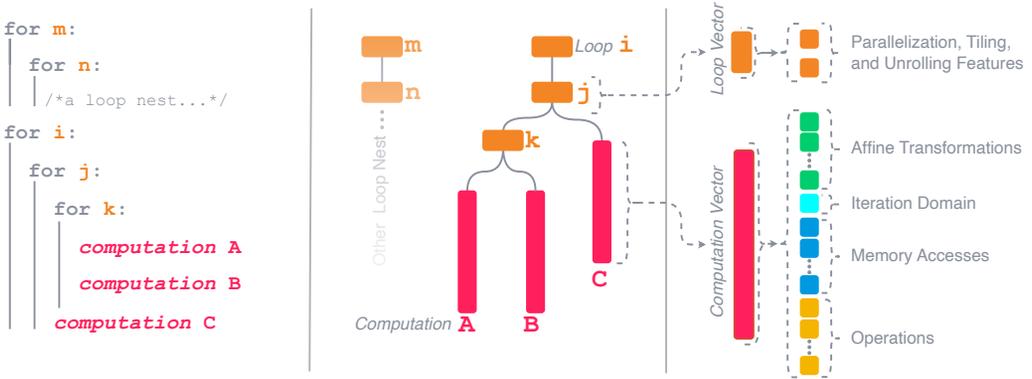


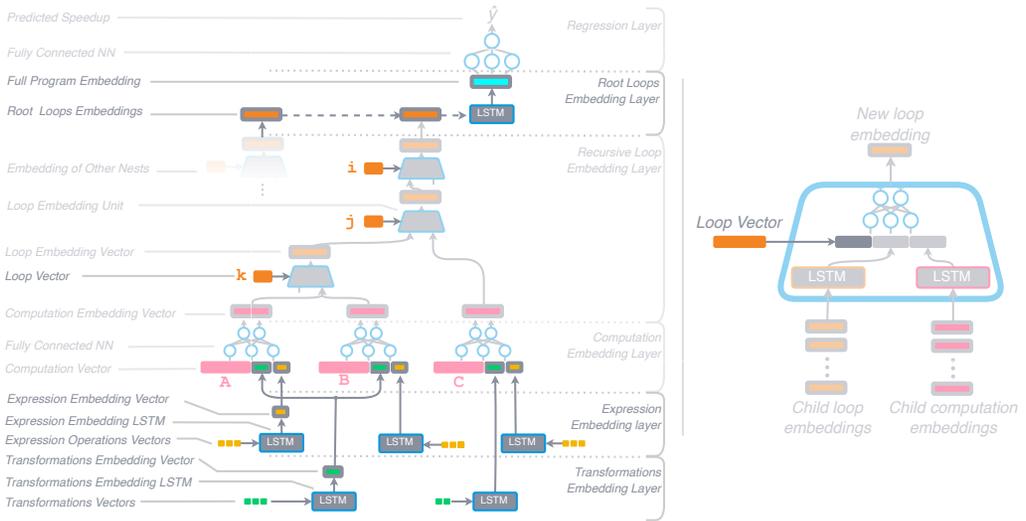
Fig. 2. Input representation example.

5.1 Input Representation

Figure 2 illustrates the input representation of LOOPPer’s cost model. The inputs of the Tiramisu model are characterized by extracting a set of high-level features from the input program and transformations. These features are extracted from a source-level representation of the program and stored in a variable-size representation. These features are organized as a sequence of tree structures that model the original hierarchy of the program’s loop nests. In each tree, the leaf nodes represent computations (statements), and the rest of the nodes represent loops. Each tree node is made of a set of features that describe the corresponding loop or computation. These features can be grouped into two classes: program features and transformation features.

