Strong-coupling critical behavior in three-dimensional lattice Abelian gauge models with charged N-component scalar fields and SO(N) symmetry

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We consider a three-dimensional lattice Abelian Higgs gauge model for a charged N-component scalar field ϕ , which is invariant under SO(N) global transformations for generic values of the parameters. We focus on the strong-coupling regime, in which the kinetic Hamiltonian term for the gauge field is a small perturbation, which is irrelevant for the critical behavior. The Hamiltonian depends on a parameter v which determines the global symmetry of the model and the symmetry of the low-temperature phases. We present renormalization-group predictions, based on a Landau-Ginzburg-Wilson effective description that relies on the identification of the appropriate order parameter and on the symmetry breaking patterns that occur at the strong-coupling model may undergo continuous transitions only for N = 2. For $v \neq 0$, i.e., in the SO(N) symmetric case, continuous transitions for N = 2, 3, 4, 6, to verify the renormalization-group predictions. Finite-size scaling analyses of the numerical data are in full agreement.

I. INTRODUCTION

scalar potential

$$V_O(\boldsymbol{\phi}) = r \, \bar{\boldsymbol{\phi}} \cdot \boldsymbol{\phi} + u \, (\bar{\boldsymbol{\phi}} \cdot \boldsymbol{\phi})^2 + v \, |\boldsymbol{\phi} \cdot \boldsymbol{\phi}|^2. \tag{1}$$

Lattice Abelian Higgs (AH) models, in which an Abelian gauge field interacts with a charged Ncomponent degenerate scalar field ϕ , provide an effective description of many collective phenomena characterized by the interplay of topological gauge excitations and scalar fluctuations [1, 2]. In particular, they provide examples of topological transitions that are not characterized by the breaking of a global symmetry [3–7]. The phase diagram of this class of systems has been extensively studied, see, e.g., Refs. [8–55], characterizing the different phases in terms of the topological properties of the gauge correlations, and identifying the possible symmetry-breaking patterns.

The global symmetry of the model and the symmetry breaking that occurs at phase transitions depend on the scalar self-interactions. Most of the investigations considered SU(N)-symmetric scalar potentials. The phase diagram and critical behaviors that occur in this class of models have been extensively investigated in the literature, see e.g., Refs. [8, 9, 31, 41, 43, 44, 46–48, 50, 52, 53, 55]. However, as discussed in Refs. [17, 54], one may also consider more complex scalar self-interactions, which are invariant under a smaller group of transformations, which preserves some irreducible permutation of the field components, to avoid transitions in which only some of the components become critical (in this case the effective theory would be of interest for the analysis of the multicritical behavior). By considering more general scalar potentials and different global symmetry groups, one is able to determine the variety of critical behaviors that can be observed in the presence of an emergent Abelian gauge symmetry in generic lattice systems.

In this work, we consider the two-parameter quartic

For v = 0 the potential is SU(N) symmetric, while for $v \neq 0$ it is only invariant under SO(N) transformations. Results for this model were presented in Ref. [54]. Here we extend, and verify numerically, the renormalizationgroup (RG) predictions obtained by identifying the order parameter and symmetry-breaking pattern at the phase transitions.

We consider a three-dimensional (3D) lattice U(1) gauge model, obtained by a straightforward discretization of the AH field theory

$$\mathcal{L} = \frac{1}{4g^2} \sum_{\mu\nu} F_{\mu\nu}^2 + \sum_{\mu} |D_{\mu}\phi|^2 + V_O(\phi).$$
(2)

We observe in passing that this gauge field theory can also be derived starting from an $O(2) \otimes O(N)$ invariant real scalar model, by gauging the O(2) global group [54]. To simplify the lattice model, we consider the limit $r \rightarrow$ $-\infty$ and $u \rightarrow \infty$ of the potential (1), keeping r/u = -2fixed, so that we can associate an N-component unitlength complex vector $\mathbf{z}_{\mathbf{x}}$ (satisfying $\bar{\mathbf{z}}_{\mathbf{x}} \cdot \mathbf{z}_{\mathbf{x}} = 1$) with each site of a cubic lattice. Concerning the gauge field, one can consider compact formulations, in which the fundamental field is a complex phase $\lambda_{\mathbf{x},\mu}$, or noncompact formulations, in which the basic gauge variable is $A_{\mathbf{x},\mu} \in \mathbb{R}$ and $\lambda_{\mathbf{x},\mu}$ is defined as $\lambda_{\mathbf{x},\mu} = e^{iA_{\mathbf{x},\mu}}$. In both cases the Hamiltonian reads [54]

$$H = H_z + \kappa K_g, \qquad (3)$$

$$H_z = -2NJ \sum_{\boldsymbol{x},\mu} \operatorname{Re} \left(\lambda_{\boldsymbol{x},\mu} \bar{\boldsymbol{z}}_{\boldsymbol{x}} \cdot \boldsymbol{z}_{\boldsymbol{x}+\hat{\mu}} \right) + v \sum_{\boldsymbol{x}} |\boldsymbol{z}_{\boldsymbol{x}} \cdot \boldsymbol{z}_{\boldsymbol{x}}|^2,$$

where $\kappa \sim g^{-2}$ is the inverse gauge coupling, and K_q is



FIG. 1: The κ -J phase diagram of the SO(N) lattice AH model with noncompact gauge fields, for $N \geq 2$ and generic values of v. Three phases are present: the small-J Coulomb (C) phase, in which the scalar field is disordered and gauge correlations are long ranged; the large-J molecular (M) and Higgs (H) ordered phases, in which the global symmetry is spontaneously broken. The results we present in this work refer to the strong-coupling CM line that starts at $\kappa = 0$.

the gauge-field Hamiltonian term, which assumes different forms in compact and noncompact formulations.

