Approximate $SU(5)$, Fine Structure Constants

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Abstract

We fit the three finestructure constants of the Standard Model with three, in first approximation theoretically estimable parameters, 1) a "unified scale",turning out *not* equal to the Planck scale and thus only estimable by a very speculative story, 2) a "number of layers" being a priori the number of families, and 3) a unified coupling related to a critical coupling on a lattice. So formally we postdict the three fine structure constants!

In the philosophy of our model there is a physically lattice theory with link variables taking values in a (or in the various) "small" representations of the Standard Model **Group**. We argue for that these representations function in first approximation as were the theory a genuine $SU(5)$ theory. Next we take into account fluctuation of the gauge fields in the lattice and obtain a correction to the a priori $SU(5)$ approximation, because of course the link fluctuations not corresponding any Standard model Lie algebra, but only to the SU(5), do not exist.

The model is a development of our old anti-grand-unification model having as its genuine gauge group, close to fundamental scale, a cross product of the standard model group $S(U(3) \times U(2))$ with itself, there being one Cartesian product factor for each family.

In these old works we included the hypotesis of "multiple point criticallity principle" which here effectively means the coupling constants be critical on the lattice. Counted relative to the Higgs scale we suggest the in our sense "unified scale" (where the deviations between the inverse fine structure constants deviate by quantum fluctuations being only from standard model groups, not SU(5)) makes up the 2/3 th power of the Planck scale relative to the Higgs scale, or better the topquarkmass scale..

1 Introduction

We[9, 10, 11, 12, 8, 13, 18, 20, 40] and others[6, 7, 4, 5] have long - long time ago - worked on fitting the fine structure constants - especially the non-abelian ones - in a model based on the main assumptions:

- **Critical Couplings at Fundamental Scale** Preferably the gauge couplings should be at some multiple critical point for a lattice theory at the“fundamental scale” . And it was in the spirit of that model, that there indeed would exist a lattice theory in Nature.
- **AntiGUT** The gauge group was at the “fundamental scale” the Cartesian product $G \times G \times ... \times G$ of the same group G with itself, one time for every family of fermions.

but mainly the Abelian coupling of $U(1)$ was not so well predicted contrary to the non-abelian ones (the attempt by Don Bennet and myself [14] got good numbers, but the theory is a bit complicated). Further Laperashvili and Das and Ryzhikh [16, 21, 5] have even united this type of model with grand unification with $SU(5)$ [16, 21]. They used also supersymmetry in their picture.

Now it is the point of the present article to also make such a combination of $SU(5)$ GUT[1, 3] and the A(nti)GUT theory(AGUT= “anti grand unification theory” meaning the type of theory with a cross product of several copies of the standard model group, e.g. one cross product factor for each family of fermions) just mentioned, but **without SUSY**. Rather we shall here seek an $SU(5)$-like “unification” **without taking the $SU(5)$ theory as really true**, but rather taking the $SU(5)$ as an **approximate symmetry** appearing, because of the link variables have a form reminding of $SU(5)$. In fact one possible argumentation is to assume, that the link variables are constructed as matrices (with dynamical matrix element with somewhat restricted movability) for a most simple and smallest faithfull representation (a sort of principle [26, 27, 28] of smallest link-representation). Another similar argumentation is to use our earlier work [26, 27, 28] telling, that one can define a concept of “small representation” so that the standard model **group**[29][1] This would, taken seriosly, tell, that it is important, that the group chosen by Nature should have small representation, and that makes it natural that the link degrees of freedom corresponds to a “small” faithfull representation of the standard model group. Then it turns out, that a typical such small representation is the one obtained by starting from the **5**-plet of $SU(5)$ and restrict to the Standard Model Group as contained in $SU(5)$. Really the standard model group $S(U(2) \times U(3))$ is even in the notation as

[1] O Raifeartaigh points out that by choosing the **group** among the set of groups, with the given Lie algebra, which is “smallest” and thus have the fewest representations, but still has the representations used by the fermions and the Higgs(es), one can claim, that one selected **the gauge group** for the used theory with its fermions etc. So a sense can be given in this way to **the Standard Model Group** and it turns out to be $S(U(2) \times U(3))$ meaning the group of 5×5 matrices composed along (and around) the diagonal a $U(2)$ and a $U(3)$ and then impose the condition - symbolized by the “S” - of the determinat of the whole 5×5 matrix being $det = 1$ gets selected as having smallest faithfull representation among all groups.

used here an obvious subgroup of just $SU(5)$, the notation of which- the 5×5 matrices - is used to write it.

In the game we proposed[26, 27, 28] to specify the Standard model group as a **group**, it turns out that a cross product of several isomorphic groups gets the same "points"(in the game of our reference [26, 27, 28] so that the AGUT model believed in the article is on a shared first place with the single Standard Model **group**) as the group itself, so a group $G_{SMG} \times G_{SMG} \times ...G_{SMG}$ would be equally favoured by the our game.

. In any case the idea is, that the link variables are in terms of the fundamental physics, that is imagined to be behind, represented by variables like in some "small" representation [26] of the standard model group, and that then this representation happens to be / naturally is effectively an $SU(5)$-representation. This means that the link variables can formally be interpreted as $SU(5)$ variables; but **in reality they are not**. (i.e. there is *no* $SU(5)$ symmetry for turning around the matrix elements in link **5**-plet, **only under the Standard Model subgroup**.) There is **no true** $SU(5)$ **theory in our model**! But we can describe the model in terminology of an $SU(5)$-theory, which is broken fundamentally. It is not broken by Higgs mechanism as in the usual SU(5)-theories (a priori at least), but other gauge fields than the ones in the standard model subgroup do not exist (in the first place). There are only gauge fields corresponding to the degrees of freedom in the standard model groups - one set for each family, however, -. (So you must imagine either, that we really have the gauge group $G_{SMG} \times G_{SMG} \times \cdots \times G_{SMG}$ with as many standrad model group factors as there are families of fermions, 3, or you imagine there to be three layers of a usual lattice, so that we have three links, where you usually have only one.)

In the very crudest approximaton for a lattice action - linear in the trace of the representation matrix, the similarity to the $SU(5)$ matrix theory is so great, that the coupling constant ratios at the fundamental lattice theory in the first approximation become just as in the GUT $SU(5)$ unification scale. However, when it now comes to perturbative corrections due to the fluctuation of the lattice theory degrees of freedom, it becomes important that the degrees of freedom present in $SU(5)$ theory, but not in the Standard model, are missing, and therefore cannot fluctuate. So the quantum corrections from the fluctuation of these - in standard model not present - degrees of freedom are lacking, and thus makes the effective couplings observed in the continuum limit get different values from what they would have gotten in a true $SU(5)$ theory. Being quantum corrections one would usually treat them perturbatively and expect them to be small. If this is indeed the case, then the usual $SU(5)$ predictions will be **approximate**! We can say that it is the main point of present article to calculate this deviation from the exact $SU(5)$ predictions to the usual picture of unifying gauge couplings. Thus the Standard Model (inverse) fine structure constants do not truly unify (at a unification scale, but we shall talk in the present paper about an "our unified scale", which is the scale at which there is unification except for our (quantum) corrections, and that we call μ_U), but we calculate the degree of lack of unification, and even make **prediction of the numerical value of the deviation.**

1.1 Character of Our Prediction(s)

The main point of the present article is really to predict the deviation from exact $SU(5)$ GUT at a certain scale μ_U at which we calculate the corrections to the exact $SU(5)$ inverse fine structure constants in the standard model as due to quantum fluctuations in the lattice theory assumed to be really physically existent at some scale. Since we predict the absolute values of the differences between the inverse finestructure constants at the scale, we have at this scale two numerical predictions, and thus we can afford to use one of these predictions at the fundamental scale to fit the scale, and we shall still have one predition left. For instance we can use the prediction at the scale, at which the ratio of the difference $1/\alpha_2(\mu_U) - 1/\alpha_{1\ SU(5)}(\mu_U)$ to the other difference $1/\alpha_{1\ SU(5)}(\mu_U) - 1/\alpha_3(\mu_U)$ shall be 2 to 3 (as our calculation implies). This is illustrated on figure 1.1, and one shall remark, that the three crossings of the inverse fine structure constant with the vertical black line on the figure at the scale about $5. * 10^{13} GeV$ has been fitted, so that the three crossings lie with the ratio $2:3$ of the two intervals. The $U(1)$ inverse fine structure constant passes in between the $SU(2)$, above it with a piece that is proportional to 2, and the $SU(3)$ line, then below it with a distance proportional to 3. But having fixed the scale μ_U this way it is still a very nontrivial prediction that e.g. the absolute difference between the $SU(2)$-crossing and the $SU(3)$-crossing is just $3\pi/2 = 4.712385$. This is illustrated on figure 1.1.

1.2 Our Rather Simple Fitting Formulas

1.2.1 The quantum corrections breaking the approximate $SU(5)$

Our formulas for fitting the three inverse finestructure constants in the Standard Model in the for $SU(5)$ adjusted notation, wherein one uses $1/\alpha_{1\ SU(5)} = 3/5 * 1/\alpha_{1\ SM} = 3/5 * cos^2\Theta_W * 1/\alpha_{EM}$ are rather simple, and concerns of course the three Standard Model fine structure by renormalization group transformed to a certain scale μ_U, which is our replacement for the unification scale (because there is of course as we know no unification scale proper unless one involves susy or something else extra). The choice of the scale μ_U is only indirectly determined in our model, and is essentially just a fitting parameter, although we in section 11 shall speculatively relate μ_U to the Planck energy scale E_{Pl} by a crude relation $\frac{\ln(\frac{E_{Pl}}{m_t})}{\ln(\frac{\mu_U}{m_t})} \approx 3/2$. Then at this

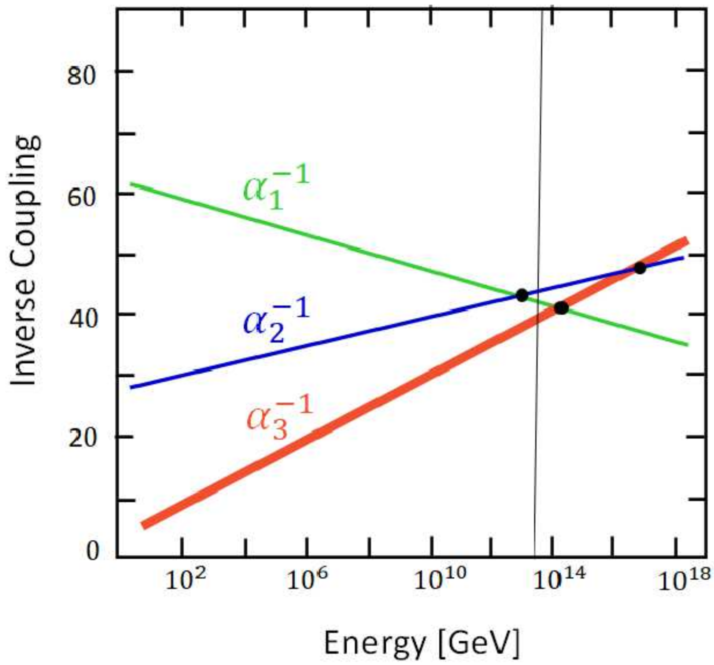


Figure 1: This is the usual graph representing the three Standard Model inverse fine structure constants with the α_1^{-1} being in the notation suitable for $SU(5)$, meaning it is 3/5 times the natural normalization, $\alpha^{-1}_{1\ SU(5)} = 3/5 * \alpha^{-1}_{1\ SM} = 3/5 * \alpha^{-1}_{EM} cos^2\Theta_W$. The vertical thin line at the energy scale $\mu_U = 5 * 10^{13} GeV$ points out “our unified scale”, which is as can be seen not really unifying the couplings, but rather is the scale where the ratio of the two independent differences, $\alpha_2^{-1} - \alpha^{-1}_{1\ SU(5)}$ and $\alpha^{-1}_{1\ SU(5)} - \alpha_3^{-1}$ have just the ratio 2/3 as our model predict at the “ our unification scale”. One may note that this “our unified scale” is actually very close to, where the three inverse couplings are nearest to each other, and in that sense an “approximate” unification scale.

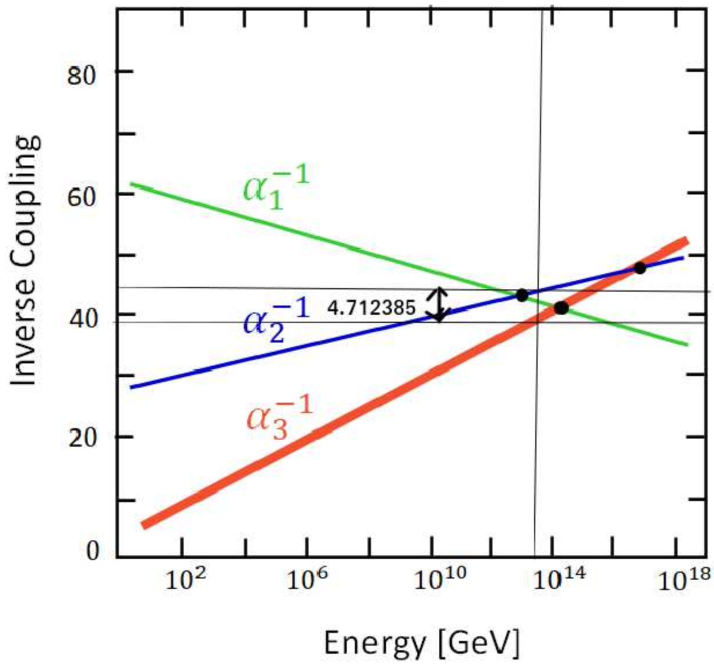


Figure 2: Same as figure 1, but now with our prediction inserted, marked as the number $4.712385 = 3 * \pi/2$, which is predicted to be at the "our unified scale"the difference $1/\alpha_2 - 1/\alpha_3$. Our prediction is, that just at horizontal thin black line at $5 * 10^{13} GeV$ corresponding to the scale μ_U, given by our fitted to the green line crossing point dividing the region between the blue and the red in the ratio 2 to 3, we shall have the difference in ordinate between the red and the blue crossing points with the vertical black being $3\pi/2$.

scale - to be fitted -

$$\frac{1}{\alpha_{1\ SU(5)}(\mu_U)} = \frac{1}{\alpha_{5\ uncor.}} - 11/5 * q \tag{1}$$

$$\frac{1}{\alpha_2}(\mu_U) = \frac{1}{\alpha_{5\ uncor.}} - 9/5 * q \tag{2}$$

$$\frac{1}{\alpha_3(\mu_U)} = \frac{1}{\alpha_{5\ uncor.}} - 14/5 * q, \tag{3}$$

where the one parameter $\frac{1}{\alpha_{5\ uncor.}}$, which could also give other names like

$$\frac{1}{\alpha_{5\ bare}} = \frac{1}{\alpha_{5\ classical}} = \frac{1}{\alpha_{5\ uncor.}}, \tag{4}$$

is our replacement for the unified inverse $SU(5)$ fine structure constant. The symbols, which we propose *uncor.* = *bare* = *classical* are to tell that this coefficient in the action functioning as the $SU(5)$ inverse coupling is without the quantum fluctuation couplings, i.e. it is uncorrected (= uncor.) or "bare". We could also define a corrected one

$$\frac{1}{\alpha_{5\ cor.}} = \frac{1}{\alpha_{5\ uncor.}} - 24/5 * q. \tag{5}$$

The other parameter q we believe to calculate in our model with its 3 families of fermions and in a Wilson lattice in a lowest order approximation:

$$q = \text{“}\#families'' * \pi/2 = 3 * \pi/2 = 4.712385. \tag{6}$$

Using this notation we could equally well use the formulation

$$\frac{1}{\alpha_{1\ SU(5)}(\mu_U)} = \frac{1}{\alpha_{5\ cor.}} + 13/5 * q \tag{7}$$

$$\frac{1}{\alpha_2(\mu_U)} = \frac{1}{\alpha_{5\ cor.}} + 3 * q \tag{8}$$

$$\frac{1}{\alpha_3(\mu_U)} = \frac{1}{\alpha_{5\ cor.}} + 2 * q. \tag{9}$$

Here in fact the quatity $\frac{1}{\alpha_{5\ cor.}(\mu_U)}$ is the in the analogous way to our treatment of the Standard Model inverse fine structure constants formally corrected $SU(5)$- inverse coupling to an effective one at the our unified scale μ_U, but of course, since there is no $SU(5)$, this is not so important, and rather formal only.

1.2.2 The Critical coupling

The requirement of the gauge couplings at the fundamental scale being just on the borderline on one or preferably more phase transitions, that are welcome to be lattice artifacts, was the basic ingredient in the wroks, of which the present one is a development[9, 10, 11, 12, 8, 13, 18, 20, 40]. In the present work with its approximate $SU(5)$ it may seem natural to require the $SU(5)$ coupling being just on the phase border for the pseudo-unified $SU(5)$ coupling as represented by $\frac{1}{\alpha_{5\ uncor.}}$. In principle the critical coupling depends on the lattice details, and it has to be calculated by lattice computer calculations, but here we have for a beginning just taken an approximate formula for the critical coupling out of our earlier works[19].

1.2.3 The "unified scale" from in lattice constant fluctuating "lattice"

The fact, that has always been a bit embarrasing for GUT theories of e.g. $SU(5)$, namely that the unified scale turns out appreciably smaller in energy than the Planck scale, is also embarrasing in our theory, and for rescuing it against this problem, we propose the speculation of a strongly fluctuating lattice. It should fluctuate in the size of the lattice constant, and we should imagine, that in various places and moments the lattice is more or less fine. We shall below see, that this kind of fluctuations can be used as an excuse for the effective scale for gravity, the Planck energy scale, and that for the Standard Model, the "our" grand unified scale (which is a replacement for the GUT scale) can deviate from each other violently. The parameter giving the our unified scale μ_U, namely the logarithm of it relative to the weak scale M_Z, namely $\ln(\frac{\mu_U}{M_Z})$ (or may be use better m_t instead of M_Z), is according to our speculation given in terms of the Planck scale, which thus is a needed input to obtain all three parameters to give the three fine structure constants.