The program features describe the original untransformed program. Such features include:

- **Iteration domain:** The original Tiramisu cost model used the upper and lower bound of each loop to describe the iteration domain of a computation. Since the scope of the original Tiramisu model was limited to programs with rectangular iteration domains, representing the bounds of loops as integer values was enough to encode the iteration domain. In LOOPPer’s model, we represent the iteration domain in its most general fashion: as an iteration domain matrix (a matrix defining the constraints over the loop iterators). We use the same format used in the classical polyhedral literature (please refer to the supplementary material for a background about the polyhedral model and the iteration domain matrix format). The iteration domain matrix format allows the representation of both rectangular and non-rectangular iteration domains. Other options exist for describing the iteration space without using the iteration domain matrix. For example, one can have an estimation of the N-dimensional rectangle that contains the non-rectangular iteration space. In section 6.3, we compare these two options and show the benefit of using this polyhedral representation.
- **Computation’s expression (right-hand side of a statement):** We represent the computation operations using a list of vectors. Each vector encodes one operation in the expression (add, sub, mul, div, max, min, etc.). We use a one-hot encoding to represent the type of operation. The vectors are ordered using a post-order traversal of the expression tree. The goal here is to capture the structure of the expression tree. The original Tiramisu cost model used a histogram of arithmetic operations (i.e., the number of occurrences of each arithmetic operation in the expression), thus omitting the actual structure of the expression. In section 6.3, we show the benefit of using this new representation.



(a) Processing the tree representation presented in Fig-
ure 2 through LOOPer's cost model.

(b) Loop embedding
unit.

Fig. 3. The architecture of LOOPer's neural network. The dim-colored elements are parts of the original Tiramisu cost model. The bright-colored parts represent our contributions to the architecture.

- A list of access matrices to represent the read and write access patterns in the statement. A numerical identifier is appended to each access matrix to indicate which memory buffer is accessed by the corresponding pattern. We use the same format used in the classical polyhedral literature (please refer to the supplementary material for background about access matrices in the polyhedral model).

The second class of features is the transformation features. These features describe the set of transformations applied to the program. We represent the affine transformations as a list of vectors where each vector represents the type and the parameters of the corresponding transformation. This list can have an arbitrary length and does not impose a particular order on the transformations. The representation of the rest of the transformations (the transformations that can only be applied once per computation, such as parallelization) is kept as tags. For instance, parallelized loops would set the parallelism tag to 1. We also explored the use of the polyhedral schedule matrix representation to represent transformations. In section 6.3, we show a comparison between our proposed representation (using a vector to represent each transformation) and that of using a schedule matrix to represent each transformation. Our proposed representation is different from the original Tiramisu cost model which used a tag for each supported type of transformation along with its parameters (if applicable). Since the Tiramisu autoscheduler did not support the application of the same transformations multiple times, this tag representation was enough for such a transformation space. However, this representation does not allow representing the application of affine transformations many times in an arbitrary order.

5.2 Model Architecture

The problem of speedup prediction can be formulated as a regression problem. Given an input program and a set of transformations, the cost model predicts the expected speedup if these transformations were to be applied. This neural network is an AST (Abstract Syntax Tree) based recursive model that combines Recurrent and Recursive Neural Networks. Figure 3 shows our proposed architecture. This architecture is inspired by the architecture of the original Tiramisu cost model with some important differences. The original model is composed of three layers only (as represented by dim-colored parts of the figure). Our proposed model architecture adds three new layers (as represented by brightly-colored parts of the figure). These layers allow the model to support our target space of programs and transformations.

We arrived to the current design of our proposed model after extensive experiments with different model architectures (feedforward, LSTM, multi-layer LSTMs, and transformers). Our proposed architecture provides the highest accuracy among all the other architectures (while still being the fastest). Using an AST-based recursive model that combines Recurrent and Recursive Neural Networks has shown to be the most effective in the context of our problem. In the following paragraphs, we will describe the purpose and function of each layer of our proposed model.

Affine Transformations Embedding layer. The role of this layer is to turn the sequences of affine transformation vectors into a learned embedding. This layer is meant to compress the variable-length sequence of transformations into a fixed-size vector containing an abstract representation of the overall combination of affine transformations. For each computation, this layer takes as input the sequence transformation vectors corresponding to that computation, processes them through an LSTM and generates an embedding vector for that combination of transformation. This layer did not exist in the original Tiramisu cost model, and its addition allows the support of the application of sequences of transformations of arbitrary length and order.