We focus on the strong-coupling regime $\kappa/J \ll 1$, in which the gauge kinetic term κK_g gives only rise to a small irrelevant perturbation. Therefore, to study the strong-coupling critical behavior, we do not need to specify the form of K_g . Actually, we can limit our analyses to the model (3) with $\kappa = 0$, neglecting the gauge term κK_g , because the critical behavior for finite (sufficiently small) values of κ is expected to be the same as along the $\kappa = 0$ line, as discussed below. As a consequence of the irrelevance of the gauge kinetic term, the critical behavior in the strong-coupling regime can be determined by considering effective Landau-Ginzburg-Wilson (LGW) theories in terms of gauge-invariant order-parameter scalar fields only.

For N > 2, the phase diagram of the noncompact AH model (3) presents two different low-temperature (large-J) phases, in which the global symmetry is spontaneously broken and that differ in the topological properties of the gauge correlations. The symmetry breaking pattern depends on the number N of components and on the Hamiltonian parameter v [54]. A sketch of the κ -J phase diagram for the noncompact AH model is shown in Fig. 1, for $N \geq 2$ and generic values of v. The κ -J phase diagram of the corresponding compact models differ substantially for sufficiently large values of κ , see, e.g., Refs. [44, 47, 48, 52]. However, the phase diagrams are qualitatively the same in the strong-coupling regime. Indeed, the behavior for $\kappa/J \ll 1$ is the same as for $\kappa = 0$ and the nature of the gauge fields is irrelevant in the latter case. Thus, to determine the critical behavior along the Coulomb-Molecular transition line reported in Fig. 1 (noncompact formulation) or along the analogous line that occurs in compact models it is enough to consider the case $\kappa = 0$.

Beside the Abelian U(1) gauge invariance, the lattice model (3) has a global SO(N) symmetry, $\phi \to S\phi$ with $S \in SO(N)$, which enlarges to SU(N) for v = 0. The global symmetry is broken at a finite-temperature disorder-order transition, whose nature depends on N and on the sign of the Hamiltonian parameter v [54].

For $\kappa = 0$ and v = 0, the lattice model (3) reduces to the SU(N) symmetric \mathbb{CP}^{N-1} model. For N = 2, it can be mapped onto an O(3)-vector model, and thus it shows a continuous transition in the Heisenberg universality class. For any $N \geq 3$ the transitions are of first order [43]. The nature of the transitions changes for $v \neq 0$, as a consequence of the smaller global symmetry of the model. As we shall see, in the presence of SO(N) invariance, continuous transitions also occur for N = 3 and N = 4, for positive values of the scalar self-interaction parameter v.

In this paper we report a numerical study of the model (3) with $\kappa = 0$. We perform Monte Carlo (MC) simulations for several values of N and determine the nature of the critical transitions using finite-size scaling (FSS) methods. The results nicely support the predictions obtained by using an effective LGW description of the system in terms of properly defined gauge-invariant order parameters. For N = 2, the O(3)-vector continuous transition at v = 0 turns into two continuous transition lines for $v \neq 0$. They belong to the Ising and XY universality class for v > 0 and v < 0, respectively. For v = 0and any $N \geq 3$ transitions are of first order. Only firstorder transitions are also expected for any N > 5 in the SO(N) invariant model. However, for N = 3 and N = 4 it is possible to observe continuous transitions for $v > v^* > 0$, where v^* is positive and corresponds to a tricritical point. For $v < v^*$ transitions are of first order. The continuous transitions belong to the O(3) vector universality class for both values of N, but the underlying mechanism is different. For N = 3 the Heisenberg behavior is a consequence of the fact that the order parameter is equivalent to a three-component real vector. For N = 4 the effective description involves two threecomponent real vectors, and the O(3) behavior follows from a nonperturbative RG analysis that shows that the interaction between these two fields is irrelevant in the critical limit.

The paper is organized as follows. In Sec. II we present the theoretical analysis of the model. In Sec. II A we summarize the general results obtained in Ref [54], while in Sec. II B we present a field-theoretical analysis of the effective LGW model appropriate to describe the SO(N)AH model for v > 0. In Sec. III we present our numerical results that confirm the theoretical predictions. Conclusions are presented in Sec. IV. The Appendix presents some technical field-theory results that are relevant for $N \ge 4$.

II. EFFECTIVE LGW DESCRIPTION OF THE TRANSITIONS

A. General arguments

Let us now review the main results on the critical behavior of the model in the strong-coupling regime obtained in Ref. [54]. The critical behavior along the strong-coupling transition line that starts at $\kappa = 0$ depends on the sign of the parameter v, which determines the symmetry breaking pattern. The symmetry of the low-temperature phases can be determined by analyzing the minima of the scalar potential (1). For v > 0, the fields corresponding to the minimum configurations can be parametrized as [54]

$$\boldsymbol{\phi} = \frac{1}{\sqrt{2}} \left(\boldsymbol{s}_1 + i \boldsymbol{s}_2 \right), \qquad \boldsymbol{s}_1 \cdot \boldsymbol{s}_2 = 0, \tag{4}$$

where s_1 and s_2 are orthogonal real vectors satisfying $|s_1| = |s_2|$. In this case the global SO(N) symmetry of the model is broken to $SO(2) \oplus O(N-2)$.

For v < 0, the minimum configurations can be parametrized as

$$\boldsymbol{\phi} = e^{i\alpha}\boldsymbol{s},\tag{5}$$

where s is a real N-component vector, and α an arbitrary phase. The SO(N) symmetry is broken to O(N-1).

