Scaling Enhancement of Photon Blockade in Output Fields

Zhi-Hao Liu[1] and Xun-Wei Xu[1,2,*]

[1] *Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, Key Laboratory for Matter Microstructure and Function of Hunan Province, Department of Physics and Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha 410081, China*

[2] *Institute of Interdisciplinary Studies, Hunan Normal University, Changsha, 410081, China*

(Dated: March 28, 2024)

Photon blockade enhancement is an exciting and promising subject that has been well studied for photons in cavities. However, whether photon blockade can be enhanced in the output fields remains largely unexplored. We show that photon blockade can be greatly enhanced in the mixing output field of a nonlinear cavity and an auxiliary (linear) cavity, where no direct coupling between the nonlinear and auxiliary cavities is needed. We uncover a biquadratic scaling relation between the second-order correlation of the photons in the output field and intracavity nonlinear interaction strength, in contrast to a quadratic scaling relation for the photons in a nonlinear cavity. We identify that this scaling enhancement of photon blockade in the output field is induced by the destructive interference between two of the paths for two photons passing through the two cavities. We then extend the theory to the experimentally feasible Jaynes-Cummings model consisting of a two-level system strongly coupled to one of the two uncoupled cavities, and also predict a biquadratic scaling law in the mixing output field. Our proposed scheme is universal and can be extended to enhance blockade in other bosonic systems.

Introduction.—Photon blockade, preventing the resonant injection/emission of more than one photon, is a pure quantum effect for single-photon generation with wide application in a myriad of quantum information protocols and technologies. Since its first proposal in an optical cavity containing strong Kerr nonlinearities [1], diverse efforts have been made to observe photon blockade experimentally and many novel mechanisms are proposed to enhance the effect. Strong coupling between light and matter at the single-photon level enabled the observation of photon blockade in experiments, including single atoms strongly coupled to an optical resonator [2–4], a quantum dot strongly coupled to a photonic crystal resonator [5, 6], and a superconducting qubit strongly coupled to a transmission line resonator [7, 8]. Different from the photon blockade induced by photon-photon repulsion based on strong nonlinearity, there is another mechanism with weak nonlinearity and the blockade is achieved by quantum interference [9–16], which has been observed for both optical [17] and microwave [18] photons. Besides, photon blockade is also predicted by nonlinear driving [19, 20] and nonlinear loss [21–24]. Moreover, photon blockade enhancement is proposed based on multimode-resonant interaction [25, 26], non-Hermitian interaction [27–30], dynamical excitation [31, 32], and coupled-resonator chain [33–35].

We note that most of the previous works focused on the photon blockade in the cavities, based on the common assumption that the photon statistics of the output field is the same as the photons in the cavities. However, this common assumption is not always true, such as we can observe photon antibunching for the reflected light but bunching for transmitted light [4, 36, 37]. Photon statistics for the mixing of two output channels has been investigated in Ref. [38], and tunable photon statistics have been observed in the mixing field of a two-photon state and a coherent field [39–41]. Nevertheless, *whether photon blockade can be enhanced in the mixing fields output from two cavities* hasn't been studied thoroughly.

In this Letter, we show that *photon blockade can be greatly enhanced in the mixing fields output from a nonlinear cavity and an auxiliary linear cavity by quantum interference*. Different from the previous works on photon blockade in weakly nonlinear photonic molecules [9–16], here the photon blockade is greatly enhanced in the output fields, and there is no direct coupling between the two cavities. We analytically identify that there is a *biquadratic scaling relation between the second-order correlation of the photons in the output field and the intracavity nonlinear interaction strength*, in contrast to a quadratic scaling law for the photons in a nonlinear cavity. Our scheme is universal and can be extended to other platforms. As an example, we consider an experimentally feasible Jaynes-Cummings (JC) model for two (uncoupled) cavities with a two-level system (TLS) coupled to one of them, and demonstrate a *biquadratic scaling relation between the second-order correlation of the photons in the output field and TLS-cavity interaction strength.*

Model.—Without loss of generality, we first consider the photon blockade in the mixing fields output from a cavity containing $\chi^{(3)}$ nonlinear medium and an auxiliary (linear) cavity [Fig. 1(a)]. The total Hamiltonian of the system in the frame rotating at the probe laser frequency ω_p can be written as ($\hbar = 1$),

$$\begin{aligned} H &= \Delta_1 a_1^\dagger a_1 + U a_1^\dagger a_1^\dagger a_1 a_1 + i\varepsilon \left(a_1^\dagger - a_1\right) \\ &\quad + \Delta_2 a_2^\dagger a_2 + i\varepsilon \left(a_2^\dagger - a_2\right), \end{aligned} \tag{1}$$

