Granular gases under resetting

Anna S. Bodrova
Moscow Institute of Electronics and Mathematics, HSE University, 123458, Moscow, Russia
(Dated: March 29, 2024)

We investigate the granular temperatures in force-free granular gases under exponential resetting. When a resetting event occurs, the granular temperature attains its initial value, while between the resetting events it cools down. We show that the granular system attains a non-equilibrium steady state value and study the dependence of average granular temperature on the resetting rate. Our theory may help to explain the behavior of non-periodically driven granular systems.

I. INTRODUCTION

A process with resetting breaks at a certain point and starts anew. Resetting has been widely studied recently and has numerous applications [1, 2]. It significantly accelerates search processes [3–5] and improves the efficiency of computer algorithms [6, 7]. The resetting process can be observed in various fields. In biology, resetting occurs in enzyme-catalyzed reactions, described in terms of Michaelis-Menten kinetics [8–10], transcription [11], and mobility of animals [12]. In economic society models, resetting may represent loss of wealth due to catastrophic events [13]. Geometric Brownian motion with stochastic resetting may be used to describe income dynamics [14, 15]. Resetting may represent eye movements while viewing and recognizing patterns [16, 17].

In the first study, resetting was considered for particles that exhibited Brownian motion [18, 19]. Subsequently, many other processes with resetting have been investigated, such as the Ornstein-Uhlenbeck process [20, 21], continuous-time random walks [22–27], Lévy flights [28, 29], Lévy walks [30], heterogeneous diffusion processes [31, 32], fractional Brownian motion [32], geometric Brownian motion [33], scaled Brownian motion [34, 35], and resetting on networks [36]. Initially, the resetting process was considered as a jump to the starting point. Later, other types of return processes were considered, such as partial resetting [37], return at constant velocity [5, 38–43], and under the action of external potential [44–46]. The latter phenomenon has been observed in experiments [47–49].

In the current study, we investigate resetting in granular systems. Examples of granular systems are numerous: stones and sand in the building industry; grains, sugar, salt, and cereals in the food industry; and powders in cosmetic production, different kinds of powders in chemical and cosmetic production [50–53]. Granular gases represent diluted granular systems [53–55], in which the typical distance between their components significantly exceeds their dimensions. Numerous examples include dust devils, large interstellar dust clouds [56], protoplanetary discs, planetary rings [57–59], and populations of asteroids [60]. In the homogeneous cooling state, the granular gases remain force-free and lose their kinetic energy during collisions [54]. To compensate for the dissipation, energy should be inserted into the system. There are

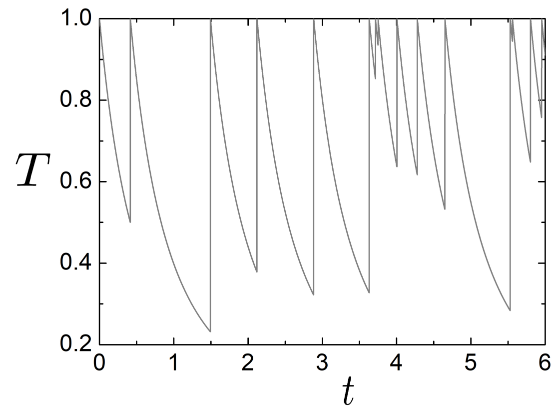

FIG. 1. A typical evolution of granular temperature $T(t)$ in granular systems with exponential resetting. The resetting rate $r = 2$, the characteristic time of the granular temperature decay $\tau_0 = 1$.

different possibilities for providing the energy supply to the granular system: vibrating [61, 62] or rotating [63] walls, external electrostatic [64] or magnetic forces [65–68]. Driving of granular gases is often described in theory and computer simulations in terms of the uniform heating [69–72]. However, energy can be inserted into the system also through the boundaries [73, 74] or as rare extreme events [75]. Between these events, the system can evolve force-free by itself. This phenomenon can be described within the framework of the resetting process. We consider Poissonian resetting, meaning that the resetting event may occur with an equal probability at any given time. During the instantaneous resetting event, the granular temperature attains its initial value, T_0. Between resetting events, the system cools homogeneously according to Haff's law [76]. We proceed as follows. In Section II, we briefly review the evolution of force-free granular gases. In Section III, we discuss the Poissonian resetting of granular temperatures. Finally, we present our conclusions in Sec. IV.

