Comment on *"Do near-threshold molecular states mix with neighboring $\bar{Q}Q$ states?"*

George Rupp[1]
[1]*Centro de Física Teórica de Partículas, Instituto Superior Técnico, Universidade de Lisboa, P-1049-001 Lisboa, Portugal*

I comment on a paper by Christoph Hanhart and Alexey Nefediev, published in Phys. Rev. D **106**, 114003 (2022). The authors discuss the interpretation of mesons close to their lowest decay threshold and present a mechanism for the formation of molecular states. The proposed formalism is then applied to the axial-vector mesons $D_{s1}(2536)$ and $D_{s1}(2460)$, presenting two scenarios for the lighter meson, namely a $D^\star K$ molecule or a compact $c\bar{s}$ state. The authors argue that the latter hypothesis requires a fine-tuning of the mixing angle between the $J^{PC}=1^{++}$ and $J^{PC}=1^{+-}$ C-parity eigenstates.

In this Comment I show that no such fine-tuning is needed, as demonstrated in an article published in Phys. Rev. D **84**, 094020 (2011), where a unitarized quark model was applied to the two C-parity eigenstates, coupled to several two-meson channels including $D^\star K$. The coupled-channel dynamics naturally leads to a mixing angle very close to the required one. Moreover, I argue that the $D_1(2420)$ and $D_1(2430)$ axial-vectors, not considered by the authors, as well as a lattice simulation in Phys. Rev. D **90**, 034510 (2014), also not mentioned by the authors, do not lend support to a molecular interpretation of $D_{s1}(2460)$. I conclude with some more general remarks about mesons coupling to S-wave thresholds.

In Ref. [1], hereafter referred to as HN22, the authors study the axial-vector (AV) charm-strange mesons $D_{s1}(2536)$ and $D_{s1}(2460)$ in the framework of the Weinberg approach to determining the nature of hadronic states. In particular, they employ this method by formulating a simple two-channel description of these and similar mesons, in which a compact QCD-based system with an unspecified quark content is coupled to one two-meson channel in an S-wave. From their analysis, the authors conclude that there are two possible scenarios for the mentioned charm-strange mesons in terms of their internal structure. In the first, so-called "strong-coupling" scenario, the $D_{s1}(2460)$ is claimed to emerge as a $D^\star K$ molecule, which largely decouples from the nearby $J^P=1^+$ quark-antiquark state, whereas the $D_{s1}(2536)$ must then be the (dominantly) $c\bar{s}$ meson. In the second, "weak-coupling" scenario, both mesons will be dominantly of the $c\bar{s}$ type, resulting from a mixture of the spectroscopic states ${}^3P_1$ ($J^{PC}=1^{++}$) and ${}^1P_1$ ($J^{PC}=1^{+-}$), as physical charm-strange mesons have no definite C-parity. The most important point of the present Comment is to rebut the claim in HN22 that the latter scenario requires a fine-tuning of the mixing angle between the ${}^3P_1$ and ${}^1P_1$ states, based on a unitary coupled-channel model calculation published in Ref. [2], to be referred to as CRB11. Further arguments against a molecular interpretation of the $D_{s1}(2460)$ will be presented as well.