Expression Embedding layer. The original Tiramisu cost model did not leverage information about the structure of the computations' expressions. Instead, it only relied on representing the memory accesses and the operations histogram in a padded fixed-size list. The purpose of this layer is to exploit the structural information from the expressions. This layer is designed to generate an embedding that represents the expressions of computations. This embedding is meant to capture information such as the type of operations (e.g., add, sub, mul, div, etc.) and their order. This layer takes a representation of the expression operations as input, passes it through an LSTM, and generates an embedding that represents the expressions.

Computation Embedding Layer. This layer is meant to merge the high-level computation features extracted for each computation with both the transformation embeddings and the expression embeddings generated by the preceding layers. This layer will then generate a computation embedding vector for each program computation. This is done by concatenating each computation vector with its corresponding transformations and expression embedding vectors and then passing the resulting vector through a fully connected neural network.

Recursive Loop Embedding Layer. At this level, the entire loop nest and the transformations affecting it get summarized into a single embedding vector. This layer recursively combines computation embeddings, loop features, and loop embeddings following the hierarchy of the original loop nests. Each recursion step generates a new loop embedding that would be fed to the following step. The root loop embedding is considered the final embedding of the entire loop nest. At each particular loop level, the embeddings of the child loops (if any) and child computations (if any) are summarized through two different LSTMs, one for the loops and the other for the computations.

The resulting vectors, along with the features of the current loop level, are merged through a fully connected neural network, generating a new loop embedding vector.

Root Loops Embedding Layer. The scope of programs of the original Tiramisu cost model was limited to single loop nests. This restriction prevents the support of a considerable portion of real-world programs. To eliminate this restriction, we introduce a Root Loops Embedding layer that is tasked to aggregate the embeddings of multiple separate loop nests. This layer uses an LSTM to generate the embedding of the entire program, given the embeddings of each root loop. This embedding contains all the relevant abstract features needed to predict the speedup.

Regression Layer. The final layer of the architecture performs the regression task, predicting the final speedup value. This layer consists of a fully connected neural network that takes as input the final embedding returned by the Root Loops Embedding Layer and outputs a single value that represents the predicted speedup.

5.3 Data Generation and Model Training

The input space of LOOPER’s model is very large since it is made of combinations of programs and transformations. Therefore, effectively training such a neural network to be accurate on such a large space requires a considerable amount of labeled data. To produce such a dataset, we generated a large corpus of transformed Tiramisu programs for which we measured and recorded the speedup. The sampling technique that we used involves a two-step process. First, we sample the program space, and then we sample the transformation space of each program. The program sampling is done by randomly generating synthetic Tiramisu programs. These programs are generated by combining sequences of common computation patterns (e.g., reductions and stencils). The transformation sampling is done by collecting the candidate transformations explored using the search technique described in Section 4. We run LOOPER’s proposed search technique on each of the synthetic programs. During the exploration, we apply each candidate schedule, compile and execute the transformed program, measure its speedup, and store it as a new data point in the dataset. We preferred this sampling approach over random sampling because the latter would include combinations of transformations that are unlikely to be encountered during real exploration, making the dataset have less useful examples.

While our model is specific to a single CPU (on which data was collected), the approach itself is hardware-independent and can be reproduced for other CPU architectures without any adaptations being required. This ease of portability is because no parts of the exploration technique, the model’s architecture, and the input characterization are hardware-dependent. In order to port LOOPER to a new CPU architecture, one only needs to generate a new dataset for the target CPU and retrain the cost model. While supporting other CPUs is possible in principle (assuming new data is generated), in this paper we are only interested in supporting a single CPU. Supporting other CPUs is left for future work.

Using the proposed data generation technique, we generated a dataset containing about 180,000 synthetic Tiramisu programs with an average of 124 schedules per program. We experimented with increasing and decreasing the average number of schedules per program (when generating the dataset). Much lower degraded accuracy. Much higher did not significantly improve accuracy. The dataset contains a total of 29 million data points. Generating such a dataset took eight months on a 15-node cluster. Each node in this cluster has a dual socket with a 12-core Intel Xeon E5-2695v2 CPU per socket, 128 GB RAM, and runs Ubuntu 18.04. This is the same hardware that we use later for the evaluation.