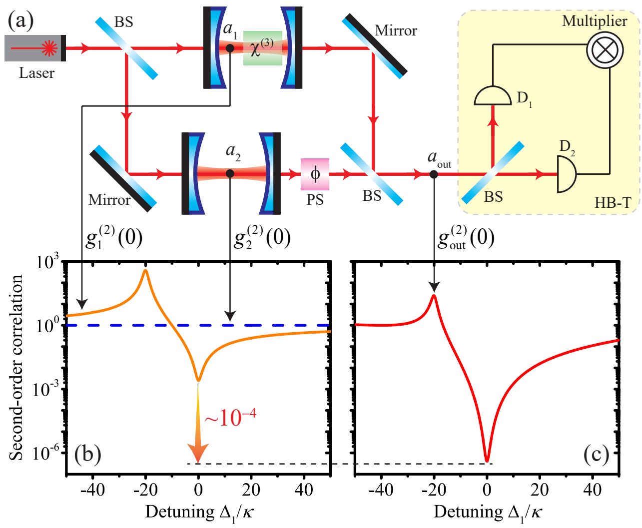

FIG. 1. (Color online) (a) A Mach–Zehnder interferometer with two cavities (a_1 and a_2) in the paths. A laser is divided into two beams by a 50/50 beam splitter (BS), and they are injected into a cavity containing $\chi^{(3)}$ nonlinear medium and an auxiliary (linear) cavity, respectively. The output fields from these two cavities mix by another BS. A phase shifter (PS) is placed in one path to induce tunable phase difference ϕ between the two paths. The second-order correlation of the output field is measured by a Hanbury-Brown-Twiss (HB-T) set-up. (b) and (c) Second-order correlations of the photons in the two cavities [$g_1^{(2)}(0)$ and $g_2^{(2)}(0)$] and the mixing output field [$g_{\rm out}^{(2)}(0)$] are plotted as functions of the detuning Δ_1/κ. The parameters are $\phi=\pi$, $U=20\kappa$, and $\delta=2U$.

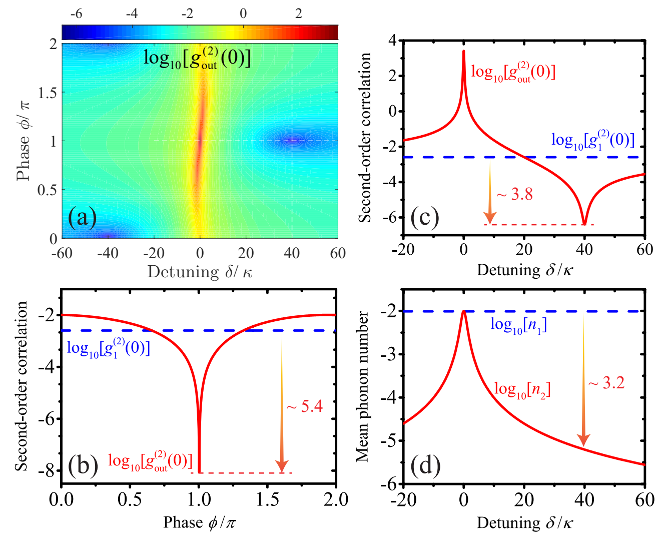

FIG. 2. (Color online) (a) The second-order correlation $\log_{10}[g_{\rm out}^{(2)}(0)]$ for different phase ϕ/π and detuning δ/κ. The second-order correlations $\log_{10}[g_{\rm out}^{(2)}(0)]$ and $\log_{10}[g_1^{(2)}(0)]$ (b) versus phase ϕ/π with $\delta=2U$ and (c) versus detuning δ/κ with $\phi=\pi$. (d) The mean photon number $\log_{10}(n_1)$ and $\log_{10}(n_2)$ versus detuning δ/κ with $\phi=\pi$. The other parameters are $\Delta_1=0$ and $U=20\kappa$.

where a_i and $a_i^\dagger$ are the annihilation and creation operators of the ith cavity with frequency ω_i $(i=1,2)$, $\Delta_i=\omega_i-\omega_p$ is the laser detuning from the cavity resonance, $\delta=\omega_2-\omega_1$ is the detuning between the two cavities, U is the nonlinear interaction strength, and ε is the pumping strength on each cavity. According to the input-output relation [42], the mixing fields $a_{\rm out}$ output from the two cavities can be described by $a_{\rm out}=(\sqrt{\kappa_1}a_1+e^{i\phi}\sqrt{\kappa_2}a_2)/\sqrt{2}-a_{\rm vac}$, where κ_i is the one-sided decay rate of the ith cavity, ϕ is the relative phase between the two output fields (tunable by using the phase shifter), and $a_{\rm vac}$ is the input vacuum field from the right-hand side of the cavities.

Photon statistics in the output field can be described by the equal-time second-order correlation function

$$
\begin{aligned}
g_{\rm out}^{(2)}(0) &= \frac{\left\langle a_{\rm out}^\dagger a_{\rm out}^\dagger a_{\rm out}a_{\rm out}\right\rangle}{\left\langle a_{\rm out}^\dagger a_{\rm out}\right\rangle^2} \\
&= \sum_{j,k,l,m=1}^{2} e^{in\phi}\sqrt{\kappa_j\kappa_k\kappa_l\kappa_m}\frac{\left\langle a_j^\dagger a_k^\dagger a_l a_m\right\rangle}{N_{\rm out}^2}, \quad (2)
\end{aligned}
$$

where $n=l+m-j-k$ and $N_{\rm out}=\kappa_1\langle a_1^\dagger a_1\rangle+\kappa_2\langle a_2^\dagger a_2\rangle+2\sqrt{\kappa_1\kappa_2}{\rm Re}(e^{i\phi}\langle a_1^\dagger a_2\rangle)$. Different from the second-order correlation function in the cavities $g_i^{(2)}(0)=\langle a_i^\dagger a_i^\dagger a_i a_i\rangle/\langle a_i^\dagger a_i\rangle^2$ $(i=1,2)$, $g_{\rm out}^{(2)}(0)$ also depends on the cross-correlation between the two cavities (i.e., $\langle a_2^\dagger a_2 a_1^\dagger a_1\rangle$, $\langle a_1^\dagger a_1^\dagger a_2 a_2\rangle$, $\langle a_1^\dagger a_1^\dagger a_1 a_2\rangle$, and $\langle a_2^\dagger a_1^\dagger a_2 a_2\rangle$), and there are phase factors $e^{in\phi}$ in front of the terms, which can be negative and induce the enhancement of photon blockade in the output field, without changing the photon statistics in the cavities.