1.2.4 Resume of the Fitting

The three parameters, with which we fit the three Standard Model fine structure constants come in our present work from rather different speculations, which though all should be sufficiently compatible, that we can have them in the same model. Here we announce, in the below table, the success of our model:

Parameter	Formula	From α's	Theory	Deviation	Section
q	q=$1/\alpha_2(\mu_U) - 1/\alpha_3(\mu_U)$	4.618201	4.712385	-0.094±0.05	3, 3.1
$1/\alpha_{5\ uncor.}(\mu_U)$	see above	51.705	45.927	5.778± 3.5	9
$\ln(\frac{\mu_U}{M_Z})$	$\ln(\frac{\mu_U}{m_t}) = \frac{2}{3} * \ln(\frac{E_{Pl\ red}}{M_t})$	26.43	24.76	1.67± 1 or 0.02	11

In the third parameter line we put a somewhat by hand taken uncertainty for the theoretical value, because the scales being diveded,the Planck scale over the scale of the three families ending at low energy taken as the M_Z scale or better top-mass m_t, is a ratio of rather illdefined concepts of scales and thus at least give an uncertainty of one unit in the natural logarithm.

Depending on how many of the stories behind the "theory" of these parameters the reader might buy as trustable the reader can decide with how many parameters, we fit the three standard model (inverse) fine structure constants. In fact the "theories" for the three different parameters are rather independent of each other, so that a selections that some are wrong and some are right would not at all be excluded.

1.3 Plan of Article

In the following section 2 we describe our, the assumption of lattice for the Standard Model **Group**, which means that it is important also, what the global structure of this group is, and not only the Lie algebra. According to O'Raifaighty [29] the global structure of group is connected with the system of allowed representations, and one thus can consider the system of repsentations for the fermions in the standard model as a strong indication for the special gauge **group** $S(U(2) \times U(3))$.

In section 3 we perform calculations of the quantum correctons meaning calculating zero-point fluctuations in plaquette variables, Taylor expanding the partition function and developping a table for the contributions of the zero point fluctuations on the continuum/effective (inverse) fine structure constants. Strictly speaking our correction depends on the type of lattice used, although we hope that it will be very little dependent. In the section 4 we at least mention the Wilson lattice action, which so to speak is the lattice we have used. In section 5 we compared to an old similar quantum fluctuation which we, Don Bennett and the present author, made many years ago in the similar model. Also we look for checking of our too crude estimation for what in lattice calculations is called tadpole improvements [31], but actually is the quantum fluctuations, we consider being the main mechanism breaking the approximate $SU(5)$, that appeared so to speak by accident, because the representation matrix in the links happened to have also $SU(5)$ symmetry, before some motions of it are restricted not to occur.

The fitting of the data - the experimentally determined fine structure constants in the Standard Model - comes in section 6, where we first determine by the requirement of the ratio of the difference between running couplings being as we predict the scale, that must be the fundamental scale in our model μ_U. It is what we can call "our unification scale" μ_U, but really of course there is no true unification, since our $SU(5)$ is only approximate. Next we compare, if the seperations at this scale is what we predict. At the end of the section we do it oppositely, as a check.

In section 9 we look at, if the coupling, say the approximate GUT one, is the critical coupling. In the works, which led up to this one, this having critical couplings were the crucial point[9, 10, 11, 12, 8, 13, 18, 20, 40]. We shall in general postpone second order calculation, but we should mention that a second order calculation is called for, see section 10, and presumably not exceedingly hard.

In section 11 we discuss the most speculative one among the parmeters in our model, which should be obtainable in an other way than just by the fitting fine-structure constants, namely the "scale of unification" μ_U. Although it is probably the most chocking result, if one would believe our model, that **the "fundamental scale μ_U is not the Planck scale**, then we shall present a speculative story on in size fluctuating lattice, that shall suggest a relation between the "fundamental scale" in our model and the Planck one (allowing them to deviate in order of magnitude).

Finally in section 12 we conclude, but also include some thoughts about the problems or suggestion for a quantum gravity, if we take the present work so serious, that we must claim that the fundamental scale for the Standard Model is the unification one, for our approximate $SU(5)$ GUT, even a bit low in energy compared to the usual unified scale. A lattice, which fluctuates even in scale in some background of a manifold or a projective space. If one could have the lattice imbedded in the continuum space wtih some symmetry including scalings, there might be a chanse of having a different way to average over fluctuations in the lattice constant size (i. e. coping with a fluctuating "fundamental scale") for the fine structure constants gauge theories and for the gravity. Such different averaging can seperate the different scales to be observed for the two groups of forces,the Standard Model ones , and gravity.

2 Our model

Our concrete model is, that we have in Nature a fundamental lattice with an energy scale μ_U crorresponding to the lattice constant $1/\mu_U$ (with $c = \hbar = 1$), the lattice being the Wilson one, say. This lattice is "tripled up" in the sense, that there is really one Wilson lattice for each family of fermions. Calling the number of families $N_{gen} = 3$ one can think of it as the genuine group being not the Standard Model Group itself SMG, but its third power $SMG \times SMG \times SMG$, the true gauge group in our model

$$\begin{aligned} G_{full} &= SMG \times SMG \times SMG && (10)\\ \text{where } SMG &= S(U(1) \times U(3)) && (11)\\ &= (\mathbf{R} \times SU(2) \times SU(3))/\mathbf{Z}_{app} && (12)\\ \text{where } \mathbf{Z}_{app} &= \{(r, U_2, U_3) | \exists n \in \mathbf{Z}[(r, U_2, U_3) = (2\pi, -\mathbf{1}, \exp(i2\pi/3)\mathbf{1})^n]\} \end{aligned}$$

Alternatively one might think of a model like this as there being three usual lattices lying parallel to each other (seperated in an extra dimension, say), It could therefore be tempting to call them "layers" of lattices.

In any case we imagine, that somehow or another the G_{full} is broken down to its diagonal subgroup, which is (isomorphic to) the standard Model group SMG. In fact this diagonal subgroup is defined as

$$
\begin{aligned}
SMG_{diag} &= \text{the subgroup of } G_{full} \text{of elements of form } (g,g,g) \\
SMG_{diag} &= \{(g,g,g) \in G_{full} = SMG \times SMG \times SMG | g \in G_{SMG}\}.
\end{aligned}
$$

(we tend to use both notations SMG and G_{SMG} for the same,so simply $SMG = G_{SMG} = S(U(2) \times U(3))$). This breaking down of G_{full} to the diagonal SMG_{diag} can easily be imagined to come about by a little bit of mixing up the different layers locally all over. ("confusion" [23, 24, 25]). In the section 11 we shall speculate a bit more complicated about the lattice structure, because we shall propose that there is even at the lattice scale diffeomorphism symmetry or at least some symmetry, like the symmetry of a projective space time containing (local or global) scalings. This then means that we imagine the lattice to fluctuate in both size and position, so that even if it is Wilson type very locally, it varies in both orientation and size of the lattice constant very strongly from place to place. If it is so, and it might be unrealistic to imagine that it is not fluctuating, if we shall have a so usual gravity theory with its reparametrisation fluctuating (as one should imagine the gauge of any gauge theory to really fluctuate [37]), e.g. the "fundamental scale" μ_U we calculate below by fitting, must be at the end considered an average value of the "fundamental scale" while the local fundamental scale fluctuates.

But apart from this story of connecting our model to gravity, the fluctuations might be ignored, and a lattice with fixed lattice constant of order $a \sim 1/\mu_U$ would be o.k. (But remember: we fit "the our unification scale" like the one in usual exact $SU(5)$ to be appeciably lower in energy than the Planckscale.)

2.1 The "small" representations used in the links and plaquettes

The crucial special assumtption for this article is to assume, that the degrees of freedom of the lattice-links representing the element of the standard model group SMG is the matrix elements of a matrix representation of this SMG on a minimal faithfull representation. It is then assumed that these matrix elements are restricted to only (be able to) move quite freely along the image of the SMG into the "small" representation used, while motion in other directions is strongly restricted (perhaps by very high potentials) but at least we shall ignore them, if there is any fluctuations, except along the standard model group, so to speak. The idea of thinking of such an imbedding is to note, that in such an imbedding we have a way of thinking of an SU(5) representation too, because the "small" reprensentation, we have in mind, is the one, that is the **5** plet reprentation of $SU(5)$. It is of course also a representation of the $SMG \subset SU(5)$. Now a really crucial point is, that we imagine, that once the SMG has been represented this $SU(5)$ simulating way, it tends to inherit an $SU(5)$ symmetry, even though our model has **no true $SU(5)$ symmetry postulated**. It is only, that it seems a bit similar in its simplest reprentaion. A bit more concretely we may say: we use, that the smoothness assumed also for the Lagrangian density as function of the plaquette- variables - which are also postulated to be formulated in 5×5 matrices - is a smoothness defined from the 5×5 matrices. When we then Taylor expand and from that look for the form of the plaquette action, we come to the trace of the 5×5 matrix just as in the usual $SU(5)$ theory. By this we have thus "sneaked in" an **approximate $SU(5)$ symmetry**. This is really the crux of matter of our model: The $SU(5)$ symmetry is **not a symmetry imposed on Nature** but rather an approximate symmetry of the way, we suggested to be the most natural way to represent the link and plaquette degrees of freedom for a model, that basically is only symmetric under the standard model group SMG. Thus there is of course already in our picture built in a breaking of the $SU(5)$ symmetry. Most importantly the **degrees of freedom from the components in the $SU(5)$ theory fields not also in the Standard Model Group SMG, are lacking.**

For us this then means, that there are no quantum fluctuations in the plaquette or link variables corresponding to these lacking degrees of freedom. The concern of the present article is to evaluate, how these lacking modes lead to lakcing some quantum corrections for the fine structure constants, and these corrections from the lacking modes of oscillations are not quite equally big for the three different Standard model gauge couplings. This is then according to us the reason for breaking in these couplings of the - of course fundamentally non-existing - $SU(5)$ symmetry.

2.2 The Plaquette Trace Action

As is usual, once you formulate your gauge theory on a lattice, you for smoothness reasons let the plaquette action typically be a linear function in the trace of the matrix representing the plaquette group element. This mainly from smoothness decided action will for the use of the reprenstation of the standard model group SMG on the **5**-plet function as if it were in $SU(5)$-thory. Actually it leads to couplings for the three sub-lie-algebras corresponding to the three Lie algebras $U(1)$, $SU(2)$, and $SU(3)$ being equal to each other in the same notation, in which they are equal in true $SU(5)$. So at first we have just from these simplicity and Taylor expansion type arguments gotten **effective $SU(5)$ symmetry**! The plaquette action, as we shall use it to give the more precise reslult including also quantum fluctuations H, takes the form

$$W_{\square} \quad = \quad ReTr\left(\exp(i(h+H))\right), \tag{13}$$

where both h and H are the Lie algebra valued fields written as represented by the representation on the **5**-plet. The h symbolize the field for which we want to estimate an effective action, we can think of it as representing a continuous field translated into the lattice and matrx formulation. On the other hand the part H should describe the quantum fluctuations, i.e. quantum mechanically of course even in a situation, in which you classically describe the situation by the field from which h has come. There is in reallity a superposition of fields configurations. That is to say, that the plaquette or link at a certain position in space time deviate appreciably from the confuguration given by h which is the "naive" translation of the ansatz field considered to the lattice. It is this deviation we call H. In first approximation - and we shall be satisfied with that - the fluctuation part H will be simply the fluctuation in vacuum.

Now it is our calculational approach to Taylor expand the trace-action (13) to include the first term, which even on the average get non-trivial contribution from the fluctuations. We shall namely in our calculation show, that it is this lowest non-trivial order term in the fluctuations, which gives the **deviation from $SU(5)$-symmetry.** And what really shall come out is, that indeed **this contribtuion also fits with the deviation from $SU(5)$-symmetry** of the (inverse) fine structure constants as measured under use of the Standard Model.

It is important for the present work to calculate, that the fluctuation in one component of H is

$$\frac{1}{2} < H^2_{one\ component} > \quad = \quad \frac{\pi}{2}\alpha(\text{in one layer lattice}) \tag{14}$$

and we shall do it in the following subsection. The reason we gave the value for 1/2 of the fluctuation, is that there is a factor 1/2 extra from the Taylor expansion, so that the counting of fluctuation contributuion in the expression $Tr(H^2h^2)$ is to be multiplied by 1/2 to give the correction to the relevant inverse fine structire constant. To the approximation that we in zeroth orde have exact $SU(5)$ we do not have to distinguish, which precise one of the various fine structure constants we shall use. This is also something, which would require a bit more thinking / calculation and we would like to postpone it for later article(see section 10).

Crucial is our Taylor expansion of the plaquette action (13),

$$\begin{aligned} W_\square &= Re(Tr(\exp(i(h+H))) \qquad (15)\\ &= ReTr[\mathbf{1}] + Re(iTr[(h+H)]) + \frac{1}{2}Re(-Tr[(h+H)^2]) + ... \\ &\quad + \frac{1}{6}Re(-iTr[(h+H)^3]) + \frac{1}{24}ReTr[(h+H)^4] + ... \end{aligned}$$

The fields both the fluctuation H and the "test" part h correspond to unitary representation matrices, are Hermitean as 5×5 matrices. Thus taking the real part removes the odd power terms, so they do not contribute, leaving in the above expansion up to 4th power,of interest, only the terms with 2nd and fourth power. Now if we are interested in the corrections to the effective (continuum) finestructure constants, we only have interest in the terms of even order in h, and thus even from the fourth ordert term we only care for those six terms in the expnansion of $(h+H)^4$, which have two h factors and two H factors. Among the a priori 2^4 =16 terms in the $(h+H)^4$ development, there are only 6 terms with h to just second power, and if the h and H commuted, these 6 terms would be identical. Indeed h and H do not commute, but when we take the average over the distribution of the fluctuations of H, it turns out that these terms after all have the same average, **as if** h and H did **commute**.

The terms to be kept for effective fine structure constant calculations purposes are:

$$\begin{aligned} W_{\Box} &= ... - \frac{1}{2} ReTr[h^2] + ... + \frac{1}{24} ReTr[HHhh(\text{in any of 6 orders})] + ... \\ \text{if commuting} &= \frac{1}{2}(ReTr[h^2] + Re\frac{1}{2}Tr[hhHH] + . \end{aligned} \tag{16}$$

In the last line we cancelled the factor 6 in the 24 by having here only one term, so this is achievable only, if the h and H "effectively" - i.e after averaging over the fluctuating H - do commute.

The full plaquette action shall have a coefficient β in front of it, of course. To connect the continuum theory with Lagrangian density

$$\mathrm{L}(x) = -\frac{1}{4g^2} F_{\mu\nu}(x) F^{\mu\nu}(x) a \tag{17}$$

$$= -\alpha^{-1}/(16\pi) * F_{\mu\nu}(x) F^{\mu\nu}(x) \tag{18}$$

we should, say, using a normalization by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] \tag{19}$$

identify for a link h_- in the μ direction

$$h_- = a^2 A_\mu \tag{20}$$

$$\text{and } h_{\Box} = \Sigma_{\text{around the box}} h_- \text{ (to linear approximation)} \tag{21}$$

Calculationally it may be most easy to avoid problems with normalization to extract the **ratio** of the second of these two terms to the first. The first of the two represent the naive(=lowest order) extraction of the continuum coupling from the lattice, while the second represents the lowest order effect of the fluctuations.

3 Extraction of Coupling Corrections

Once we have decided to look for the ratio of the second order and the fourth order terms in the Taylor expansion of the Plaquette action (16), we should be able to extract the *relative* correction due to inclusion of the quantum fluctuations by just putting in some ansatz for fields alone meaning a set up of one of the three standard model sub-group fields at a time, and even the normalization (of h) is then not important for this relative size of the two terms, while the size of the fluctuations have to be calculated, though.

Now we want to estimate the three Standard model finestructure constants - or rather their ratios - by putting on a "test field" which for the plaquette action, on which we think, is denoted $h = h_{\square}$, and if we think of a purely spatial plaquette, is really a magnetic field of that plaquette. This magnetic field is thought upon in the notation with the coupling constant absored into the field, so that the action actually has an inverse finestructure constant contained as factor to compensate the absorbed charge-factor e_0 say,

$$S = ... + \sum_{plaquettes} \frac{1}{2\pi\alpha_0} * ReTr(U(\square)) \quad (22)$$

$$\text{or continuum } S \propto \int \frac{1}{16\pi\alpha_0} F_{\mu\nu}F^{\mu\nu} d^4x. \quad (23)$$

(see section 4 for why just $\frac{1}{2\pi\alpha}$ in front of the $ReTr(U_{\square})$.)

Thus the inverse fine structure constant are found from how the action (or say the magnetic energy) varies approximately linearly with the square of the test field imposed h^2. If the fluctuation field was $SU(5)$-invariant - as it would of course be in a theory without any breaking of the SU(5)-symmetry, the three fine structure constants in the "SU(5)" invariant notation, which is wellknown to deviate from the more natural one by the replacement:

$$\left.\frac{1}{\alpha_1}\right|_{natural} = \left.\frac{1}{\alpha_1}\right|_{SU(5)} * \frac{5}{3}, \quad (24)$$

would be equal to each other all three.

The test-fields, we shall use, and which for the non-abelian groups $SU(2)$ and $SU(3)$ corresponds to the coupling definitions

$$S = \int (-\frac{1}{4e_2^2}\frac{1}{2}Tr_{matrix,2\times 2}F_{\mu\nu}F^{\mu\nu}) - \frac{1}{4e_3^2}\frac{1}{2}Tr_{matrix,3\times 3}(F_{\mu\nu}F^{\mu\nu}) + ...)d^4x$$

could be

$$\text{For } SU(2): h_{SU(2)} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{25}$$

$$\text{for } SU(3): h_{SU(3)} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{26}$$

$$\text{for U(1)}: h_{U(1)} = \frac{1}{\sqrt{30}} \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & = & -2 \end{bmatrix} \tag{27}$$

All the three proposed test-matrices h have been normalized, so that their squares

$$h^2_{U(1)} = \frac{1}{30} * diag(9, 9, 4, 4, 4) \tag{28}$$

$$h^2_{SU(2)} = \frac{1}{2} diag(1, 1, 0.0.0) \tag{29}$$

$$h^2_{SU(3)} = \frac{1}{2} diag(0.0., 1, 1, 0) \tag{30}$$

become of trace equal to unity

$$Tr(h^2_{U(1)}) = 1 \tag{31}$$

$$Tr(h^2_{SU(2)}) = 1 \tag{32}$$

$$Tr(h^2_{SU(3)}) = 1. \tag{33}$$

It is this normalization that ensures that the three couplings all become equal in the exact $SU(5)$ limit. (From the unbroken symmetry under the Standard model group it will not matter which component under one of the three standard model groups is used as test- field, as long as it is a combination of the components of just that one of the three groups $U(1)$, $SU(2)$ and $SU(3)$.) These fields h are meant to be added to the already fluctuating field, but not to flutuate themselves, and then dividing the thereby achieved (magnetic) energy increase or action decrease we shall obtain (apart from a constant factor) the inverse finestructure constant for the subgroup of the Standard Model in question.