To characterize the spontaneous breaking of the SO(N) symmetry, two different order parameters were introduced,

$$R_{L,\boldsymbol{x}}^{ab} = \frac{1}{2} (\bar{z}_{\boldsymbol{x}}^{a} z_{\boldsymbol{x}}^{b} + \bar{z}_{\boldsymbol{x}}^{b} z_{\boldsymbol{x}}^{a}) - \frac{1}{N} \delta^{ab}, \qquad (6)$$

$$T_{L,\boldsymbol{x}}^{ab} = \frac{1}{2i} (\bar{z}_{\boldsymbol{x}}^a z_{\boldsymbol{x}}^b - \bar{z}_{\boldsymbol{x}}^b z_{\boldsymbol{x}}^a), \qquad (7)$$

which transform under two different representations of the SO(N) group. Their behavior depends on the sign of v. For v < 0, $R_{L,\boldsymbol{x}}^{ab}$ condenses in the ordered phase, while $T_{L,\boldsymbol{x}}^{ab}$ vanishes. For v > 0 and N = 2, $T_{L,\boldsymbol{x}}^{ab}$ condenses, while $R_{L,\boldsymbol{x}}^{ab}$ vanishes. Finally, for v > 0 and $N \ge 3$, both order parameters condense in the ordered phase.

For sufficiently small values of κ along the CM transition line (or along the corresponding line in compact models), gauge fluctuations are not expected to play an active role at the transition. Indeed, the gauge properties of the two small- κ phases are the same: gauge modes are long ranged and charged excitations are confined in both of them. Therefore, the transition should be uniquely driven by the breaking of the global symmetry. Thus, an effective description of the critical universal behavior can be obtained by considering a LGW theory for an appropriate gauge-invariant scalar order parameter that condenses at the transition, without considering the gauge fields [43, 44, 48].

For v < 0 the relevant order parameter [54] is $R_{L,x}^{ab}$. The antisymmetric operator $T_{L,x}^{ab}$ is expected to be disordered on both sides of the transition. Since $R_{L,x}^{ab}$ is a real symmetric operator, we expect the small- κ transitions to be described by a LGW for a real symmetric traceless $N \times N$ matrix field $\Phi^{ab}(\boldsymbol{x})$, that represents a coarse-grained average of $R_{L,\boldsymbol{x}}^{ab}$ over a large, but finite, lattice domain. The corresponding LGW Lagrangian is obtained by considering all monomials in $\Phi^{ab}(\boldsymbol{x})$ that are allowed by the global SO(N) symmetry up to fourth order. We obtain

$$\mathcal{L}_{\Phi} = \operatorname{Tr}(\partial_{\mu}\Phi)^{2} + r\operatorname{Tr}\Phi^{2} + s\operatorname{tr}\Phi^{3} \qquad (8)$$
$$+ u (\operatorname{Tr}\Phi^{2})^{2} + v\operatorname{Tr}\Phi^{4}.$$

For N = 2, we can parametrize the field as

$$\Phi = \begin{pmatrix} \phi_1 & \phi_2 \\ \phi_2 & -\phi_1 \end{pmatrix} . \tag{9}$$

It follows that $(\Phi^2)^{ab} = (\phi_1^2 + \phi_2^2)\delta^{ab}$, the cubic term vanishes, and the two quartic terms are equivalent. The resulting LGW theory is equivalent to that of the O(2)symmetric vector model. Thus, we predict continuous transitions to belong to the XY universality class. On the other hand, for $N \ge 3$ the cubic Φ^3 term is generally present. This is usually considered as the indication that phase transitions are of first order, as one can easily infer using mean-field arguments. We expect this behavior to hold for any v < 0, up to v = 0, where we recover the SU(N)-invariant CP^{N-1} model, whose transition is continuous for N = 2, in the O(3) vector universality class, and of first order for any $N \ge 3$ [43, 46].

As discussed in Ref. [54], for v > 0 the relevant order parameter is the antisymmetric tensor field $T_{L,\boldsymbol{x}}^{ab}$. We shall therefore consider the LGW model for an antisymmetric $N \times N$ real field $\Psi^{ab}(\boldsymbol{x})$, which represents the coarse-grained average of $T_{L,\boldsymbol{x}}^{ab}$. The corresponding LGW Lagrangian reads

$$\mathcal{L}_{\Psi} = \operatorname{Tr} \partial_{\mu} \Psi^{t} \partial_{\mu} \Psi + r \operatorname{Tr} \Psi^{t} \Psi + u \left(\operatorname{Tr} \Psi^{t} \Psi \right)^{2} + w \operatorname{Tr} \left(\Psi^{t} \Psi \right)^{2}, \qquad (10)$$

where $\Psi^t = -\Psi$ is the transpose of Ψ . Note that the cubic term is absent because $\operatorname{Tr}\Psi^n = 0$ for any odd n. As discussed in Ref. [54], also the operator $R^{ab}_{L,\boldsymbol{x}}$ is expected to be critical at transitions with v > 0 for any $N \geq 3$. The analysis of the behavior for $v \to \infty$ shows that in this limit we have the relation

$$R_L^{ab} = -a \left[(T_L^2)^{ab} - \frac{\delta^{ab}}{N} \operatorname{Tr} T_L^2 \right], \qquad (11)$$

where a is a positive constant. In the LGW formalism, this implies that R_L^{ab} has the same critical behavior as

$$\mathcal{R}^{ab} = (\Psi^2)^{ab} - \frac{\delta^{ab}}{N} \text{Tr}\Psi^2.$$
 (12)

This relation should hold for any continuous transition with v > 0.

For N = 2 and N = 3 the LGW Lagrangian (10) can be simplified [54, 56]. For N = 2 we can write Ψ^{ab} in

terms of a single real scalar field ϕ defined by $\Psi^{ab} = \epsilon^{ab}\phi$. The two quartic terms are equivalent, and we obtain the LGW model for a real scalar field. Continuous transitions are therefore expected to belong to the Ising universality class. Note that $\mathcal{R}^{ab} = 0$ in this case, which implies that R_L^{ab} is not critical for N = 2.