The dynamics of the system are governed by the master equation [43] $d\rho/dt=-i[H,\rho]+\sum_{i=1,2}\kappa_i\left(2a_i\rho a_i^\dagger-a_i^\dagger a_i\rho-\rho a_i^\dagger a_i\right)$, where ρ is the density matrix of the system. Without loss of generality, we set $\kappa_1=\kappa_2=\kappa$ and rescale other parameters by this quantity, such as $\varepsilon=\kappa/10$ for weak pumping.

Photon Blockade Enhancement.—To demonstrate the photon blockade enhancement more clearly, the second-order correlation functions for photons in the two cavities $g_i^{(2)}(0)$ and in the output field $g_{\rm out}^{(2)}(0)$ are shown in Fig. 1(b) and 1(c) with the same ordinate scale. As expected, the photon statistics in the two uncoupled cavities are independent of each other: strong photon blockade $g_1^{(2)}(0)\approx 2.54\times10^{-3}$ in the cavity a_1 for strong enharmonicity ($U=20\kappa$), and no photon blockade $g_2^{(2)}(0)=1$ in the cavity a_2 without nonlinearity. Surprisingly, a much stronger photon blockade is obtained in the mixing output field $a_{\rm out}$ for $[g_{\rm out}^{(2)}(0)/g_1^{(2)}(0)]\sim10^{-4}$ at resonant frequency ($\Delta_1=0$).

Figure 2(a) is a color plot of $\log_{10}[g_{\rm out}^{(2)}(0)]$ as a function of the phase ϕ/π and detuning δ/κ, for $\Delta_1 = 0$ and $U = 20\kappa$. The minimum of $\log_{10}[g^{(2)}_{\rm out}(0)]$ is reached for $\phi \approx \pi$ and $\delta \approx 2U$ (or $\phi \approx 0$ and $\delta \approx -2U$). Two cuts taken from the color plot for $\delta = 2U$ and $\phi = \pi$ are shown in Figs. 2(b) and 2(c), respectively. The photon blockade is enhanced, i.e. $g^{(2)}_{\rm out}(0) < g^{(2)}_1(0)$, in the regime of $0.66\pi < \phi < 1.33\pi$, and the minimal value of $g^{(2)}_{\rm out}(0)$ is about 5.4 orders smaller than $g^{(2)}_1(0)$ at $\phi \approx 0.996\pi$ [Fig. 2(b)]. Moreover, $g^{(2)}_{\rm out}(0)$ also strongly depends on the detuning δ/κ between the two cavities, and it is about 3.8 orders smaller than $g^{(2)}_1(0)$ at $\delta \approx 2U$ [Fig. 2(c)]. These results suggested that quantum interference might be responsible for the great enhancement of photon blockade in the output field.

In addition, the mean photon numbers in the cavities ($n_i = \langle a_i^\dagger a_i\rangle$) are plotted as functions of the detuning δ/κ in Fig. 2(d). Under the optimal condition $\delta \approx 2U$, the single photon generation in the auxiliary cavity a_2 is suppressed seriously ($n_2/n_1 \approx 10^{-3.2}$) for large detuning $\delta \gg \kappa$. Hence, almost all of the single photons in the output field is emitted from the cavity a_1. However, the auxiliary cavity a_2 provides another path for two photons passing through the whole system, which is the key ingredient for the enhancement of photon blockade in the output field as discussed bellow. By the way, the mean photon number in the auxiliary cavity a_2 is almost the same as the one in cavity a_1 ($n_1 \approx n_2$) under the resonant condition $\delta = 0$, and they cancel each other at $\phi = \pi$ for destructive interference, which induces a strong bunching effect [$g^{(2)}_{\rm out}(0) \gg 1$] [14, 40] in the output field [Fig. 2(c)].

Biquadratic scaling.—In order to understand the origin of the giant enhancement of photon blockade in the output field, we derive the expressions of the second-order correlations [$g^{(2)}_{\rm out}(0)$ and $g^{(2)}_1(0)$] analytically [44]. Here, including the effect of optical decay, an effective Hamiltonian $H_{\rm eff} = H - i\kappa(a_1^\dagger a_1 + a_2^\dagger a_2)$ is introduced according to the quantum-trajectory method [45]. Under weak driving condition ($\varepsilon \ll \kappa$), the wave function on a Fock-state basis can be truncated to the two-photon manifold as: $|\psi\rangle = \sum_{n_2=0}^{2-n_1}\sum_{n_1=0}^{2} C_{n_1n_2}|n_1,n_2\rangle$. Here, $|n_1,n_2\rangle$ represents the Fock state of n_1 photons in cavity a_1 and n_2 photons in cavity a_2, with the probability amplitude $C_{n_1n_2}$. The expression of $C_{n_1n_2}$ can be obtained by solving the Shrödinger equation $d|\psi\rangle/dt = -iH_{\rm eff}|\psi\rangle$ in the steady states.