3.1 Difference between our Approximate $SU(5)$ and usual $SU(5)$.

In the very first approximation - the $SU(5)$-invariant one - there is the same amount of fluctuation in all the 24 compponents of the $SU(5)$-Lie algebra, actually each of them have the average of the field squared for one component $1/2* < H^2_{\text{one component}} >= \frac{\pi}{2} * \alpha_5$. **But in the philosophy, that only the Standard model components really exist, we must in our model only have fluctuations in these components.**

The difference between our model, in which there truly speaking only is gauge symmetry by the Standard Model, and not even fields corresponding to the full $SU(5)$, and the usual $SU(5)$ theory comes in by **restricting the fluctuation field H in our model to only fluctuate in Standard Model degrees of freedom.**

Actually the Lie algebra components, which are in the $SU(5)$-Lie-algebra but not in the Standard model one, can be in the notation, we have chosen here (27), be represented by the matrix element being put to zero in the following matrix 5×5:

$$\begin{bmatrix} \cdot & \cdot & 0 & 0 & 0 \\ \cdot & \cdot & 0 & 0 & 0 \\ 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot \end{bmatrix}$$

I.e. the difference between our model and the $SU(5)$ symmetric model is, that the fluctuation in the vacuum fields on the 2 times 6 points in this matrix marked by the 0 's is suppressed in our model, while in the $SU(5)$ symmetric H the fluctuation is the same size in all the matrix element except for the detail that the trace of H is restricted to be zero,

$$tr(H) \quad = \quad 0. \tag{34}$$

In both usual $SU(5)$ and ours the trace is zero,but the 12 element marked with zero are restricted from fluctuating only in our model.

The technique to estimate what happens when one puts up in a region a smooth continium field is simply, that we add the field due to the continuum field, F say, translated to the matrix h to the fluctuating field H. That is to say we consider the configuration:

$$U(\square) \quad = \quad \exp(i(H + h)), \tag{35}$$

then to extract magnetic energy or the action of the plaquette, we assume the usual type of real part of the trace action:

$$S_{plaquette} \quad \propto \quad ReTr(U(\square)), \tag{36}$$

and look for the terms in the action change, which is of second order in the continuum extra field representing the continuum field. The coefficient to this second order h^2 to give the action change due to the continuum field is simply proportional to the inverse fine structure constant for the type of field we used.

3.2 Expansion of $\exp(i(H+h))$

The Taylor expansion of the exponential is wellknown and we only have to keep the terms of second order in h, and we shall not go further than to second order in H, so we only need to expand to fourth order in the sum $H+h$.

In fact we generelly have

$$\begin{aligned} ReTr(\exp(i(H+h)) &= ReTr(\mathbf{1}) + \frac{1}{2}ReTr((i(H+h))^2) + \frac{1}{24}ReTr((i(H+h))^4), \\ &\quad \text{(odd powers give zero).} \end{aligned} \tag{37}$$

Dropping but the h^2 order terms we get

$$\begin{aligned} S_{plaquette}|_{h^2-part} &= ReTr(U(\Box))|_{h^2-part} && (38) \\ &= \frac{1}{2}ReTr(h^2) + \frac{1}{24} * 6ReTr(h^2H^2) && (39) \\ &\quad \text{(provided that } h \text{ and } H \text{ commute)} \\ \text{Otherwise :} &= \frac{1}{2}ReTr(h^2) + \frac{1}{24} * \Big(4ReTr(h^2H^2) + 2ReTr(hHhH)\Big) \\ &= \frac{1}{2}ReTr(h^2) + \frac{1}{6}ReTr(h^2H^2) + \frac{1}{12}ReTr(hHhH). && (40) \end{aligned}$$

3.3 Classification of Fluctuations

For the presentation of the calculation of the quantum fluctuation corrections to the three different fine structure constants in the Standard Model, we divide the fluctuations into four classes. Have in mind that in crudest approximation the vacuum fluctuations in the $SU(5)$ symmetric approximation consists of independent fluctuations after all the 24 basis vectors in a basis for the $SU(5)$ Lie algebra. Imaginig having chosen this basis so that the 12 basis vectors are also basis vectors for the three sub Lie algebras corresponding to the three Standard Model groups, we can divide the fluctuation into four sets, denoted symbolically by H_1 for the fluctuation in the single mode of the $U(1)$ subgroup, H_2 for the fluctuation in the $SU(2)$ degrees of freedom, and H_3 for the $SU(3)$ fluctuations, and then for us the most interesting class H_{int}, namely those remaining fluctuations in the $SU(5)$ Lie algebra, which do not fall into any of the three welknown subgroups of $SU(5)$ in the Standard model, and which in our model are declared not to exist in Nature and thus must be removed. I.e. these fluctuations under the name H_{int} are put to zero. With such a classification we can divide the fourth order term into a series in principle of 3×4 combinations. In fact we can ask for any of the three finestructure constants for which we want to calculate the quantum fluctuation corrections, what the contribution is from one of any of the four fluctuation classses, H_1, H_2, H_3, and H_{int}.

3.4 Calculation description

We want to calculate the shift in the three inverse fine structure constants of the Standrd Model by first calculate the relative changes $\frac{\Delta\alpha_i^{-1}}{\alpha_i^{-1}}$ of these inverse finestructure constants $1/\alpha_i$ for $i = 1, 2, 3$ denoting respctively the subgroups $U(1)$, $SU(2)$, and $SU(3)$. Since we are now computing the "correction" after the very lowest order approximation is considered to be exact $SU(5)$ symmetry, we can in principle be careless with which finstructure constants we use in this calculation, when performed at the unification point of energy scale, because at this scale at zeroth approximation all three and even the α_5 are equal.

We shall first caculate the shifts $\Delta\alpha_i^{-1}(\mu_U)$ from their relative shifts. For this we need the very important $1/2* < H^2_{\text{one component}} > = \frac{\pi}{2}\alpha_5$ (but it is here we can be careless to our approximation with which α_1 you replace this $\alpha_5(\mu_u)$), and the factor $\frac{\pi}{2}$ is explained below in section 4.

Thus the shift of the inverse fine structure constant becomes

$$\Delta\frac{1}{\alpha_i(\mu_U)} = \frac{1}{\alpha_i(\mu_U)} * \frac{ReTr(H^2h_i^2)}{2ReTr(h_i^2)}(\text{for effective commutativity}) \tag{41}$$

$$= \frac{1}{\alpha_i(\mu_U)} * < H^2_{\text{one component}} > * \frac{ReTr(H^2h_i^2)}{2ReTr(h_i^2)* < H^2_{\text{one component}} >}$$

$$= \frac{\pi}{2} * \frac{ReTr(H^2h_i^2)}{2ReTr(h_i^2)* < H^2_{\text{one component}} >}. \tag{42}$$

One can think of the fraction $\frac{ReTr(H^2h_i^2)}{2ReTr(h_i^2)*<H^2_{\text{one component}}>}$ as a kind of counting how many components of the fluctuation contribute to the correction of the ith inverse fine structure constant,

$$\text{"Eff. \# } < H^2 > \text{ contributions"} \overset{=}{def} \frac{ReTr(H^2h_i^2)}{2ReTr(h_i^2)* < H^2_{\text{one component}} >} \tag{43}$$

$$= \sum_{j=1,2,3} \text{"Eff. \# } < H^2 > \text{ contributions"}|_{H_j}.$$

Here of course

$$\text{"Eff. \# } < H^2 > \text{ contributions"}|_{H_j} \overset{=}{def} < \frac{ReTr(H_j^2h_i^2)}{2ReTr(h_i^2)* < H^2_{\text{one component}} >} >$$

From the H_i	α_1^{-1} $h_{U(1)}$	α_2^{-1} $h_{su(2)}$	α_3^{-1} $h_{SU(3)}$
H_1	$\frac{2*81+3*16}{900}$ =7/30	$\frac{2*9}{2*30}$ =3/10	$\frac{4}{30}$ =2/15
H_2	$\frac{3*9*2}{3*30}$ =9/10	$\frac{2*3}{2*2}$ =3/2	0 =0
H_3	$\frac{4*3*8}{3*30}$ =16/15	0 =0	$\frac{8}{3}$ =8/3
sum	11/5	9/5	14/5
H_{int}	$\frac{54+24}{30}$ =13/5	3 =3	2 =2
check	24/5	24/5	24/5
half s. half H_{int}	11/10 13/10	9/10 3/2	7/5 1

Table 1: Table of the numbers $\frac{ReTr(H_i^2 h_j^2)}{2*ReTr(h_j^2)<H_i^2>}$ first without the explicit denominator 2, but then at the very two lowest lines the half is taken for sum of the contribution from the Standard Model group fluctuations and for the ones from the H_{int} which is missing in the standard model.

If we include into this sum also the H_{int} fluctuations, we get the corrections under unbroken $SU(5)$ and in this case the sum of these " Eff. $\# < H^2 >$ contributions" should for all three inverse fine structure constants be 24/5. There are 24 components for full $SU(5)$, but in order to contribute to the trace Tr a factor 1 you need 5 1's (along the diagonal).

3.5 The table

By a little thinking of, that we want the average of these fluctuations which are independent, except along the diagonal, and that elements in the matrix related by permuting column number with row number are strongly correlated as must be the case to ensure hermiticity of the fluctuating fields $H = H^\dagger$, we find out that one gets the same result whatever the order in the matrix product, so that effectively h and H commute after all.

Let us now list a table these "'Eff. $\# < H^2 >$ contributions" and their calculations:

The numbers in this table are easily obtained when having in mind when the trace is of the form $Tr(H^2 h^2)$ because we can then simply evaluate the traces by using the following diagonal matrices:

$$< H_1^2 > \;=\; \frac{1}{30} * diag(9,9,4,4,4) \tag{44}$$
$$< Tr(H_1^2) > \;=\; 1 \tag{45}$$
$$< H_2^2 > \;=\; \frac{3}{2} * diag(1,1,0,0,0) \tag{46}$$
$$< Tr(H_2^2) > \;=\; 3 \tag{47}$$
$$< H_3^2 > \;=\; \frac{8}{3} diag(0,0,1,1,1) \tag{48}$$
$$< Tr(H_3^2) > \;=\; 8 \tag{49}$$
$$< H_{int}^2 > \;=\; diag(3,3,2,2,2) \tag{50}$$
$$< Tr(H_{int}^2) > \;=\; 12 \tag{51}$$

combined with the squares of the ansatz matrices

$$h_{U(1)}^2 = \frac{1}{30} diag(9,9,4,4,4) \tag{52}$$
$$Tr(h_{u(1)}^2) = 1 \tag{53}$$
$$h_{SU(2)}^2 = diag(1/2,1/2,0,0,0) \tag{54}$$
$$tr(h_{SU(2)}^2) = 1 \tag{55}$$
$$h_{SU(3)}^2 = diag(0,0,1/2,1/2,0) \tag{56}$$
$$Tr(h_{SU(3)}^2) = 1 \tag{57}$$

3.6 The Problem with commutation

The above multiplication to make the table is o.k. if the h's and H's indeed commute. Effectively, however, we can show that by the averaging, we do end up as if they commuted:

The h's, i.e. the ansatz matrices, we can simply choose diagonal, because that is just to select an appropriate basis vector for the group one wants. If the fluctuation field is a diagonal one it is then indeed commuting, but if we consider an off-diagonal component of an H_i field, then we can argue that it leads to a product of the two diagonal elements in the h and this leads in the special cases we consider to taking trace of an h which is zero. So in pracsis it is as if we had commutation, almost by accident.

4 Wilson Action

We shall use the notation for the single layer (our model has three layers corresponding to three families) Wilson lattice, being related to a continuum theory(we here leave the gauge group open) and with the charge absorbed into the field $F^{\mu\nu}(x)$ (containing magnetic $\vec{B}$ and electric part $\vec{E}$ with their g absorbed):

If we use a notation, in which the $A_\mu(x)$ gauge fields are already Lie-algabra valued fields - or for our $U(N)$ groups of interest here equivalently matrices - and thus can define basis-vector matrices λ_a and T_a so that

$$\begin{aligned} A_\mu(x) &= (\Sigma) A_\mu^a \frac{\lambda_a}{2} && (58) \\ &= (\Sigma) A_\mu^a T_a && (59) \\ \text{where, say, for off-diagonal } \lambda_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} && (60) \\ \lambda_2 &= \begin{bmatrix} 0 & -i & 0 & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} && (61) \\ \text{and with normalization } Tr(\lambda_a \lambda_b) &= 2\delta_{ab} && (62) \\ \text{and } Tr(T_a T_b) &= 1/2 * \delta_{ab} && (63) \end{aligned}$$

you can by interpreting the $A_\mu(x)$ fields as representation in some representation R construct unitary matrices in the crude continuum limit identification

$$U_\mu(x) = \exp(iaA_\mu(x)) \qquad (64)$$

in the usual way require the

$$\begin{aligned} S_{Wilson}[U] &= -\frac{\beta}{2N} \Sigma_\Box (W_\Box + W_\Box^*) && (65) \\ &= \frac{a^4\beta}{4N} \int \frac{d^4x}{a^4} tr F_{\mu\nu} F_{\nu\mu} + ... && (66) \\ \text{where } W_\Box &= tr(U_\mu(x) U_\nu(x+\hat{\mu}) U^\dagger(x+\hat{\nu}) U^\dagger(x)) && (67) \\ &= tr(\text{oredered product around the plaquette } \Box) \end{aligned}$$

obtain using (17) $S = \int -\frac{1}{4g^2} F_{\mu\nu} F^{\nu\mu} d^4x$ the relation

$$\begin{aligned} \frac{\beta}{2N} &= \frac{1}{g^2}. && (68) \\ \text{or } \frac{\beta}{N} &= \frac{1}{2\pi\alpha}. && (69) \end{aligned}$$

And this leads to that the fluctuating part $H = (\Sigma)H^aT_a = (\Sigma)H^a\frac{\lambda_a}{2}$ of the exponent in the plaquette variable

$$U_\Box = \exp(i\Sigma H^a \frac{\lambda_a}{2}) \quad (70)$$

goes into the action with

$$\Sigma_\Box \frac{\beta}{N} Retr \exp(i\Sigma H^a \frac{\lambda_a}{2}) \quad (71)$$

$$= \Sigma_\Box \frac{1}{2\pi\alpha} Retr \exp(i\Sigma H^a \frac{\lambda_a}{2}) \quad (72)$$

$$\underset{\text{second o.}}{\approx} \frac{1}{2\pi\alpha} \Sigma_\Box Retr(-\frac{1}{2}(\Sigma_a H^a \frac{\lambda_a}{2})^2) \quad (73)$$

$$= \frac{1}{4\pi\alpha} \Sigma_\Box \Sigma_a (H^a)^2/2 \quad (74)$$

$$= \Sigma_{\Box\, a} \frac{1}{8\pi\alpha}(H^a)^2 \quad (75)$$

So if the plaquettes were not coupled - what they though are - then in the partition function / the Euclidean path integral which is

$$Z = \int DU \exp(-\beta S[U]) \quad (76)$$

$$\approx \Pi_{\Box\, a} \exp(-\frac{1}{8\pi\alpha} * (H^a)^2) \quad (77)$$

where DU is the Haar measure, the fluctuation of a plaquette variable (exponent) H^a would be given as $< (H^a)^2 >$ (no summation) $=8\pi\alpha/2$ (when restriction between the plaquette variables were neglected), since $\frac{\int x^2 \exp(-Kx^2)dx}{\int \exp(-Kx^2)dx} = 1/(2K)$. But of course they are connected so that there are only half the plaquette variables,which are independent. This can actually be seen to lead to that the distribution of the partition function distribtuion become twise as narrow measure in the square H^a average: So in the lattice partition faction or the Euklideanized path integral the fluctuation is

$$< (H^a)^2 > \text{(no summation)} = 8\pi\alpha/2/2 = 2\pi\alpha. \quad (78)$$

We here used that the plaquette variables, say $H^a(\Box)$ for the different plaquettes $\Box$ are not independently integrated over. On the contrary for each cube in the lattice there is a constraint which linearized means that the sum of six plaquaette variable for the plaquettes around the cube is restricted to be zero. Since there in 4 dimensions are 6 plaquettes per site and 4 cubes, this restriction would in first go mean that there per site were only 2 independent plaquette variables, but that is, however, not true, because there is a constraint between the four cube-constraints on the plaquettes. So in reality there is per site 3 independent constrains on 6 a priori plaquette variables. This gives that the average of the square $(H^a)^2$ of a (Gaussian distributed) plaquaette variable get reduced by a factor 6 to (6-3) meaning a factor 2. Simplifying to just 2 variables to get restricted to 1 independent we could just think of a Gaussian distribution about the origo in a plane, and that we then restrict the at first two dimensional to a diagonal - a single dimension - being a restriction symmetric between the two orinal variables thought of as the coordinates. Then the restricted distribtuion on the symmetric diagonal would project into one of the coordinate axes with the average of the saquare diminished by a factor 2.

The meaning of our basis choice for defining our lattice variables H^a could be illustrated by asking, what is now the calculated average of the square of an off diagonal element in the 5×5 matrix. E.g. for matrix element row 1 column 2 we get

$$\begin{aligned} < |H_{\text{row 1 column 2}}|^2 > &= < (H^1/2)^2 + (H^2/2)^2 > && (79) \\ &= 1/2 * 2\pi\alpha = \pi\alpha. && (80) \end{aligned}$$

It is such an - most easy off diagonal element we denote by $H_{\text{one component}}$ and its numerical average square is thus for **one layer**

$$\begin{aligned} < |H_{\text{one component}}|^2 > |_{\text{one layer}} &= \pi\alpha. && (81) \\ \text{Want } \frac{1}{2} < |H_{\text{one component}}|^2 > |_{\text{one layer}} &= \pi/2 * \alpha. && (82) \end{aligned}$$

The reason we want this half of the average square of the matrix element in the 5×5 matrix, is that in the Taylor expansion (39) has a factor 2 deviation between the two terms,which we shall compare.

