Is γ_{KLS}-generalized statistical field theory complete?

Paulo R. S. Carvalho*
Departamento de Física, Universidade Federal do Piauí, 64049-550, Teresina, PI, Brazil

In this Letter we introduce some field-theoretic approach for computing the critical properties of γ_{KLS}-generalized systems undergoing continuous phase transitions, namely γ_{KLS}-statistical field theory. From this new approach emerges the new generalized $\mathrm{O}(N)_{\gamma_{KLS}}$ universality class, which is capable of encompassing nonconventional critical exponents for real imperfect systems known as manganites not described by standard statistical field theory. We compare the generalized results with those obtained from measurements in manganites. The agreement was satisfactory, where the relative errors are $< 5\%$ for the most of manganites used. Although the present approach describes the aforementioned nonconventional critical indices, we show that it is not complete. For example, it does not explain the results for some other manganites, being explained only for nonextensive statistical field theory recently introduced in literature. So, γ_{KLS}-statistical field theory has to be discarded for statistical mechanics generalization purposes.

I. INTRODUCTION

Recently, a field-theoretic renormalization group method for describing the unconventional critical behavior of some real imperfect systems known as manganites [1–20] was proposed [21]. These systems are complex and present a large number of strong interacting particles, defects, impurities, inhomogeneities, nonlinearities, competition etc. Such systems can not be understood by employing standard Gibbs-Boltzmann statistical field theory (SFT) formulated by Kenneth Wilson [22]. Rather, we have to employ some generalized version of Wilson field-theoretic renormalization group, namely nonextensive statistical field theory (NSFT) [21]. As it is known, there are many attempts of generalizing Gibbs-Boltzmann statistical mechanics [23–27]. In this direction, we have to check what of these proposed generalizations satisfy to the all requirements needed for a consistent generalized theory. Some of these conditions are: they have to emerge from a maximum principle and a trace-form entropy, present decisivity, maximality, concavity, Lesche stability, positivity, continuity, symmetry and expansibility. Once all these conditions are satisfied, the corresponding proposed generalized theory has to be applied to all experimental situations not described by the standard theory. If there is, at least, one experimental situation in which some proposed generalized statistics does not describe the referred experimental results, this statistics is not general enough to represent a generalized theory. Then it has to be discarded as one candidate to generalize statistical mechanics. In fact, recently it was shown that it was the case for Kaniadakis attempt for generalizing statistical mechanics [28], *i. e.*, this statistics, namely Kaniadakis statistics [24] does not describe the set of unconventional critical exponents values for some manganites as NSFT does [21].

The aim of this Letter is to search if there are some field-theoretic renormalization group generalizations satisfying all the requirements aforementioned other than NSFT (parameterized by the parameter q encoding some effective interaction [21]). For that, we employ the γ_{KLS}-generalized statistics proposed in Ref. [26]. It is parameterized by the parameter γ [26, 27]. In the present context of critical exponents, we have to avoid to represent different concepts with the same letter, for example the γ parameter and the susceptibility critical exponent γ. Then we have to use γ_{KLS} instead γ for representing that parameter. We will call such a theory as γ_{KLS}-generalized statistical field theory or γ_{KLS}-SFT for short. The range of γ_{KLS} is $-1/2 < \gamma_{KLS} < 1/2$ [26, 27]. We expect to recover the nongeneralized critical exponents values obtained by Kenneth Wilson [22] in the limit $\gamma_{KLS} \to 0$. Also we expect the emergence of some γ_{KLS}-generalized universality class for γ_{KLS}-generalized Ising-like systems, namely the $\mathrm{O}(N)_{\gamma_{KLS}}$ one. Now the critical indices will depend on the dimension d, N and symmetry of some N-component order parameter, if the interactions of the constituents of the systems are of short- or long-range type and γ_{KLS}. From the physical interpretation of the results, it will emerge the corresponding physical interpretation of the γ_{KLS} parameter. For a similar field-theoretic approach, see Ref. [29].

II. γ_{KLS}-SFT

We introduce the γ_{KLS}-SFT by defining its generating functional by

$$Z[J] = \mathcal{N}^{-1} \exp_{\gamma_{KLS}} \left[- \int d^d x \mathcal{L}_{int} \left(\frac{\delta}{\delta J(x)} \right) \right] \times \int \exp \left[\frac{1}{2} \int d^d x d^d x' J(x) G_0(x - x') J(x') \right] \quad (1)$$

* prscarvalho@ufpi.edu.br

where

$$e_{\gamma_{KLS}}^{-x} = \left[\left(\frac{1+\sqrt{1+4\gamma_{KLS}^{3}x^{3}}}{2}\right)^{\frac{1}{3}} + \left(\frac{1-\sqrt{1+4\gamma_{KLS}^{3}x^{3}}}{2}\right)^{\frac{1}{3}}\right]^{\frac{1}{\gamma_{KLS}}}, \qquad (2)$$

is the γ_{KLS}-exponential function [26, 27]. We can determine the constant $\mathcal{N}$ from the condition $Z[J=0]=1$. Now we can compute the static γ_{KLS}-generalized critical exponents for $\mathrm{O}(N)_{\gamma_{KLS}}$ universality class for γ_{KLS}-ϕ^4 theory through six distinct and independent methods in $d=4-\epsilon$ dimensions. The two independent γ_{KLS}-generalized indices valid for all loop levels are given by

$$\eta_{\gamma_{KLS}} = \eta + \frac{\gamma_{KLS}}{1-\gamma_{KLS}}\frac{(N+2)\epsilon^2}{2(N+8)^2}, \qquad (3)$$

$$\nu_{\gamma_{KLS}} = \nu + \frac{\gamma_{KLS}}{1-\gamma_{KLS}}\frac{(N+2)\epsilon}{4(N+8)}. \qquad (4)$$

The η and ν indices are the corresponding nongeneralized critical indices valid for all loop orders. The corresponding dynamic γ_{KLS}-generalized critical index is given by

$$z_{\gamma_{KLS}} = z + \frac{\gamma_{KLS}}{1-\gamma_{KLS}}\frac{[6\ln(4/3)-1](N+2)}{2(N+8)^2}\epsilon^2, \qquad (5)$$

where z is the nongeneralized index value valid for all loop levels.