Under the conditions for photon blockade enhancement ($\phi = \pi$, $\Delta_1 = 0$, and $\delta = 2U \gg \kappa$), the second-order correlation function can be written as

$$g^{(2)}_{\rm out}(0) \approx \frac{2\kappa^2}{N^2_{\rm out}} \left\{ \left|C_{20} - \sqrt{2}C_{11}\right|^2 + |C_{02}|^2 - 2{\rm Re}\left[\left(\sqrt{2}C^*_{11} - C^*_{20}\right)C_{02}\right]\right\}, \quad (3)$$

where $N_{\rm out} \approx \kappa[|C_{10}|^2 + |C_{01}|^2 - 2{\rm Re}\,(C_{01}C^*_{10})$. The probability amplitudes for two-photon states in the steady state are approximately given by

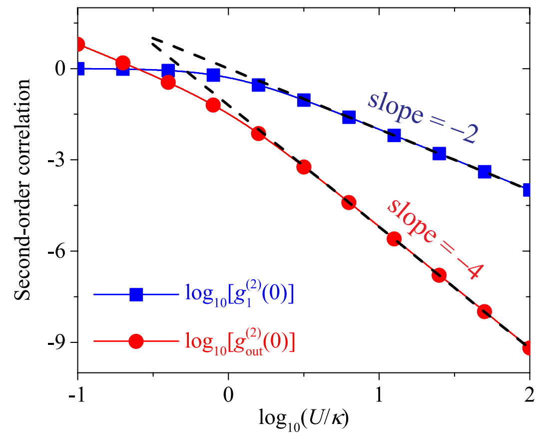

FIG. 3. (Color online) The second-order correlations $\log_{10}[g^{(2)}_{\rm out}(0)]$ and $\log_{10}[g^{(2)}_1(0)]$ versus the nonlinear interaction strength $\log_{10}(U/\kappa)$ with $\Delta_1 = 0$, $\phi = \pi$, and $\delta = 2U$.

$$C_{20} \approx \frac{-i}{U - i\kappa}\frac{\varepsilon^2}{\sqrt{2}\kappa}, \qquad C_{02} \approx -\frac{\sqrt{2}}{2U - i\kappa}\frac{\varepsilon^2}{4U},$$
$$C_{11} \approx \frac{-i}{U - i\kappa}\left(\frac{\varepsilon^2}{2\kappa} - \frac{i\varepsilon^2}{4U}\right) \quad (4)$$

with $|C_{20}| \approx \sqrt{2}|C_{11}| \gg |C_{02}|$. The first and last terms inside the curly brace of Eq. (3) are canceled out by the destructive interference between C_{20} and C_{11}. Then the second-order correlation function in the output field is approximately given by

$$g^{(2)}_{\rm out}(0) \approx \frac{1}{16}\left(\frac{\kappa}{U}\right)^4. \quad (5)$$

The second-order correlation of the photons in the output field depends on the strength of the nonlinear interaction with a biquadratic scaling law, which is different from the second-order correlation of photons in the cavity a_1, i.e., $g^{(2)}_1(0) \approx (\kappa/U)^2$, with a quadratic scaling law.

Both $\log_{10}[g^{(2)}_{\rm out}(0)]$ and $\log_{10}[g^{(2)}_1(0)]$, obtained by solving the master equation numerically, are plotted as functions of $\log_{10}(U/\kappa)$ in Fig. 3. In the strong nonlinear regime $U/\kappa \gg 1$, the slope of $\log_{10}[g^{(2)}_{\rm out}(0)]$ versus $\log_{10}(U/\kappa)$ is -4, which is much larger than the slope of -2 for $\log_{10}[g^{(2)}_1(0)]$ versus $\log_{10}(U/\kappa)$. The numerical results agree well with the analytical expressions in the strong nonlinear regime (black dashed lines in Fig. 3). Thus, the scheme we proposed can not only greatly enhance photon blockade by several orders, but also change the scaling exponent of the second-order correlation on the nonlinear interaction strength from -2 to -4.

JC model.—The scheme for photon blockade enhancement in output field is universal, which can be extended to other optical platforms with anharmonic energy levels, such as the JC model [46]. The strong coupling between a single cavity and a TLS has been realized decades ago [47–50], and photon blockade was demonstrated in a large number of experiments based on JC model [2–8]. Here, we demonstrate a scaling enhancement of photon blockade in the mixing field output from two (uncoupled) cavities with a TLS strongly coupled to one of them.

The scheme can be extended to the JC model just by replacing the $\chi^{(3)}$ nonlinear medium [Fig. 1(a)] by a TLS [Fig. 4(a)], and the system is described by

$$\begin{aligned} H_{\mathrm{JC}} &= \Delta_1 a_1^\dagger a_1 + \Delta_a \sigma_+ \sigma_- + g\left(a_1^\dagger \sigma_- + \sigma_+ a_1\right) \\ &\quad + \Delta_2 a_2^\dagger a_2 + i\varepsilon\left(a_1^\dagger + a_2^\dagger - \mathrm{H.c.}\right), \end{aligned} \tag{6}$$

where σ_+ and σ_- are the raising and lowering operators of the TLS with transition frequency ω_a, $\Delta_a = \omega_a - \omega_p$ is the laser detuning from the TLS, and g is the TLS-cavity coupling strength. We assume that the TLS is resonant with the cavity ($\Delta_a = \Delta_1 = \Delta$), and the decay rate of the TLS is $\kappa_a = 2\kappa$.

In order to confirm the applicability of our scheme in JC model, we perform a fully numerical simulation of the second-order correlation in the output field based on the master equation. $\log_{10}[g^{(2)}_{\mathrm{out}}(0)]$ is plotted as a function of ϕ/π and Δ_2/κ in Fig. 4(b), for $g = 20\kappa$ and $\Delta = -g$. The minimum of $\log_{10}[g^{(2)}_{\mathrm{out}}(0)]$ appears around $\phi = \pi$ and $\Delta_2 \approx -13.3\kappa \approx -2g/3$. From the cuts of the color scale plot shown in Figs. 4(c) and 4(d), $g^{(2)}_{\mathrm{out}}(0)$ is about 4.3 orders smaller than $g_1^{(2)}(0)$ at $\phi = 0.99\pi$ with $\Delta_2 = -2g/3$ [Fig. 4(c)], and about 3 orders smaller than $g_1^{(2)}(0)$ at $\Delta_2 \approx -13.38\kappa$ with $\phi = \pi$ [Fig. 4(d)].