Using this dataset, we trained LOOPER’s neural network to predict speedups by minimizing the MAPE (Mean Absolute Percentage Error) loss between the real and the predicted values. Training

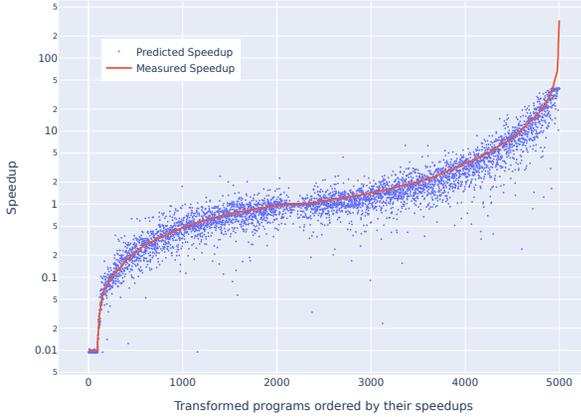


Fig. 4. Predicted speedups compared to measured speedups

time on this dataset is 35 hours for 500 epochs. The model was trained on machine equipped with an AMD EPYC 7742 64-Core Processor, 1TB of RAM, and an NVIDIA A100-SXM-80GB GPU.

6 EVALUATION

In this section, we demonstrate and analyze the performance of LOOPPer in contrast to state-of-the-art autoschedulers. We will first evaluate LOOPPer’s cost model in isolation to assess its reliability as an objective function (section 6.1.) We then evaluate LOOPPer’s efficiency (as a whole system) in optimizing code (section 6.2). To do so, we compare the speedups that LOOPPer achieves to other autoschedulers, namely Pluto, Pluto+, and the Tiramisu autoscheduler. In the same section we also evaluate LOOPPer’s search module in isolation (by using the ground truth measurements to guide the exploration) and we discuss the trade-off between the speed and performance of LOOPPer. We then conclude the evaluation by providing an ablation study on the model’s architecture to justify our design choices (section 6.3).

Experimental Setup. We performed the evaluation on an Ubuntu 22.04.3 system running on a dual-socket 12-core Intel Xeon E5-2695v2 CPU equipped with 128 GB of RAM.

6.1 Cost Model Evaluation

We evaluate the prediction accuracy of LOOPPer’s cost model by computing relevant metrics on a test set of unseen programs. To train and evaluate our model, we split our dataset into a training set (80%) and a validation set (20%). To ensure the soundness of our testing, we make sure no program is repeated in both the validation and the training set. The metrics that we use to evaluate the cost model are MAPE, Spearman’s rank correlation coefficient, and nDCG score.

MAPE. The first metric we use for the evaluation is the MAPE (Mean Absolute Percentage Error) score between the ground-truth speedups and the model’s predictions. The MAPE of our cost model is 26.7% on the validation set, which represents 20% of the whole dataset.

Note that the MAPE of LOOPPer’s cost model is higher than the one presented in [9]. This is due to the significant expansion of the input space both in terms of supported transformations and supported programs. In the upcoming sections, we will show that, even with a higher MAPE, LOOPPer still outperforms [9] and gets comparable performance to Pluto.