III. COMPARISON BETWEEN THEORETIC AND EXPERIMENTAL RESULTS

We compare the results of Eqs. (3)-(4) and scaling relations among them with those obtained from experiments for some manganites in Tables I-II. We use the values $\eta=0.061(8)$, $\nu=0.689(2)$, $\beta=0.365(3)$, $\gamma=1.336(4)$ [30] and $\eta=0.030(4)$, $\nu=0.630(1)$, $\beta=0.325(2)$, $\gamma=1.241(2)$ [30] for nongeneralized Heinsenberg and Ising, respectively.

We observe in Tables I-II that experimental values obtained from experiments for various manganites ($\gamma_{KLS}\neq 0$) are distinct from those nongeneralized ones ($\gamma_{KLS}=0$). The agreement is satisfactory, withing some margin of error of $<5\%$ for the most of manganites. Then, the critical behavior of manganites is nonconventional [1–20] and can not be described by conventional SFT [22]. They belong to both γ_{KLS}-Heinsenberg ($N=3$) and γ_{KLS}-Ising ($N=1$) universality classes, respectively. The γ_{KLS}-generalizaed critical exponents (temperature) values increase (decrease, see Table III) with increasing of γ_{KLS}.

TABLE I. Static γ_{KLS}-generalized critical exponents ($\gamma_{KLS}\neq 0$) to 3d ($\epsilon=1$) Heisenberg ($N=3$) systems, obtained from experiment through Modified Arrott (MA) plots [31], Kouvel-Fisher (KF) method [32] and γ_{KLS}-SFT of this work.

γ_{KLS}-Heisenberg	$\beta_{\gamma_{KLS}}$	$\gamma_{\gamma_{KLS}}$
$La_{0.67}Sr_{0.33}MnO_3$[33]MAP	0.333(8)	1.325(1)
γ_{KLS} = -0.5 This work	0.343(3)	1.267(7)
$Pr_{0.77}Pb_{0.23}MnO_3$[3]MAP	0.343(5)	1.357(20)
$Pr_{0.77}Pb_{0.23}MnO_3$[3]KF	0.344(1)	1.352(6)
γ_{KLS} = -0.30 This work	0.350(3)	1.289(7)
$AMnO_3$[4]MAP	0.355(7)	1.326(2)
$AMnO_3$[4]KF	0.344(5)	1.335(2)
γ_{KLS} = -0.15 This work	0.357(3)	1.309(7)
$Nd_{0.7}Pb_{0.3}MnO_3$[5]MAP	0.361(13)	1.325(1)
$Nd_{0.7}Pb_{0.3}MnO_3$[5]KF	0.361(5)	1.314(1)
γ_{KLS} = -0.10 This work	0.360(3)	1.318(7)
$LaTi_{0.2}Mn_{0.8}O_3$[6]KF	0.359(4)	1.280(10)
γ_{KLS} = -0.10 This work	0.360(3)	1.318(7)
$La_{0.67}Sr_{0.33}Mn_{0.95}V_{0.05}O_3$[33]MAP	0.358(5)	1.381(4)
γ_{KLS} = -0.10 This work	0.360(3)	1.318(7)
γ_{KLS} = 0.00 [7]	0.365(3)	1.336(4)
$Nd_{0.85}Pb_{0.15}MnO_3$[5]MAP	0.372(1)	1.340(30)
$Nd_{0.85}Pb_{0.15}MnO_3$[5]KF	0.372(4)	1.347(1)
γ_{KLS} = 0.05 This work	0.369(3)	1.362(7)
$Nd_{0.6}Pb_{0.4}MnO_3$[9]KF	0.374(6)	1.329(3)
γ_{KLS} = 0.05 This work	0.369(3)	1.362(7)
$La_{0.67}Sr_{0.33}Mn_{0.95}V_{0.15}O_3$[33]MAP	0.375(3)	1.355(6)
γ_{KLS} = 0.05 This work	0.369(3)	1.362(7)
$La_{0.67}Ba_{0.22}Sr_{0.11}MnO_3$[10]MAP	0.378(3)	1.388(1)
$La_{0.67}Ba_{0.22}Sr_{0.11}MnO_3$[10]KF	0.386(6)	1.393(4)
γ_{KLS} = 0.10 This work	0.377(3)	1.347(7)
$LaTi_{0.95}Mn_{0.05}O_3$[6]KF	0.378(7)	1.290(20)
γ_{KLS} = 0.10 This work	0.377(3)	1.347(7)
$LaTi_{0.9}Mn_{0.1}O_3$[6]KF	0.375(5)	1.250(20)
γ_{KLS} = 0.10 This work	0.377(3)	1.347(7)
$LaTi_{0.85}Mn_{0.15}O_3$[6]KF	0.376(3)	1.240(10)
γ_{KLS} = 0.10 This work	0.377(3)	1.347(7)
$La_{0.67}Ca_{0.33}Mn_{0.9}Ga_{0.1}O_3$[11]MAP	0.380(2)	1.365(8)
$La_{0.67}Ca_{0.33}Mn_{0.9}Ga_{0.1}O_3$[11]KF	0.387(6)	1.362(2)
γ_{KLS} = 0.15 This work	0.378(3)	1.372(7)
$La_{0.67}Ba_{0.22}Sr_{0.11}Mn_{0.9}Fe_{0.1}O_3$[10]MAP	0.398(2)	1.251(5)
$La_{0.67}Ba_{0.22}Sr_{0.11}Mn_{0.9}Fe_{0.1}O_3$[10]KF	0.395(3)	1.247(3)
γ_{KLS} = 0.30 This work	0.395(3)	1.424(7)
$La_{0.67}Ba_{0.22}Sr_{0.11}Mn_{0.8}Fe_{0.2}O_3$[10]MAP	0.411(1)	1.241(4)
$La_{0.67}Ba_{0.22}Sr_{0.11}Mn_{0.8}Fe_{0.2}O_3$[10]KF	0.394(3)	1.292(3)
γ_{KLS} = 0.30 This work	0.395(3)	1.424(7)
$Pr_{0.7}Pb_{0.3}MnO_3$[3]MAP	0.404(6)	1.354(20)
$Pr_{0.7}Pb_{0.3}MnO_3$[3]KF	0.404(1)	1.357(6)
γ_{KLS} = 0.35 This work	0.402(3)	1.446(7)
$La_{0.75}(Sr,Ca)_{0.25}Mn_{0.9}Ga_{0.1}O_3$[10]MAP	0.420(5)	1.221(2)
$La_{0.75}(Sr,Ca)_{0.25}Mn_{0.9}Ga_{0.1}O_3$[10]KF	0.428(5)	1.286(4)
γ_{KLS} = 0.45 This work	0.421(3)	1.503(7)
$Pr_{0.5}Sr_{0.5}MnO_3$[12]MAP	0.443(2)	1.339(6)
$Pr_{0.5}Sr_{0.5}MnO_3$[12]KF	0.448(9)	1.334(10)
γ_{KLS} = 0.50 This work	0.434(3)	1.540(7)