II. HAFF'S LAW

The granular temperature is one of the most important parameters in the description of granular gases. This is defined in terms of the mean kinetic energy [54]:

$$\frac{3}{2}nT(t) = \frac{m\langle v^2\rangle}{2} = \int d\mathbf{v} f(\mathbf{v},t)\,\frac{mv^2}{2} \tag{1}$$

Here, m is the mass of the granular particle, and $f(\mathbf{v},t)$ is the velocity distribution function, which is assumed here for simplicity to be Maxwellian, despite the slight deviations from the Maxwellian form obtained in both theory [77–83] and experiments [84]. Owing to the dissipative collisions, the granular temperature in the granular gas gradually decreases. The evolution of granular temperature occurs according to the following differential equation:

$$\frac{dT(t)}{dt} = -T(t)\xi(t) \tag{2}$$

Here the cooling rate is equal to

$$\xi(t) = \frac{4}{3}\left(1-\varepsilon^2\right) n\sigma^2\sqrt{\frac{\pi T}{m}} \tag{3}$$

The restitution coefficient ε quantifies the dissipative losses during the collision of granular particles as follows [53, 54]:

$$\varepsilon = \left|\frac{(\mathbf{v}'_{ki}\cdot\mathbf{e})}{(\mathbf{v}_{ki}\cdot\mathbf{e})}\right|\,. \tag{4}$$

Here, $\mathbf{v}_{ki} = \mathbf{v}_k - \mathbf{v}_i$ and $\mathbf{v}'_{ki} = \mathbf{v}'_k - \mathbf{v}'_i$ are the relative velocities before and after a collision, respectively, and $\mathbf{e}$ is a unit vector directed along the inter-center vector at the collision instant. For simplicity the restitution coefficient is assumed to be constant [54], although it depends on the relative velocity of the colliding particles [85, 86]. The post-collision velocities $\mathbf{v}'_k$ and $\mathbf{v}'_i$ are related to the pre-collision velocities $\mathbf{v}_k$ and $\mathbf{v}_i$ as follows [54]:

$$\mathbf{v}'_{k/i} = \mathbf{v}_{k/i} \mp \frac{1+\varepsilon}{2}(\mathbf{v}_{ki}\cdot\mathbf{e})\mathbf{e}\,. \tag{5}$$

The differential equation (Eq. 2) with the cooling rate (Eq. 3) can be solved explicitly, and the temperature obeys Haff's law [76]:

$$T(t) = T_0\left(1+\frac{t}{\tau_0}\right)^{-2}, \tag{6}$$

where $T_0 = T(0)$ is the initial granular temperature. The inverse temperature relaxation time is equal to half of the cooling rate at the initial time, $\tau_0^{-1} = \xi(0)/2$.

III. RESULTS AND DISCUSSION

Now let us assume that the Poissonian resetting occurs with the waiting time distribution

$$\psi(t) = re^{-rt} \tag{7}$$

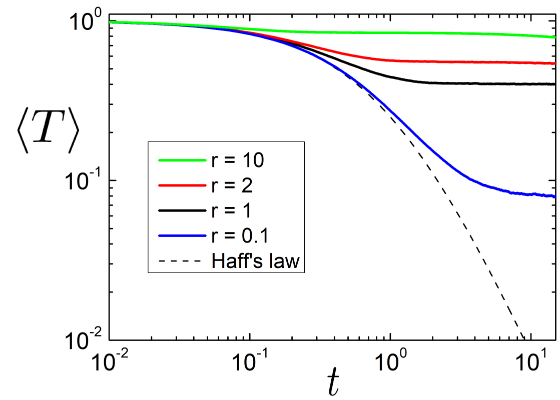

FIG. 2. Time evolution of the granular temperature $T(t)$ averaged over $N = 10^5$ systems. The characteristic time of the granular temperature decay $\tau_0 = 1$. Dashed line shows the evolution of a granular temperature of a force-free granular system without resetting according to the Haff's law.