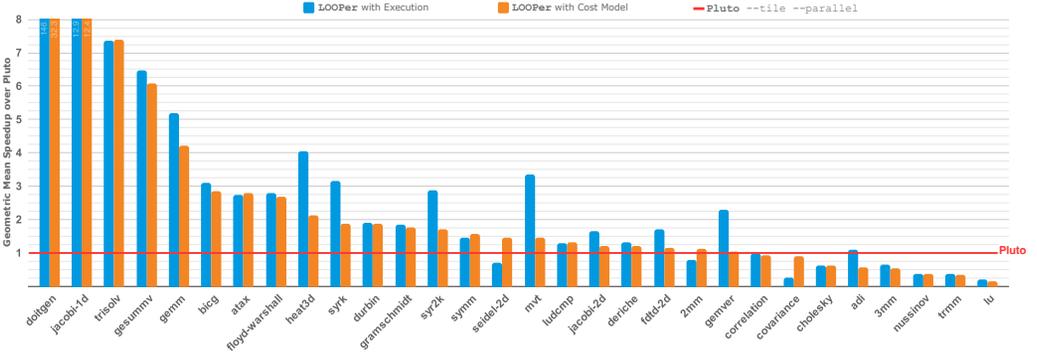


Fig. 5. Speedups of LOOPer when using the cost model and when executing the optimized programs compared to Pluto. The speedups are aggregated by geometric mean over the five sizes of each benchmark. The benchmarks are sorted by descending order of LOOPer’s speedups.

Spearman’s Rank Correlation Coefficient. This metric evaluates the correlation between the predicted and the measured speedups. It is a value between -1 and 1 where a value of 1 means perfect positive correlation. Spearman’s rank correlation coefficient of LOOPer’s cost model is 0.74 , which shows a significant correlation between the predicted and the measured speedups.

nDCG. We also used nDCG (Normalized Discounted Cumulative Gain) to measure the ranking quality of our model. The nDCG score ranges from 0 to 1 , with higher values indicating better ranking ability. We calculate the nDCG score for each program in the validation set and then take the average to represent the model’s performance. The average nDCG on all the programs in the validation is 0.94 , which shows that the order of the predicted speedups is nearly ideal.

Comparing Predicted and Measured Speedups. Figure 4 compares predicted and measured speedups. We use a random subset of the test set consisting of 5000 transformed programs. To simplify the visualization, we sort the programs based on their speedups in ascending order. The predicted speedups are close to the measured ones, as shown in the figure.

6.2 Evaluation of the Autoscheduler

Evaluation Benchmarks. To evaluate LOOPer’s autoscheduling performance, we use the PolyBench benchmark suite [27], the gold standard benchmark used to evaluate polyhedral compilers. PolyBench consists of 30 benchmarks that are extracted from various computing areas, including linear algebra, stencils, physics simulation, etc. We used version $4.2.1^3$. For each benchmark, we used the five different problem sizes that PolyBench defines (MINI, SMALL, MEDIUM, LARGE, and EXTRALARGE). We use the default Polybench data types. To simplify the presentation of the results, we take the geometric mean of the speedups obtained on all five sizes for each benchmark. Note that benchmark programs were not used for training the cost model. LOOPer’s cost model is exclusively trained on randomly synthesized programs as explained in section 5.3.

6.2.1 Comparison with Pluto. In this section, we compare LOOPer with the Pluto autoscheduler [15]. Figure 5 shows the speedups for both on the Polybench benchmarks. We used Pluto with the options `-parallel -tile` to enable parallelism and tiling. We used default tile sizes for Pluto. The baseline for calculating speedups is the execution time obtained using Pluto. The column “LOOPer

³www.github.com/MatthiasJReisinger/PolyBenchC-4.2.1

with *Cost Model*" represents the speedups found by LOOPPer using the cost model to guide the space exploration (i.e., our proposed approach). The results presented are achieved using a beam of size 3 in LOOPPer's search method.

LOOPPer outperforms Pluto in 21 benchmarks out of 30 achieving a median speedup of 1.38 \times and a geometric mean of 1.34 \times . This is mainly due to the fact that LOOPPer uses a more precise cost model, taking into consideration multiple factors (all of which are data-driven). Pluto, in contrast, uses a linear objective function that tries to minimize the distance between producer and consumer statements, which helps in maximizing data locality and outermost parallelism but does not take into consideration other factors.