TABLE II. Static γ_{KLS}-generalized critical exponents ($\gamma_{KLS} \neq 0$) to 3d ($\epsilon = 1$) Ising ($N = 1$) systems, obtained from experiment through Modified Arrott (MA) plots [31], Kouvel-Fisher (KF) method [32] and γ_{KLS}-SFT of this work.

γ_{KLS}-Ising	$\beta_{\gamma_{KLS}}$	$\gamma_{\gamma_{KLS}}$
$La_{0.8}Sr_{0.2}MnO_3$[13]	0.290(10)	
γ_{KLS} = -0.35 This work	0.311(1)	1.201(3)
$Nd_{0.55}Sr_{0.45}Mn_{0.98}Ga_{0.02}O_3$[14]	0.308(10)	1.197
γ_{KLS} = -0.35 This work	0.311(1)	1.201(3)
$Pr_{0.6}Sr_{0.4}MnO_3$[15]MAP	0.315(0)	1.095(7)
$Pr_{0.6}Sr_{0.4}MnO_3$[15]KF	0.312(2)	1.106(5)
γ_{KLS} = -0.15 This work	0.318(1)	1.221(3)
$La_{0.8}Ca_{0.2}MnO_3$[16]KF	0.316(7)	1.081(36)
γ_{KLS} = -0.15 This work	0.318(1)	1.221(3)
$La_{0.7}Ca_{0.3}Mn_{0.85}Ni_{0.15}O_3$[17]MAP	0.320(9)	0.990(82)
γ_{KLS} = -0.05 This work	0.322(1)	1.234(3)
$Nd_{0.6}Sr_{0.4}MnO_3$[18]MAP	0.320(6)	1.239(2)
$Nd_{0.6}Sr_{0.4}MnO_3$[18]KF	0.323(2)	1.235(4)
γ_{KLS} = -0.05 This work	0.322(1)	1.234(3)
$Nd_{0.6}Sr_{0.4}MnO_3$[15]MAP	0.321(3)	1.183(17)
$Nd_{0.6}Sr_{0.4}MnO_3$[15]KF	0.308(4)	1.172(11)
γ_{KLS} = -0.05 This work	0.322(1)	1.234(3)
$Nd_{0.67}Ba_{0.33}MnO_3$[19]MAP	0.325(4)	1.248(19)
$Nd_{0.67}Ba_{0.33}MnO_3$[19]KF	0.326(5)	1.244(33)
γ_{KLS} = -0.05 This work	0.322(1)	1.234(3)
γ_{KLS} = 0.00 [7]	0.325(2)	1.241(2)
$La_{0.65}Bi_{0.05}Sr_{0.3}MnO_3$[20]MAP	0.335(3)	1.207(20)
$La_{0.65}Bi_{0.05}Sr_{0.3}MnO_3$[20]KF	0.316(7)	1.164(20)
γ_{KLS} = 0.10 This work	0.330(1)	1.258(3)
$La_{0.65}Bi_{0.05}Sr_{0.3}Mn_{0.94}Ga_{0.06}O_3$[20]MAP	0.334(4)	1.192(8)
$La_{0.65}Bi_{0.05}Sr_{0.3}Mn_{0.94}Ga_{0.06}O_3$[20]KF	0.307(8)	1.138(5)
γ_{KLS} = 0.10 This work	0.330(1)	1.258(3)

TABLE III. γ_{KLS}-generalized critical temperature ($\gamma_{KLS} \neq 0$) to 3d ($\epsilon = 1$) Heisenberg ($N = 3$) systems, obtained from experiment through Modified Arrott (MA) plots [31] and Kouvel-Fisher (KF) methods.