4.1 Our Relative Correction

In the calculation of the relative correction to the inverse exact $SU(5)$ fine structure constants we need the ratio of the two terms (39) and the correction term comes from the Taylor expansion as

$$\begin{aligned} \text{“ correction term”} &= \frac{1}{4} * tr(h^2 H^2)(\text{if commuting effectively}) \\ \text{while the corresponding “uncorrected”} &= \frac{1}{2} tr(h^2). && (83) \end{aligned}$$

4.2 On Table

Use the numbers from thee table being just traces of the products of the diagonal matrices, which are noramlized so that their traces are 1 for the h^2 and the dimension of the Lie Group for the H_i^2 - normalizes the difference $\frac{1}{\alpha_2} - \frac{1}{\alpha_3}$ to one “unit” ignoring yet the factor 3 of number of families, and the hereby absorbed denominator 2, being

$$\text{The “unit”} = \frac{\pi}{2} \qquad (84)$$

now in the notation with "Re Tr" (in which it would at first have been π. So the prediction will be that the difference at the unifying scale of the two nonabelian inverse fine strucutre constants - which had number 1 (when the explicit 1/2 not included - will be $\frac{\pi}{2}$ for only one family, but $3*\pi/2$ for three families.

5 Compare with Old Work with Bennett,and with Computer works

Since it is so crucial for our prediction that we calculate the absolute size of the quantum correction,our $q = 3 * \pi/2$ correctly and that it is indeed such a quantum correction effect,we shall here compare it to an old work with Don Bennett, though only calculating this correction for simple groups SU(3) and SU(2), but it checks the absolute size. That the physics of this type of quantum correction works even with a background of an extensive computer calculation is seen in the next subsection 5.1 In my old work with Don Bennett [13] arXiv:hep-ph/9311321v1 "Predictions for Nonabelian Fine Structure Constants from Multicriticality" we in fact presented the same correction, which we use here and even had the normalization included and used that the correction to the inverse fine structure constants are

$$\begin{aligned} \frac{1}{\alpha} &\rightarrow \frac{1}{\alpha}(1 - C_f \pi \alpha) && (85) \\ &= \frac{1}{\alpha} - C_f \pi && (86) \end{aligned}$$

where C_f means the quadratic Cassimir in the fundamental representation of the group in question. In fact we find in this article:

$$\begin{aligned} C_f^{SU(2)} &= \frac{3}{4} && (87) \\ C_f^{SU(3)} &= \frac{4}{3} && (88) \end{aligned}$$

5.1 Tadpole correction calculations

In fact the quatity $< H_i^2 >$, which is so crucial to us to get estimated, is a quantity needed to make the so called tad-pole improvements for lattice calculations[30]. In the calculation by Niyazi eta. [31] we find some computer study, that also reach the quantity u_0 defined by

$$u_0^4 = \left\langle \frac{1}{N} Tr(U_p(\Box)) \right\rangle, \quad (89)$$

or as being the average value in the fluctuating lattice (in vaccum) for a link variable. They present as a result of their numerical studies in a region of

β's around $\beta = 7.5$ in their nottation meaning $1/\alpha_3 = 7.5/5 * 2\pi = 9.42477$:

$$u_0(\beta) \quad = \quad 0.87010 + 0.03721\Delta\beta - 0.01223(\Delta\beta)^2. \tag{90}$$

where $\Delta\beta = \beta - 7.3$.

On the basis of the crudest approximations as speculated in our section 4 we expect the $u_0(\beta)$ to be of the form

$$\begin{aligned}
u_0^4(\beta) &= 1 - \frac{C}{\beta} && (91)\\
\text{needing then } C &= 7.3 * (1 - 0.87010^4) && (92)\\
&= 7.3 * (1 - 0.057316) && (93)\\
&= 3.1159. && (94)\\
\text{If so, shift } \Delta\frac{1}{\alpha_3} &= C/3 * 2\pi * (1 - \frac{2*4}{20}) && (95)\\
&= C * 2\pi/5 && (96)\\
&= 3.9155 && (97)\\
&\approx 4.1888 && (98)\\
&= 8/3 * \pi/2 && (99)
\end{aligned}$$

(here the correction factor comes from our (68) , correction for a $N_c = 3$ in the notation of Niyazi et a.,and correction because the continuum coupling - the α_3 -gets a contribution from a lattice action term with double plaquettes having a coefficient $\beta/20$ in first approximation and contribtuiing 8 times as much as the "main Wilson term") If the inverse β type fitting here is correct, then the derivative being the coefficiient on the second term $0.03721\Delta\beta$ should be

$$\begin{aligned}
\frac{d}{d\beta}u_0(\beta) &= \frac{d}{d\beta}\sqrt[4]{1 - \frac{C}{\beta}} && (100)\\
&= \frac{1}{4}(1 - \frac{C}{\beta})^{-3/4} * (\frac{C}{\beta^2}) && (101)\\
&= \frac{1}{4}u_0^{-3} * C/\beta^2 && (102)\\
&= \frac{1}{4}C/7.3^2/0.87010^3 && (103)\\
&= C * 0.00712 && (104)\\
&= 0.022190. && (105)
\end{aligned}$$

This is a little bit lower than the 0.03721.

From formula (2) in reference [31] we see that Niayzi et al. uses the N included in the action explicitely so that for $SU(3)$ their $\beta = 3\beta_{without\ the N}$,so e.g. the $\beta = 7.3$ where they worked would mean in the notation without the N included in the definition $7.3/3 = 2.4333$. Then since in the usual notation which Niayzy et al.seems to use one has e.g. according to [32] $\beta = \frac{2N_c}{g_s^2}$ implying

$$\frac{1}{\alpha_3} = \frac{4\pi}{g_s^2} = 2\pi\beta \text{with no } N_c \text{ notation} \tag{106}$$

But there is a further point in extracting the fine structure constant used in the work by Nyaizi et al: They use L'́uscher-Weisz action which even in the large $\beta = \beta_{pl}$ limit has an extra term consisting of double plachette actions with a coefficient which according to [33] is given by the

$$\begin{aligned}
\beta_{rt} &= -\frac{\beta_{pl}}{20u_0^2} * (1 + 0.4905\alpha_3) && (107)\\
S[U] &= \beta_{pl}\Sigma_{rt}\frac{1}{3}ReTr(1 - U_{pl}) && (108)\\
&+ \beta_{rt}\Sigma_{rt}\frac{1}{3}Re(1 - U_{rt}) && (109)\\
&+ \beta_{pg}\Sigma_{pg}\frac{1}{3}ReTr(1 - U_{pg}) && (110)\\
\text{so } \beta_{eff}|_{lowest\ order} &= \beta_{pl} * (1 - \frac{1}{20} * 4 * 2) && (111)\\
&= \beta_{pl} * \frac{3}{5} && (112)
\end{aligned}$$

So this would mean we shall use (69), but with β/N put to $3/5 * \beta_{pl}/3$. The case $\beta = 7.3$ in the notation of [31] corresponds then to

$$\begin{aligned}
2\pi * 2.433333 &= \frac{1}{\alpha_3} && (113)\\
\text{giving } 1/\alpha_3 &= 15.2890 \text{ forgetting the } 3/5 \\
\text{so the } u_0^4 &= 0.87010^4 = 0.573161057 && (114)\\
\text{will correct by} 15.2890 * (1 - 0.87010^4) &= 6.25597 && (115)\\
\text{which should be } \pi/2 * 8/3 &= 4.18878 && (116)
\end{aligned}$$

However,when we now remember the inclusion of the effect of the double plaquette term at least in the weak coupling limit giving the factor 3/5, then instead to Niyazi et al. 's $\beta = 7.3$:

$$\begin{aligned}
\beta_{true} &= 7.3/3 * 3/5 && (117)\\
&= 7.3/5 && (118)\\
&= 1.46 && (119)\\
\text{giving } \frac{1}{\alpha_3} &= 2\pi\beta_{true} && (120)\\
&= 9.1734 && (121)\\
\text{and shift by } 9.1734 * (1 - 0.87010^4) &= 3.91556 && (122)\\
\text{again compare to } 8/3 * \pi/2 &= 4.188787 && (123)
\end{aligned}$$

Now there is very little difference,so that we can consider it, that this extraction from the calculation of the u_0 became a test of our calculation of the correction from loop corrections being so crucial for the present work.

Let us take yet an example namely $\beta_{Nyaizi} = 7.7$,it gives $\beta = \beta_{Nyaizi}/3 * 3/5 = 1.54$ and $1/\alpha_3 = 2\pi * 1.54 = 9.6761$. Now we had for 7.7, $u_0 = 0.8803$ and thus $1 - u_0^4 = 1 - 0.8803^4 = 0.399486$ giving the change of the 9.6761 by 3.8655. Still close to 4.1887 (But I do not like it got further away from this 4.1887, when the coupling got weaker, because we expect our values exact in the weak coupling limit).

6 Fitting

The first step in our fitting of our model is to calculate the "unifying" scale μ_u, at which the ratios between the differences between the inverse fine structure constants for the three subgroups of the Standard Model group is the one predicted from our calculation of the quantum fluctuation corrections. In fact the three inverse fine structure constants shall lie on the number axis as the numbers $(2, 13/5, 3)$ corresponding to the subgroups $(SU(3), U(1), SU(2)$, where we have chosen the $SU(5)$-normalisation for the $U(1)$-finstructure constant. The relation is expressed in terms of the two independent differences, that can be formed. Let us , e.g., say

$$\frac{\frac{1}{\alpha_2} - \frac{1}{\alpha_{1\ SU(5)}}}{3 - 13/5} = \frac{\frac{1}{\alpha_{1\ SU(5)}} - \frac{1}{\alpha_3}}{13/5 - 2} \tag{124}$$

$$\Rightarrow \frac{1}{\alpha_2} - \frac{1}{\alpha_{1\ SU(5)}} = \frac{2}{3} * (\frac{1}{\alpha_{1\ SU(5)}} - \frac{1}{\alpha_3}) \tag{125}$$

$$\Rightarrow \frac{1}{\alpha_2} - \frac{5}{3} * \frac{1}{\alpha_{1\ SU(5)}} + 2/3 * \frac{1}{\alpha_3} = 0 \tag{126}$$

Expressing the $\frac{1}{\alpha_i}$'s as

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi} \ln\left(\frac{\mu}{M_Z}\right) + ... \tag{127}$$

$$\text{with } b_i^{SM} = (41/10, -19/6, -7), \tag{128}$$

this relation for the $\alpha_i(\mu_u)$'s is written for the M_Z-scale finestructure constants as

$$\frac{1}{\alpha_2(M_Z)} - \frac{5}{3} * \frac{1}{\alpha_{1\ SU(5)}(M_Z)} + 2/3 * \frac{1}{\alpha_3(M_Z)} = (b_2 - \frac{5}{3}b_1 + 2/3 * b_3)/(2\pi) * \ln\left(\frac{\mu_u}{M_Z}\right).$$

Inserting the values obtained for the M_Z inverse fine structure constants this becomes:

$$\begin{aligned}
29.57 \pm 0.06\% - \frac{5}{3} * 59.00 \pm 0.02\% + \frac{2}{3} * 8.446 \pm 0.6\% &= \frac{-19/6 - 5/3 * 41/10 + 2/3 * (-7)}{2\pi} * \\
&* \quad \ln \frac{\mu_u}{M_Z} \\
-63.10 &= -44/3/6.2832 \ln \frac{\mu_u}{M_Z} & (129) \\
\Rightarrow \ln \frac{\mu_u}{M_Z} &= 27.03 & (130) \\
\Rightarrow \frac{\mu_u}{M_Z} &= 5.482 * 10^{11} & (131) \\
\text{Using } M_Z &= 91.1876 GeV & (132) \\
\text{thus } \mu_u &= 5.00 * 10^{13} & (133)
\end{aligned}$$

6.1 Table for inverse fine structure constants and our fitting

In the table 6.1 we go through the calculation of first determine the our unification scale by requiring the ratios of the two relative deviations from true $SU(5)$ symmetry to be in the ratio required by our model. This we have shown to be done by requiring the linear combination of the three inverse finestructure constants at this unifying scale to make zero the linear combination of the inverse fine structure constants having the coefficients $(-5/3, 1, 2/3)$ for respectively $(1/\alpha_{1\ SU(5)}, 1/\alpha_2, 1/\alpha_3)$. As a check of our model we work out, by correcting for the quantum fluctuations in the inverse fine structure constant, to reproduce the two $1/\alpha_5$'s, namely the one without quantum corrections - the bare $SU(5)$ inverse fine structure constant - and the 'èffective " $SU(5)$ inverse fine structure constant which has been corrected for these quantum corrections. The test is that these two formal$SU(5)$ (inverse) couplings shall be the same whichever of the three standard model fine structure constants are uused for the calculation of them, provided our model agrees with the data used.

6.2 Values at the μ_u-scale

What we are really interested in is the magnitude of the deviation from $SU(5)$ being accurate at the our "unified scale" μ_u, and we should like to develop the expression for this deviation in terms of the original variables at M_Z even. But to get an overview it is better first obtain the deviations by simply calculating the three inverse finstructure constants at the our

	$1/\alpha_{1\ SM}$	$1/\alpha_{1\ SU(5)}$	$1/\alpha_2$	$1/\alpha_3$
Formula	$1/\alpha_{EM} cos^2\Theta_W$	$3/5 * 1/\alpha_{EM} cos^2\theta_W$	$1/\alpha_{EM} * sin^2\Theta_W$	α_3^{-1}
Start $\#'s$	$127.916 * 0.76884$	$\frac{3}{5} * 127.916 * 0.76884$	$127.916 * 0.23116$	0.1184^{-1}
Value Uncertainty	98.347 ± 0.02	59.008 ± 0.013	29.569 ± 0.017	8.446 ± 0.05
Coefficient		-5/3	1	2/3
Contribution Uncertainty		-98.347 ± 0.02	29.569 ± 0.017	5.631 ± 0.034
 Sum Uncertainty	**SUM:** -63.147 ± 0.04			
b's	41/6	41/10	-19/6	-7
b-contribution		-5/3*41/10 = -41/6	1*(-19/6) = -19/6	2/3*(-7) =-14/3
Sum	(-41-19-28)/6 =-44/3			
b-contr./2π	-2.33420017	-1.087559696	-0.503991079	-0.742723695
$\ln(\frac{\mu_U}{M_Z})$ Uncertainty	**Ratio:** $\frac{-63.147}{-2.33420017}$ =**27.053** ± 0.02			
Scale μ_U Uncertainty	$5.116 * 10^{13} GeV$ $\pm 0.1 * 10^{13} GeV$			
b's/2π		0.652535818	-0.503991079	-1.114085543
$\ln(\frac{\mu_U}{M_Z}) * \frac{b's}{2\pi}$ Uncertainty		17.653 ± 0.01	-13.634 ± 0.01	-30.139 ± 0.02
Value at μ_U Uncertainty		41.355 ± 0.017	43.203 ±0.02	38.585 ± 0.05
Pred.to $1/\alpha_{5\ bare}$		3*11/5*π/2 =10.367247	3*9/5*π/2 =8.482293	3*14/5*π/2 =13.194678
$1/\alpha_{5\ bare}$ Uncertainty		51.722322462 ± 0.017	51.685853652 ± 0.02	51.780040772 ± 0.05
Pred. to$1/\alpha_{5\ cont}$		3*13/5*π/2 = 12.252201	3*3*π/2 =14.137155	3*2*π/2 =9.42477
$1/\alpha_{5\ cont}$ Uncertainty		29.103 ± 0.017	29.066 ± 0.02	29.161 ± 0.05
 Average Deviations	**Average:** 29.092	 w=35 0.0107	 w=25 -0.0258	 w=4 0.0683

"unification scale" μ_u:

$$\begin{aligned}
\frac{1}{\alpha_{1\ SU(5)}(\mu_u)} &= 59.00 \pm 0.02 - 0.65254 * 27.0566 && (134)\\
&= 59.00 - 17.66 && (135)\\
&= 41.34 && (136)\\
\frac{1}{\alpha_2(\mu_u)} &= 29.57 + 0.50399 * 27.0566 && (137)\\
&= 29.57 + 13.64 && (138)\\
43.21 & && (139)\\
\frac{1}{\alpha_3(\mu_u)} &= 8.446 + 1.11409 * 27.0566 && (140)\\
&= 8.446 + 30.143 && (141)\\
&= 38.59 && (142)
\end{aligned}$$

We may note down the differences and check that they are in the right ratio:

$$\begin{aligned}
\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_{1\ SU(5)}(\mu_u)} &= 43.21 - 41.34 && (143)\\
&= 1.87. && (144)\\
\frac{1}{\alpha_{1\ SU(5)}(\mu_u)} - \frac{1}{\alpha_3(\mu_u)} &= 41.34 - 38.59 && (145)\\
&= 2.75 && (146)\\
\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_3(\mu_u)} &= 43.21 - 38.59 && (147)\\
&= 4.62 && (148)
\end{aligned}$$

The test now is if

$$\begin{aligned}
2/5 * 4.62 &\stackrel{?}{=} 1.87 && (149)\\
\text{In fact } 2/5 * 4.62 &= 1.85 && (150)\\
\text{and } 3/5 * 4.62 &\stackrel{?}{=} 2.75 && (151)\\
\text{In fact } 3/5 * 4.62 &= 2.77 && (152)
\end{aligned}$$

Now our question is how big is this 4.62 in units of $\pi/2 = 1.5708$. We find

$$\begin{aligned}
\frac{\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_3(\mu_u)}}{\pi/2} & && (153)\\
&= \frac{4.62}{\pi/2} && (154)\\
&= 2.94 \approx 3 = \#families! && (155)
\end{aligned}$$

This is remarkably close to 3, the number of families! (with an order of magnitude uncertainty ± 0.1 it the inverse finestructure constants, a deviation of only 0.06 is very good!) This is in itself a remarkable coincidence, in spirit with our old work stories about critical inverse finestructure constants getting multiplied by the number of families, because of the antiGUt theory behind.

Corresponding to this spacing, we can now with the above calculations being used find in fact two $SU(5)$ inverse couplings, namely one before the effect of the quantum fluctuations $< H^2 >$ of H are taken into account and one after they are taken into account for the - in our theory non-existent - whole $SU(5)$.