One of the factors that Pluto ignores but our model considers is the extent of each loop which is encoded in the iteration domain matrix presented in section 5. Taking into consideration the extent of a loop is important when deciding whether to apply parallelization or not. LOOPPer only parallelizes if the benefit of parallelization outweighs its overhead. This decision is completely data-driven as we did not need to engineer LOOPPer to handle this particular case. Pluto, in contrast, always parallelizes the outermost loop when this is legal, which is not always the optimal strategy. The effect of this strategy is important in small problem sizes. For example, in the *seidel2d* benchmark, Pluto applied parallelization after skewing, whereas our method applies only skewing without parallelization; this yielded a schedule that is 1.4 \times faster than Pluto's. A similar issue is found in *trisolv*, where LOOPPer opts out of applying any transformation since it does not find any useful ones in the space. Pluto, however, still chooses to apply parallelization even in the smaller sizes. This leads Pluto to degrade the performance of the initial program. In contrast, LOOPPer outputs a program that is 7 \times faster (up to 123 \times faster for the MINI size in *trisolv*).

Pluto outperforms LOOPPer in 7 out of 30 benchmarks. This is due to either a model misprediction that misguides the search or to the fact that Pluto leverages transformations that we do not explore.

In the *syrk_LARGE* benchmark, for example, our model wrongly predicted that adding reversal and tiling would improve the parallelization of the program. Although this is a good schedule (a speed up of 100 \times), Pluto outperformed LOOPPer and outputted a transformation that is 1.7 \times faster than ours. Another advantage to Pluto is that it applies some transformations that were not reached by LOOPPer. For example, in the two benchmarks *nussinov* and *cholesky*, LOOPPer didn't apply any transformations for all the sizes, whereas Pluto was able to modify the code through affine transformations, enable parallelization, and outperform our system. Enabling parallelization in these two specific cases would require transformations like loop distribution, which are not currently supported by LOOPPer but are set to be added in future work.

The full list of results and raw execution times are provided in the supplementary materials.

6.2.2 Comparison with Pluto+. We also compare LOOPPer with Pluto+ [14], an improved version of the Pluto compiler. The results for Pluto+ vs. LOOPPer are close to the ones presented in the previous section (Sec. 6.2.1), with the geometric mean on all of PolyBench being 1.2 \times and a median of 1.34 \times . In our experiments, Pluto+ gives a 15% performance improvement over Pluto in PolyBench, which is not enough to bridge the gap between LOOPPer and Pluto.

6.2.3 Restricting the Tiling Parameters for LOOPPer. One hypothesis we had for why LOOPPer outperforms Pluto is that our system explores more tiling parameters (32, 64, 128), whereas Pluto does not. To ensure that this is not the only factor making the difference between Pluto and LOOPPer, we disable the exploration of different tiling parameters in LOOPPer and set the tiling factors to 32 (the same default parameters used by Pluto). Our experiments show that even when we add this limitation to LOOPPer, we still obtain a geometric mean of 1.31 \times compared to Pluto.

6.2.4 Comparison with Measurement-guided Exploration. In this section, we compare the performance of LOOPer using two different evaluation methods: first, using the cost model to evaluate each candidate in the search tree, and second, by compiling and executing those candidates to get their execution time (i.e. ground-truth speedup measurements). Figure 5 shows the speedups obtained by the two methods using a beam of size 3. The column "LOOPer with Execution" shows the results of using LOOPer with execution instead of using the cost model.

Results obtained by execution (a perfect model) represent the maximum speedups LOOPer could achieve with the current search method. The better our cost model is, the closer we are to the ideal speedups. The downside of guiding search with execution is that it requires compiling and executing every candidate transformation encountered during the search. This significantly slows down the search and can render it impractical for large programs or large search spaces. Using a cost model provides a compromise between the search time and the quality of the schedules found.

In many benchmarks, the autoscheduler guided by the cost model is able to achieve comparable results to the autoscheduler guided by execution, with a median ratio of 0.79 between the two (speedup by model/speedup by execution). This disparity is justified by the fact that the cost model's predictions are not perfect and this can mislead the search into solutions with lower quality. This can be seen in the *gemver* benchmark, where the speedup obtained by LOOPer represents only 42% of the speedups obtained by execution.

In some cases like *seidel2d_LARGE*, LOOPer when guided with the cost model, achieves a higher speedup than the measurements-guided LOOPer. The greediness of beam search prohibits the measurements-guided exploration from reaching some profitable transformations that the model-guided exploration was able to reach. The cost model misranks schedules that have close ground-truth speedups leading it to explore paths not explored by the measurements-guided exploration.