γ_{KLS}-Heisenberg	$\beta_{\gamma_{KLS}}$	$T_{c,\gamma_{KLS}}(K)$
$La_{0.67}Sr_{0.33}MnO_3$[33]MAP	0.333(8)	350
γ_{KLS} = -0.5 This work	0.343(3)	
$La_{0.67}Sr_{0.33}Mn_{0.95}V_{0.05}O_3$[33]MAP	0.358(5)	326
γ_{KLS} = -0.10 This work	0.360(3)	
$La_{0.67}Sr_{0.33}Mn_{0.95}V_{0.10}O_3$[33]MAP	0.367(3)	301
γ_{KLS} = 0.05 This work	0.369(3)	
$La_{0.67}Sr_{0.33}Mn_{0.95}V_{0.15}O_3$[33]MAP	0.375(3)	290
γ_{KLS} = 0.10 This work	0.377(3)	
$La_{0.67}Ba_{0.22}Sr_{0.11}MnO_3$[10]MAP	0.378(3)	343
$La_{0.67}Ba_{0.22}Sr_{0.11}MnO_3$[10]KF	0.386(6)	342
γ_{KLS} = 0.10 This work	0.377(3)	
$La_{0.67}Ba_{0.22}Sr_{0.11}Mn_{0.9}Fe_{0.1}O_3$[10]MAP	0.398(2)	191
$La_{0.67}Ba_{0.22}Sr_{0.11}Mn_{0.9}Fe_{0.1}O_3$[10]KF	0.395(3)	189
γ_{KLS} = 0.30 This work	0.395(3)	
$La_{0.67}Ba_{0.22}Sr_{0.11}Mn_{0.8}Fe_{0.2}O_3$[10]MAP	0.411(1)	130
$La_{0.67}Ba_{0.22}Sr_{0.11}Mn_{0.8}Fe_{0.2}O_3$[10]KF	0.394(3)	139
γ_{KLS} = 0.30 This work	0.395(3)	
$La_{0.7}Ba_{0.3}MnO_3$[2]MAP	0.341(3)	339
$La_{0.7}Ba_{0.3}MnO_3$[2]KF	0.357(160)	340
γ_{KLS} = -0.50 This work	0.343(3)	
$La_{0.6}Pr_{0.1}Ba_{0.3}MnO_3$[2]MAP	0.396(16)	321
$La_{0.6}Pr_{0.1}Ba_{0.3}MnO_3$[2]KF	0.391(2)	321
γ_{KLS} = 0.30 This work	0.395(3)	
$La_{0.5}Pr_{0.2}Ba_{0.3}MnO_3$[2]MAP	0.494(10)	304
$La_{0.5}Pr_{0.2}Ba_{0.3}MnO_3$[2]KF	0.491(23)	304
γ_{KLS} = 0.50 This work	0.434(3)	

We interpret these results as follows: a given physical quantity, near the transition point, diverges. How much it diverges is measured by its associated critical exponent. In the case of susceptibility, for example, its inverse furnishes a measure of how much the material is susceptible to the changes in magnetic field. Higher (lower) values of the γ critical index indicates more (less) susceptible or weaker interacting (stronger) systems. These facts are in agreement with the form of the effective energy of the system, which can be obtained by the some expansion around $\gamma_{KLS} \approx 0$. In fact, taking the leading contribution to the energy of the system as E, in units of $k_B T$, from $e_{\gamma_{KLS}}^{-E} \approx e^{-E}\left(1 - \frac{1}{2}\gamma_{KLS}E^2\right) \approx e^{-\left(E + \frac{1}{2}\gamma_{KLS}E^2\right)}$. So the effective energy $E + \frac{1}{2}\gamma_{KLS}E^2$ increases with the increasing of γ_{KLS}. Then higher (lower) values of γ_{KLS} represent systems interacting weaker (stronger) or more (less) susceptible and thus possessing higher (lower) values of their critical exponents. Also higher (lower) values of γ_{KLS} give higher (lower) values of E and then we have to furnish less (more) energy to attain the respective critical transition temperature so the critical transition temperatures assume lower (higher) values and decrease. Although γ_{KLS}-*SFT* can explain the results for the Tables I-II, it does not explain the results of Table IV for some materiais, being explained only for NSFT (see Table IV) of Ref. [21]. Then, γ_{KLS}-*SFT* must be discarded as one trying to generalize statistical mechanics. For our knowledge, only NSFT of Ref. [21] remains a fully consistent or complete generalized formulation of statistical field theory.

IV. OTHER γ_{KLS}-MODELS

Now that γ_{KLS}-SFT has been validated experimentally, we display the γ_{KLS}-critical indices for other models.

A. Both γ_{KLS}-percolation and γ_{KLS}-Yang-lee edge singularity

In the case of both γ_{KLS}-percolation [38] ($\alpha = -1$ and $\beta = -2$) and γ_{KLS}-Yang-Lee edge singularity [38] ($\alpha = -1$ and $\beta = -1$) in dimensions $d = 6 - \epsilon$ we can write,

$$\eta_{\gamma_{KLS}} = \eta - \frac{\gamma_{KLS}}{1 - \gamma_{KLS}} \frac{8\alpha\beta}{3(\alpha - 4\beta)[\alpha - 4\beta(1 - 3\gamma_{KLS})/(1 - \gamma_{KLS})]}\epsilon, \quad (6)$$

TABLE IV. Static critical exponents to 3d ($\epsilon = 1$) Heinsenberg ($N = 3$) systems, obtained from experiment through Modified Arrott (MA) plots [31], Kouvel-Fisher (KF) method [32] and NSFT ($q \neq 0$) of Ref. [21].

Heisenberg	β	γ
$La_{0.7}Sr_{0.3}MnO_{0.97}Ni_{0.03}O_3$[34]	0.468(6)	1.010(21)
γ_{KLS} = - This work	-	-
$q = 0.40$ [21]	0.469(4)	1.640(8)
$La_{0.7}Sr_{0.3}Mn_{0.94}Co_{0.06}O_3$[35]	0.478(13)	1.165(27)
γ_{KLS} = - This work	-	-
$q = 0.39$ [21]	0.474(4)	1.653(8)
$La_{0.7}Sr_{0.3}Mn_{0.92}Co_{0.08}O_3$[35]	0.483(18)	1.112(28)
γ_{KLS} = - This work	-	-
$q = 0.37$ [21]	0.484(4)	1.680(8)
$La_{0.7}Sr_{0.3}Mn_{0.90}Co_{0.10}O_3$[35]	0.487(16)	1.109(63)
γ_{KLS} = - This work	-	-
$q = 0.36$ [21]	0.489(4)	1.695(8)
$La_{0.67}Ca_{0.33}Mn_{0.95}Fe_{0.05}O_3$[36]	0.550(10)	1.0246(3)
γ_{KLS} = - This work	-	-
$q = 0.27$ [21]	0.556(4)	1.876(9)
$La_{0.67}Ca_{0.33}Mn_{0.90}Cr_{0.10}O_3$[37]	0.555(6)	1.170(40)
γ_{KLS} = - This work	-	-
$q = 0.27$ [21]	0.556(4)	1.876(9)
$La_{0.67}Ca_{0.33}Mn_{0.75}Cr_{0.25}O_3$[37]	0.680(10)	1.090(30)
γ_{KLS} = - This work	-	-
$q = 0.19$ [21]	0.674(5)	2.172(10)