For N = 3 we can write $\Psi^{ab}(x)$ in terms of a single three-component vector as $\Psi^{ab} = \epsilon^{abc}\phi^c$, where ϵ^{abc} is the completely antisymmetric tensor. Again, the quartic terms are equivalent and we obtain the O(3) vector LGW Hamiltonian. Thus, continuous transitions should belong to the O(3) vector universality class. As for the operator \mathcal{R}^{ab} , we obtain

$$\mathcal{R}^{ab} = \phi^a \phi^b - \frac{1}{3} \delta^{ab} \phi^2.$$
 (13)

This relation implies that R_L^{ab} should have the same critical behavior as the spin-two operator in the Heisenberg model.

No simplifications occur for $N \geq 4$. To determine the critical behavior one should therefore study the RG flow of the model (10) in the space of the quartic couplings u and w. As discussed in the Appendix, two different types of symmetry breakings are possible in model (10), depending on the sign of w. An ordered phase with $SO(2)\oplus O(N-2)$ symmetry is obtained for w < 0. Therefore, continuous transitions for v > 0 are only possible if the LGW field theory admits a stable fixed point with w < 0.

B. Field-theory analysis of the effective LGW model for v < 0

In this Section we perform a field-theory analysis of the RG flow in the model with Lagrangian (10) for $N \ge 4$, with the purpose of studying the possible existence of stable RG fixed points with w < 0. For this purpose we consider the ϵ -expansion approach and we compute the β -functions at two-loop order. We obtain

$$\begin{aligned} \beta_u(u,w) &= -\epsilon u + \frac{1}{12}(N^2 - N + 16)u^2 + \frac{1}{4}w^2 \\ &+ \frac{1}{6}(2N - 1)uw - \frac{1}{24}(3N^2 - 3N + 28)u^3 \\ &- \frac{11}{36}(2N - 1)u^2w - \frac{1}{288}(5N^2 - 5N + 164)uw^2 \\ &- \frac{1}{48}(2N - 1)w^3, \end{aligned}$$
(14)

$$\beta_w(u,w) = -\epsilon w + 2uw + \frac{1}{12}(2N-1)w^2 -\frac{1}{72}(5N^2 - 5N + 164)u^2w - \frac{11}{36}(2N-1)uw^2 -\frac{1}{96}(N^2 - N + 20)w^3.$$
(15)

The one-loop terms of these series agree with the oneloop series computed in Ref. [56]. At one loop, beside the trivial fixed point u = w = 0, the β functions always have a zero on the w = 0 axis. This fixed point corresponds to an O(K) invariant [where K = N(N-1)/2] theory and is always unstable. Indeed, the w term is a spin-four perturbation of the fixed point, which is always relevant for $N \ge 4$ [57–59]. Two additional fixed points are present, but only for relatively small values of N; more precisely, for

$$N < N^*(\epsilon) \approx \frac{1}{4}(2 + 3\sqrt{22}) - \frac{9\epsilon}{16\sqrt{22}} \approx 4.018 - 0.120\epsilon,$$
(16)

with corrections of order ϵ^2 . Given the small negative correction term, it seems plausible to assume that $N^* < 5$ in three dimensions ($\epsilon = 1$), which implies that no stable fixed points exist for $N \ge 5$. We thus predict transitions to be of first order for any $N \ge 5$.

Let us now discuss the model with N = 4. In this case the antisymmetric tensor Ψ^{ab} transforms under a reducible representation of the SO(4) group. It is therefore convenient to parametrize Ψ^{ab} in terms of two threecomponent vectors ϕ_1^e and ϕ_2^e (e = 1, 2, 3) that transform irreducibly:

$$\Psi^{ef} = \frac{1}{2} \sum_{g} \epsilon^{efg} (\phi_1^g - \phi_2^g), \quad \Psi^{4f} = \frac{1}{2} (\phi_1^f + \phi_2^f), \quad (17)$$

for e, f, g = 1, 2, 3. In terms of these two fields we obtain the Lagrangian

$$\mathcal{L}_{\phi} = \frac{1}{2} \sum_{i=1}^{2} [(\partial_{\mu}\phi_i)^2 + r\phi_i^2] + (u + \frac{3}{4}w)(\phi_1^2 + \phi_2^2)^2 - \frac{w}{2}(\phi_1^4 + \phi_2^4). \quad (18)$$

This model is known in the literature as MN model [60–65] and represents the most general model in which M N-component real vector fields (M = 2 and N = 3 in our case) interact symmetrically.

Beside the unstable fixed point with w = 0, the model admits a second simple fixed point that corresponds to two noninteracting O(3) vector fields. Indeed, since for u + 3w/4 = 0 the two vector fields decouple, there is a fixed point with

$$u = \frac{3}{2} U_{O(3)}^* \qquad w = -2U_{O(3)}^*, \tag{19}$$

where $U_{O(3)}^* > 0$ is the fixed point of the O(3) Lagrangian

$$L_{O(3)} = \frac{1}{2} (\partial_{\mu} \varphi)^2 + \frac{r}{2} \varphi^2 + U(\varphi^2)^2.$$
 (20)

It is easy to prove nonperturbatively that this fixed point is stable. Indeed, the RG dimension of the perturbation is $y_p = 2/\nu_{O(3)} - d = \alpha_{O(3)}/\nu_{O(3)}$, as it corresponds to an energy-energy interaction between the two scalar fields. Since $\alpha_{O(3)} < 0$ in the O(3) model, the interaction is irrelevant and thus the fixed point is stable. The fixed point lies in the region w < 0 and is therefore relevant for the model with v > 0. Thus, we predict that continuous transitions for N = 4 belong to the O(3) universality class. Note, however, that y_p is very small, $y_p \approx -0.19$, and thus we expect slowly decaying scaling corrections to the critical behavior.

