Critical Behavior of Non-Hermitian Kondo effect in Pseudogap System

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The combination of non-Hermitian physics and strong correlation can yield numerous novel and intriguing effects. A previous study on the non-Hermitian Kondo model in ultra-cold atoms reports the reversion of the renormalization group flow. In this work, We investigate the non-Hermitian Kondo effect in system with special form of density of sates $\rho(\omega) \sim |\omega|^r (r > 0)$, which is called pseudogap system. We find that when $r < 1/2$, our conclusion from perturbative renormalization group theory aligns well with previous studies on the traditional pseudogap Kondo problem. In the case of r being equal to $1/2$, a fixed point with reverse property appears in the renormalization group flow. When r is lager than $1/2$, an unstable fixed point appears on the complex plane of the parameter space. Additionally, we validate the conclusions around renormalization group for the $r < 1/2$ interval using the large N expansion method.

I. INTRODUCTION

The Hermitian nature of the Hamiltonian is one of the key assumptions in quantum mechanics. It ensures both probability conservation in isolated quantum systems and the real-valuedness of the energy expectation value of a quantum system. However, in practical quantum systems, there are often exchanges of energy, particles, and information with the environment. Consequently, open quantum systems can often be described using non-Hermitian Hamiltonians [1–3]. Studies have shown that non-Hermitian physics can lead to many novel conclusions, such as the non-Hermitian skin effect[4–6], spontaneous PT symmetry breaking resulting in imaginary parts in particle spectra[7], and some novel topological states[8]. Subsequently, researchers also investigated some non-Hermitian strongly correlated electron models[9–13]. In cold atom experiments, the long-lived metastable excited states of alkaline earth atoms can act as local magnetic moments, while their ground states play the role of conduction electrons[14–17]. In Ref[9], the authors consider the two-body losses/gains induced by inelastic scattering between the ground and excited states, leading to the non-Hermitian Kondo effect.Through an analysis based on perturbative renormalization group theory, they found that due to the introduction of non-Hermitian terms, the renormalization group flow forms a fixed point with a reverse property at the origin of coordinates. That is, the renormalization group flow of the system originates from the origin and returns to it.

At low temperature, magnetic impurities strongly affect the properties of any electron liquid. The "Kondo effect" [18, 19] is characterized by a temperature scale T_K: When the temperature (T) is larger than T_K, the electrons of the host materials are only weakly scattered by the impurity; for $T < T_K$ the (antiferromagnetic) coupling grows nonperturbatively and leads to the formation of a many-body singlet with the electron liquid, which completely screens the impurity magnetic moment.

Furthermore, materials exhibiting pseudogap density of states(DOS) near the Fermi surface have attracted significant attention, such as Dirac and Weyl semimetals, whose the DOS typically scales proportionally to the rth power of energy. In these materials, the conduction and valence bands touch at specific points in the Brillouin zone, known as Dirac points and Weyl points, respectively. Excitations of electrons near these points behave similarly to massless Dirac fermions. Additionally, in d-wave superconductors, due to the particular form of the energy gap function, the superconducting gap vanishes at certain nodes, where excitations with Dirac cone-like dispersion also occur. These materials often exhibit numerous novel properties due to their unique band structures. Furthermore, their response to localized magnetic moments has also garnered attention[20–32]. In Ref.[21], the author initially investigated the coupling effects between the pseudogap system and localized magnetic moments based on poor man's scaling and mean-field theory. The results revealed that there exists an critical parameter controlling the phase transition between decoupled and strong coupled phase under zero tempurature. Later, in Ref.[27], the authors studied the phase diagram of the pseudogap Kondo problem using numerical renormalization group theory. They found that for systems satisfying particle-hole symmetry, an unstable fixed point appeared on the renormalization group flow when $r < \frac{1}{2}$, denoted as J_c. They termed it the symmetric critical(SCR) point, which separates the local moment(LM) phase from the symmetric strong coupling(SSC) phase. When the Kondo coupling coefficient $J < J_c$, the renormalization group flow moves towards the LM phase, whereas when $J > J_c$, it flows towards the SSC phase.This conclusion aligns well with the results obtained from mean-field theory.When $r > \frac{1}{2}$, the SCR point merges with the SSC phase. In this case, regardless of the strength of the coupling coefficient, the system does not exhibit a strong coupling phase.