Here r is the constant rate of the resetting events, that is, with probability rdt resetting event occurs during the time interval $(t, t+dt)$. Thus, the average time between resetting events is equal to $1/r$. The survival probability $\Psi(t)$ is defined as the probability that no resetting event occurs between zero and t,

$$\Psi(t) = 1 - \int_0^t \psi(t')dt' = e^{-rt}\,. \tag{8}$$

We now perform simulations of a granular system with resetting. Initially, the evolution of granular temperature occurs according to Haff's law (Eq. 6). If the resetting event occurs at a constant rate r, the granular temperature takes the initial value $T_0 = 1$. Subsequently, it starts to decrease again. A typical evolution of such a system is shown in Fig. 1. After calculating the average over $N \gg 1$ systems with such an evolution, we obtain the average granular temperature $\langle T(t)\rangle$, as depicted in Fig. 2. Now, let us derive it analytically:

$$\langle T(t)\rangle = T_0\left(1+\frac{t}{\tau_0}\right)^{-2} e^{-rt} + r\int_0^t T_0\left(1+\frac{\tau}{\tau_0}\right)^{-2} e^{-r\tau}d\tau \tag{9}$$

Here, the first term accounts for the realizations where no resetting occurs up to observation time t. The second term accounts for the case where the resetting event occurs at time $t-\tau$, and after that no resetting occurs between $t-\tau$ and t. The evolution of the granular temperature according to Eq. (9) is in good agreement with the simulation results shown in Fig. 2. At the beginning of the evolution, it decreases according to Haff's law; at time $t \sim 1/r$ it reaches its equilibrium value T_{eq}. If $\tau_0 = 1/r$, the characteristic mean time between the

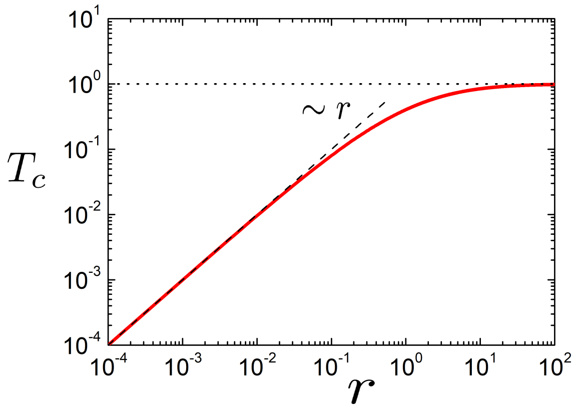

FIG. 3. Dependence of the steady-state value of granular temperature T_c on the resetting rate r, Eq. (10). The dashed line shows linear dependence at short times (Eq. 11), $T_c \sim r$. At large r the steasy-state granular temperature tends to the initial value $T_0 = 1$, depicted with a dotted line

resetting events and the temperature relaxation time are equal, and equilibrium is reached at $t \simeq 1$. At $r \to 0$ the mean time between the resetting events tends to infinity and the granular temperature evolves freely according to Haff's law (Eq. 6). If the resetting events are very frequent, $r \gg \tau_0$, the temperature remains constant, close to its initial value, $T \simeq T_0$. At long times $t \gg 1/r$ the probability that no resetting events have occurred tends to zero, and the first term in Eq. (9) can be neglected.

Apparently the system reaches a non-equilibrium steady state, and the average granular temperature reaches a constant value T_c. It can be formally derived by setting $t \to \infty$ in Eq. (9)

$$\begin{aligned} T_c &= T_0\, r \int\limits_0^\infty \left(1 + \frac{\tau}{\tau_0}\right)^{-2} e^{-r\tau} d\tau = \\ &= rT_0 \left(1 - r\Gamma\left(0, r\right) e^r\right) \end{aligned} \tag{10}$$

Here, $\Gamma(0, r)$ is the incomplete gamma function. At $r \to 0$ the equilibrium average granular temperature scales as

$$T_c = T_0 \left(r + (\gamma + \log r)\, r^2 + o(r^2)\right) \tag{11}$$

with γ being the Euler's constant. At $r \to \infty$ Eq. (10) becomes

$$T_c = T_0 \left(1 - \frac{2}{r} + \frac{6}{r^2} + o\left(\frac{1}{r^2}\right)\right) \tag{12}$$

The dependence of the equilibrium granular temperature T_c on the resetting rate r is shown in Fig. 3. At low resetting rates, it increases linearly according to Eq. (11), and at high resetting rates, it remains close to the initial value $T_0 = 1$. In the latter case the temperature does not have sufficient time to cool between subsequent resetting events.