$$\nu_{\gamma_{KLS}}^{-1} = \nu^{-1} - \frac{\gamma_{KLS}}{1-\gamma_{KLS}} \frac{40\alpha\beta}{3(\alpha - 4\beta)[\alpha - 4\beta(1-3\gamma_{KLS})/(1-\gamma_{KLS})]}\epsilon, \tag{7}$$

$$\omega_{\gamma_{KLS}} = \omega. \tag{8}$$

For γ_{KLS}-Yang-Lee edge singularity, the $\eta_{\gamma_{KLS}}$ and $\nu_{\gamma_{KLS}}$ indices are dependent [38, 39]. They are related through $\nu_{\gamma_{KLS}}^{-1} = (d - 2 + \eta_{\gamma_{KLS}})/2$ [38, 39]. Thus from the value of $\eta_{\gamma_{KLS}}$, by employing the values $\alpha = -1$ and $\beta = -1$, we compute $\nu_{\gamma_{KLS}}$. The remaining ones can be obtained from the scaling relations among them [40].

B. γ_{KLS}-ϕ^6 theory

The N-component γ_{KLS}-ϕ^6 theory in $d = 3 - \epsilon$ [41, 42] describes γ_{KLS}-tricritical points to $N = 2$ and corresponds to ^{3}He-^{4}He mixtures or antiferromagnets in the presence of a strong external field (metamagnets) [43]. The γ_{KLS}-critical exponents assume the form

$$\eta_{\gamma_{KLS}} = \eta + \frac{\gamma_{KLS}}{1-\gamma_{KLS}} \frac{(N+2)(N+4)}{12(3N+22)^2}\epsilon^2, \tag{9}$$

$$\nu_{\gamma_{KLS}} = \nu + \frac{\gamma_{KLS}}{1-\gamma_{KLS}} \frac{(N+2)(N+4)}{3(3N+22)^2}\epsilon^2. \tag{10}$$

Actually, η and ν are known up to six-loop order [43].

C. γ_{KLS}-long-range systems

The γ_{KLS}-long-range critical exponents for N-component γ_{KLS}-Ising-like models whit interactions decaying as $1/r_{ij}^{d+\sigma}$ can be defined at three different sectors [44]. The γ_{KLS}-critical indices, in $d = 2\sigma - \varepsilon$, are given by

$$\eta_{\sigma,\,\gamma_{KLS}} = \begin{cases} \eta_{\gamma_{KLS}} & \text{if} \quad \sigma > 2 - \eta_{\gamma_{KLS}} \\ \eta_\sigma & \text{if} \quad d/2 < \sigma < 2 - \eta_{\gamma_{KLS}} \\ 0 & \text{if} \quad \sigma < d/2, \end{cases}$$

$$\nu_{\sigma,\,\gamma_{KLS}} = \begin{cases} \nu_{\gamma_{KLS}} & \text{if} \quad \sigma > 2 - \eta_{\gamma_{KLS}} \\ \nu_\sigma + \frac{\gamma_{KLS}}{1-\gamma_{KLS}}\frac{(N+2)}{\sigma^2(N+8)}\epsilon & \text{if} \quad d/2 < \sigma < 2- \\ \eta_{\gamma_{KLS}} & \\ 1/2 & \text{if} \quad \sigma < d/2, \end{cases}$$

where $\eta_\sigma = 2 - \sigma$ for any loop order [45, 46], $\eta_{\gamma_{KLS}}$ and $\nu_{\gamma_{KLS}}$ are the short-range γ_{KLS}-critical indices of Eqs. (3)-(4), and ν_σ has been evaluated up to two-loop order [47] in the interval $d/2 < \sigma < 2 - \eta_{\gamma_{KLS}}$.

D. γ_{KLS}-Gross-Neveu model

From the γ_{KLS}-Gross-Neveu model in $d = 2 + \epsilon$ [48] we can study both ψ and $\bar{\psi}$ Dirac massive fermions of mass $\mathcal{M}$. The γ_{KLS}-critial exponents are

$$\eta_{\psi,\,\gamma_{KLS}} = \eta_\psi + \frac{\gamma_{KLS}}{1-\gamma_{KLS}} \frac{(2N-1)}{8(N-1)^2}\epsilon^2, \tag{11}$$

$$\eta_{\mathcal{M},\,\gamma_{KLS}} = \eta_{\mathcal{M}} + \frac{\gamma_{KLS}}{1-\gamma_{KLS}} \frac{\epsilon}{2(N-1)}, \quad \nu_{\gamma_{KLS}} = \nu, \tag{12}$$

where, up to now, the nongeneralized critical exponents have been evaluated up to four-loop level [49].

E. γ_{KLS}-uniaxial systems with strong dipolar forces

γ_{KLS}-uniaxial systems with strong dipolar forces in the z-direction are described by the following Hamiltonian [50]

$$\mathcal{H} = -\sum_{\vec{x}\vec{x}'}\sum_{\mu\nu} S_{\vec{x}}^{\mu} S_{\vec{x}'}^{\nu} \left(V_{\mu\nu}(\vec{x} - \vec{x}') + \frac{\gamma\partial_z\partial_{z'}}{|\vec{x} - \vec{x}'|^{d-2}} \right), \tag{13}$$

where $V_{\mu\nu}(\vec{x})$ is the short-range potential and γ is a parameter for controlling the dipolar forces intensity. In in $d = 3 - \epsilon$ dimensions, the γ_{KLS}-critical exponents are given by

$$\eta_{\gamma_{KLS}} = \eta + \frac{\gamma_{KLS}}{1-\gamma_{KLS}} \frac{4(N+2)}{9(N+8)^2}\epsilon^2, \tag{14}$$

$$\nu_{\gamma_{KLS}}^{-1} = \nu^{-1} - \frac{\gamma_{KLS}}{1-\gamma_{KLS}}\frac{(N+2)}{(N+8)}\epsilon, \tag{15}$$

where, η and ν are displayed in Ref. [50] up to two-loop level.