6.3 The $SU(5)$ unification couplings

Using the table 6.1 we find, that using as unit $1/\alpha_2(\mu_u) - 1/\alpha_3(\mu_u) = 4.62 \approx 3\pi/2$, the two a bit different inverse unified couplings $1/\alpha_{5\ bare}$ and $1/\alpha_{5\ cont}$ (for $SU(5)$ formally) at our unification scale μ_u are given as

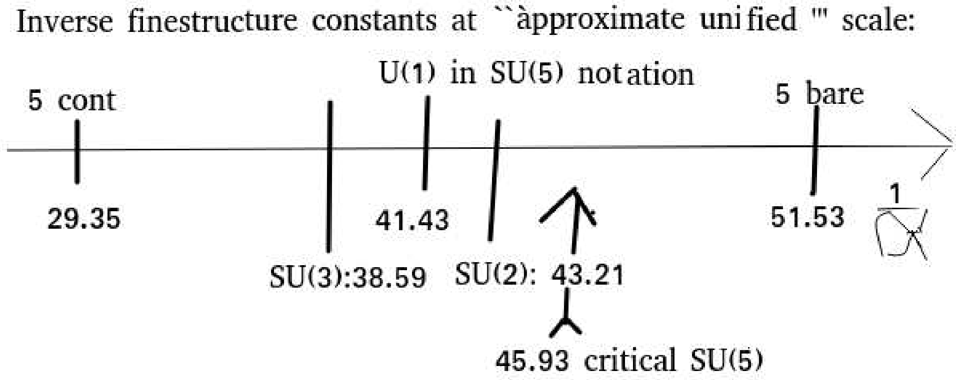
The "bare":

$$
\begin{aligned}
1/\alpha_{5\ bare} &= 1/\alpha_{1\ SU(5)}(\mu_U) + 11/5 * 4.62 = 41.34 + 10.164 = 51.504 \\
\text{or} &\quad 1/\alpha_2(\mu_u) + 9/5 * 4.62 = 43.21 + 8.316 = 51.526 \qquad (156) \\
\text{or} &\quad 1/\alpha_3(\mu_U) + 14/5 * 4.62 = 38.59 + 12.936 = 51.526 \qquad (157)
\end{aligned}
$$

The corrected:

$$
\begin{aligned}
1/\alpha_{5\ cont}(\mu_U) &= 1/\alpha_{1\ SU(5)}(\mu_U) - 13/5 * 4.62 = 41.34 - 12.012 = 29.328 \\
\text{or} &\quad 1/\alpha_2(\mu_U) - 3 * 4.62 = 43.21 - 13.86 = 29.35 \qquad (158) \\
\text{or} &\quad 1/\alpha_3(\mu_U) - 2 * 4.62 = 38.59 - 9.24 = 29.35 \qquad (159)
\end{aligned}
$$

(We used in this table the "experimental" value $q = 4.62$ but it would have made only very little difference to use the theoretical value $q = 3 * \pi/2$, because our agreement is so good)



7 What says our result about Original variables?

Our remarkable result is that at the "unified scale " μ_u for our *approximate* $SU(5)$ the difference between say $\frac{1}{\alpha_2(\mu_u)}$ and $\frac{1}{\alpha_3(\mu_u)}$ is just the number of families N_{gen} *times the 'unit"* $\frac{\pi}{2}$. It is so to speak the deviation from proper $SU(5)$ symmetry, which seems remarkably to be an integer - the number of families - times the "unit"$\frac{\pi}{2}$, which denotes the amount of shift in an inverse α per unit of quantum fluctuations in the lattice theory of the theory in question.

For testing and for illustrating, that there is truly a content in our prediction, we want now to rewrite this result in terms of the M_Z-scale quantities:

Let us begin to write down the difference, that should have the remarkable value $N_{gen} * \frac{\pi}{2}$ (where N_{gen} is the number of families):

$$\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_3(\mu_u)} \quad = \quad \frac{1}{\alpha_2(M_Z)} - \frac{1}{\alpha_3(M_Z)} - \frac{b_2 - b_3}{2\pi} \ln \frac{\mu_u}{M_Z},$$

where now

$$\ln\frac{\mu_u}{M_Z} = \frac{1/\alpha_2(M_Z) - 5/3 * 1/\alpha_{1\ SU(5)}(M_Z) + 2/3 * 1/\alpha_3(M_Z)}{\frac{b_2 - 5/3*b_1 + 2/3*b_3}{2\pi}}$$

so that

$$\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_3(\mu_u)} = \frac{1}{\alpha_2(M_z)} - \frac{1}{\alpha_3(M_Z)} - $$
$$-\frac{b_2 - b_3}{b_2 - 5/3 * b_1 + 2/3 b_3} *$$
$$*(1/\alpha_2(M_Z) - 5/3 * 1/\alpha_{1\ SU(5)}(M_Z) + 2/3 * 1/\alpha_3(M_Z)).$$

Here the ratio of the b_i's becomes:

$$\frac{b_2 - b_3}{b_2 - 5/3 * b_1 + 2/3 * b_3} = \frac{-19/6 - (-7)}{-19/6 - 5/3 * (41/10) + 2/3 * (-7)}$$
$$= \frac{-190 + 420}{-190 - 5/3(+246) + 2/3 * (-420}$$
$$= \frac{-570 + 1260}{-570 - 1230 - 840} \quad (160)$$
$$= \frac{690}{2640} \quad (161)$$
$$= \frac{23}{88} \quad (162)$$

$$Numerically$$

$$\frac{-3.166 + 7.000}{-3.166 - 5/3 * 4.100 - 2/3 * 7.000} \quad (163)$$
$$= \frac{3.834}{-3.166 - 6.8333 - 4.666}$$
$$= \frac{3.834}{-14.6653} \quad (164)$$
$$= -0.26143(\text{agree with}\frac{23}{88}) \quad (165)$$
$$(166)$$

Our difference is

$$\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_3(\mu_u)} = (\frac{111}{88\alpha_2} - \frac{115}{264\alpha_{1\ SU(5)}} - \frac{218}{264\alpha_3})|_{M_Z}$$
$$= \left((\frac{111}{88} + 3/5 * \frac{115}{264})\frac{1}{\alpha_{EM}} sin^2\Theta - 3/5 * \frac{115}{264}\frac{1}{\alpha_{EM}} - \frac{218}{264} * \frac{1}{\alpha_3}\right)|_{M_Z}$$
$$= \left(\frac{1}{\alpha_{EM}} * (\frac{333 + 69}{264} * sin^2\Theta - \frac{115}{264}) - \frac{218}{264} * \frac{1}{\alpha_3}\right)|_{M_Z}$$
$$= \left(\frac{1}{\alpha_{EM}} * (\frac{402}{264} * sin^2\Theta - 3/5\frac{115}{264}) - \frac{218}{264} * \frac{1}{\alpha_3}\right)|_{M_Z}$$
$$= \left(\frac{1}{\alpha_{EM}} * (\frac{201}{132} sin^2\Theta - \frac{69}{264}) - \frac{218}{264} * \frac{1}{\alpha_3}\right)|_{M_Z}$$

7.1 Calculating Our difference $\frac{1}{\alpha_2(\mu_u)} - \frac{1}{\alpha_3(\mu_u)}$ from M_Z scale Data.

Let us use

$$\begin{aligned} \frac{1}{\alpha_{EM}(M_Z)} &= 127.916 \pm 0.015 && (167)\\ sin^2\Theta &= 0.23116 \pm 0.00013 && (168)\\ \alpha_3(M_Z) &= 0.1184 \pm 0.0007 && (169) \end{aligned}$$

Then our difference becomes:

$$\begin{aligned} \text{“}difference'' &= \frac{1}{\alpha_2(\mu_U)} - \frac{1}{\alpha_3(\mu_U)} && (170)\\ &= \left(\frac{1}{\alpha_{EM}} * (\frac{201}{132} sin^2\Theta - \frac{69}{264}) - \frac{109}{132} * \frac{1}{\alpha_3}\right)|_{M_Z} \\ &= ((127.916 \pm 0.015) * (\frac{201}{132} * (0.23116 \pm 0.00013) - \frac{69}{264}) - \\ &- \frac{109}{132} * \frac{1}{0.1184 \pm 0.0007}) && (171)\\ &= 4.6187 \pm 0.0014(\text{from } \alpha_{EM}) \pm 0.025(\text{from } sin^2\theta) \pm 0.041(\text{from } \alpha_3) \\ &\stackrel{?}{=} 3 * \pi/2 = 4.7124 && (172)\\ deviation &= 0.0937 \pm 0.046 && (173)\\ deviation \quad & is\ about \quad 2 s.d. && (174) \end{aligned}$$

If you would like to blame all our deviation on the strong α_3, we would get, that in stead of the used 0.1184 a number 2.3 standard deviation higher, meaning the replacement,

$$\begin{aligned} \alpha_3(M_Z) = 0.1184 \pm 0.0007 \quad & \rightarrow \quad 0.1200 && (175)\\ \text{A strengthning by } 0.0016 \quad & meaning \quad 2.3 s.d. && (176) \end{aligned}$$

8 Alternative Way of Calculating

As an alternative - or check - we could impose our predicted values for the differences of the inverse fine structure constants and in that way obtain a “unification scale” μ_U. If our model is right then fitting the “unification scale” to the different differences between the three inverse fine structure constants in the standard model should lead to the *same* “unification scale”.

Let us as the first example take the difference $\frac{1}{\alpha_2(\mu)} - \frac{1}{\alpha_{1\ SU(5)}}$. At the

M_Z scale we have:

$$difference_{21} = \left(\frac{1}{\alpha_2} - \frac{1}{\alpha_{1\ SU(5)}}\right)|_{M_Z} \quad (177)$$
$$= \left(\frac{1}{\alpha_{EM}(M_Z)} * sin^2\Theta_W - \frac{3}{5} * \frac{1}{\alpha_{EM}} * cos^2\Theta_W\right)|_{M_Z} \quad (178)$$
$$= (-\frac{3}{5} + \frac{8}{5} sin^2\Theta_W) * \frac{1}{\alpha_{EM}} \quad (179)$$
$$= (-3/5 + 8/5 * (0.23116 \pm 0.00013)) * (127.916 \pm 0.015) \quad (180)$$
$$= (-0.230144 \pm 0.00020) * (127.916 \pm 0.015) \quad (181)$$
$$= -29.4390999 \pm 0.02 \quad (182)$$

The slope by renorm group of this difference is

$$\frac{b_2 - b_{1\ SU(5)}}{2\pi} = \frac{-19/6 - 41/10}{2\pi} \quad (183)$$
$$= \frac{-95 - 123}{2\pi * 30} \quad (184)$$
$$= \frac{-218}{60\pi} \quad (185)$$
$$= 1.156526 \quad (186)$$

Now our model - with its quantum fluctuations - says that at the "unified scale" of interest in our model the difference, 2 to 1, shall have run to

$$\text{"difference"}_{2\ to\ 1} = (3 - 13/5) * 3 * \frac{\pi}{2} \quad (187)$$
$$= 2/5 * 3 * \pi/2 \quad (188)$$
$$= 1.884954. \quad (189)$$

So the ratio of the our "unified scale" to the M_Z-scale has the logarithm

$$\ln(\frac{\mu_U}{M_Z}) = \frac{1.884954 - (-29.4390999)}{1.156526} \quad (190)$$
$$= \frac{31.32405}{1.156526} \quad (191)$$
$$= 27.084608474 \pm 0.02 \quad (192)$$
$$\frac{\mu_U}{M_Z} = 5.79023 * 10^{11} \quad (193)$$
$$\text{"unifying scale" } \mu_u = 5.27997 * 10^{13} GeV \pm 10^{12} GeV \quad (194)$$

We earlier got by different calculation 27.0566 giving with $M_Z = 91.1876 GeV$ that the "unifying scale" $5.134 * 10^{12} GeV$.

Note that the difference between the two different fits to the $\ln(\frac{\mu_U}{M_Z})$ deviate by just 0.03 while the predicted quantity 1.88 we used would give rise to a contribution to this logarith of the order of 1.6, which is more than 50 times larger.So we can claim that the prediction works well to about 2% accuracy.

In the table 2 we have collected similar calculations for the other two differences, too.

8.1 The 2 minus 3 case

We could estimate the same "unification scale" μ_U logarithm $\ln(\frac{\mu_U}{M_Z})$ similarly using another difference predicted as e.g. $(\frac{1}{\alpha_2}-\frac{1}{\alpha_3})|_{\mu_U} = 1*3*\pi/2 = 4.712385$.

At the M_Z-scale we have

$$\begin{aligned}
\frac{1}{\alpha_2(M_Z)}-\frac{1}{\alpha_3(M_Z)} &= \frac{1}{\alpha_{EM}(M_Z)}*sin^2\Theta_W-\frac{1}{\alpha_3(M_Z)} \\
&= (127.916\pm 0.015)*(0.23116\pm 0.00015)-\frac{1}{0.1184\pm 0.0001} \quad (195) \\
&= (29.5691\pm 0.003)-8.4459\pm 0.01 \quad (196) \\
&= 21.1232\pm 0.01, \quad (197) \\
\text{but at } \mu_U \text{ we predict : } (\frac{1}{\alpha_2}-\frac{1}{\alpha_3})|_{\mu_U} &= 1*3*\pi/2 \quad (198) \\
&= 4.712385 \quad (199) \\
\text{Running needed: } \text{“}run\ need'' &= 21.1232-4.72385 \quad (200) \\
&= 16.3993 \quad (201)
\end{aligned}$$

and this difference run with the renorm group by the rate

$$\begin{aligned}
\frac{d(1/\alpha_2-1/\alpha_3)}{d\mu} &= \frac{19/6-7}{2\pi} \quad (202) \\
&= \frac{-23}{6*2\pi} \quad (203) \\
&= -0.610094. \quad (204)
\end{aligned}$$

So the natural logarith of ratio

$$\begin{aligned}
\ln(\frac{\mu_U}{M_Z}) &= \frac{16.3993}{0.610094} \quad (205) \\
&= 26.8800\pm 0.02 \quad (206)
\end{aligned}$$

This is to be compared with the 27.085 ± 0.02 from above, and deviate by about 0.20, which with an uncertainty for the difference between the two numbers put to 0.03 would be $7 s.d.$. But note that even with this not so impressive number of standard deviations the deviation of 0.20 is compared to the number $4.7123/0.6101 = 7.725$ corresponding to our prediction of the value at the unified scale, about 30 times as small. So our theory works in that sense to 3% accuracy.

8.2 Superfluous case Difference 1 to 3

Although it is just related to the twoo foregoing let us explicitely for check also calculate what the our requirement for the difference 1to 3 means

$$\begin{aligned}
\frac{1}{\alpha_{1\ SU(5)}(M_z)} - \frac{1}{\alpha_3(M_Z)} &= \frac{1}{\alpha_{EM}(M_Z)} * cos\Theta_W(M_Z) * 3/5 - \frac{1}{\alpha_3(M_Z)} && (207\\
&= (127.916 \pm 0.015) * (1 - 0.23116 \pm 0.00013) * 3/5 - 1/(0.1184 \pm 0.0007\\
&= 59.0082 \pm 0.02 - (8.4459 \pm 0.7\%) && (208\\
&= 50.5623 \pm 0.021. && (209
\end{aligned}$$

Then

$$\begin{aligned}
\ln(\frac{\mu_U}{M_Z}) &= \frac{(50.5623 - 3/5 * 3 * \pi/2) * 2\pi}{41/10 + 7}\\
&= \frac{47.7349 * 2\pi}{111/10} && (210)\\
&= 27.0204 \pm 0.01 && (211)
\end{aligned}$$

8.3 Table

By accident the average of the three values for $\ln(\frac{\mu_U}{M_Z})$ turns out to be exactly 27.00 within our uncertainty. The 11th line in the table gives the deviation from this average relative to the part of the $\ln(\frac{\mu_U}{M_Z})$, which is due to our prediction value, so it give the order of magnitude of the failure of our prediction relatively. Remark, that even the biggest of these three deviation measures relative to our predictions is **only 0.052** meaning that even this deviation is only so well fitting by accident in one out of 24 cases.