IV. CONCLUSIONS

Granular systems with resets were investigated. We assumed that during resetting, the temperature was instantly reset to the initial value. This may occur when the container with the granular material is shaken from time to time, a magnetic force is applied, or if energy is instantly inserted into a granular system in a different way. The granular system tends to be in a non-equilibrium steady state with an average constant value of the granular temperature. This constant value decreases with increasing of the average interval between resetting events. Our results may be helpful for understanding non-periodically driven granular systems.

V. ACKNOWLEDGEMENT

A.S.B. thanks Erez Aghion for fruitful discussions.

[1] M. R Evans, S. N. Majumdar, and G. Schehr, *J. Phys. A* **53**, 193001 (2020).
[2] S. Gupta and A. M. Jayannavar, *Front. Phys.* **10**, 789097 (2022).
[3] A.V. Chechkin, I.M. Sokolov, *Phys. Rev. Lett.* **121**, 050601 (2018).
[4] A. Pal and S. Reuveni, *Phys. Rev. Lett.* **118**, 030603 (2017).
[5] A. Pal, Ł. Kuśmierz, S. Reuveni, *Phys. Rev. Research* **2**, 043174 (2020).
[6] A. Montanari and R. Zecchina, *Phys. Rev. Lett.* **88**, 178701 (2002).
[7] J. H. Lorenz, *PLoS One*, **11** 043202 (2016).
[8] S. Reuveni, M. Urbach, and J. Klafter, *Proc. Natl. Akad. Sci. U.S.A.* **111**, 4391 (2014).
[9] T. Rotbart, S. Reuveni, M. Urbakh, *Phys. Rev. E* **92**, 060101(R) (2015).
[10] T. Robin, S. Reuveni, M. Urbakh, *Nat. Commun.* **9**, 779 (2018).
[11] É. Roldán, A. Lisica, D. Sánchez-Taltavull, and S.W. Grill, *Phys. Rev. E* **93**, 062411 (2016).
[12] D. Boyer, C. Solis-Salas, *Phys. Rev. Lett.* **112**, 240601 (2014).
[13] I. Santra, *Europhys. Lett.* **137**, 52001 (2022).
[14] P. Jolakoski, A. Pal, T. Sandev, L. Kocarev, R. Metzler, V. Stojkoski, *Chaos, Solitons & Fractals* **175**, 113921 (2023).
[15] V. Stojkoski, P. Jolakoski, A. Pal, T. Sandev, L. Kocarev, and R. Metzler, *Phil. Trans. R. Soc. A* **380** 20210157 (2022).