In CRB11, both axial-vector charm-strange ($c\bar{s}$) and charm-light ($c\bar{q}$) mesons were studied in the context of a fully unitary and analytic multichannel model, with quark-antiquark as well as meson-meson channels. In either case, the two spectroscopic quark-antiquark eigenstates ${}^3P_1$ and ${}^1P_1$ are coupled to all OZI-allowed vector-pseudoscalar and vector-vector channels. Of these, only the $D^\star K$ and $D^\star\pi$ channels are kinematically open for the corresponding bare $1^{+\pm}$ $c\bar{s}$ and $c\bar{q}$ states, respectively, However, all other channels also contribute to the real part of the physical masses through their loops. Owing to the difference in couplings of the ${}^3P_1$ and ${}^1P_1$ components to the two-meson channels, their assumed degeneracy in the CRB11 model is lifted upon allowing interaction with the two-meson channels. In both cases, the S-matrix pole of the resulting mixture of ${}^3P_1$ and ${}^1P_1$ states that is dominantly ${}^3P_1$ shifts much more than the orthogonal combination. Focusing for the moment on the $c\bar{s}$ system, we observe an extremely small shift of the mostly ${}^1P_1$ pole, giving rise to a $D^\star K$ decay width of the order of 1 MeV, as well as a real part of the energy close to the mass of the $D_{s1}(2536)$ meson. On the other hand, the dominantly ${}^3P_1$ pole shifts downward by almost 100 MeV and so below the $D^\star K$ threshold, with a resulting mass close to that of the $D_{s1}(2460)$ and zero width in the OZI-only approximation. The two pole trajectories as a function of the overall model coupling λ are shown in Fig. 1, reprinted from CRB11. The conclusion is that the coupled-channel dynamics automatically leads to a mixing angle very close to the ideal one, i.e., $\cos\Theta_{\mathrm{AV}}\approx\sqrt{2/3}\Rightarrow\Theta_{\mathrm{AV}}\approx 35.3°$. (Note that in the conventions of HN22 the ideal mixing angle is given by 90° minus the angle defined in CRB11.) So absolutely no fine-tuning of this angle is needed, as already concluded more generally in Ref. [3] by diagonalizing a Hamiltonian for two degenerate bare quark-antiquark states coupled to the continuum. Also the model of Ref. [4], based on a chiral quark-pion Lagrangian, supports the interpretation of the $D_{s1}(2536)$ and $D_{s1}(2460)$ as in CRB11 and Ref. [3], albeit without generating the corresponding mixing angle dynamically.

Turning now our attention to the $c\bar{q}$ system and the PDG [5] mesons $D_1(2420)$ and $D_1(2430)$, not considered in HN22, the coupled-channel dynamics employed in CRB11 works equally well, leading to a $D_1(2420)$ state much narrower than one would expect for a meson decay-

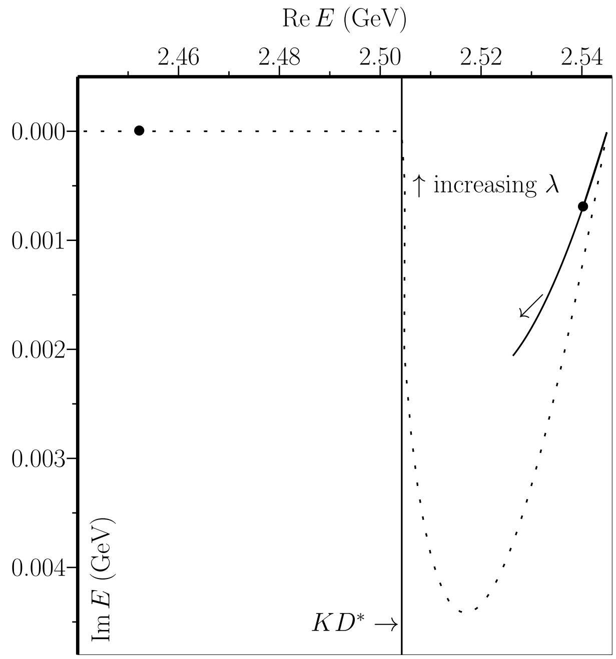

FIG. 1. S-matrix pole trajectories of the $D_{s1}(2536)$ (solid line) and $D_{s1}(2460)$ (dotted line) as a function of the overall coupling λ. Dots represent the physical pole positions for the same value of λ. Figure reprinted from FIG. 3 of Ref. [2].