F. γ_{KLS}-spherical model

The γ_{KLS}-spherical model [51] can be obtained by taking the limit $N \rightarrow \infty$ [52] of the O$(N)_{\gamma_{KLS}}$ model of the present Letter. After taking this limit, we obtain in $d = 4 - \epsilon$

$$\eta_{\gamma_{KLS}} = \eta, \qquad \nu_{\gamma_{KLS}} = \nu + \frac{\gamma_{KLS}}{1-\gamma_{KLS}}\frac{\epsilon}{4}, \tag{16}$$

where the corresponding nongeneralized values are exact, namely $\eta = 0$ and $\nu = 1/(2-\epsilon)$ [22].

G. γ_{KLS}-Lifshitz critical points

The γ_{KLS}-Lifshitz points [53–66] are composed of the m-axial Lifshitz points [54] an their generalized forms for the higher character cases [55]. For the latter, there are $d-m$-, m_2-,..., m_n-dimensional vectors, respectively. The generic higher character γ_{KLS}-Lifshitz anisotropic (treated both in the orthogonal approximation) and isotropic (computed in both orthogonal approximation and exactly) indices are given by

$$\eta_{n,\,\gamma_{KLS}} = \eta_n + \frac{\gamma_{KLS}}{1-\gamma_{KLS}} n\frac{(N+2)}{2(N+8)^2}\epsilon_n^2, \tag{17}$$

$$\nu_{n,\,\gamma_{KLS}} = \nu_n + \frac{\gamma_{KLS}}{1-\gamma_{KLS}}\frac{(N+2)}{4n(N+8)}\epsilon_n, \tag{18}$$

where $\epsilon_L = 4 + \sum_{n=2}^{L}[(n-1)/n]m_n - d$,

$$\eta_{n,\,\gamma_{KLS}} = \eta_n + \frac{\gamma_{KLS}}{1-\gamma_{KLS}}\frac{(N+2)}{2n(N+8)^2}\epsilon_n^2, \tag{19}$$

$$\nu_{n,\,\gamma_{KLS}} = \nu_n + \frac{\gamma_{KLS}}{1-\gamma}\frac{(N+2)}{4n^2(N+8)}\epsilon_n, \tag{20}$$

where $\epsilon_L = 4n - d$,

$$\eta_{n,\,\gamma} = \eta_n + \frac{\gamma}{1-\gamma}\frac{(-1)^{n+1}\Gamma(2n)^2(N+2)}{\Gamma(n+1)\Gamma(3n)(N+8)^2}\epsilon_n^2, \tag{21}$$

$$\nu_{n,\,\gamma} = \nu_n + \frac{\gamma}{1-\gamma}\frac{(N+2)}{4n^2(N+8)}\epsilon_n. \tag{22}$$

In Ref. [55], the γ_{KLS}-critical indices were obtained up to next-to-leading order.

H. Long-range γ_{KLS}-$\lambda\phi^3$ theory

The long-range γ_{KLS}-$\lambda\phi^3$ [67] critical indices in $d = 3\sigma - \varepsilon$ are given by

$$\eta_{\sigma,\,\gamma_{KLS}} = \eta_\sigma, \qquad \nu_{\sigma,\,\gamma_{KLS}}^{-1} = \nu_\sigma^{-1} - \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{\alpha}{\beta}\epsilon, \tag{23}$$

The values of the constants α and β are -1, -1 and -1, -2 for the nongeneralized Yang-Lee edge singularity and percolation [38], respectively. The nongeneralized critical exponent $\eta_\sigma = 2-\sigma$ is exact [67]. Then the index $\eta_{\sigma,\,\gamma_{KLS}}$ is exact. nongeneralized exponents were obtained up to two-loop level in Ref. [67].

I. γ_{KLS}-multicritical points

The γ_{KLS}-multicritical points of order k [68] whose critical dimension is $d_c = \frac{2k}{k-1}$ possess the following γ_{KLS}-critical exponents, namely

$$\eta_{\gamma_{KLS}} = \eta + \frac{\gamma_{KLS}}{1-\gamma_{KLS}} 4(k-1)^2\frac{(k)!^6}{(2k!)^3}\epsilon^2. \tag{24}$$

The nongeneralized index η was obtained in Ref. [68] up to $2k-2$-loop level.

J. γ_{KLS}-Gross-Neveu-Yukawa model

With the γ_{KLS}-Gross-Neveu-Yukawa model [69], in $d = 4-\epsilon$, we can study the interaction between one scalar field ϕ and N massless Dirac fermions ψ and $\bar{\psi}$. The γ_{KLS}-critical indices are given by

$$\eta_{\psi,\,\gamma_{KLS}} = \eta_\psi + \gamma_{KLS}\frac{2\epsilon}{(2N+3)[(2N+1)(1-\gamma_{KLS})+2(1-3\gamma_{KLS})]}, \tag{25}$$

$$\eta_{\phi,\,\gamma_{KLS}} = \eta_\phi + \gamma_{KLS}\frac{8N\epsilon}{(2N+3)[(2N+1)(1-\gamma_{KLS})+2(1-3\gamma_{KLS})]}, \tag{26}$$

$$\nu_{\gamma_{KLS}}^{-1} = \nu^{-1} - \frac{A_{N,\,\gamma_{KLS}}}{(2N+3)[(2N+1)(1-\gamma_{KLS})+2(1-3\gamma_{KLS})]}\epsilon, \tag{27}$$

$$A_{N,\,\gamma_{KLS}} = (2N+3)[R_{N,\,\gamma_{KLS}}/6(1-\gamma_{KLS}) + 2N(1-\gamma_{KLS})] - [(2N+1)(1-\gamma_{KLS}) + 2(1-3\gamma_{KLS})](R_N/6 + 2N), \tag{28}$$

$$R_{N,\,\gamma_{KLS}} = -[(2N-3)(1-\gamma_{KLS}) + 4\gamma_{KLS}] + \sqrt{[(2N-3)(1-\gamma_{KLS}) + 4\gamma_{KLS}]^2 + 144N/(1-\gamma_{KLS})}, \tag{29}$$

$$R_N = \lim_{\gamma_{KLS}\to 0} R_{N,\,\gamma_{KLS}}. \tag{30}$$

In Ref. [70], η_ψ, η_ϕ and ν were evaluated up to four-loop level.