[16] D. Noton, L. Stark, *Vis Res* **11** 929 (1971).
[17] M.P. Eckstein, *J. Vis.* **11** 14 (2011).
[18] M. R. Evans and S. N. Majumdar, *Phys. Rev. Lett.* **106**, 160601 (2011).
[19] S. N. Majumdar, S. Sabhapandit, G. Schehr, *Phys. Rev. E* **91**, 052131 (2015).
[20] A. Pal, *Phys. Rev. E* **91**, 012113 (2015).
[21] P. Trajanovski, P. Jolakoski, K. Zelenkovski, A. Iomin, L. Kocarev, and T. Sandev, *Phys. Rev. E* **107**, 054129 (2023)
[22] A.S. Bodrova, I.M. Sokolov, *Phys. Rev. E* **101**, 062117 (2020).
[23] M. Montero and J. Villarroel, *Phys. Rev. E* **87**, 012116 (2013).
[24] V. Méndez and D. Campos, *Phys. Rev. E* **93**, 022106 (2016).
[25] V.P. Shkilev, *Phys. Rev. E* **96**, 012126 (2017).
[26] M. Montero, A. Mas-Puigdellósas, J. Villarroel. *Eur. Phys. J. B* **90**, 176 (2017).
[27] Ł. Kuśmierz, E. Gudowska-Nowak, *Phys. Rev. E* **99**, 052116 (2019).
[28] L. Kuśmierz, S. N. Majumdar, S. Sabhapandit, and G. Schehr, *Phys. Rev. Lett.* **113**, 220602 (2014).
[29] L. Kuśmierz, E. Gudowska-Nowak, *Phys. Rev. E* **92**, 052127 (2015).
[30] T. Zhou, P. Xu, W. Deng, *Phys. Rev. Research* **2**, 013103.
[31] T. Sandev, V. Domazetoski, L. Kocarev, R. Metzler, and A. Chechkin, *J. Phys. A: Math. Theor.* **55** 074003 (2022)
[32] W. Wang, A.G. Cherstvy, H. Kantz, R. Metzler, and I.M. Sokolov, *Phys. Rev. E* **104** 024105 (2021)
[33] D. Vinod, A.G. Cherstvy, W. Wang, R. Metzler, and I.M. Sokolov, *Phys. Rev. E* **105** L012106 (2022)
[34] A.S. Bodrova, A.V. Chechkin, I.M. Sokolov, *Phys. Rev. E* **100**, 012120 (2019).
[35] A.S. Bodrova, A.V. Chechkin, I.M. Sokolov, *Phys. Rev. E* **100**, 012119 (2019).
[36] A. P. Riascos, D. Boyer, P. Herringer, and J. L. Mateos, *Phys. Rev. E* **101** 062147 (2020)
[37] M. Dahlenburg, A. V. Chechkin, R. Schumer, and R. Metzler, *Phys. Rev. E* **103** 052123 (2021)
[38] A.S. Bodrova, I.M. Sokolov, *Phys. Rev. E* **101**, 052130 (2020).
[39] A.S. Bodrova, I.M. Sokolov, *Phys. Rev. E* **102**, 032129 (2020).
[40] A. Pal, Ł. Kuśmierz, S. Reuveni, *New J. Phys.* **21**, 113024 (2019).
[41] A. Pal, Ł. Kuśmierz, S. Reuveni, *Phys. Rev. E* **100**, 040101(R) (2019).
[42] A. Masó-Puigdellosas, D. Campos, and V. Mendez, *Phys. Rev. E* 100, 042104 (2019).
[43] M. Radice, *J. Phys. A: Math. Theor.* **55**, 224002 (2022).
[44] D. Gupta, C.A. Plata, A. Kundu, and A. Pal, *J. Phys. A: Math. Theor.* **54**, 025003 (2020).
[45] D. Gupta, C.A. Plata, A. Pal, and A. Kundu, *J. Stat. Mech.*, 043202 (2021).
[46] I. Santra, S. Das and S. K. Nath, *J. Phys. A: Math. Theor.* **54**, 334001 (2021).
[47] O. Tal-Friedman, A. Pal, A. Sekhon, S. Reuveni, and Y. Roichman, *J. Phys. Chem. Lett.* **11**, 7350 (2020).
[48] B. Besga, A. Bovon, A. Petrosyan, S. N. Majumdar, and S. Ciliberto, *Phys. Rev. Res.* **2**, 032029(R) (2020).
[49] F. Faisant, B. Besga, A. Petrosyan, S. Ciliberto, and S. N. Majumdar, *J. Stat. Mech.* **2021**, 113203 (2021).
[50] H. Hinrichsen, D. E. Wolf *The Physics of Granular Media.* (Berlin: Wiley, 2004).
[51] J. Duran, *Sands, Powders and Grains.* Berlin: Springer-Verlag, (2000).