ing to $D^\star\pi$ in an S-wave and a lot of phase space, as well as a very broad $D_1(2430)$ resonance. Note that in the latter case the pole does not move below the — much lower lying — $D^\star\pi$ threshold, which can be understood as a consequence of an effective Adler zero in this channel. As for the broad $D_1(2430)$ resonance, it would be very difficult to understand as a meson-meson molecule in the philosophy of HN22, which immediately raises the question why the $c\bar{s}$ and $c\bar{q}$ systems should be dealt with differently in that approach. In contrast, the formalism in CRB11 allows to understand the complete pattern of $D_{s1}(2460)$, $D_{s1}(2536)$, $D_1(2430)$, and $D_1(2420)$ masses and widths — 8 observables in total — with only two adjustable parameters, one of which already strongly constrained by prior model applications. Moreover, in neither case is a fine-tuning of the ${}^3P_1$-${}^1P_1$mixing angle necessary.

Let us now see what insight on AV charm mesons is provided by the lattice. In Ref. [6], $J^P = 0^+$, 1^+, and 2^+ charm-strange mesons were studied by including both quark-antiquark and two-meson interpolating fields in the simulations. In the AV case, a $c\bar{s}$ state appears below the $D^\star K$ threshold even in the single-hadron approach, that is, without including $D^\star K$ interpolators. However, the inclusion of $D^\star K$ scattering operators significantly improves the signal and a clear identification of the state with the $D_{s1}(2460)$ can be made. Furthermore, a narrow $D_{s1}(2536)$ is found in the same simulation. Therefore, interpreting the $D_{s1}(2460)$ as a molecular $D^\star K$ state largely decoupled from $c\bar{s}$ is not supported by this lattice calculation. As for the $D_1(2420)$ and $D_1(2430)$ mesons, members of the same lattice collaboration [7] carried out a similar simulation as in Ref. [6], obtaining results chiefly in agreement with the earlier findings in CRB11, just like in the $c\bar{s}$ case [6]. Furthermore, a recent lattice computation [8] by members of the Hadron Spectrum Collaboration generally confirmed the results on the $D_1(2420)$ and $D_1(2430)$ found in Ref. [7]. Also note the remark in a very recent lattice talk [9] about these two mesons:

> *"A stable spectrum requires both $q\bar{q}$- and meson-meson-like operators!"*

Some further observations are due concerning a couple of other enigmatic mesons mentioned by the authors of HN22. Most notably there is the remarkable $J^{PC} = 1^{++}$ charmonium state $\chi_{c1}(3872)$ [5], also still called $X(3872)$, which lies practically on top of the S-wave $\bar{D}^{\star 0}D^0$ threshold and so has been frequently considered a molecular candidate. In Ref. [10], members of the same lattice collaboration as in Refs. [6, 7] employed $c\bar{c}$, $\bar{D}^{\star 0}D^0$, and even tetraquark interpolators to describe the $X(3872)$. Their definitely most important conclusion was that the state does not survive if no $c\bar{c}$ interpolators are included, with $\bar{D}^{\star 0}D^0$ also being important in order to obtain a clear signal close to the experimentally observed meson. The additionally included tetraquark interpolators turn out to be practically immaterial in the simulations. These lattice results lend support to earlier momentum-space [11] and coordinate-space [12] (Ref. [13] in HN22) coupled-channel model calculations. Also note that the multichannel calculation in Ref. [13], including several S-wave and D-wave two-meson channels, reveals an $X(3872)$ wave function dominated by a pronounced $c\bar{c}$ core, despite an overall $\bar{D}^{\star 0}D^0$ probability of about 65% due to the extremely long tail of this wave-function component owing to the tiny binding energy.