In this paper, we investigate critical behavior of the non-Hermitian Kondo effect in pseudogap systems to study the response of pseudogap materials to localized magnetic moments in open systems. Through perturbative renormalization group studies, we find that for systems possessing particle-hole symmetry, the phase diagram of the system closely resembles the Hermitian case when $r < 1/2$. In the case with $r = 1/2$, the strongly coupled fixed point exhibits a kind of reverse property. However, when $r > 1/2$, a new unstable fixed point appears in the complex plane. This new critical behavior is entirely different from the behavior in the Hermitian case, demonstrating that non-Hermiticity can lead to more novel properties in strongly correlated models.Subsequently, we applied the large N expansion method and defined the residual function of the mean-field self-consistent equation. We find nontrivial solutions of the self-consistent equation on the real axis for $J > J_c$, thereby verifying the conclusion of the perturbative renormalization group for $r < 1/2$.

II. MODEL

In this work, we start with Hamiltonian of Non-Hermitian Kondo impurity coupled with the conduction electrons with pseudogap DOS. this will be combined by two parts:

$$H = H_c + H_K. \tag{1}$$

In the above expression, the first term represents the conduction electron component, and its specific form is as follows.

$$H_c = \sum_{\mathbf{k},\sigma} \epsilon_k c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}, \tag{2}$$

where $c_{\mathbf{k}\sigma}$ is the annihilation operator of conduction electrons. And the DOS of these electrons has such special form:

$$\rho(\omega) = \begin{cases} C|\omega|^r & \text{if } |\omega| \le D \\ 0 & \text{else }, \end{cases} \tag{3}$$

where ω and D respectively represent the energy and bandwidth of the conduction electrons and C is proportional coefficient which we will be set to one for the convenience of future discussions.

The second term describes the non-Hermitian Kondo interaction between impurity spin and conduction electrons spin

$$H_K = J\vec{S}_{imp} \cdot \vec{s}(0), \tag{4}$$

where$J = J_r + iJ_i$ is a complex number, $\vec{S}$ is impurity spin and $\vec{s}(0) = c^\dagger_{\mathbf{k}\sigma}\sigma_{\sigma\sigma'}c_{\mathbf{k}\sigma'}$ is the spin of conduction electrons at the point of local electrons.

III. RENORMALIZATION GROUP

We calculate the beta function using perturbation renormalization group theory to third order. We obtain the following RG equation:

$$\frac{dj}{d\ln D} = rj - j^2 + \frac{j^3}{2}, \tag{5}$$

where $j = \rho(D)J$ is the dimensionless coupling constant. By analysis , we find that there exits three different construction of the diagram of RG flow, as shown in 1

In the interval $0 < r < \frac{1}{2}$, as illustrated in the Figure.1(a), we can see that there are two stable fixed points on the real axis, located at the origin and (1.5, 0) respectively. These correspond to the LM phase and the SSC phase. In between, at the point (0.5, 0), there is an unstable fixed point corresponding to the SCR phase, which governs the phase transition between the LM phase and SSC phase. This result aligns with the traditional pseudogap Kondo effect conclusions. The specific form of the critical point j_c is as follows:

$$j_c = 1 - \sqrt{1-2r} \approx r + \mathcal{O}(r^2). \tag{6}$$

When $r = \frac{1}{2}$, as is shown in Figure.1(b), the critical point of the SCR phase and the SSC phase merge. This results in the appearance of a fixed point with reverse property, as mentioned in Ref[9], at the current fixed point of the strongly coupled phase. In other words, the renormalization group flow starts from the point (1.0, 0) and returns to the same point.