The $\ln(\frac{\mu_U}{M_Z}) = 27.00$ correspond to that the "unification scale in our model"

$$\begin{aligned}
\frac{\mu_U}{M_Z} &= 5.32 * 10^{11} && (212)\\
\text{and } \mu_U &= 4.85 * 10^{13} GeV && (213)
\end{aligned}$$

9 Critical Coupling

Now we have without using but the lattice theory philosophy - see the old works and [41], also connection to our several phase speculations [42] - reached to an understanding in our picture of the deviations from the $SU(5)$ symmetry. It would of course be natural first to look for, if the unifying coupling should be the critical one for $SU(5)$ corrected of course for the factor that is the number of families. This is though not at all obviously the correct thing to do in our philosophy, because we have in the philosophy of the present article no true $SU(5)$ theory. It is only approximate, but

1.		$1/\alpha_2 - 1/\alpha_{1\ SU5}$	$\pm$	$1/\alpha_2 - 1/\alpha_3$	$\pm$	$1/\alpha_{1\ SU5} - 1/\alpha_3$	$\pm$
2.	$dif\|_{M_Z}$	-29.4390	0.03	21.1232	0.05	50.5623	0.05
3.	$dif\|_{\mu_U\ pred.}$	$2/5*3*\pi/2$		$1*3*\pi/2$		$3/5*3*\pi/2$	
4.		=1.88495		=4.71239		=2.82743	
5.	dist to run	31.32405		-16.3993		-47.7349	
6.	Run rate	$\frac{19/6+41/10}{2\pi}$		$\frac{19/6-7}{2\pi}$		$\frac{-41/10-7}{2\pi}$	
7.		=1.156526		=-0.6101		=-1.76662	
8.	$\ln(\frac{\mu_U}{M_Z})$	27.0846	0.03	26.8797	0.1	27.02046	0.03
	as av.+dev.	27.04+0.0446		27.04-0.1203		27.04-0.01954	
9.	$\|\frac{dif\|_{\mu_U\ pred.}}{\text{Run rate}}\|$	$\|\frac{1.88}{1.15}\|$		$\|\frac{4.71}{-0.610}\|$		$\|\frac{2.827}{-1.7666}\|$	
10.		=1.629		=7.724		= 1.6005	
11.	rel.dev.	0.052		-0.015		0.011	
12.	$\ln(\frac{\mu_U}{M_Z})$	1.5		1.6		0.6	
	s.d.f. av.						
13.	d. fr. 27.03	0.05		-0.15		-0.01	

Table 2: Table of results for three - not indenpent - ways of using the by us predicted differences between the running inverse fine structure constants at “unfied scale in our model” (which is the scale at which the three running differences should be equal to the numbers in line 3 or 4. These predictions are to be fullfilled at this “unified scale” which using each of the three differences is written in line 8, and the success of our model is really that these three numbers agree. They deviate from their average 27.04 by the numbers of standard deviations (s.d.) given in line 12. The “small” deviations agree within accuracy. But more important is to compare these deviations from the common average to the ratios given in line 9. which should be the contribution from our prediction numbers translated into the numbers in $\ln(\frac{\mu_U}{M_Z}$, which we gave in line 8. Here it turns out that the deviations from the average of the three numbers as written in line 12 in terms of standard deviations, when compared to these predictions divided by the running rate are relatively small as seen in line 11. In fact these numbers in line 11 are at most of the order of 1/20, while the two smaller ones of them are only of the order of 1/70. This means that our prediction values turned out correctly to better than 5%. A similar conclusion would be reached by instead of the average of the three $\ln(\frac{\mu_U}{M_Z})$ fits using the value of the $\ln(\frac{\mu_U}{M_Z})$ fitted by directly insisting on the ratio of the differences of th the inverse fine structure constants being the one we require. This insisting on the ratio of the differences directly lead to 27.03, which is only deviating by 0.01 from the average here in the table which was 27.04,when wieighting with uncertaitties wereused in evaluating the average (the naive average is rather 27.00) . The difference o.o3 is only 1.5 s.d. and quite small compared the to the predictions corresponding shifts in the $\ln(\frac{\mu_U}{M_Z})$ as seen in line 9.or 10. Again this fact ensures that our agreement although not perfect (yet) is remarkably good.

lacks half of the degrees of freedom. nevertheless let us for first orientation look for comparing the expression for the $SU(5)$ critical coupling given by Laperashvili, Ryzhikh, and Das [16, 18]

$$\alpha_{N\,crit}^{-1} = \frac{N}{2}\sqrt{\frac{N+1}{N-1}}\alpha_{U(1)crit}^{-1} \tag{214}$$

where we for the critical $U(1)$ coupling take the lattice value for Wilson and Villain actions:

$$\alpha_{crit}^{lat} \approx 0.2 \pm 0.015. \tag{215}$$

This gives

$$\alpha_{5\,crit}^{-1} = 0.2^{-1} * 5/2 * \sqrt{3/2} = 5 * 5/2 * 1.2247 \tag{216}$$

$$= 15.309 \tag{217}$$

With the family factor $N_{gen} = 3$ this would let us expect $15.309 * 3 = 45.927$ to be compared with the estimates from data above. (Presumably) the value to compare with is the 51.5 for the unified coupling not corrected by the quantum fluctuations, which we considered so much in this paper. Now we must remember, that the $U(1)$-critical coupling was $0.2 \pm 0,015$ meaning 7.5% uncertainty. These 7.5% means ± 3.45 for the 46 we predict. So the "experimental" 51.5 from our fit is only off by $\frac{5.5}{3.45} = 1.6$ s.d.. If there is an uncertainty in the critical coupling formula, we used, in addition to the one from the uncertainty in the critical coupling for $U(1)$, then the deviation in standard deviations will be even smaller than the 1.6.

So formally we must count the hypotesis, that indeed the critical inverse unified finestructure constant should be just 3 times the critical one, is very successfull! One should have in mind, that in reality the "the critial finstructure constant" is not quite well defined, because it depends on the details of the lattice theory.

If we accept this agreement, we can say, that we fitted all three Standar Model fine structure constants with **only the unification scale**, i.e. **one** paramter. The unification value of the fine structure constant for the $SU(5)$ was determined by the "critcallity".

Actually we shall even below in section 11 claim that we can relate the approximate unification scale - the lacking parameter to predict at this stage in the article - to the top-mass and the Planck scale,so that at the end we shall have predicted all three parameters.

9.1 More thoughts on the Critical Coupling, Unified Coupling

Thinking a bit deeper: We should really not take a formula for the $SU(5)$ critical coupling without correction, because we have been claiming all through the article that in our model the $SU(5)$ symmetry and all its degrees and freedom do not exist. Rather we should look for correcting the number for the critical α_5 to the critial coupling for the lattice standard Model group coupling:

Very crudely we think of the critial coupling for groups like the ones we look at to be the transition between two phases described as

- 1. An essentially classical phase, wherein the coupling is so weak - i.e. $1/\alpha$ so large, that at the scale we consider (the lattice links cale) all the plaquette variables are so close to unity, that the quantum effects can be considered just perturbations, but that basically we have the classical theory working.

- 2. A "confined" phase, in which we rather have that to first approximation the plaquette variables are distributed uniformly all over the group volume,as the Haar measure, we could say. Of course it will be still be more likely to find the plaquette variables closer to the unit element in the group until the inverse coupling $1/\alpha$ reaches zero. But now it is the variation of the probability density over the group that is the "small" perturbation.

If the standard model group lie as a **dence network** inside the $SU(5)$ in the **5**-plet vector representation space, then the a bit smeared volume of the standard model group would be similar to that of $SU(5)$ proper, and the value of the (inverse) fine structure constant, at which one or the other one of the two approximations above will shift their dominance (i.e. the critical value), will be (roughly) the same as for full $SU(5)$. But of course the denceness of the net formed by the standard model group is not perfect, and thus it will require that one goes to a somewhat stronger coupling (i.e. smaller inverse $1/\alpha$) to give the " confinement phase" enough weight in the partition function to (barely) compete with the "classical phase". So thus we expect

$$\frac{1}{\alpha_{SMG\,crit}} \leq \frac{1}{\alpha_{5\,crit}} \textbf{but only a bit}. \tag{218}$$

But now we have - to be fair - to remember that the standard model group, never had the quantum fluctuating degrees of freedom, which the full $SU(5)$ lattice gauge theory has. It lacks at least the 12 degrees of freedom, we refered to by H_{int} in our calculation. So going from the standard model "total" coupling, if such a thing existed, to the various subgroups $SU(2)$, $SU(3)$, *and* $U(1)$ would not corresond to taking away so many fluctuations as, if one went from the full $SU(5)$. So the critical $\frac{1}{\alpha_{crit\,SMG}}$ should not be identified with the above fitted $\frac{1}{\alpha_{5\,bare}}$, but rather with an inverse fine structure constant of a type, that shall not have had it fluctuations in the set H_{int} type ones removed,as we did in our formalism when constructing this “bare” inverse $SU(5)$ fine structure constant. So what we should rather identify as the implimentation of the critical coupling assumption, is to say, that a “fitted” $\frac{1}{\alpha_{SMG}}$ is the one you get by not counting that the refered to by H_{int} modes be included, but only the other ones, is to be identified by the $3 * \frac{1}{\alpha_{smg\ crit}}$ which by (218) is - actually only a bit - smaller than $\frac{1}{\alpha_{5\ bare\ crit}}$. The “fitted” quantity $\frac{1}{\alpha_{SMG}}$ comes actually very close to be an average of the three inverse fine structure constants from the standard model,which is rather expected, since it is the standard model genuine gauge group. Then if the dence net-work with which the standard model group G_{SMG} covers the $SU(5)$, there will only be little difference between the two sides in (218) and we now expect, that the average of the three standard model group inverse fine structure constants at “our unification scale” say essentially being $\frac{1}{\alpha_{SMG}}$ shall be a bit smaller than the critical SMG inverse fine structure constant times the 3, which again is just a bit smaller than the 3 times the critical inverse fine structure constant for $SU(5)$:

$$
\begin{aligned}
41.34 &= \frac{1}{\alpha_{1\ SU(5)}(\mu_U)} (\text{taken as average } 1/\alpha_i) && (219)\\
&\approx \frac{1}{\alpha_{SMG}(\mu_U)} && (220)\\
&= 3 * \frac{1}{\alpha_{SMG\ ccrit}} && (221)\\
&\overset{<}{\text{a bit}}\ 3 * \frac{1}{\alpha_{5\ bare\ crit}} && (222)\\
&= 3 * 15.3 = 45.9 && (223)
\end{aligned}
$$

10 Crude Second Order Calculation

We did in principle the above calculations only up to first order approximation in a preturbative scheme, in which the 0th order approximation is the exact $SU(5)$, wherein all three standard fine structure constants are equal to each other, and the first order approximation is the one, in which our corrections are considered small of first order, so that the squares of the corrections can be considered negligible. The numerical order of the first order quantities are

$$
\begin{aligned}
\text{“first order size”} &\approx \frac{\alpha}{1 \text{ or } 3 * \pi/2} && (224)\\
&\approx 1/10. && (225)
\end{aligned}
$$

One unneccesary ignorance of second order terms, which are expected to be of the order $(1/10)^2$ times the main term is, that we above let the α appearing as a factor in the $< H^2 >$'s cancel with the $1/\alpha$ whichever among the $1/\alpha_i$'s we meet. Actually it was tempting to think, that by using this lucky trick of getting rid of the parameters in the estimate of our corrections, we were likely actually to get a better result with respect to agreeing for our calculations. Once we look for accuracies of the second order, there may be more corrections, such as the H distribtuion being not even just Gaussian, and the whole program of doing the second order deserves a further article. Here we shall only make a very crude attempt to estimate the effect of seeing what α (among the three one say) comes into which of the fluctuations $< H^2 >$. We shall make the assumption that the α to be used for an H_i where it is the fuctuation in one of the basis vectors for the subgroup i of the $SU(5)$, is α_i. Then we see from table 3.5 that we have had relativley good luck by letting the two α's cancel each other, because the mostly contrbuting $< H_i^2 >$ to the correction for the inverse fine structure constant $1/\alpha_j$, for the standard model subgroup denoted j is actually mostly $i = j$ itself. In fact e.g. according to the table 3.5 the correction to

$$\text{Fraction of SU(2)-inverse coupling not } H_2 \, \frac{3/10}{3/2} = \frac{1}{10} \tag{226}$$

$$\text{Fraction of SU(3)-inverse coupling not in } H_3 \, \frac{2/15}{8/3} = \frac{1}{20} \tag{227}$$

For the $U(1)$ inverse fine structure constant the dominant cotrbutuion to the corrections comes from the two nonabelian groups, i.e. from H_2 and H_3, but it has a bigger number from the H_1 than any of the other two groups, namely 7/30. But since the $U(1)$ coupling correction is so mixed, to take all the same α is not so bad.

In any case it looks that it is only about 1/10 of the correction for the SU(2) coupling and 1/20 for the SU(3) coupling, that would be changed by being a bit more carefull with which α to use. The change to the more correct α to use would thus increase difference $1/\alpha_2 - 1/\alpha_3$ percentwise by

$$\begin{aligned}
\text{Decrease of } 1/\alpha_2 - 1/\alpha_3 &= \frac{1/10 + 1/20}{2} * 4.7/2/40 && (228)\\
&= 3/40 * 0.06 && (229)\\
&= 0.045 \text{ relatively} && (230)
\end{aligned}$$

This is now to be compared with the deviation of of the $3 * \pi/2 = 4.712385$ from the number in (148) which is 4.62 and thus smaller than or prediction $3 * \pi/2 = 4.712385$ by 0.09 which relatively is 0.0190. This agrees only modulo a factor 2.

The observed by renorm group developping the fine structure constants to the "our unification scale" defined from the ratios of the two independent differences of inverse couplings to be 2:3 was 4.62, i.e. smaller than the theoretical 4.71, but now the effect of pushing the inverse finstructure constants predicted down from their starting point in the SU(5)-symmetric limit $1/\alpha_{5\ naive}$ is getting increased for the $SU(2)$-inverse fine structure constant, because for that the changed H_1 contribution is getting increased by our second order correction because the $\alpha_{1\ SU(5)}$ is correctly stonger than what we used at first. For the $1/\alpha_3$ oppsitely the $1/\alpha_{1\ SU(5)}$ is above the $1/\alpha_3$ at the "our unification " so that for the $1/\alpha_3$ the H_1 contribution corresponds to a weaker $1/\alpha_{1\ SU(5)}$ thus giving a lower suppression compared to naive inverse SU(5) coupling, $1/\alpha_{5\ naive}$. Thus the theoretical 4.712385 should be deminshed - since the 3- inverse coupling goes up by the correction and 2-inverse coupling down - relatively by the 0.045. But that would bring the theoretical number to 4.50, close to the 4.62.

The deviation from the only to first order result of the number gotten by fitting is of the order of magnitude of the second order estimate. So it is important to estimate this second order approach more carefully.

11 Speculative Relation to Planck Scale

A major problem and surprize comming, if one takes our suggestion of truly existing lattice at the approximate or ours unification scale $\mu_U = 5.18 * 10^{13} GeV$ seriously is, that it suggests a "fundamental" scale **quite different from the Planck scale**. To seek a way out of this problem we propose to think of a **fluctuating lattice even in size of the lattice constant** in the sense that we speculate, that the general theory of relativity is still perturbatively treatable and rather well understood already - so that no completely speculated quatum gravity theory is needed at the μ_U scale - so that the whole lattice structure must be in a quantum superpostion state invariant under the reparametrization group from the general relativity. That is to say, with the philosophy, that there is very big quantum fluctuations in the gauge and taking the diffeomorphism of reparametrization symmetry as the gauge symmetry of general relativity, we must take it that the world is in a superposition of all the possible deformations of the lattice - needed for our model for the approximate GUT $SU(5)$ - achieved by reparametrizations. That is to say, that in a typical component in this superpostion somewhere we find a very small lattice constant and somewherewe find a very big one, so that lattice cannot be exactly a Wilson one e.g.. But locally it could still be close to a Wilson lattice. Then of course the lattice constant value suggested by our parameter μ_U as lattice constant $a \approx 1/\mu_U$ could only be true in an average sense:

$$\mu_U \quad = \quad \mathbf{Average}\frac{1}{a}, \tag{231}$$

where a is some local, or may be better single link, lattice constant, i.e. length of the link in the metric of the general relativity, which should still be perturbatively treatable in the range around $1/a \approx 5.18 * 10^{13} GeV$ (which is a small energy relative to the Planck scale).

So the physical model, in which we developped our more primitive lattice model, is in the rest of the article further developped into **some presumably more chaotic lattice theory (a kind of glass), in which the degree of fineness varies from region to region and you find links of all possible sizes, and at least approximate diffeomorphism invariant structure of the lattice.** It is of course only approximately diffeomrphisminvariant by being in superposition of having different fineness of the lattice at any place. From the approximate diffeomorphism invariant structure of the lattice model in this section we cannot avoid, that the density of links of the length around a has to vary approximately like

$$\begin{aligned}\text{“density”}(\ln(a))d\ln(a) &= P(\ln(a) < \ln\text{“link length”} < \ln(a) + d\ln(a)) \\ &= a^{-4}d\ln(a), \qquad (232)\end{aligned}$$

where $P(\ln(a) < \ln\text{“link length”} < \ln(a) + d\ln(a))$ is the probablity of finding a random link taken out of our “chaotic lattice” within the scale in logartihm from $\ln(a) < \ln(\text{lattice constant }) < \ln(a) + d\ln(a)$. A similar distribtion of the sizes of the plaquettes found in the “chaotic lattice” of this section, would also have a factor in the density going as the fourth power of the inverse plaquette side size.

There is actually a divergence problem with this “chaotic lattice” as we speculate it: If indeed this density distribution should be fully true the probablity of finding links of a specific order of magnitude would need to be zero and all the contribtuion would come from infinitely small links or infinitely long link. So we have to imagine that there finally must be some cut offs for very long - not so important - and for very short links at least.

To have approximate diffeomorphism symmetry and thus also approximate scale-invariance we should have at most a very slowly varying weight factor depending on the logarithm of the say link length - but only very weakly breaing the scale symmetry in the range of scales we consider relevant, meaning scales between the Planck scale and macroscopic scales.

But if we scall be concrete wewould propose aGaussian weighting as a function of the logarithm of say the link length. Near the peak in the Gaussian such a Guassian weighting is only very weakly breaking the scaling invariance, but for very large or very small scales the Gaussian distribution of the weighting in the logarithm is enoermous. But somehow we hope that for very small or very big link length we have got the cut off effectively and there are anyway so little chanse for the links having that size that it does not matter so much. But I think we need a cut off in this style of being smooth for some “relevant” region and then very drastically cutting off in the scales ofvery small a (i.e. high energies) becuase ifwe did not have the strong cut off somewhere, then attempting to play simultaneous with the extra factor $(1/a)^4$ for the Standard model approximate $SU(5)$ and an other extra factor $(1/a)^6$ for describeing the generalrelativity Einstein-Hilbert action would unavoidably lead to severe divergensies.

We could say that the proposed Gaussian as function of the logarithm very robust by being able to cut off at the ends large and big scales any polynomial extra factor. Then in addition to in this way be able to cope with any power extra factor, it can be claimed in the appropriate region of scales to be rather flat, so that is not at such scales drastically breaking scale symmetry.

With this very special "cut off" assumption, it might be felt needed tomake at least a little bit of propaganda for it: Once we preferably should have had invariance under scalings in size,it is suggested that we need a slowly varying weight as function of the logarithm of the scale. We also like at the end a robust cut off that can cut off anything polynomial say and then an exponential of a smooth function

$$\text{``}weight'' \quad \sim \quad \exp(f(\ln(1/a)) \tag{233}$$

is suggested. But then the Gaussian -which may not be socrucial exactly - is gotten by Taylor expanding the function f around the maximum, which is of course the most important region. One could as propaganda also say, that the cut-off proposed represents a weak coupling to the metric tensor of gravity.

Then depending on whether you have a factor a^{-4} as for the inverse fine structure constants or a factor a^{-6} as for the gravitational κ the weighted maximum in the over scale logarithm intergal, will have somewhat different central values, i.e. central logarithms of scales.

These centers of the contributing distrbutions will be the effective lattice scales for the different weightings. So we can indeed get the μ_U weighted with a^{-4} and the gravitational scale being the central one for weight a^{-6} become different by orders of magnitude. If we just at first give a name to scale μ_0 which one gets with weight 1, then in the Taylor expansion lowest order approximation the drag shifting will be in the ratio $6:4$, so that

$$\ln(\frac{E_{Pl}}{\mu_0}) \quad = \quad 6/4\ln(\frac{\mu_U}{\mu_0}). \tag{234}$$

(whether one shall use the formal Planck constant just made by dimensional arguments from the Newton constant G or some reduced one with an extra factor 8π extracted might be discussed, but may be just considered an uncertainty)

11.1 Averaging over our "chaotic lattice"

When we have some part of the continuum lagrangian like the $\frac{2\pi}{\alpha}F_{\mu\nu}F^{\mu\nu}d^4x$, then the contribution to it in the lattice theory - our chaotic one or just a usual Wilson lattice - come from individual plaquettes or whatever combination of the lattice ingredients, that contribute, but you get therefore a bigger contribution the more of these contributing objects there are per hypercubic unit volume to the coefficient in the of the continuum lagrangian density.