In order to understand the enhancement of photon blockade in the output field [44], we derive the expression of $g^{(2)}_{\mathrm{out}}(0)$ by using the effective Hamiltonian $H_{\mathrm{JC,eff}} = H_{\mathrm{JC}} - i\kappa(a_1^\dagger a_1 + a_2^\dagger a_2 + \sigma_+\sigma_-)$ and wave function $|\varphi\rangle = \sum_{n_2=0}^{2-n_1}\sum_{n_1=0}^{2} C_{gn_1n_2}|g,n_1,n_2\rangle + \sum_{n_2=0}^{1-n_1}\sum_{n_1=0}^{1} C_{en_1n_2}|e,n_1,n_2\rangle$. Here, $|g,n_1,n_2\rangle$ ($|e,n_1,n_2\rangle$) denotes the Fock state of n_1 photons in cavity a_1, n_2 photons in cavity a_2, and the TLS in the ground (excited) state, with the probability amplitude $C_{gn_1n_2}$ ($C_{en_1n_2}$). The optimal condition $\Delta_2 = -2g/3$ for photon blockade in the output field is obtained by setting $C_{g02} \approx \sqrt{2}C_{g11}$, and they are canceled out by the destructive interference in the output field at phase difference $\phi = \pi$. Thus the second-order correlation function in the output field becomes [44]

$$g^{(2)}_{\mathrm{out}}(0) \approx 16\left(\frac{\kappa}{g}\right)^4. \tag{7}$$

We also have the second-order correlation in cavity a_1 as $g_1^{(2)}(0) \approx 36(\kappa/g)^2$ for single-mode JC model, and they [dash lines in Fig. 4(e)] agree well with numerical results in the strong coupling regime. Similar to the case for the cavity containing $\chi^{(3)}$ nonlinear medium, the scheme with JC model can also enhance photon blockade in the output field by several orders, and change the scaling exponent of the second-order correlation on the strength of the TLS-cavity interaction from -2 to -4.

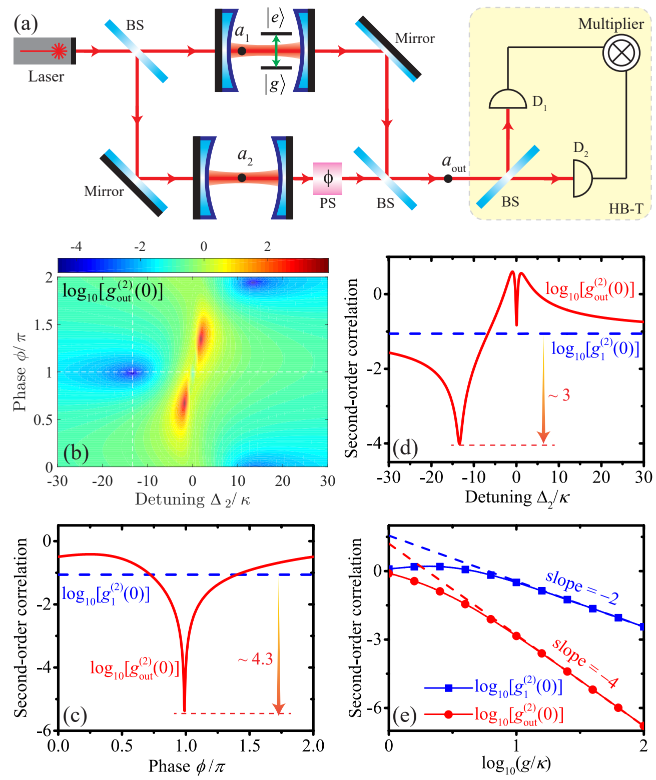

FIG. 4. (Color online) Photon blockade enhancement in the mixing output field of two cavities with a TLS in one of them. (a) Sketch of the proposed scheme with a TLS (excited state $|e\rangle$ and ground state $|g\rangle$) in cavity a_1. (b) The second-order correlation $\log_{10}[g^{(2)}_{\mathrm{out}}(0)]$ for different phase ϕ/π and detuning Δ_2/κ. The second-order correlations $\log_{10}[g^{(2)}_{\mathrm{out}}(0)]$ and $\log_{10}[g_1^{(2)}(0)]$ (c) versus phase ϕ/π with $\Delta_2 = -2g/3$ and (d) versus detuning Δ_2/κ with $\phi = \pi$. (e) The second-order correlations $\log_{10}[g^{(2)}_{\mathrm{out}}(0)]$ and $\log_{10}[g_1^{(2)}(0)]$ versus the interaction strength $\log_{10}(g/\kappa)$ with $\phi = \pi$, $\Delta_1 = -g$ and $\Delta_2 = -2g/3$. The other parameters are $g = 20\kappa$ and $\kappa_a = 2\kappa$.

Conclusions.—In conclusion, we have proposed a scheme to achieve scaling enhancement of photon blockade in the mixing field output from a nonlinear cavity (in the strong nonlinear regime) and an auxiliary (linear) cavity. We identify that the probability for two photons in the output field can be significantly inhibited by the quantum interference between two of the paths for two photons passing through the whole system, leading to a biquadratic scaling relation between the second-order correlation of the photons in the output field and intracavity nonlinear interaction strength, in contrast to a quadratic scaling relation for the photons in a nonlinear cavity. The scheme for photon blockade enhancement is not only achievable in the cavity containing $\chi^{(3)}$ nonlinearity [51] and TLS [52, 53] discussed in this Letter, but also applicable in cavities with $\chi^{(2)}$ nonlinearity [54, 55] and optomechanical interactions [56–62]. Furthermore, our scheme can be directly extended to enhance phonon blockade [63–66], magnon blockade [67–71], and polariton blockade [72–76].