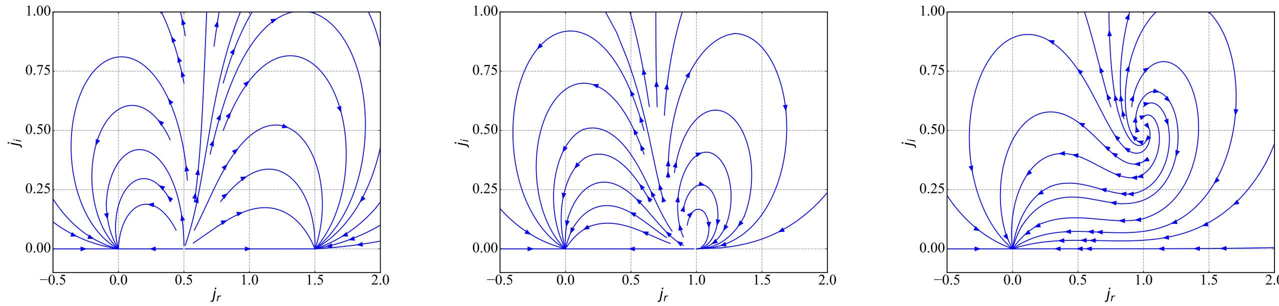
If r is greater than $\frac{1}{2}$, the renormalization group flow is depicted in Figure1(c). For the Hermitian case in this interval, there should be no presence of the SSC phase in the system, let alone a quantum phase transition between SSC and LM phases. However, surprisingly, from the graph, we observe the emergence of a new unstable fixed point on the complex plane. The renormalization group flow of the system moves from this point to the origin, indicating the onset of a new critical phenomenon within this range.

IV. LARGE N EXPANSION

As a preparation for employing the large N expansion technique, we first extend the Kondo model Hamiltonian to a model with SU(N) symmetry, commonly referred to as the Coqblin-Schrieffer model[33]:

$$H_{CS} = H_c - \frac{J}{N}\sum_{\mathbf{k},\alpha}\sum_{\mathbf{k}',\beta} c^\dagger_{\mathbf{k}\alpha} f_\alpha f^\dagger_\beta c_{\mathbf{k}'\beta}, \tag{7}$$

where $\alpha, \beta = 1, 2, \ldots, N$ is the extended spin quantum number of electrons and J is the complex Kondo coupling parameter.



(a) (b) (c)

FIG. 1. RG flow diagrams of the non-Hermitian pseudogap Kondo model in different values of r:(a)r=0.375, (b)r=0.5,(c)r=0.625

Subsequently, in the language of path integrals, it is required to introduce a Lagrange multiplier λ to enforce the single occupancy condition of local electrons $n_f = \sum_\alpha f_\alpha^\dagger f_\alpha = 1$.

The next step involves using the Hubbard-Stratonovich transformation to rewrite the four-operator interaction terms in Eq.7 into the following form:

$$-\frac{J}{N}\sum_{\mathbf{k},\alpha}\sum_{\mathbf{k}',\beta} c^\dagger_{\mathbf{k}\alpha} f_\alpha f^\dagger_\beta c_{\mathbf{k}'\beta} \to \sum_{\mathbf{k},\alpha}(\bar{V} c^\dagger_{\mathbf{k}\alpha} f_\alpha + h.c.) + N\frac{|V|^2}{J}, \tag{8}$$

where $\bar{V} = \bar{V}_{\mathbf{k}} = -\frac{J}{N}\sum_\alpha \left\langle c^\dagger_{\mathbf{k}\alpha} f_\alpha \right\rangle$ is the static boson field describing the coupling between the local electrons and conduction electrons.