To determine the critical behavior of \mathcal{R}^{ab} defined in Eq. (12), we express it in terms of ϕ_1 and ϕ_2 . We obtain

$$\mathcal{R}^{ef} = -\frac{1}{2} (\phi_1^e \phi_2^f + \phi_1^f \phi_2^e) + \frac{1}{2} \delta^{ef} \phi_1 \cdot \phi_2, \qquad (21)$$
$$\mathcal{R}^{44} = -\frac{1}{2} \phi_1 \cdot \phi_2, \qquad \mathcal{R}^{4e} = -\frac{1}{2} \sum_{fg} \epsilon^{efg} \phi_1^f \phi_2^g,$$

where e, f, g run from 1 to 3. These relations show that R_L^{ab} behaves as the product of two independent O(3) vector fields.

C. Summary

The previous analysis and the results of Ref. [54] allow us to predict the behavior of the model in the strong coupling regime $\kappa \ll 1$. For N = 2 we expect Ising transitions for v > 0 and XY transitions for v < 0. The line with v = 0 is a multicritical line where the symmetry group enlarges to O(3) and we observe the same critical behavior as in the CP^1 model.

For N = 3 and N = 4, we expect first-order transitions for v < 0 (no stable fixed points exist in the LGW effective theory) and also for v = 0, as in the CP^{N-1} model [43]. For v > 0 continuous transitions are possible, belonging to the O(3) universality class in both cases (but with slowly decaying scaling corrections for N = 4). Since for v = 0 transitions are expected to be of first order as in the CP^{N-1} model [43], it is natural to expect first-order transitions also for small positive values of v. As a consequence, we predict the existence of a tricritical positive value v^* , such that the transition is in the Heisenberg universality class for $v > v^*$ and of first order for $v < v^*$.

Finally, for $N \ge 5$ no stable fixed points occur in the LGW RG flow and thus we expect transitions to be of first order in all cases.

III. NUMERICAL RESULTS

In this section we present numerical Monte Carlo (MC) results, with the purpose of verifying the predictions of the previous Section. We consider the model with $\kappa = 0$ and partition function

$$Z = \sum_{\{z,\lambda\}} e^{-H_z(z,\lambda)},\tag{22}$$

(we set $\beta = 1/T = 1$) and perform several runs by varying J around the critical point for N = 2, 3, 4, and 6. We consider cubic lattices of size L^3 with periodic boundary conditions and use a combination of Metropolis and, for the gauge field λ , microcanonical updates.¹

A. Observables and finite-size scaling relations

To characterize the critical behavior we consider correlations of the order parameters. We consider the twopoint correlation function of the operator R_L^{ab} ,

$$G_R(\boldsymbol{x} - \boldsymbol{y}) = \sum_{ab} \langle R^{ab}_{L,\boldsymbol{x}} R^{ba}_{L,\boldsymbol{y}} \rangle, \qquad (23)$$

and the analogous quantity $G_T(\boldsymbol{x} - \boldsymbol{y})$ for T_L^{ab} . Then, we define the Fourier transform

$$\widetilde{G}_{\#}(\boldsymbol{p}) = \frac{1}{V} \sum_{\boldsymbol{x}-\boldsymbol{y}} e^{i\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{y})} G_{\#}(\boldsymbol{x},\boldsymbol{y})$$
(24)

 $\left(V~{\rm is~the~volume}\right)$ of the two correlation functions. The corresponding susceptibilities and correlation lengths are defined as

$$\chi_{\#} = \widetilde{G}_{\#}(\mathbf{0}), \tag{25}$$

$$\xi_{\#}^{2} \equiv \frac{1}{4\sin^{2}(\pi/L)} \frac{G_{\#}(\mathbf{0}) - G_{\#}(\mathbf{p}_{m})}{\widetilde{G}_{\#}(\mathbf{p}_{m})}, \qquad (26)$$

where $p_m = (2\pi/L, 0, 0)$.

In our FSS analysis we use RG invariant quantities. We consider

$$R_{\xi,\#} = \xi_{\#}/L \tag{27}$$

and the Binder parameters. We define B_R as

$$B_R = \frac{\langle \mu_{2,R}^2 \rangle}{\langle \mu_{2,R} \rangle^2}, \qquad \mu_{2,R} = \sum_{\boldsymbol{xy}} \sum_{ab} R_{L,\boldsymbol{x}}^{ab} R_{L,\boldsymbol{y}}^{ba}. \tag{28}$$

The definition of B_T is analogous.

For N = 4, we also consider the operators

$$\phi_{\pm}^{A} = T_{L}^{A4} \pm \frac{1}{2} \sum_{BC} \epsilon^{ABC} T_{L}^{BC}, \qquad (29)$$

where all indices run from 1 to 3. As already discussed, these two quantities transform irreducibly under SO(4) rotations. The correlation functions

$$G_{\phi,\pm}(\boldsymbol{x}-\boldsymbol{y}) = \sum_{A} \langle \phi^{A}_{\pm,\boldsymbol{x}} \phi^{A}_{\pm,\boldsymbol{y}} \rangle \tag{30}$$

¹ For \boldsymbol{z} we use Metropolis updates with two different proposals: a) we select two components i, j and perform a real rotation, $z'_i = z_i \cos \alpha + z_j \sin \alpha$, $z'_i = -z_i \sin \alpha + z_j \cos \alpha$; b) we select a single component and propose $z'_i = e^{i\alpha} z_i$. For λ_{μ} , we consider a Metropolis update with $\lambda'_{\mu} = e^{i\alpha} \lambda_{\mu}$. In all cases α is chosen in an interval $[-\theta, \theta]$, where θ guarantees an acceptance of approximately 40% (different values of θ are used in the three cases above). For λ we also use a microcanonical update. If $F = z_{\boldsymbol{x}} \cdot \bar{z}_{\boldsymbol{x}+\mu}$, we perform the update $\lambda'_{\boldsymbol{x},\mu} = \bar{\lambda}_{\boldsymbol{x},\mu} F/\overline{F}$.