Another famous meson is the $D^\star_{s0}(2317)$ charm-strange scalar, which has also been often considered a molecule or a tetraquark state. In Ref. [14], members of the lattice collaboration of Refs. [6, 7, 10] described this scalar meson on a combined basis of $c\bar{s}$ and DK operators, in much the same way as the $D_{s1}(2460)$ with $c\bar{s}$ and $D^\star K$ operators in Ref. [6], finding a state below the DK threshold compatible with the physical $D^\star_{s0}(2317)$. The inclusion of both types of interpolating fields is crucial to obtain energy levels with small statistical uncertainties. The results of this simulation support the earlier coupled-channel modeling of this meson in Ref. [16]. In another and more recent lattice simulation [15], including tetraquark interpolating fields besides $c\bar{s}$ and DK, the $D^\star_{s0}(2317)$ emerges as a state below the DK threshold that is mostly of a quark-antiquark type, with a small DK component. The tetraquark interpolators turn out to be essentially irrelevant.

Now, one should realize the special features of non-exotic S-wave meson-meson scattering, in which $q\bar{q}$ states couple very strongly to two-meson channels. Most significantly, the light scalar mesons have been shown to appear naturally as extra and dynamically generated resonances in unitary coupled-channel models formulated in coordinate space [17] and momentum space [18], besides the regular quark-model scalars in the ballpark of 1.3–1.5 GeV. In that respect, let us quote from HN22:

> *"Some bare poles appear below and some above the $\varphi\bar{\varphi}$ threshold. In the regime of small coupling, the poles lying above the threshold get shifted to the complex plane and then, as the coupling increases, their trajectories bend and reapproach the real axis. Such a behaviour of the poles was previously discussed in the literature — see, for example, Refs. [25,42,43]."*

Now, the authors of HN22 mentioning their Ref. [42] (Ref. [18] in the present Comment) in the quoted remark are mistaken, because for small coupling the poles of the $f_0(500)$ and $K_0^\star(700)$ lie very deep in the complex energy plane, while those of the $f_0(980)$ and $a_0(980)$ can also not be traced back to the real axis in that limit [18]. This phenomenon of generating meson resonances via nonperturbative coupled-channel dynamics in S-wave scattering [19] is not exclusive to the light scalars, but can also be seen in the case of, for example, the $D_{s0}^\star(2317)$ [16] and the $X(3872)$ [11, 12]. To make life even more complicated, the dynamically generated $X(3872)$ pole was shown in Ref. [12] to interchange its identity with an intrinsic pole for small changes in a model parameter, while hardly influencing the resulting pole position. Therefore, identifying an S-wave meson resonance as dynamically generated or intrinsic can sometimes be very cumbersome, which adds to the many complexities of meson spectroscopy.

In conclusion, I point out that all the mesons mentioned in the foregoing share the property of coupling in an S-wave to their lowest or dominant meson-meson decay threshold. What does distinguish them, though, is the precise locations of those thresholds, which depend on the quantum numbers and constituent quark masses in the decay products. So my assessment is that the proximity of their masses to their lowest (or dominant) S-wave two-meson decay thresholds, with the $X(3872)$ being an extreme case, is to some extent accidental yet not entirely, in view of the undeniable role that such S-wave thresholds play in locking [20] or even generating [17] S-matrix poles.

Nevertheless, more definite conclusions can only be drawn from precise experimental data, as the authors of HN22 suggest in the case of the $D_{s1}(2460)$. However, the problem with their proposal to measure the theoretically estimated width of the isospin-violating decay $D_{s1}(2460)^+ \to D_s^{\star+}\pi^0$, of the order of 100 keV in the molecular scenario, is not only a challenge to experiment, as the authors themselves admit. Because also the theoretical estimates, both in the molecular and $c\bar{s}$ scenarios, are based on several assumptions, not to speak of the inevitable $D^\star K$ loop contributions in the (dominantly) $c\bar{s}$ case as well. So I suggest electromagnetic decays like $D_{s1}(2460)^+ \to D_s^{\star+}\gamma$ and $D_{s1}(2460)^+ \to D_{s0}^\star(2317)^+\gamma$ as better candidates [5] to probe the structure of $D_{s1}(2460)$, just like for the $X(3872)$ [13, 21].