K. γ_{KLS}-short- and long-range directed percolation

Consider γ_{KLS}-short- and γ_{KLS}-long-range directed percolation [71, 72] in $d = 4-\epsilon$ and $d = 2\sigma - \varepsilon$ dimensions, respectively. Then we obtain

$$\eta_{\gamma_{KLS}} = \eta - \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{\epsilon}{3}, \tag{31}$$

$$\eta_{\sigma,\,\gamma_{KLS}} = \eta_\sigma - \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{2\varepsilon}{7}, \tag{32}$$

$$\nu_{\gamma_{KLS}} = \nu + \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{\epsilon}{8}, \tag{33}$$

$$\nu_{\sigma,\,\gamma_{KLS}} = \nu_\sigma + \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{4\varepsilon}{7\sigma^2}, \tag{34}$$

$$z_{\gamma_{KLS}} = z - \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{\epsilon}{6}, \tag{35}$$

$$z_{\sigma,\,\gamma_{KLS}} = z_\sigma - \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{2\varepsilon}{7}. \tag{36}$$

The standard critical exponents have been computed up to two- and one-loop level in Ref. [71], respectively.

L. γ_{KLS}-short- and long-range dynamic isotropic percolation

Now consider γ_{KLS}-short- and γ_{KLS}-long-range dynamic isotropic percolation [71, 72] at $d = 6-\epsilon$ and $d = 3\sigma - \varepsilon$ dimensions, respectively. Their critical exponents are given by

$$\eta_{\gamma_{KLS}} = \eta - \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{2\epsilon}{21}, \tag{37}$$

$$\eta_{\sigma,\,\gamma_{KLS}} = \eta_\sigma - \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{3\varepsilon}{4}, \tag{38}$$

$$\nu_{\gamma_{KLS}} = \nu + \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{5\epsilon}{42}, \tag{39}$$

$$\nu_{\sigma,\,\gamma_{KLS}} = \nu_\sigma + \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{\varepsilon}{2\sigma^2}, \tag{40}$$

$$z_{\gamma_{KLS}} = z - \frac{\gamma_{KLS}}{1-3\gamma_{KLS}}\frac{\epsilon}{3}, \tag{41}$$

$$z_{\sigma,\,\gamma_{KLS}} = z_\sigma - \frac{\gamma_{KLS}}{1-3\gamma}\frac{3\varepsilon}{8}. \tag{42}$$

The nongeneralized indices were computed up to two- and one-loop level in Ref. [71], respectively.

V. CONCLUSIONS

In summary, we have introduced some field-theoretic approach for evaluated the critical properties of γ_{KLS}-generalized systems undergoing continuous phase transitions, called γ_{KLS}-statistical field theory. Although it was capable of encompassing nonconventional critical exponents for real imperfect systems not described by standard statistical field theory, it is not complete. For example, it does not explain the results of Table IV for some other manganites, being explained for NSFT of Ref. [21] (there might be alternative models capable of accurately representing the experimental data). So, it has to be discarded for statistical mechanics generalization purposes.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

ACKNOWLEDGMENTS

PRSC would like to thank the Brazilian funding agencies CAPES and CNPq (Grant: Produtividade 307982/2019-0) for financial support.