satisfy $G_{\phi,+}(\boldsymbol{x}) = G_{\phi,-}(\boldsymbol{x})$ and $G_T(\boldsymbol{x}) = -G_{\phi,+}(\boldsymbol{x}) - G_{\phi,-}(\boldsymbol{x})$. In particular, the correlation length computed using $G_{\phi,\pm}(\boldsymbol{x})$ is the same as ξ_T . The Binder parameter is instead different. We define

$$B_{\phi} = \frac{1}{2} \frac{\langle \mu_{2,+}^2 \rangle}{\langle \mu_{2,+} \rangle^2} + \frac{1}{2} \frac{\langle \mu_{2,-}^2 \rangle}{\langle \mu_{2,-} \rangle^2} \qquad \mu_{2,\pm} = \sum_{\boldsymbol{x}\boldsymbol{y}} \sum_A \phi_{\pm,\boldsymbol{x}}^A \phi_{\pm,\boldsymbol{y}}^A.$$
(31)

At continuous transitions, in the FSS limit, the Binder parameter as well as any renormalization-group invariant quantity R scales as

$$R(J,L) \approx f_R(X) + L^{-\omega} f_{c,R}(X), \quad X = (J - J_c) L^{1/\nu},$$
(32)

where ω is the leading correction-to-scaling exponent, and J_c gives the position of the critical point. Relation (32) can also be written as

$$R(\beta, L) = F_R(R_{\xi}) + L^{-\omega} F_{c,R}(R_{\xi}) + \dots$$
(33)

where $F_R(x)$ is universal—it only depends on the universality class, the boundary conditions, and the lattice shape—and $F_{c,R}(x)$ is universal apart from a multiplicative constant. Relation (33) will play an important role to identify the universality class: To verify that the models belong to the Ising, XY, and Heisenberg universality classes, as predicted above, we will compare the curves $F_R(R_{\xi})$ computed in the present model with those computed in the corresponding N-vector model with the same boundary conditions. If the identification is correct, the data we obtain here should converge towards the corresponding N-vector curves as L increases.

Critical exponents can also be obtained from the FSS analysis. The exponent ν can be obtained by fitting the data to Eq. (32). The exponent η instead can be obtained by fitting the susceptibility data to

$$\chi = L^{2-\eta} [G_{\chi}(X) + O(L^{-\omega})], \qquad (34)$$

where X is defined in Eq. (32). Numerically, however, it is more convenient to fit the data to

$$\chi = L^{2-\eta} [\tilde{G}_{\chi}(R_{\xi}) + O(L^{-\omega})], \qquad (35)$$

since these fits do not require any knowledge of ν and J_c .

B. Strong-coupling critical behavior for N = 2

To determine the critical behavior for N = 2, we have performed MC simulations at $\kappa = 0$, varying J. We have only considered relatively small lattice sizes ($L \leq$ 16), as their results are already sufficient to confirm quite precisely the predictions of the previous Section.

First, we set v = 10. We observe a critical transition for $J \approx 0.37$, which we expect to be an Ising transition. To verify it, in the upper panel of Fig. 2 we report the Binder parameter B_T versus $R_{\xi,T}$ and compare the data with the curve computed in the Ising model. We observe



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FIG. 2: Top: Plot of B_T versus $R_{\xi,T}$ for v = 10; Bottom: Plot of B_R versus $R_{\xi,R}$ for v = -10. In both cases N = 2and $\kappa = 0$. The continuous curves have been computed in the Ising model (upper panel) and in the XY model (lower panel). The relative error on the curves is approximately of 0.5%.

good scaling, in spite of the fact that lattices are quite small. To further confirm the predictions, we fit B_T and $R_{\xi,T}$ to Eq. (32). Parametrizing the universal curve with a polynomial, we obtain $\nu = 0.61(3)$ and $\nu = 0.64(2)$ from the analysis of B_T and $R_{\xi,T}$, respectively, in good agreement with the Ising result [66] $\nu_I = 0.629971(4)$. Finally, to determine J_c precisely, we repeat the fits fixing ν to the Ising value, obtaining $J_c = 0.3741(5)$.

An analogous analysis has been performed for v = -10. In the lower panel of Fig. 2 we report the Binder parameter B_R versus $R_{\xi,R}$ and compare the data with the curve computed in the XY model. Again, we observe good agreement confirming the LGW prediction. To estimate J_c we have fitted the two RG invariant ratios to Eq. (32), fixing $\nu = \nu_{XY} = 0.6717(1)$ [67–69]. We obtain $J_c = 0.5633(3)$.



FIG. 3: Top: plot of B_T versus $R_{\xi,T}$ for different values of L; Bottom: plot of B_R versus $R_{\xi,R}$. Data for $\kappa = 0, v = 10$, and N = 3. The continuous curves have been computed in the Heisenberg O(3) vector model. In the top panel we report the curve for vector (spin-1) observables; in the lower panel we report the curve for tensor (spin-2) observables (the relative error on these curves is approximately 0.5%).

C. Strong-coupling critical behavior for N = 3

For N = 3 we have performed a numerical analysis for v = 10 and $\kappa = 0$ to verify the predicted behavior. A priori, the transition is expected to be either of first order (this occurs if $v < v^*$, where v^* is the tricritical point), or continuous in the Heisenberg universality class. The numerical results are consistent with an O(3) continuous transition. Indeed, if we plot the Binder parameter B_T versus $R_{\xi,T}$, the results fall quite precisely on the corresponding universal curve for vector correlations in the Heisenberg model, see the upper panel of Fig. 3. As an additional check, we have fitted the estimates of B_T and $R_{\xi,T}$ to Eq. (32), obtaining $\nu = 0.73(2)$, which is consistent with the accurate estimate [70] $\nu = 0.71164(10)$ for the Heisenberg universality class, see also Refs. [66, 71, 72]. To determine J_c , we have repeated the fits fixing ν to the O(3) value, obtaining $J_c = 0.4479(3).$



FIG. 4: Plot of B_{ϕ} versus $R_{\xi,T}$ (top) and of B_R versus $R_{\xi,R}$ (bottom). Data for $\kappa = 0$, v = 10, and N = 4. The continuous curve in the upper panel has been computed in the Heisenberg O(3) model, using vector (spin-1) correlations. The relative error on the curve is approximately 0.5%.