6.2.5 The Search Speed Trade-off. Using a cost model to evaluate different optimization candidates in the search space instead of compiling and running those programs is useful for two reasons: first, this allows faster space exploration. This is mainly because predicting speedups using the cost model is faster than compiling and running programs (especially for programs that have large inputs). The second reason is that in some contexts, cross-compilation is necessary, and compiling on the target machine is not easy; therefore, allowing the compiler to optimize code even in the absence of access to the target machine is desirable.

In this section, we compare the search time obtained with a model-guided exploration of the space and a measurement-guided exploration (i.e., with real measurements after compiling and executing transformed programs). To do so, we run LOOPer with both evaluation methods and with the same beam size (a beam size of 3) on the entire PolyBench set, and we record the search times for each program. We find that the model-guided LOOPer is, on average, 1288× faster than the measurement-guided LOOPer while delivering comparable results as shown previously (section 6.2.4). This is justifiable by the fact that the inference time of our model is 32ms on average (inference on CPU) whereas compiling and executing a candidate can be orders of magnitude slower. Taking the benchmark *2mm_LARGE* as an example, the search space exploration time using our method is 25.416s. Using the measurement-guided exploration, the search time is more than 7 hours, which means that the search speed has improved by almost 1098×.

The full list of raw search times is provided in the supplementary materials.

6.2.6 Comparison with the Tiramisu autoscheduler. In this section, we compare LOOPer with the Tiramisu autoscheduler described in [9]. We use eight benchmarks from PolyBench for this comparison because the Tiramisu autoscheduler does not support the rest of PolyBench due to two main limitations: not supporting non-rectangular iteration domains and not supporting programs

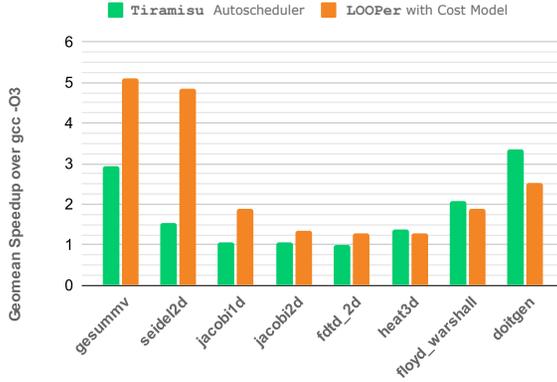


Fig. 6. Speedups of the best schedules found by LOOPer using the cost model compared to the Tiramisu autoscheduler. The speedups are aggregated by geometric mean over the five sizes of each benchmark that the Tiramisu autoscheduler supports.

with multiple loop nests. For the case of the *gemm* benchmark, it is supposed to be supported by the Tiramisu but the autoscheduler crashes when we try this particular benchmark.

With a geometric mean speedup of $1.59\times$ and a median speedup of $1.56\times$, LOOPer outperforms Tiramisu in 5 out of 8 benchmarks and achieves comparable performance in the three remaining benchmarks, as shown in figure 6. The difference in speedups is due to the fact that LOOPer considers a larger space of affine loop transformations. It supports the application of shifting, loop fusion at any loop level, and the application of multiple skewings, reversals, and interchanges. In contrast, the Tiramisu autoscheduler does not support shifting, skewing, and reversal and can apply interchange only once. It also does not support the application of loop fusion at any loop level.

In the benchmarks, loop skewing is the main affine transformation applied by LOOPer and not applied by Tiramisu’s autoscheduler. It is applied multiple times in some cases. Skewing not only improves data locality but also enables parallelism. A clear example is the Seidel 2D benchmark, where parallelization is not legal without skewing. In this specific benchmark, we outperform the Tiramisu autoscheduler on all sizes, and the maximum speedup obtained is $17.3\times$ compared to Tiramisu in the XLARGE size.