---

[1] C. Hanhart and A. Nefediev, Phys. Rev. D **106**, 114003 (2022) [arXiv:2209.10165 [hep-ph]].

[2] S. Coito, G. Rupp, and E. van Beveren, Phys. Rev. D **84**, 094020 (2011) [arXiv:1106.2760 [hep-ph]].

[3] E. van Beveren and G. Rupp, Eur. Phys. J. C **32**, 493 (2004) [arXiv:hep-ph/0306051 [hep-ph]].

[4] A. M. Badalian, Y. A. Simonov, and M. A. Trusov, Phys. Rev. D **77**, 074017 (2008) [arXiv:0712.3943 [hep-ph]].

[5] R. L. Workman *et al.* [Particle Data Group], PTEP **2022**, 083C01 (2022).

[6] C. B. Lang, L. Leskovec, D. Mohler, S. Prelovsek, and R. M. Woloshyn, Phys. Rev. D **90**, 034510 (2014) [arXiv:1403.8103 [hep-lat]].

[7] D. Mohler, S. Prelovsek, and R. M. Woloshyn, Phys. Rev. D **87**, 034501 (2013) [arXiv:1208.4059 [hep-lat]].

[8] Nicolas Lang and David J. Wilson [Hadron Spectrum Collaboration], Phys. Rev. Lett. **129** (2022) 252001 [arXiv:2205.05026 [hep-ph]].

[9] Nicolas Lang, *"Charmed meson resonances from Lattice QCD"*, Talk at Workshop Excited QCD 2024, Benasque, Spain, 14–20 January 2024; see page 20: https://www.benasque.org/2024eqcd/talks_contr/185_5-talk.pdf

[10] M. Padmanath, C. B. Lang, and S. Prelovsek, Phys. Rev. D **92**, 034501 (2015) [arXiv:1503.03257 [hep-lat]].

[11] S. Coito, G. Rupp, and E. van Beveren, Eur. Phys. J. C **71**, 1762 (2011) [arXiv:1008.5100 [hep-ph]].

[12] S. Coito, G. Rupp, and E. van Beveren, Eur. Phys. J. C **73**, 2351 (2013) [arXiv:1212.0648 [hep-ph]].

[13] M. Cardoso, G. Rupp, and E. van Beveren, Eur. Phys. J. C **75**, 26 (2015) [arXiv:1411.1654 [hep-ph]].

[14] D. Mohler, C. B. Lang, L. Leskovec, S. Prelovsek, and R. M. Woloshyn, Phys. Rev. Lett. **111**, 222001 (2013) [arXiv:1308.3175 [hep-lat]].

[15] C. Alexandrou, J. Berlin, J. Finkenrath, T. Leontiou, and M. Wagner, Phys. Rev. D **101**, 034502 (2020) [arXiv:1911.08435 [hep-lat]].

[16] E. van Beveren and G. Rupp, Phys. Rev. Lett. **91**, 012003 (2003) [arXiv:hep-ph/0305035].

[17] E. van Beveren, T. A. Rijken, K. Metzger, C. Dullemond, G. Rupp, and J. E. Ribeiro, Z. Phys. C **30**, 615 (1986) [arXiv:0710.4067 [hep-ph]].

[18] E. van Beveren, D. V. Bugg, F. Kleefeld, and G. Rupp, Phys. Lett. B **641**, 265 (2006) [arXiv:hep-ph/0606022 [hep-ph]].

[19] J. A. Oller and E. Oset, Nucl. Phys. A **620**, 438 (1997) [Erratum-ibid **652**, 407 (1999)] [arXiv:hep-ph/9702314 [hep-ph]].

[20] D. V. Bugg, J. Phys. G **35**, 075005 (2008) [arXiv:0802.0934 [hep-ph]].

[21] A. M. Badalian, V. D. Orlovsky, Y. A. Simonov, and B. L. G. Bakker, Phys. Rev. D **85**, 114002 (2012) [arXiv:1202.4882 [hep-ph]].