As we discussed in Sec. II A, the correlations of the field $R_{L,x}$ should behave as the correlations of the spin-two operator (it is defined as $\Sigma^{ab} = \sigma^a \sigma^b - \delta^{ab}/3$, where σ^a is the 3-component Heisenberg spin) in the O(3) model. To verify this prediction, in the lower panel of Fig. 3 we report B_R versus ξ_R/L , together with the Heisenberg scaling curve for B_{Σ} versus ξ_{Σ}/L , where the latter quantities are computed from correlations of the spin-two operator Σ^{ab} . We observe a reasonable agreement. Tiny deviations are observed for intermediate values of $R_{\xi,R}$, presumably the result of corrections to scaling.

D. Strong-coupling critical behavior for N = 4

For N = 4 we have investigated the critical behavior for v = 10 and $\kappa = 0$. If we plot the Binder parameters B_R , B_T , and B_{ϕ} versus the $R_{\xi,R}$ and $R_{\xi,T}$ we observe good scaling, indicating that the transition is continuous, see Fig. 4. To verify the arguments of Sec. II B and, in particular, whether the critical behavior belongs to



FIG. 5: Plot of $L^{-2+\eta_T}\chi_T$ versus $R_{\xi,T}$ (top) and of $L^{-2+\eta_R}\chi_R$ versus $R_{\xi,R}$ (bottom). Data for $\kappa = 0$, v = 10, and N = 4. We set $\eta_T = \eta_H$ and $\eta_R = 1 + 2\eta_H$, where η_H is the vector susceptibility exponent in the Heisenberg O(3) model: $\eta_H = 0.0362$.

the Heisenberg universality class, we compare the plot of B_{ϕ} versus $R_{\xi,T}$ with the corresponding curve computed in the Heisenberg model, see the upper panel of Fig. 4. The numerical data are close to the Heisenberg curve, although some systematic deviations are clearly visible, especially for intermediate values of $R_{\xi,T}$, i.e., close to the critical point. These small deviations can be easily explained by the presence of slowly decaying scaling corrections due to the $\phi_1^2 \phi_2^2$ in the LGW approach. They decay very slowly, as $L^{-0.19}$, making it very difficult to observe the asymptotic behavior. For instance, to reduce scaling corrections by a factor of two, one should increase the lattice size be a factor of 38, which is is clearly not feasible.

To provide additional evidence for the correctness of the LGW predictions, we consider the susceptibilities χ_R and χ_T . The susceptibility χ_T should scale as the magnetic susceptibility in the Heisenberg model. Therefore, data should scale as in Eq. (35) with [66, 70–72] $\eta_T = \eta_H = 0.0362(1)$. This prediction is verified in Fig. 5. Data scale very well, as predicted. A second



FIG. 6: Top: Distribution of the energy E for v = 10, J = 0.453, L = 12. Bottom: Distribution of the energy E for v = -10, J = 0.3606, L = 6. Results for N = 6 and $\kappa = 0$.

important consistency check is provided by the analysis of χ_R . The arguments of Sec. II B indicate that R_L behaves as the product of two independent O(3) vector fields. This implies that $G_T(\boldsymbol{x})$ has the same critical behavior as $G_H(\boldsymbol{x})^2$, where $G_H(\boldsymbol{x}) = \langle \sigma_{\mathbf{0}} \cdot \sigma_{\boldsymbol{x}} \rangle$ is the vector correlation function in the Heisenberg model (σ is the fundamental variable in the Heisenberg model). At the critical point, $G_H(\boldsymbol{x})$ scales as $|\boldsymbol{x}|^{-1-\eta_H}$. Therefore, we have

$$\chi_T \sim \int d^3x \, G_H(\boldsymbol{x})^2 \sim \int^L r^2 dr \, r^{-2-2\eta_H} \sim L^{1-2\eta_H}.$$
(36)

It follows that χ_R scales as in Eq. (35) with $\eta_R = 1 + 2\eta_H = 1.0724(1)$. This prediction is tested in Fig. 5. Again, data scale quite well, confirming the LGW predictions.

E. Strong-coupling critical behavior for N = 6

For N = 6 we expect first-order transitions for all values of v, For $\kappa = 0$ and N = 7, Ref. [46] observed a very

strong metastability already on lattices of size L = 12. Thus, we have performed simulations on small lattices to be able to identify metastability effects. We have considered two values of v, v = 10 and v = -10. In both cases we observe a bimodal distribution of the energy in some interval of values of J. In Fig. 6 we show the probability distribution of

$$E = \frac{1}{3L^3} \sum_{\boldsymbol{x}\mu} \bar{\boldsymbol{z}}_{\boldsymbol{x}} \cdot \boldsymbol{z}_{\boldsymbol{x}+\hat{\mu}} \lambda_{\boldsymbol{x},\mu}, \qquad (37)$$

for two specific values of J. Data show a clear two-peak structure with a large latent heat, confirming the first-order nature of the transitions.

IV. CONCLUSIONS

In this work we discuss the critical behavior of lattice Abelian gauge models in which the fundamental field is an N-component complex vector, and which are symmetric under SO(N) transformations, focusing on the behavior in the strong gauge-coupling regime. A detailed analysis of the low-temperature configurations, combined with general LGW arguments allowed Ref. [54] to make precise conjectures on the nature of the low- κ transitions in this class of models. In particular, while SU(N) symmetric models may undergo continuous transitions only for N = 2 in the strong-coupling regime, in SO(N) symmetric models continuous transitions (in the Heisenberg universality class) are also possible for N = 3, provided that the Hamiltonian parameters are such that the symmetry breaking pattern at the transition is $SO(3) \to SO(2) \oplus \mathbb{Z}_2$. For models with Hamiltonian (3) this occurs for v > 0.