LOOPer inaccurately predicted some transformations as beneficial in cases where simple transformations would have been enough. This has allowed Tiramisu to get better speedups in benchmarks such as *heat3d*, *floyd_warshall*, and *doitgen*. These problems are due to the fact that the cost model of the Tiramisu autoscheduler is more accurate. It has a MAPE of 16% compared to 26.7% for our model. This difference can be explained by the fact that LOOPer’s model covers a significantly larger space of transformations and programs, making predicting speedups much harder.

6.3 Ablation Study for the Cost Model

To justify our design choices with regard to the model architecture and input representation, we present three experiments as an ablation study of our contributions to the model.

6.3.1 Iteration Space Representation. In our proposed cost model, we represent the iteration domain using the polyhedral representation of iteration domains (Sec. 5). In this representation, the iteration domain is represented as a set of constraints over the loop iterators and provides a precise representation of the shape of the iteration domain. Another simpler way to represent

non-rectangular iteration domains is to compute an N-dimensional rectangle that surrounds the non-rectangular iteration domain. While this is an approximation of the original iteration domain, one advantage in this case is that the representation is smaller. When comparing these two choices, using the polyhedral representation of the iteration domain proved to be better. The MAPE error when the polyhedral representation was used was 4.6% lower.

6.3.2 Transformation Representation. Another design choice concerns the representation of affine transformation. We explore two representations for affine transformations. The first is the one we presented in Sec. 5 and which represents each transformation as a unique vector with its parameters. The second uses the polyhedral schedule matrix representation to represent transformations. In our experiments, both representations perform similarly in terms of validation error. We chose to use the vector-based representation for two reasons: first, it uses less space, and second, it is easier for experts unfamiliar with the polyhedral model.

6.3.3 Expression Representation. In the original Tiramisu model, the statement expressions were represented using a histogram of 4 operations (addition, multiplication, subtraction, and division) i.e., the count of each operation. In this work, we introduced a new representation for expressions that captures the order of operations that is more general and includes more information about the statement. Our proposed representation of expressions reduces the validation MAPE error by 1%.

6.4 Summary of Evaluation

The proposed deep-learning based cost model achieves a MAPE of 26.7% on a test set of unseen programs. On standard benchmarks, LOOPer achieves a geometric mean speedup of 1.34× compared to Pluto. This is mainly due to the fact that LOOPer’s cost model is data-driven and, therefore takes into consideration factors Pluto does not. LOOPer achieves a geometric mean speedup of 1.59× compared to the Tiramisu autoscheduler. This is mainly due to the fact that Tiramisu does not support affine transformations such as skewing and shifting. LOOPer also supports all of the PolyBench benchmarks, while the Tiramisu autoscheduler supports only nine benchmarks out of thirty. Current cases where LOOPer has a lower speedup than existing compilers are either due to model mispredictions or to transformations not being reachable by our search method. This motivates more research on improving the accuracy of the model and expanding the search space.

7 LIMITATIONS AND FUTURE WORK

While LOOPer has shown promising results compared to state-of-the-art, it still has limitations we plan to address. Currently, the search space we cover does not include loop distribution. In addition, while certain affine transformations (skewing, interchange, and reversal) are explored in an arbitrary order, the other transformations are explored in a fixed order. Future research directions will focus on addressing these limitations and improving the model’s accuracy.

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9 CONCLUSION

In this paper, we presented LOOPer: a novel polyhedral autoscheduler that explores affine transformations using a deep-learning based cost model. Compared to state-of-the-art, this autoscheduler covers a large subset of affine transformations, allowing the application of complex sequences of polyhedral transformations. It also supports programs that have multiple loop nests as well as

programs that have rectangular and non-rectangular iteration domains, allowing the optimization of a larger set of programs.

LOOPer shows that it is possible to use deep learning to predict speedups for a large space of affine transformations and programs, opening the door for more research about using deep-learning based cost models.

The proposed cost model has a MAPE of 26.7% and achieves a median speedup of 1.38× over Pluto, a state-of-the-art polyhedral compiler. It achieves a median speedup of 1.56× over the Tiramisu autoscheduler, mainly because it can explore a larger space of affine transformations.

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