Finally, taking the limit $N \to \infty$, the results of the path integral are entirely determined by the saddle points of the integrand. In summary, we ultimately obtain the following approximation for the free energy F in large N limit:

$$\begin{aligned} F = &-\frac{N}{\pi}\int_{-D}^{D} \frac{d\omega}{e^{\beta\omega}+1}\tan^{-1}\left[\frac{\pi|\omega|^r|V|^2}{-\omega+\lambda-|V|^2\mathcal{P}(\omega)}\right] \\ &+ N\frac{|V|^2}{J} - \lambda, \end{aligned} \tag{9}$$

where $\mathcal{P}(\omega) = \mathcal{P}\int \frac{d\epsilon|\epsilon|^r}{\omega-\epsilon} = \pi|\omega|^r\tan(\frac{\pi r}{2})\mathrm{sgn}(\omega)$ representing the principal value integral.In the saddle-point approximation, the parameter of free energy should satisfy the following self-consistent relation:

$$\begin{aligned} \frac{\partial F}{\partial \bar{V}} &= 0, \\ \frac{\partial F}{\partial \lambda} &= 0. \end{aligned} \tag{10}$$

At the zero-temperature limit, the specific form of these two equations are as follows:

$$\int_{-D}^{0} d\omega \frac{V|\omega|^r(-\omega+\lambda)}{(-\omega+\lambda-|V|^2\mathcal{P}(\omega))^2+\pi|\omega|^r|V|^2} - \frac{V}{J} = 0, \tag{11}$$

$$\int_{-D}^{0} d\omega \frac{|V|^2|\omega|^r}{(-\omega+\lambda-|V|^2\mathcal{P}(\omega))^2+\pi|\omega|^r|V|^2} - \frac{1}{N} = 0. \tag{12}$$

Since J is a complex number, Lagrange multiplier must be a complex number to make the above equations valid, i.e. $\lambda = \lambda_r + i\lambda_i$.Observing this equation system composed of two equations, we can immediately deduce that $V = 0$ is a solution to the system in the large N limit. Since all interactions are regulated by the bosonic field V, if this trivial solution is stable, it implies the decoupling of impurities from conduction electrons, corresponding to the LM phase. Therefore, the existence of a non-trivial solution to the equations signifies the presence of a phase transition from weak to strong coupling.

To discuss the non-trivial solution of the equations, we eliminate V from Eq.11. Assuming the existence of a non-trivial solution with small $|V|^2$ and λ near the critical point, we set $|V|^2$, λ_r and λ_i to be equal to 0, after eliminating. And this yields the expression for the critical Kondo coupling coefficient J_c on real axis:

$$J_c = \frac{r}{D^r}. \tag{13}$$

This result aligns with Eq.6 obtained from the renormalization group theory in the interval where $r < \frac{1}{2}$.

To analyze the properties of the system of equations, we define the residual function

$$Re(J_r, J_i, |V|^2, \lambda_r, \lambda_i, D, N) = \sqrt{|f_1-1|^2+|f_2-1|^2}, \tag{14}$$

where

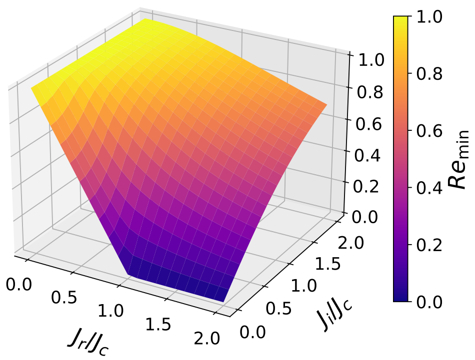
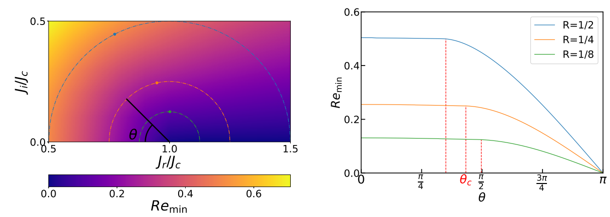