This work is supported by the National Natural Science Foundation of China (NSFC) (Grants No. 12064010 and No. 12247105), the science and technology innovation Program of Hunan Province (Grant No. 2022RC1203), and Hunan provincial major sci-tech program (Grant No. 2023ZJ1010).

* xwxu@hunnu.edu.cn

[1] A. Imamoğlu, H. Schmidt, G. Woods, and M. Deutsch, Strongly interacting photons in a nonlinear cavity, Phys. Rev. Lett. **79**, 1467 (1997).
[2] K. M. Birnbaum, A. Boca, R. Miller, A. D. Boozer, T. E. Northup, and H. J. Kimble, Photon blockade in an optical cavity with one trapped atom, Nature (London) **436**, 87 (2005).
[3] B. Dayan, A. S. Parkins, T. Aoki, E. P. Ostby, K. J. Vahala, and H. J. Kimble, A Photon Turnstile Dynamically Regulated by One Atom, Science **319**, 1062 (2008).
[4] T. Aoki, A. S. Parkins, D. J. Alton, C. A. Regal, B. Dayan, E. Ostby, K. J. Vahala, and H. J. Kimble, Efficient routing of single photons by one atom and a microtoroidal cavity, Phys. Rev. Lett. **102**, 083601 (2009).
[5] A. Faraon, I. Fushman, D. Englund, N. Stoltz, P. Petroff, and J. Vučković, Coherent generation of nonclassical light on a chip via photon-induced tunneling and blockade, Nature Phys. **4**, 859 (2008).
[6] A. Reinhard, T. Volz, M. Winger, A. Badolato, K. J. Hennessy, E. L. Hu, and A. Imamoğlu, Strongly correlated photons on a chip, Nature Photon. **6**, 93 (2012).
[7] C. Lang, D. Bozyigit, C. Eichler, L. Steffen, J. M. Fink, A. A. Abdumalikov, M. Baur, S. Filipp, M. P. da Silva, A. Blais, and A. Wallraff, Observation of resonant photon blockade at microwave frequencies using correlation function measurements, Phys. Rev. Lett. **106**, 243601 (2011).
[8] A. J. Hoffman, S. J. Srinivasan, S. Schmidt, L. Spietz, J. Aumentado, H. E. Türeci, and A. A. Houck, Dispersive photon blockade in a superconducting circuit, Phys. Rev. Lett. **107**, 053602 (2011).
[9] T. C. H. Liew and V. Savona, Single photons from coupled quantum modes, Phys. Rev. Lett. **104**, 183601 (2010).
[10] M. Bamba, A. Imamoğlu, I. Carusotto, and C. Ciuti, Origin of strong photon antibunching in weakly nonlinear photonic molecules, Phys. Rev. A **83**, 021802 (2011).
[11] A. Majumdar, M. Bajcsy, A. Rundquist, and J. Vučković, Loss-enabled sub-poissonian light generation in a bimodal nanocavity, Phys. Rev. Lett. **108**, 183601 (2012).
[12] X.-W. Xu and Y.-J. Li, Antibunching photons in a cavity coupled to an optomechanical system, J. Phys. B: At. Mol. Opt. Phys. **46**, 035502 (2013).
[13] X.-W. Xu and Y. Li, Strongly correlated two-photon transport in a one-dimensional waveguide coupled to a weakly nonlinear cavity, Phys. Rev. A **90**, 033832 (2014).
[14] X.-W. Xu and Y. Li, Tunable photon statistics in weakly nonlinear photonic molecules, Phys. Rev. A **90**, 043822 (2014).
[15] Y. H. Zhou, H. Z. Shen, and X. X. Yi, Unconventional photon blockade with second-order nonlinearity, Phys. Rev. A **92**, 023838 (2015).
[16] H. Flayac and V. Savona, Unconventional photon blockade, Phys. Rev. A **96**, 053810 (2017).
[17] H. J. Snijders, J. A. Frey, J. Norman, H. Flayac, V. Savona, A. C. Gossard, J. E. Bowers, M. P. van Exter, D. Bouwmeester, and W. Löffler, Observation of the unconventional photon blockade, Phys. Rev. Lett. **121**, 043601 (2018).
[18] C. Vaneph, A. Morvan, G. Aiello, M. Féchant, M. Aprili, J. Gabelli, and J. Estève, Observation of the unconventional photon blockade in the microwave domain, Phys. Rev. Lett. **121**, 043602 (2018).
[19] A. Lingenfelter, D. Roberts, and A. A. Clerk, Unconditional fock state generation using arbitrarily weak photonic nonlinearities, Sci. Adv. **7**, eabj1916 (2021).
[20] Y.-X. Ma and P.-B. Li, Deterministic generation of phononic fock states via weak nonlinearities, Phys. Rev. A **108**, 053709 (2023).
[21] Y.-H. Zhou, X.-Y. Zhang, T. Liu, Q.-C. Wu, Z.-C. Shi, H.-Z. Shen, and C.-P. Yang, Environmentally induced photon blockade via two-photon absorption, Phys. Rev. Appl. **18**, 064009 (2022).
[22] X. Su, J.-S. Tang, and K. Xia, Nonlinear dissipation-induced photon blockade, Phys. Rev. A **106**, 063707 (2022).
[23] A. Ben-Asher, A. I. Fernández-Domínguez, and J. Feist, Non-hermitian anharmonicity induces single-photon emission, Phys. Rev. Lett. **130**, 243601 (2023).
[24] Y. Zuo, R. Huang, L.-M. Kuang, X.-W. Xu, and H. Jing, Loss-induced suppression, revival, and switch of photon blockade, Phys. Rev. A **106**, 043715 (2022).
[25] K. Stannigel, P. Komar, S. J. M. Habraken, S. D. Bennett, M. D. Lukin, P. Zoller, and P. Rabl, Optomechanical quantum information processing with photons and phonons, Phys. Rev. Lett. **109**, 013603 (2012).
[26] M. Ludwig, A. H. Safavi-Naeini, O. Painter, and F. Marquardt, Enhanced quantum nonlinearities in a two-mode optomechanical system, Phys. Rev. Lett. **109**, 063601 (2012).
[27] R. Huang, Ş. K. Özdemir, J.-Q. Liao, F. Minganti, L.-M. Kuang, F. Nori, and H. Jing, Exceptional photon blockade: Engineering photon blockade with chiral exceptional points, Laser Photon. Rev. **16**, 2100430 (2022).
[28] J. Li, R. Yu, and Y. Wu, Proposal for enhanced photon blockade in parity-time-symmetric coupled microcavities, Phys. Rev. A **92**, 053837 (2015).
[29] Y. H. Zhou, H. Z. Shen, X. Y. Zhang, and X. X. Yi, Zero eigenvalues of a photon blockade induced by a non-hermitian hamiltonian with a gain cavity, Phys. Rev. A **97**, 043819 (2018).
[30] J. Y. Sun and H. Z. Shen, Photon blockade in non-hermitian optomechanical systems with nonreciprocal couplings, Phys. Rev. A **107**, 043715 (2023).
[31] S. Ghosh and T. C. H. Liew, Dynamical blockade in a single-mode bosonic system, Phys. Rev. Lett. **123**, 013602 (2019).
[32] M. Li, Y.-L. Zhang, S.-H. Wu, C.-H. Dong, X.-B. Zou, G.-C. Guo, and C.-L. Zou, Single-mode photon blockade enhanced by bi-tone drive, Phys. Rev. Lett. **129**, 043601