Actually we can use simple dimensional arguments to see how the average of the continuum Lagrangian coefficient comes about:

For the inverse fine structure constants you simply get a contribution to the action from each plaquette independent of its size (provided you let the β weighting the plaquette in the action be the same whichever the size of the plaquette, especially with our philosophy that it should be critical such *beta* independent of the size is suggested.). So in terms of an integral over the logarithm of the inverse size, say $1/a$, of the laticce constant or link-length we have

$$1/\alpha \propto \int (1/a)^4 \text{“cut off weight ”} d\ln(1/a) \tag{235}$$
$$\propto \int (1/a)^3 \text{“cut off weight” } d(1/a) \tag{236}$$
$$\propto (1/a)^4|_{\text{at peak for } (1/a)^4 \text{ *weight}} \tag{237}$$

But gravity, extra $1/a^2$:

$$\kappa \propto \int (1/a)^4 * (1/a)^2 \text{“cut off weight”} d\ln(1/a) \tag{238}$$
$$\propto \int (1/a)^5 \text{“cut off weight”} d(1/a) \tag{239}$$
$$\propto (1/a)^6|_{\text{at peak for } (1/a)^6 \text{*weight}} \tag{240}$$

So we see that we **predict** from the “chaotic lattice ” model with its approximate scale invariance, by an essentially dimensional argument, that there shall be different effective lattice scales for the Yang Mills theories μ_U, and for gravity. (But it is of course dependent on our Gaussian in log in some sense special cut off, although it is suggestive.)

In the figure 11.4 we illustrate, how we after having inserted a strong cut off implementing weight get a distribtuion in the logarithm $\ln(1/a)$ of the scale with a broad peak, (which we imagine Gaussian, in this log, in first approximation).

The main point is that the dominant or peak value for the distributions depend on the exact distribution, and that the one for gravity has got an extra factor $(1/a)^2$. For the Standard Model gauge couplings this peak scale is only of relevance via the renomalization group, while for gravity the very size of the (inverse) coupling κ (also) depends on the peak value for the (logarithm of) $1/a$.

It should be clarified, that it is only because of some “phenomenologically” added “ cut off weight ” factor that we at all mannage to get a peaking distribution instead of some nonsence divergent one, just increasing monotomously. So the picture we propose is really much dependent on there being some cut off of this type, and this cut off has to be considered some sort of "new physics", even though we escape from assuming many details about it, except that it is smooth in the logarithm of the scale and sufficiently strong to cause the convergence (preferably exponential in form, but with a low coefficient on the function, say $f(\ln(1/a)) =$ "small number" $*\,(\ln(1/a) - const.)^2$, in the exponant.).

Let us now suppose that including this "'new physics" weight there is scale, which we call μ_0 for which the density of plaquettes or links counted per **link-size volume** is maximal. Then if we do not put the factor $(1/a)^4$ or $(1/a)^6$ on as we did above, then the peak of the so to speak just "weight" would be at μ_0 or we should say $\ln(\mu_0)$, when thinking of the plotting with $\ln(1/a)$ along the abscissa as in figure 11.4.

Now in the approximation of the "weight" distribtuion being Gaussian in the logarithmic scale and noticing that the extra factors $(1/a)^4$ and $(1/a)^6$ from the logarithmic abscissa point of view are linear terms in the exponent $4\ln(1/a)$ and $6\ln(1/a)$ which will shift the peak from $\ln(\mu_0)$ by amonts proportional torespectively 4 and 6, we see that

$$\frac{\ln(\frac{E_{Pl}}{\mu_0})}{\ln(\frac{\mu_U}{\mu_0})} = \frac{6}{4} = \frac{3}{2} \tag{241}$$

11.2 On the Maximum before the Powers in $1/a$ Factors

In seeking to guess, what to take for the maximum density scale μ_0, when no extra factor like the $(1/a)^4$ or $(1/a)^6$, we should have in mind that the density of plaquettes in a volume (in four space) of size like the plaquette or link is indeed, what we called the number of "layers", which again were identified with the number of families, or at least this density of plaquettes in the range associated with a plaquette is proportional to the number of layers.

Since we identify by our hypotesis the number of layers with the number of families, we take the number of layers at different scales to reflect the number of families being present as fermions with negligible mass at the various scales. That is to say, that in the range of scales of the quark and (charged) lepton masses we have region of scales where as one goes down in energy loose more and more families. With such a philosophy of counting only the effectively massless fermions at the scale we may - using a table like table 11.2 - extrapolate to a scale with maximal number of families and take that as μ_0; we could take it close to the mass of the mostmassive quark or lepton, the top. Actually as seen in table 4 putting $\mu_0 = m_t$ the top quark mass is close to make our prediction (241) be satisfied. Fitting to make our prediction (241) be exactwould require a slightly higher in energy scale μ_0.

We consider this close to agreement as success for explaining theoretically the "unification scale" μ_U of our approximate $SU(5)$.

Name	Mass	$\ln(Mass/GeV)$	Sums etc.
Quarks:			
up	2.16 MeV	-6.137	
down	4.67MeV	-5.367	
strange	93.4MeV	-2.371	
charme	1.27GeV	0.239	
bottom	4.18GeV	1.430	
top	172.5GeV	5.150	
sum quraks		-7.055	-1.176
"average"	309 MeV	-1.176	
electron	0.5109989461MeV	-7.055	
muon	105.6583745MeV	-2.248	
tau	1776.86MeV	0.575	
sum leptons		-9.252	-3.084
"average"	45.78 MeV	-3.084	
av. weight 2:1	163 MeV	-1.812	

Table 3: Here we just listed the charged quarks and leptons exposing their masses and the natural logarithms of the latter with the purpose of very crudely use them to extrapolate to scale μ_0 at which the number of at that scale effectively massless flavours would be maximal. This scale μ_0 is presumably very close to the top-mass, since just above m_t all the quarks and leptons are effectively massless. But how high above we shall expect the maximum for the purpose of our lattice remain speculations.

11.3 Ambiguity of Concept of Planck Energy Scale, reduced ?

In reduced Planck units, the Planck energy $1.22 * 10^{19} GeV$ from unreduced Planck units is divided by $\sqrt{8\pi} = 5.01325$ so as to get

$$\begin{aligned} E_{Pl\ red} &= 1.22 * 10^{19} GeV/5.013225 && (242)\\ &= 2.4335 * 10^{18} GeV && (243) \end{aligned}$$

Now, however, we must ask: what is it that gives us a scale in the sence the studies of the running couplings tells us? The ratio of the reduced Planck energy $2.43{*}10^{18} GeV$ relative to the logarithmically averaged charged lepton masses $m_{average} = 163 MeV$ is

$$\begin{aligned} \frac{2.43 * 10^{18} GeV}{0.163 GeV} &= 1.4930 * 10^{19} && (244)\\ \text{and has } \ln(\frac{E_{Pl\ red}}{m_{av.ch.fermions}}) &= 44.15 && (245)\\ \text{Further: } \frac{m_Z}{m_{av.ch.fermions}} &= \frac{91.1876 GeV}{163 MeV} && (246)\\ &= 559.4 && (247)\\ \text{and has } \ln(\frac{M_Z}{m_{av.ch.fermions}} &= 6.327 && (248)\\ \text{So for “our” scale } \ln(\frac{\mu_U}{m_{av.ch.fermions}}) &= 27.05 + 6.327 && (249)\\ &= 33.38 && (250)\\ \text{Thus the ratio } \frac{\ln(\frac{E_{Pl\ red}}{m_{av.cg.fermions}})}{\ln(\frac{\mu_U}{m_{av.ch.fermions}})} &= \frac{44.15}{33.38} && (251)\\ &= 1.323. && (252) \end{aligned}$$

Had we not used the reduced Planck energy, but the usual one, we would have got the logarithmic distance from the quark and charged lepton mass scale to the Planck one $\ln(\sqrt{8\pi}) = 1.612$ bigger, so that it would go from the 44.15 up to 44.15 + 1.612 =45.76. Then we would get the ratio changed to

$$\begin{aligned} \frac{\ln(\frac{E_{pl}}{m_{av.ch.fermions}})}{\ln(\frac{\mu_U}{m_{av.ch.fermions}})} &= \frac{44.15 + 1.61}{33.38} && (253)\\ &= \frac{45.76}{33.38} && (254)\\ &= 1.371 && (255) \end{aligned}$$

In fact we think, we can argue for, that this latter choice is not the correct one, because the 8π or 4π usually comes from the differnce in the coefficient to a Coulomb field and the charge appearing in the field theory action. When we have just used the fermion masses without any 4π-like correction we associate it with the simple relation $m = g_y \phi >$, while if I would like the Yukawa-field aruond the Higgs particle I would get a $1/(4\pi)$ factor in. So the simple masses correspond we could say to the Yukawa coupling g_y being used for unit, and not the alternative $g_y/(4\pi)$. So to speak

$$G \sim \frac{g_y}{4\pi} \tag{256}$$

$$(4\pi \text{or} 8\pi)G \sim g_y \text{ and thus also } m \tag{257}$$

This argues for, that the reduced $E_{Pl\,red}$ was the right one to use not to introduce unjustified extra factors.

We could also have argued that the nice scheme of the Standard Model with its gauge fields and three families is spoiled, when going down in energy already at the Higgs scale, so that we should not come up with this logarithmically averaged fermion masses, but just use the very Z^0 mass M_Z instead, then our ratio would be a bit simpler to compute:

$$\frac{\ln(\frac{E_{Pl\,red}}{M_Z})}{\ln(\frac{\mu_U}{M_Z})} = \frac{-1.612 + \ln(\frac{1.22*10^{19}GeV}{91.1876GeV})}{27.05} \tag{258}$$

$$= \frac{-1.612 + 39.43}{27.05} \tag{259}$$

$$= 1.398 \tag{260}$$

11.4 Table of Combinations

The most important outcome of the fluctuating-size-of-links lattice, we propose, is that it gives us te possibility of having a Planck scale very different from te “unification scale” and still claim a “fundamental” lattice at the unification scale. But we would of course like to see, if the order of magnitudes are at all thinkable. We therefore in figure **??** illustrate how we imagine a smooth Gaussian distribution in the logarith of the link length say.

Description of figure11.4: Here the number densities of links or of plaquettes, in a small length range of say a percent counted or weighted in different ways. The curve “original” is for counting this number density as the number in 4-cube of size proportional to the link length range which is being counted. In the two other curves the “original” density has been weighted with respectively the inverse fourth power of the link-length a and the sixth power. For all three curves it is the logarithm of the density, which is plotted and a Gaussian behavior as function of the logarithm of the inverse length of the link is assumed as suggestive example. Plotted with logarithmic ordinate of course a Gaussian distribution looks like a downward pointing parabola, and the three curves are meant to be such downward pointing parbolas. It is trivial algebra to see that weighting the density counted the

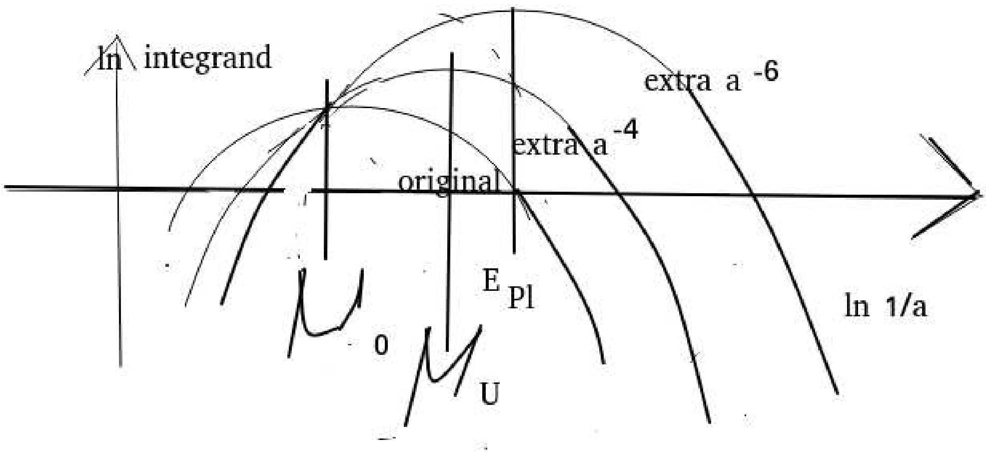


Figure 3: As function of the logarithm of the scale - say given as energy being the inverse of the link length $1/a$ we give here the 1) density of links perlinks-length to the fourth, 2) this density multiplied by a^{-4},and that is the contribution to the Lagrangian dentity for Yang Mills theories, 3) the first density multiplied by a^{-6}, and that is the density of contribution to the Einstein Hilbert Lagrangian density. See also the text. In our approximation we assume these densities to be Gaussian,and with the logarithmic ordinate these Gaussians are parabolas pointing downwards.

"original" way by further respectively $(1/a)^4$ and $(1/a)^6$, the logarithms of which are linear in $\ln(1/a)$, just leads to displacements of the p parbolas, but leave their shapes the same. For the fine structure constants or say our approximate $SU(5)$ it is the total number of plaquettes equivalent to the weighting with $(1/a)^4$ that counts, and the effective lattice link-size for our approximate $SU(5)$ model should thus be the tip of the distribution with the "extra factor a^{-4}". The abscissa of this tip is therefore marked by the symbol μ_U (with a μ written by the curve progarm). Because the Einstein Hilbert action has a dimension 2 different behavior from the just counting plaquettes, it is the abscissa of the tip of the parabola, which had an a^{-6} weighting relative to the "original", which means the effective lattice link-size for the extraction of the Planck scale E_{Pl} energy. One shall note from figure or the trivial algebra that denoting the abscissa for the peak of the "original" by μ_0 then the pushing of this tip energy scale by the two different linear extra terms in the logarithm by the a^{-4} and a^{-6} respectively makes displacements in the dominant (energy)scale by terms in the logarithm being in the ration $4 : 6 = 2 : 3$. This means the prediction

$$\frac{\ln(\frac{\mu_U}{\mu_0})}{\ln(\frac{E_{Pl}}{\mu_0})} = \frac{4}{6} = \frac{2}{3}. \tag{261}$$

Name	μ_0	E_{Pl} $=1.22 * 10^{19} GeV$	$E_{Pl\,red}$ $= 2.34 * 10^{18} GeV$
Z^0 mass	M_Z =91.1876GeV	1.4579	1.3968
Av. fermion mass	$m_{av.fermions}$ =163 MeV	1.3711	1.3216
Top quark	m_t =172.52 GeV	1.4689	1.4079
Fitted μ_0	$\mu_{0\,best}$ =24.231 TeV	1.5769	1.5 (exact)

Table 4: **Table with $\mu_U = 5.1 * 10^{13}$ GeV of** $\frac{\ln \text{“gravity scale”}}{\ln \text{“unified scale”}} = \frac{\ln(\frac{E_{Pl\ or\ Plred}}{\mu_0})}{\ln(\frac{\mu_U}{\mu_0})}$

But we have to guess e.g $\mu_0 = M_Z$ or $\mu_0 = m_t$ to use this.

11.5 Variants of the Relation Planck Scale to “our unified”

In fact the scales μ_0 and also the “Planck scale” do not come in precisely fromour physics and are at best order of magnitudes wise determined. The μ_0 scale should be where the effective number of families is having maximum, but honestly this effective number of families is 3 from the top-mass and up to infinity? So we only know $\mu_0 \geq m_t$. And for the Planck scale we shallin factclaim that it would be expected that the reduced Planck scale (including the often associated 8π to G before using dimensional arguments to construct an energy scale) is actually more reasonable to use.

11.6 How well agrees Our Relation?

In the table 4 we therefore combine our fit above obtained value of the “our unification scale” $\mu_U = 5.1 * 10^{13}$ GeV with some reasonable suggestions for the two less welldefined scales μ_0 and the “Planck scale”.

11.7 Fitted μ_0

Alternative to just guessing on good ideas of what our scale μ_0 at which the density of the size of the scale is maximal counting with its own link length as unit we can simply fit, what we would like this scale to be and then if possible build up a story of, what it should be of that order. Such a fitting of the scale μ_0 would simply mean, that we solve the equation of our prediction, say

$$\begin{aligned}
\ln(\frac{E_{Pl\,red}}{\mu_0}) &= \frac{3}{2} * \ln(\frac{\mu_U}{\mu_0}) && (262)\\
\text{formally giving:} \frac{1}{2}\ln(\mu_0) &= \frac{3}{2}\ln(\mu_U) - \ln(E_{Pl\,red}) && (263)\\
&= 31.563 * 3/2 - 42.2967(\text{using GeV}) && (264)\\
&= 5.048 && (265)\\
\text{So: } \ln(\mu_0) &= 2 * 5.048(\text{in GeV used}) && (266)\\
&= 10.095(\text{with GeV}) && (267)\\
\Rightarrow \mu_0 &= 24.23 TeV. && (268)
\end{aligned}$$

The choice of μ_0 that would make prediction perfect was as seen from the table and our calculation $24TeV$, which is higher than the top quark mass only by a factor $\frac{24.23}{172.25GeV} = 7.11$. Very speculatively one could attempt to construct some fitting of the density as a function of the scale fermions still findable as effectively massles at the scales considered. Above the top mass of course the effective number of massless Fermions at the scale correspond to 3 families, but as one asks below the top qaurk mass there is a 1/3 or 1/4 of a family missing, and one could claim that just below the top mass we have some 3-1/4 or 3-1/3 families left, and then crudely estimate in this spirit, if one could fit the (non-integer) number of families to a function reaching a bit above the top-mass a maximum of 3 families. Then this maximum in a curve taken as a function of the logartihm of the scale would have its maximum with value 3 families very close to our wanted $\mu_0 = 24.23TeV$. (most wellcome to make gravity scale match our model).

Such a very crude and somewhat arbitrary extrapolation might be this:

Firts have in mind that the derivative of the number of "effectively massless" fermion families as function of the logarithm of the scale is given by the "density of the fermions with mass at that scale"="density of species".