FIG. 2. The surface of the function $Re_{\min}$ with respect to the variations of parameters J_r and J_i for $r = 0.4$, $D = 10^4$, and $N = 10^4$.

$$f_1 = J \int_{-D}^{0} d\omega \frac{|\omega|^r(-\omega+\lambda)}{(-\omega+\lambda-|V|^2\mathcal{P}(\omega))^2+\pi|\omega|^r|V|^2},$$
$$f_2 = N \int_{-D}^{0} d\omega \frac{|V|^2|\omega|^r}{(-\omega+\lambda-|V|^2\mathcal{P}(\omega))^2+\pi|\omega|^r|V|^2}. \tag{15}$$

By adjusting parameters $|V|^2$, λ_r and λ_i, we can find the minimum value $Re_{\min}$ of Re for each Kondo coupling coefficient parameter, thereby obtaining the surface of the function $Re_{\min}(J_r, J_i)$ as it varies across the entire parameter space in the complex plane. Figure.2 illustrates the variation of $Re_{\min}$ for $r = 0.4$, $D = 10^4$, and $N = 10^4$. For points where the function $Re_{\min} = 0$, nontrivial solutions to our equations exist; conversely, for points where $Re_{\min} \neq 0$, non-trivial solutions do not exist. The result reveals that within the interval $0 < r < 1$, the function $Re_{\min}$ has zeros on the real axis for $J > J_c$. This indicates the existence of non-trivial solutions to the self-consistent equations within this region.

Furthermore, by employing the function $Re_{\min}$, we can also discern the critical behavior of the system on the complex plane.Through calculations, we obtained the functional relationship of $Re_{\min}$ with respect to the central angle θ on semicircles of different radii centered at the point J_c, as shown in Figure 3. Their variation is depicted in Figure 3(b). It can be observed that there exists a critical angle θ_c: when the central angle $\theta < \theta_c$, the variation function is almost a horizontal line, implying that the system is in the same universal class with LM phase; however, when the central angle $\theta > \theta_c$, the decreasing trend of the function begins to increase, reaching zero at $\theta = \pi$, indicating the system being in a strong coupling phase within this range. The critical points corresponding to the critical angles at different radii are indicated by dots in Figure 3(a). It can be seen that near



(a) (b)

FIG. 3. Illustrating the variation of $Re_{\min}$ with the central angle θ at different radii $R = 1/2$(blue curve), $R = 1/4$(orange curve), $R = 1/8$(green curve). It can be observed that in (b) there exists a critical angle θ_c for each radius. When $\theta > \theta_c$, the rate of decrease of $Re_{\min}$ increases. And In (a), the positions of the critical angles of different radii in the distribution of $Re_{\min}$ are indicated by dots

the point J_c, the critical points are located just above J_c, and as the radius increases gradually, the position of the critical points approaches the imaginary axis. This suggests inaccuracies in the results of mean-field theory away from the vicinity of J_c.

In summary, we first obtained the phase diagram of the non-Hermitian Kondo effect in pseudogap system through perturbative renormalization group theory. The result reveals that when $r < 1/2$, there exists unstable fixed point (SCR) on the real axis, controlling the phase transition between LM phase and SSC phase ; when $r = 1/2$, the SCR fixed point merges with the SSC phase, forming a fixed point with reverse property; when $r > 1/2$, an unstable fixed point appears on the complex plane. Subsequently, we further validated the conclusion of the perturbative renormalization group in the $r < 1/2$ interval through the large N expansion method, and obtained critical property of the system on the complex plane through analysis. However, for $r > 1/2$, the appearance of unstable fixed points on the complex plane raises the question of their physical significance, which requires further investigation and exploration. Due to the rapid advancement in non-Hermitian physics in recent years, it is foreseeable that the combination of non-Hermitian and strongly correlated physics will inevitably give rise to more novel physical phenomena.

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