[52] H. J. Herrmann, J.-P. Hovi and S. Luding, *Physics of Dry Granular Media.* Dordrecht: NATO ASI Series, Kluwer (1998).
[53] H. Jaeger, S. Nagel, & R. Behringer, *Rev. Mod. Phys.* **68**, 1259 (1996).
[54] N. V. Brilliantov, T. Pöschel, *Kinetic theory of Granular Gases.* Oxford: Oxford University Press (2004).
[55] A. Mehta, in *Granular Physics*, Cambridge University Press (2011);
[56] G. Winnewiser, G.C. Pelz *The physics and chemistry of interstellar molecular clouds.* Proceedings of the 2nd Cologne-Zermatt Symposium Held at Zermatt, Switzerland, Springer (1993).
[57] R. Greenberg, A. Brahic, *Planetary rings.* University of Arizona Press, Tucson (1984).
[58] F. G. Bridges, A. Hatzes and D. N. C. Lin, *Nature* **309**, 333 (1984).
[59] N.V. Brilliantov, P. L. Krapivsky, A. Bodrova, F. Spahn, H. Hayakawa, V. Stadnichuk, J. Schmidt, *Proc. Natl. Acad. Sci. USA* **112**, 9536 (2015).
[60] D. Hestroffer, P. Sánchez, L. Staron et al. *Astron. Astrophys. Rev.* **27**, 6 (2019).
[61] R. D. Wildman and D. J. Parker, *Phys. Rev. Lett.*, **88**, 064301 (2002).
[62] A. Prevost, D. A. Egolf and J. S. Urbach, *Phys. Rev. Lett.*, **89**, 084301 (2002).
[63] O. Zik, D. Levine, S.G. Lipson, S. Shtrikman and J. Stavans, *Phys. Rev. Lett.*, **73**, 644 (1994).
[64] I. S. Aranson and J. S. Olafsen, *Phys. Rev. E*, **66**, 061302 (2002).
[65] A. Snezhko, I. S. Aranson and W.-K. Kwok, *Phys. Rev. Lett.*, **94**, 108002 (2005).
[66] C. C. Maass, N. Isert, G. Maret and C. M. Aegerter, *Phys. Rev. Lett.*, **100**, 248001 (2008).
[67] M. Adachi, P. Yu, and M. Sperl, *npj Microgravity* **5**, 19 (2019).
[68] M. Adachi, M. Balter, X. Cheng, et al. *Microgravity Sci. Technol.* **33**, 11 (2021).
[69] J. M. Montanero and A. Santos, *Granular Matter* **2**, 53 (2000).
[70] R. Cafiero, S. Luding, *Phys. A* **280**, 142 (2000).
[71] A. Megías, A. Santos, *Granular Matter* **21**, 49 (2019).
[72] V.V. Prasad, R. Rajesh, *J. Stat. Phys.* **176**, 1409 (2019).
[73] J. S. van Zon, and F. C. MacKintosh, *Phys. Rev. Lett.* **93**, 038001 (2004).
[74] J. S. van Zon, and F. C. MacKintosh, *Phys. Rev. E* **72**, 051301 (2005).
[75] W. Kang, J. Machta1 and E. Ben-Naim, *Europhys. Lett.* **91**, 34002 (2010).
[76] P. K. Haff, *J. Fluid Mech.*, **134**, 401 (1983).
[77] A. Goldshtein and M. Shapiro, *J. Fluid Mech.* **282**, 75 (1995).
[78] T. P. C. van Noije and M. H. Ernst, *Gran. Mat.* **1**, 57 (1998).
[79] M. Huthmann, J. A. Orza, and R. Brito, *Gran. Mat.* **2**, 189 (2000).
[80] N. V. Brilliantov and T. Pöschel, *Europhys. Lett.* **74**, 424 (2006).
[81] A.S. Bodrova, N.V. Brilliantov, *Physica A* **388**, 3315 (2009).

[82] A.S. Bodrova, N.V. Brilliantov, *Mosc. Univ. Phys. Bull.* **64**, 128 (2009).
[83] A.K. Dubey, A.S. Bodrova, S. Puri, N.V. Brilliantov, *Phys. Rev. E* **87**, 062202 (2013).
[84] P. Yu, M. Schröter, M. Sperl, *Phys. Rev. Lett.*, **124** (20), 208007 (2020).
[85] R. Ramirez, T. Pöschel, N.V. Brilliantov, T. Schwager, *Phys. Rev. E* **60**, 4465 (1999).
[86] T. Schwager, T. Pöschel, *Phys. Rev. E* **78**, 051304 (2008).