In this work we extend the theoretical analysis to values N satisfying $N \ge 4$, focusing on the case v > 0, that was not considered in Ref. [54]. We perform a fieldtheoretical analysis of the model, determining the RG flow of the renormalized parameters close to four dimensions, using the ϵ expansion approach. For $N \ge 5$ no stable fixed points are present, indicating that the transitions in the strong-coupling regime must be always of first order. For N = 4, we can perform a nonperturbative analysis of the RG flow, that allows us to prove the existence of a stable fixed point, corresponding to two decoupled Heisenberg critical behaviors. Thus, for N = 4continuous transitions are possible for v > 0, again in the Heisenberg universality class.

The theoretical predictions of Ref. [54] and those presented here rely on several crucial assumptions. In particular, they assume that an effective description can be obtained by considering the two order parameters reported in Eq. (6) and (7) $(T_L^{ab}$ for v > 0 and R_L^{ab} for v < 0), and the corresponding LGW theory. To verify the correctness of these assumptions, we have performed numerical simulations. For N = 2 we observe an Ising transition and an XY transition for v = 10 and v = -10. Heisenberg transitions are observed for v = 10 both for N = 3 and N = 4—in the latter case with significant scaling corrections, in agreement with theory, that predicts corrections decaying as $L^{-0.19}$ with the size L of the system. For N = 6 transitions are of first order for v = 10 and v = -10. The FSS analysis of the MC data therefore fully confirms the general scenario.

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Appendix A: Mean-field analysis of the Landau-Ginzburg-Wilson model for an antisymmetric tensor

We now determine the symmetry breaking patterns for the LGW theory with Lagrangian (10). For this purpose it is enough to consider the model in the mean-field approximation, i.e., to determine the minima of the meanfield Hamiltonian

$$H_{MF} = r \operatorname{Tr} \Psi^t \Psi + u (\operatorname{Tr} \Psi^t \Psi)^2 + w \operatorname{Tr} (\Psi^t \Psi)^2.$$
(A1)

As the Hamiltonian is SO(N) invariant, we can use this symmetry to simplify the analysis. We will now show that every real antisymmetric matrix A of rank N can be written as $A = VA_BV^t$, where $V \in SO(N)$ and A_B is a block-diagonal antisymmetric matrix. If N is even, we can write (M = N/2)

$$A_B = \operatorname{diag}(A_1, \dots, A_M), \tag{A2}$$

where the matrices A_i are antisymmetric and twodimensional. If N is odd, we have instead (M = (N - 1)/2)

$$A_B = \operatorname{diag}(A_1, \dots, A_M, 0). \tag{A3}$$

To prove this result, note that the nonvanishing eigenvalues of an antisymmetric matrix are purely imaginary. Since A is also real, they must appear in complexconjugate pairs. Therefore, if N is even the eigenvalues are $\{ia_1, -ia_1, ia_2, -ia_2, \ldots\}$. If N is odd one eigenvalue is necessarily zero. Since the matrix $A^tA = -A^2$ is symmetric, it can be diagonalized by using an orthogonal matrix. Therefore, there exists an orthogonal matrix V such that

diag
$$(-a_1^2, -a_1^1, -a_2^2, -a_2^2, \ldots) =$$

= $VA^2V^t = (VAV^t)(VAV^t).$ (A4)

Now consider an eigenvector v of A^2 . It is trivial to show that Av is also an eigenvector of A^2 with the same eigenvalue. If all eigenvalues a_i are distinct, this relation implies that VAV^t has necessarily the block-diagonal structure (A2) or (A3). If not all eigenvalues are distinct, we can still choose V so that the block structure holds. It is interesting to note that a two-dimensional antisymmetric matrix has the form

$$\begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix} \tag{A5}$$

and thus it is determined by its eigenvalues $\pm ia$, up to a sign.

We are now in position to discuss the minima of the mean-field Hamiltonian. If $M = \lfloor N/2 \rfloor$, modulo SO(N) transformations we can take Ψ in block-diagonal form so that

$$\Psi^{t}\Psi = \operatorname{diag}(a_{1}^{2}, a_{1}^{2}, a_{2}^{2}, a_{2}^{2}, \dots, a_{M}^{2}, a_{M}^{2}, (0)), \qquad (A6)$$

where the last 0 occurs only for odd N. We should the refore determine the minima of

$$H_{MF} = 2r \sum_{i} a_i^2 + 4u \left(\sum_{i} a_i^2\right)^2 + 2w \sum_{i} a_i^4.$$
 (A7)

For r > 0, the minimum corresponds to $a_i = 0$ for all *i*: this is the disordered phase. For r < 0, we should distinguish two cases:

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(i) For w < 0, a minimum configuration corresponds to $a_1 = a, a_2, \ldots, a_M = 0$, with

$$a^{2} = -\frac{r}{2(2u+w)}, \quad H_{MF,\min} = -\frac{r^{2}}{2(2u+w)}.$$
 (A8)

The configuration is invariant under $SO(2) \oplus O(N-2)$ transformations (note that two-dimensional antisymmetric matrices are invariant under SO(2) transformations). This is the relevant phase for the model with v > 0.

(ii) For w > 0 the minimum corresponds to $a_1, \ldots, a_M = a$ with

$$a^{2} = -\frac{r}{2(2Mu+w)}, \quad H_{MF,\min} = -\frac{Mr^{2}}{2(2Mu+w)},$$
(A9)

which is invariant under the compact symplectic transformation group USp(2M). If N is odd there is an additional \mathbb{Z}_2 invariance.

Note that this calculation also provides the stability conditions for the quartic potential [56], 2u + w > 0 and 2Mu + w > 0.

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