(2022).
[33] Y. Wang, W. Verstraelen, B. Zhang, T. C. H. Liew, and Y. D. Chong, Giant enhancement of unconventional photon blockade in a dimer chain, Phys. Rev. Lett. **127**, 240402 (2021).
[34] J. Li, C. ming Hu, and Y. Yang, Enhancement of photon blockade via topological edge states, arXiv:2311.11431 (2023) [quant-ph].
[35] Z.-G. Lu, Y. Wu, and X.-Y. Lü, Chiral interaction induced near-perfect photon blockade, arXiv:2402.09000 (2024) [quant-ph].
[36] J.-Q. Liao and C. K. Law, Correlated two-photon transport in a one-dimensional waveguide side-coupled to a nonlinear cavity, Phys. Rev. A **82**, 053836 (2010).
[37] T. Shi, S. Fan, and C. P. Sun, Two-photon transport in a waveguide coupled to a cavity in a two-level system, Phys. Rev. A **84**, 063803 (2011).
[38] H. Flayac and V. Savona, Input-output theory of the unconventional photon blockade, Phys. Rev. A **88**, 033836 (2013).
[39] Y. J. Lu and Z. Y. Ou, Observation of nonclassical photon statistics due to quantum interference, Phys. Rev. Lett. **88**, 023601 (2001).
[40] E. Zubizarreta Casalengua, J. C. López Carreño, F. P. Laussy, and E. d. Valle, Conventional and Unconventional Photon Statistics, Laser Photonics Rev. **14**, 1900279 (2020).
[41] E. Zubizarreta Casalengua, J. C. López Carreño, F. P. Laussy, and E. del Valle, Tuning photon statistics with coherent fields, Phys. Rev. A **101**, 063824 (2020).
[42] C. W. Gardiner and M. J. Collett, Input and output in damped quantum systems: Quantum stochastic differential equations and the master equation, Phys. Rev. A **31**, 3761 (1985).
[43] H. Carmichael, *An Open Systems Approach to Quantum Optics* (Springer-Verlag Berlin Heidelberg, 1993).
[44] See Supplemental Material at http://link.aps.org/ supplemental/XXX, which includes the analytical calculations on the second-order correlation fucntions in the mixing fields output from two (uncoupled) cavities with: (§1) $\chi^{(3)}$ nonlinearity; (§2) TLS-cavity interaction, in one of them.
[45] M. B. Plenio and P. L. Knight, The quantum-jump approach to dissipative dynamics in quantum optics, Rev. Mod. Phys. **70**, 101 (1998).
[46] E. Jaynes and F. Cummings, Comparison of quantum and semiclassical radiation theories with application to the beam maser, Proc. IEEE **51**, 89 (1963).
[47] R. J. Thompson, G. Rempe, and H. J. Kimble, Observation of normal-mode splitting for an atom in an optical cavity, Phys. Rev. Lett. **68**, 1132 (1992).
[48] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics, Nature (London) **431**, 162 (2004).
[49] J. P. Reithmaier, G. Sek, A. Löffler, C. Hofmann, S. Kuhn, S. Reitzenstein, L. V. Keldysh, V. D. Kulakovskii, T. L. Reinecke, and A. Forchel, Strong coupling in a single quantum dot-semiconductor microcavity system, Nature (London) **432**, 197 (2004).
[50] T. Yoshie, A. Scherer, J. Hendrickson, G. Khitrova, H. M. Gibbs, G. Rupper, C. Ell, O. B. Shchekin, and D. G. Deppe, Vacuum rabi splitting with a single quantum dot in a photonic crystal nanocavity, Nature (London) **432**, 200 (2004).
[51] J.-h. Chen, X. Shen, S.-J. Tang, Q.-T. Cao, Q. Gong, and Y.-F. Xiao, Microcavity nonlinear optics with an organically functionalized surface, Phys. Rev. Lett. **123**, 173902 (2019).
[52] H. Walther, B. T. H. Varcoe, B.-G. Englert, and T. Becker, Cavity quantum electrodynamics, Rep. Prog. Phys. **69**, 1325 (2006).
[53] X. Gu, A. F. Kockum, A. Miranowicz, Y.-X. Liu, and F. Nori, Microwave photonics with superconducting quantum circuits, Phys. Rep. **718-719**, 1 (2017).
[54] X. Zhang, Q.-T. Cao, Z. Wang, Y.-x. Liu, C.-W. Qiu, L. Yang, Q. Gong, and Y.-F. Xiao, Symmetry-breaking-induced nonlinear optics at a microcavity surface, Nature Photon. **13**, 21 (2019).
[55] J. Lu, M. Li, C.-L. Zou, A. A. Sayem, and H. X. Tang, Toward 1% single-photon anharmonicity with periodically poled lithium niobate microring resonators, Optica **7**, 1654 (2020).
[56] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, Rev. Mod. Phys. **86**, 1391 (2014).
[57] P. Rabl, Photon blockade effect in optomechanical systems, Phys. Rev. Lett. **107**, 063601 (2011).
[58] A. Nunnenkamp, K. Børkje, and S. M. Girvin, Single-photon optomechanics, Phys. Rev. Lett. **107**, 063602 (2011).
[59] J.-Q. Liao and F. Nori, Photon blockade in quadratically coupled optomechanical systems, Phys. Rev. A **88**, 023853 (2013).
[60] A. Kronwald, M. Ludwig, and F. Marquardt, Full photon statistics of a light beam transmitted through an optomechanical system, Phys. Rev. A **87**, 013847 (2013).
[61] X.-W. Xu, Y.-J. Li, and Y.-x. Liu, Photon-induced tunneling in optomechanical systems, Phys. Rev. A **87**, 025803 (2013).
[62] S. R. Das, S. Majumder, S. K. Sahu, U. Singhal, T. Bera, and V. Singh, Instabilities near ultrastrong coupling in a microwave optomechanical cavity, Phys. Rev. Lett. **131**, 067001 (2023).
[63] Y.-x. Liu, A. Miranowicz, Y. B. Gao, J. c. v. Bajer, C. P. Sun, and F. Nori, Qubit-induced phonon blockade as a signature of quantum behavior in nanomechanical resonators, Phys. Rev. A **82**, 032101 (2010).
[64] X.-W. Xu, A.-X. Chen, and Y.-x. Liu, Phonon blockade in a nanomechanical resonator resonantly coupled to a qubit, Phys. Rev. A **94**, 063853 (2016).
[65] S. Debnath, N. M. Linke, S.-T. Wang, C. Figgatt, K. A. Landsman, L.-M. Duan, and C. Monroe, Observation of hopping and blockade of bosons in a trapped ion spin chain, Phys. Rev. Lett. **120**, 073001 (2018).
[66] Y. S. S. Patil, J. Yu, S. Frazier, Y. Wang, K. Johnson, J. Fox, J. Reichel, and J. G. E. Harris, Measuring high-order phonon correlations in an optomechanical resonator, Phys. Rev. Lett. **128**, 183601 (2022).
[67] X. Zhang, C. L. Zou, L. Jiang, and H. X. Tang, Cavity magnomechanics, Sci. Adv. **2**, e1501286 (2016).
[68] Z.-X. Liu, H. Xiong, and Y. Wu, Magnon blockade in a hybrid ferromagnet-superconductor quantum system, Phys. Rev. B **100**, 134421 (2019).
[69] H. Y. Yuan and R. A. Duine, Magnon antibunching in a nanomagnet, Phys. Rev. B **102**, 100402 (2020).
[70] J.-k. Xie, S.-l. Ma, and F.-l. Li, Quantum-interference-