On a logarithmic mass scale there is a "density of species" of 1 over a scale dististance $\ln \frac{m_t}{m_b} = 6.02$, meaning density = 0.166 per e-factor. The center of this interval is the geometric mean of 172.25 GeV and 4.180 GeV, which is 26.83 GeV. Next take the interval beteen the b-mass and the s-mass which has length $\ln(\frac{4.180}{0.095}) = 3.784$ and contains two species quarks and 3 species fermions, if we include the τ-lepton. This means for this region around $\sqrt{4.180 * 0.095 GeV} = 0.630$ GeV we have the density of quark species $2/3.784 = 0.5285$. This density is 3.184 times bigger than in the interval between t and b. If the deviation from the maximal number 3 of the number of families at the scale being seen as massless were varying with the logarithm of the scale quadratically counted out from some center value of the scale, then the slope of it, being in fact the "density of species" would vary linearly, and we should just extrapolate the density to the point, where it passes zero to find the maximum point for the formal number of massless families. In logarithmthe distance between the two points we considered is $\ln(\frac{26.83}{0.630}) = 3.75$,and the linear extrapolation leads to the zero for the slope point displaced upward from the 26.83 by the exponential of 3.75/(3.184-1)= 1.717, and that gives 149.4 GeV. It is actually very close to the top-mass. Now we see, that if we take the difference between the two possibilities we mention in the table for the Planck scale as an estimate of the uncertainty of only order of magnitude numbers, then this uncertainty for the ratio given in the table is of order 0.06. But the best of the points actually for the top mass as μ_0 deviates only by 0.03 from the predicted 1.5, so we must say that we shouls take it as agreement within expected uncertainty.

In any case we have shown how a fluctuating lattice size speculatively can solve our problem, that the unification scale is quite different from the Planck energy scale,in spite of that we want a common lattice to describe them both.

12 Conclusion

We have succeeded in constructing a lattice model picture, in which we fit the three fine structure constants in the Standard Model by **three parameters, which are with limited accuracy predicted by various assumptions of the model**. What we consider the most important is, that we suggest, that the way with smallest representation used as the link variables for the Standard Model **group** - understood as the global group structure $S(U(2) \times U(3))$ in the O'Raifeartaigh sense [29] and not only the Lie algebra - is in fact links, that also could have been an $SU(5)$ representation, and therefore the model obtains an **approximate** $SU(5)$-symmetry, when we imposed the usual trace-action. We then took it, that this first approximation $SU(5)$-symmetry of the classically treated simple trace action was broken by quantum fluctuations, which are of course only present for those fluctuations, which are true standard model group degrees of freedom, while the degrees of freedom which are only in $SU(5)$, but not in the Standard model group, of course do not contribute quantum corrections to correct the finestructure constants in our model, wherein they do not exist. It is this quantum correction breaking the $SU(5)$ symmetry (The $SU(5)$ relation between the couplings is only valid in the classical approximation) that brings the deviations from $SU(5)$ GUT theories **without help from additions as susy**, and indeed we "predict" in our model not only ratios of the shifts caused by the quantum fluctuation for the three different standard modelinverse fine structure constants, but also **the absolute size of the corrections**. So even, if we used say the ratio of the corrections to fit the pseudo-unified scale, or let us say "our unification scale" μ_U, then it is still a prediction, that we know the **size of the correction** from precise $SU(5)$ unification. This prediction - it must be admitted though - contains a factor

3 being the number of families. Really it is the number of parallel lattices supposed to exist in Nature, that is 3, so the connection to the number of families is, that there would be by assumption one layer (one of the Wilson lattices lying in parallel) for each family of fermions. (Each family its own "layer".)

The success of this predicting the **deviation** from GUT by quantum corrections fits actually the to experiment fitted fine structure constants at say the M_Z (Z^0 -mass scale) **within uncertainties**! And this is quite remarkable, because these uncertainties for the three inverse finstructure constants in the Standard Model are much smaller by a factor of the order of 50 than the corrections due to the quantum fluctuations, we predicted.

It is due to the high accuracy, with which the fine structure constants are - now a days - known, that we can find so good agreement compared to our quantum corrections, because these corrections are indeed about 10 times smaller than the typical inverse fine structure constant, which is of order 40, while our correction are of the order of 1 times the important "unit" for our corrections $3 * \pi/2 = 4.7124$. In fact we predict e.g. the difference between the inverse fine structure constants at the "our unification scale " (μ_U) such as

$$1/\alpha_2(\mu_u) - 1\alpha_3(\mu_u) \quad \text{“}predicted'' \quad 3 * \frac{\pi}{2} = 4.7124 \tag{269}$$

$$\text{turned out: } 1/\alpha_2(\mu_u) - 1\alpha_3(\mu_u) \quad \text{“}fitted'' \quad 4.62. \tag{270}$$

and the uncertainty in these inverse fine structure such as e.g. the $1/\alpha_3$ is ± 0.05, so the deviation of 0.09 is only 1.8 s. d.(s.d.= standard deviations), and if we count two similar numbers the estimated uncertainty would be $\pm\sqrt{2} * 0.05 = \pm 0.07$ and we would have 1.3 s.d. Our deviation and uncertainty are of the order of a factor 52 smaller than the quantity of deviation 4.62, which we found!

It would in itself be interesting just to leave the two further parameters, namely the unified coupling - for the $SU(5)$ - and the scale of this approximate unification, because we would even then have an interesting relation between the fine structure constants.

12.1 The further two parameters

But we have also formally manged to find assumptions, so that these two further parameters are fitted within the now somewhat smaller accuracies:

- The Unified Coupling as Critical coupling

  We mannaged to be allowed to claim, that the unified coupling is indeed the critical coupling for the non-exitent $SU(5)$ in our model. So in a way there is the little worry with this prediction: that it is for the $SU(5)$ lattice gauge thoery, we use the critical coupling, but this

$SU(5)$ theory is not truly present in our model. One should possibly replace the $SU(5)$ critical couplng by one for a modified $SU(5)$ with the degrees of freedom cut down to those of the Standard Model -like it is in our model - but such a correction would make the critical coupling be stronger(i.e. lower $1/\alpha_{5\ crit}$), and that would make the fitting with this critical coupling being the unified one worse prediction. So after such improvement our unified coupling prediction would not work so well, if this was all we did. But if one starts from a standard model group critical coupling, then one should not make the quantumcorrections as if it were a full $SU(5)$. When we also correct the quantum correction to be for the Standard Model Group, then it actually seems to agree better.

- Relation of the Unified Scale to the Planck scale

  Our story behind our formally within errors relating in our model our unified scale - at which our corrections are to be applied - to the Planck scale may be a bit too much made up with guesses to be truly convincing. Thus this part of the work should rather than being an attempt to find a third predicted parameter, namely the unification scale - what it though also is -, be taken as a needed story for rescuing our model against a severe problem: Our unification scale μ_U should as the lattice scale be the fundamental scale in our model. But that is not so good, because this "unification energy scale" is much lower than the presumably fundamental scale of gravity,the Planck energy scale?

12.2 Problem with Planck scale in our Model

The problem with the Planck scale comesabout like this:

It is not surprising that this unified scale turns out, like in all GUT-theories, to be appreciably smaller than the Planck scale, and in our theory it is even compared to usual unification a bit small:

$$\mu_U \quad = \quad 5.13 * 10^{13} GeV. \tag{271}$$

However, the real problem is that we suggest to have a lattice that is taken **seriously to exist in Nature,** and wewould seemingly loose ordinary continuum manifold physicsfor smaller distances than $1/\mu_U$ and the seemingly approximate well working generel relativity taken classically at such scales, would be already to be considered as aquantum gravity, and in addition we would find it a priori non-attracktive to have several (two) fundamentalscales(μ_U and the Planck energy scale).

12.3 Gravity has to be "Weak" on Fundamental Scale

This may bring us some message about gravity: We have to invent a story of the kind, that gravity is for some reason very weak compared to the fundamental scale expectation. Our above described model namely has as its philosophy, that the unified scale - which remains low compared to the Planck scale in energy - is to be the " fundamental scale"! You might speculatively think about, that the $g^{\mu\nu}$ (with upper indices) has appeared as kind of spontaneous breaking of e.g. diffeomorphism symmetry, and and thus has a chance to be small (often one finds reltively small spontaneously breaking fields, otherwise it would not be so common with low temperature super conductivity, that it was a big sensation to find high temperature super conductivity). If this $g^{\mu\nu}$ is small compared the our fundamental lattice, then compared to this lattice the $g_{\mu\nu}$ with lower indices will be large and thus the length say of a lattice link would be big. This bigness would be bigness compared to the Planck constant and so getting $g^{\mu\nu}$ by some spontaneous breaking story would help bringing about the lack of coincidence of our fundamental scale with the Planck one[22].

Although this idea of having $g^{\mu\nu}$ representing a smontaneous symmetry break down and being "small" for that reason, seems attrcktive to me, we shall in this article rather seek to solve the problem with the Planck scale being different from "the our unified one" μ_U by the idea of fluctuating lattice link size described in next subsection.

12.4 Fluctuating Lattice Scale

A priori it seems somewhat embarrasing, that our theory taken seriously wants a fundamental scale with lattice already at the approximately unification scale $5 * 10^{13} GeV$, while we a priori would expect the fundamenntal scale at the Planck scale, especially for the gravity itself, when we even seek to uphold a principle of critical coupling constants . If a lattice gravity should have in one sense or another a critical coupling, then the lattice should be of the Planck scale lattice constant roughly. The speculation solution, that almost has to be needed is, that of the in scale fluctuating lattice like this or something similar:

At around the "unifying scale" the gravitational fields must behave classically to a very good approximation, except though that a gauge degree of freedom would tend to fluctuate infinitely (actually Ninomiya Förster and myself[37] even would let such strong quantum fluctuations be the reason for the exact gauge symmetry.) because there is lacking terms in the Lagrangian, that can keep the gauge to a fixed one, except the by hand put in gauge fixing terms, but they are of course not physical.

This then means, that we must think in a gravity containing theory as the lattice fluctuating being dense with small lattice constant somewhere in the

Riemann space-time and large somewhere else. In that case we must imagine that the "observed" lattice scale (as for our model the $5.13 * 10^{13} GeV$) will be some appropriate average over a highly fluctuating lattice constant size. We would expect the local lattice scale to fluctuate with a distribution that would be an approximately flat distribution in the logarithme of the lattice constant, because the diffeomorphism group contains scalings and the Haar measure for a pure scaling symmetry subgroup would suggest a smooth in logarithm distribtution. But now, while the averaging of the Yang Mills Lagrangiam over a distribution of scales with a smooth distribution in the logarithm would be weighted in slowly varying way, the gravity action, the Einstein Hilbert one varies with a power law with the scale of the lattice, if you, as we had success with, assumed a critial coupling. This would then lead to that the average size of the lattice link or plaquette structures contributing dominantly to gravity action would be much smaller than the ones contributing to the Yang Mills fields action.

This could suggest a mechanism for the seeming fundamental scale (= lattice constant size scale) for gravity would be much higher in energy than for the Yang Mills theories.

A fluctuating lattice might provide a natural explanation for the much smaller Planck length than length scale at the Yang Mill.

12.5 Baryon Non-conservation ?

Our theory is in danger of inheriting baryon decay in analogy to the usual $SU(5)$ grand unification theories, but at least the gauge particles in the $SU(5)$ theory which are not in one of the standard modelgroups also are supposed not to exist in our scheme, so the obvious diagram with an exchange of such an $SU(5)$ gauge particle is missing in our model. Actually it is in our model some four fermion interaction, that could give the baryon violation, but such an interaction would have a dimension similar to that of the Einstein Hilbert action, and thus the interaction of such a type violating baryon number conservation would be suppressed as a term in Lagrangian of high order with Planck energy as the energy unit. At least that is, what happens in our model, just using our cut off scheme as we did with gravity (fluctuating lattice scale). Whether our Gaussian in log weighting can be assumed sufficiently consistently to suppress the baryonnumber violation suffiently to cope with bounds on proton decay may deserve study in a later work, but at first it looks like working and giving sufficient suppression. .

12.6 Is Approximate Scale Invariance a Dirty Assumption?

Of course, when we claim, what we in this fluctuating lattice model claim, that we have on the one hand an at least approximate scale invariance; but nevertheless, that this symmetry is broken so much, that the size distribution of the numbers of lattice links, or of lattice plaquettes, has maximum at some finite scales - even in order of magnitude - depending on the exact weighting, this sounds a bit dangerous, and can only be true approximately. The meaning is e.g., that if you include some extra power of the (inverse) link length $(1/a)^n$, it can shift maximum in the size distribution from e.g. "the 0ur unifying scale" to the Planck scale. It also involves some physical effect or principle, that performs the needed very strong suppression of links or plaquettes being stronger and stronger the smaller the link or plaquette.

At first it looks like breaking reparametrization invariance in general relativity, does not sound nice. But we must postpone this problem just having now admitted, that there is a problem, that would need more detailed modelling, and that most likely such improved models would be too complicated to be believable.

12.7 Our progress compared to our earlier works

One way of looking at the progress of the present work is to think of it as an updated version of the work by Don Bennett and myself[14], which seeks to get all the three fine structure constant from criticallity at Planck scale and the antiGUT type of model with the gauge group being a cross product of 3 isomorphic Standard Model groups. But in the old works we had to help by extra assumptions the $U(1)$ fine structure constant. In the present article this helping the $U(1)$ has been replaced by the approximate $SU(5)$, so that it seems more natural, and not so specially just making some story for $U(1)$ alone.

12.8 Outlook

12.8.1 The Dream of Exact Formula for α_{EM}

Of course behind such fittings of finestructure constants is the holy gral dream of finding the mathematical formula for the (electrodynamics) fine structure constant, because that is so well known - many decimals - that it contains so much infromation[39] that one could hope to justify a theory to be correct, if it fitted the fine structure constant in a sufficiently simple way (with the many decimals). A work like the present would suggest restrictions on the form of the formula for the fine structure constant, and thereby make an a bit more complicated formula be acceptable as convincingly right provided it were of the right form.

But to make a formula without from phenomenology included expressions possible we would of course need to have the Higgs and the fermion masses connected, and for the time being the usual philosophy is, that the Higgs scale is a pure mystery and, that it needs a solution of the hirarchy problem to be possible at all. Some different philosophy e.g. a coupling of the weak scale or Higgs scale to the development of the renorm group (for e.g. the top quark mass) is needed, one example is our [35, 34] applying the complex action theory also in [36].

12.8.2 Could the See-saw Scale be identified with Our Unifcation Scale?

It is characteristic of the our unified scale μ_U for the only approximate that it is a bit to the low side in energy to even unified scales in other models (especially if it is models with susy), and further it is the spirit of our model that since our unification scale is a lattice scale - or some dominating average in a fluctuating lattice link size -. It is only $5.13 * 10^{13} GeV$. So it puts us in the direction of asking if the see-saw mass scale could be the same as our unification scale?

The neatrino mass square differences are for the atmospheric netrino mass square difference and the solar one

$$\Delta m_A^2 \approx 1.4 * 10^{-3} eV^2 \text{ to } 3.3 * 10^{-3} eV^2 \quad (272)$$

$$\Delta m_{sol}^2 \approx 7.3 * 10^{-5} eV^2 \text{ to } 9.1 * 10^{-5} eV^2 \quad (273)$$

indicating masses of the order of magnitudes (4 to 5) $* 10^{-2} eV$ and $3 * 10^{-3} eV$. With say a typical charged fermion mass in the Standard Model being of mass $1 GeV$, you would expect by dimensional arguments a see saw neutrino mass of the order

$$\text{"see saw scale"} \approx \frac{(1GeV)^2}{10^{-2} eV} \quad (274)$$

$$= 10^{11} GeV \quad (275)$$

$$\text{Not so far from our} \mu_U = 5.13 * 10^{13} GeV. \quad (276)$$

If we take it that the spread in the charged fermion masses from the electron mass $0.5 * 10^{-3} GeV$ and the top quark $174 GeV$ implies that our typical charged fermion mass shall be considered to have 2 to 3 orders of magnitude uncertaity, implying by the squaring in going to the see-saw mass a doubling in the numbers of orders of magnitude, then the see-saw scale is

$$\text{"see saw scale"} \approx 10^{11} GeV * 10^{\pm 5} \quad (277)$$

$$\text{having inside errors } \mu_U = 5.13 * 10^{13} GeV. \quad (278)$$

So if we believe in a lattice already at the $5.13 * 10^{13} GeV$, we can look for replacement of the see-saw neutrinos by some lattice effects.

12.8.3 Small Hierarchy by the Charges from $G_{SMG} \times \cdots \times G_{SMG}$

If our model were right one would look for understanding the charged fermion masses along the lines of our old work with Yasutaka Takanishi and Colin Froggatt [40], while the neutrino oscillations would be related to the lattice of effective lattice scale only $5.13 * 10^{13} GeV$.

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I remember, that Svend Erik Rugh was the first to tell me, that in fact the SU(5) GUT coupling was crittical (for SUSY-GUT) the unified coupling inverted is down to $1/\alpha_5(10^{16}GeV) \sim 25$ and closer to just one of the above mentioned critical (217), $15.3 \pm 7.5\%$, but without the factor 3.

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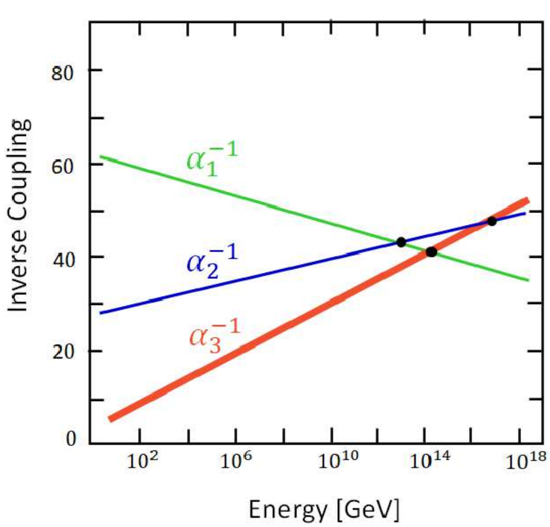
80
60
40
20
0
Inverse Coupling
α_1^{-1}
α_2^{-1}
α_3^{-1}
10^2
10^6
10^{10}
10^{14}
10^{18}
Energy [GeV]