enhanced magnon blockade in an yttrium-iron-garnet sphere coupled to superconducting circuits, Phys. Rev. A **101**, 042331 (2020).

[71] Z.-y. Jin and J. Jing, Magnon blockade in magnon-qubit systems, Phys. Rev. A **108**, 053702 (2023).

[72] A. Verger, C. Ciuti, and I. Carusotto, Polariton quantum blockade in a photonic dot, Phys. Rev. B **73**, 193306 (2006).

[73] S.-N. Huai, Y.-L. Liu, Y. Zhang, and Y.-x. Liu, Mechanically modulated emission spectra and blockade of polaritons in a hybrid semiconductor-optomechanical system, Phys. Rev. A **98**, 033825 (2018).

[74] E. V. Denning, M. Wubs, N. Stenger, J. Mørk, and P. T. Kristensen, Cavity-induced exciton localization and polariton blockade in two-dimensional semiconductors coupled to an electromagnetic resonator, Phys. Rev. Res. **4**, L012020 (2022).

[75] G. Munoz-Matutano, A. Wood, M. Johnsson, X. Vidal, B. Q. Baragiola, A. Reinhard, A. Lemaître, J. Bloch, A. Amo, G. Nogues, *et al.*, Emergence of quantum correlations from interacting fibre-cavity polaritons, Nature Mater. **18**, 213 (2019).

[76] A. Delteil, T. Fink, A. Schade, S. Höfling, C. Schneider, and A. İmamoğlu, Towards polariton blockade of confined exciton–polaritons, Nature Mater. **18**, 219 (2019).