[1] D. Turki, Z. K. Ghouri, S. Al-Meer, K. Elsaid, M. I. Ahmad, A. Easa, G. Remenyi, S. Mahmood, E. K. Hlil, M. Ellouze, and F. Elhalouani, Magnetochemistry **3**, 28 (2017).
[2] J. Khelifi, A. Tozri, E. Dhahri, and E. Hlil, J. Magn. Magn. Mater. **349**, 149 (2014).
[3] B. Padmanabhan, H. L. Bhat, S. Elizabeth, S. Rößler, U. K. Rößler, K. Dörr, and K. H. Müller, Phys. Rev. B **75**, 024419 (2007).
[4] A. Omri, A. Tozri, M. Bejar, E. Dhahri, and E. Hlil, J. Magn. Magn. Mater. **324**, 3122 (2012).
[5] N. Ghosh, S. Rößler, U. K. Rößler, K. Nenkov, S. Elizabeth, H. L. Bhat, K. Dörr, and K.-H. Müller, J. Phys. Condens. Matter **18**, 557 (2005).
[6] J. Yang and Y. P. Lee, Appl. Phys. Lett. **91**, 142512 (2007).
[7] S. Ghodhbane, A. Dhahri, N. Dhahri, E. Hlil, J. Dhahri, M. Alhabradi, and M. Zaidi, J. Alloys Compd. **580**, 558 (2013).
[8] A. Berger, G. Campillo, P. Vivas, J. E. Pearson, and S. D. Bader, J. Appl. Phys. **91**, 8393 (2002).
[9] M. Sahana, U. K. Rössler, N. Ghosh, S. Elizabeth, H. L. Bhat, K. Dörr, D. Eckert, M. Wolf, and K.-H. Müller, Phys. Rev. B **68**, 144408 (2003).
[10] F. Ben Jemaa, S. H. Mahmood, M. Ellouze, E. K. Hlil, and F. Halouani, J. Mater. Sci. **49**, 6883 (2014).
[11] S. Rößler, U. K. Rößler, K. Nenkov, D. Eckert, S. M. Yusuf, K. Dörr, and K.-H. Müller, Phys. Rev. B **70**, 104417 (2004).
[12] A. K. Pramanik and A. Banerjee, Phys. Rev. B **79**, 214426 (2009).
[13] L. Vasiliu-Doloca, J. W. Lynn, Y. M. Mukovskii, A. A. Arsenov, and D. A. Shulyatev, J. Appl. Phys. **11**, 7342 (2007).
[14] B. Yu, W. Sun, J. Fan, X. Lan, W. Zhang, Y. Zhu, H. Han, L. Zhang, L. Ling, and H. Yang, Mater. Res. Bull. **99**, 393 (2018).
[15] A. Oleaga, A. Salazar, M. Ciomaga Hatnean, and G. Balakrishnan, Phys. Rev. B **92**, 024409 (2015).
[16] P. Zhang, P. Lampen, T. Phan, S. Yu, T. Thanh, N. Dan, V. Lam, H. Srikanth, and M. Phan, J. Magn. Magn. Mater. **348**, 146 (2013).
[17] T.-L. Phan, Q. Tran, P. Thanh, P. Yen, T. Thanh, and S. Yu, Solid State Commun. **184**, 40 (2014).
[18] M. Jeddi, J. Massoudi, H. Gharsallah, S. I. Ahmed, E. Dhahri, and E. K. Hlil, RSC Adv. **11**, 7238 (2021).
[19] S. Hcini, M. Boudard, S. Zemni, and M. Oumezzine, Ceram. Int. **41**, 2042 (2015).
[20] M. Baazaoui, M. Oumezzine, and W. Cheikhrouhou-Koubaa, Phys. Solid State **62**, 278 (2014).
[21] P. R. S. Carvalho, Phys. Lett. B **838**, 137683 (2023).
[22] K. G. Wilson and M. E. Fisher, Phys. Rev. Lett. **28**, 240 (1972).
[23] C. Tsallis, J. Stat. Phys. **52**, 479 (1988).
[24] G. Kaniadakis, Physica A **296**, 405 (2001).
[25] C. Beck and E. Cohen, Physica A **322**, 267 (2003).
[26] G. Kaniadakis, M. Lissia, and A. M. Scarfone, Phys. Rev. E **71**, 046128 (2005).
[27] G. Kaniadakis, Eur. Phys. J. B **70**, 3 (2009).
[28] T. F. A. Alves, J. F. S. Neto, F. W. S. Lima, G. A. Alves, and P. R. S. Carvalho, Phys. Lett. B **843**, 138005 (2023).
[29] A. Plastino and M. C. Rocca, Chin. Phys. C **42**, 053102 (2018).
[30] M. Phan, V. Franco, N. Bingham, H. Srikanth, N. Hur, and S. Yu, J. Alloys Compd. **508**, 238 (2010).
[31] A. Arrott and J. E. Noakes, Phys. Rev. Lett. **19**, 786 (1967).
[32] J. S. Kouvel and M. E. Fisher, Phys. Rev. **136**, A1626 (1964).
[33] S. Mnefgui, N. Zaidi, A. Dhahri, E. Hlil, and J. Dhahri, J. Solid State Chem. **215**, 193 (2014).
[34] D. Ginting, D. Nanto, Y. Zhang, S. Yu, and T. Phan, Phys. B: Condens. Matter **412**, 17 (2013).
[35] T.-L. Phan, T. Thanh, and S. Yu, J. Alloys Compd. **615**, S247 (2014).
[36] P. Nisha, S. Savitha Pillai, A. Darbandi, M. R. Varma, K. Suresh, and H. Hahn, Mater. Chem. Phys. **136**, 66 (2012).
[37] P. Nisha, S. Savitha Pillai, M. R. Varma, and K. Suresh, Solid State Sci. **14**, 40 (2012).
[38] O. F. de Alcantara Bonfim, J. E. Kirkham, and A. J. McKane, J. Phys. A **14**, 2391 (1981).
[39] L. Zambelli and O. Zanusso, Phys. Rev. D **95**, 085001 (2017).
[40] M. Borinsky, J. A. Gracey, M. V. Kompaniets, and O. Schnetz, Phys. Rev. D **103**, 116024 (2021).
[41] M. Stephen and J. McCauley, Phys. Lett. A **44**, 89 (1973).
[42] J. Hager and L. Schäfer, Phys. Rev. E **60**, 2071 (1999).
[43] J. S. Hager, J. Phys. A **35**, 2703 (2002).
[44] E. Brezin, G. Parisi, and F. Ricci-Tersenghi, J. Stat. Phys. **157**, 855 (2014).
[45] M. Lohmann, G. Slade, and B. C. Lallace, J. Stat. Phys. **169**, 1132 (2017).
[46] G. Slade, Commun. Math. Phys. **358**, 343 (2018).
[47] M. E. Fisher, S.-k. Ma, and B. G. Nickel, Phys. Rev. Lett. **29**, 917 (1972).
[48] D. J. Gross and A. Neveu, Phys. Rev. D **10**, 3235 (1974).
[49] J. A. Gracey, T. Luthe, and Y. Schröder, Phys. Rev. D **94**, 125028 (2016).
[50] E. Brézin and J. Zinn-Justin, Phys. Rev. B **13**, 251 (1976).
[51] T. H. Berlin and M. Kac, Phys. Rev. **86**, 821 (1952).
[52] H. E. Stanley, Phys. Rev. **176**, 718 (1968).
[53] R. M. Hornreich, M. Luban, and S. Shtrikman, Phys. Rev. Lett. **35**, 1678 (1975).
[54] M. M. Leite, Phys. Rev. B **67**, 104415 (2003).
[55] M. M. Leite, Phys. Rev. B **72**, 224432 (2005).
[56] L. C. de Albuquerque and M. M. Leite, J. Phys. A **34**, L327 (2001).
[57] M. M. Leite, Phys. Lett. A **326**, 281 (2004).
[58] M. M. Leite, Phys. Rev. B **61**, 14691 (2000).
[59] M. M. Leite, Phys. Rev. B **68**, 052408 (2003).
[60] C. F. Farias and M. M. Leite, J. Stat. Phys. **148**, 972 (2012).
[61] J. B. da Silva Jr. and M. M. Leite, J. Math. Phys. **53**, 043303 (2012).
[62] M. V. S. Santos and M. M. Leite, J. Phys. Conf. Ser. **490**, 012232 (2014).
[63] M. I. de Sena and M. M. Leite, J. Phys. Conf. Ser. **574**, 012170 (2015).

[64] M. V. S. Santos, J. B. da Silva Jr, and M. M. Leite, Eur. Phys. J. Plus **134**, 372 (2019).
[65] M. V. S. Santos, J. B. da Silva Jr, and M. M. Leite, Eur. Phys. J. Plus **134**, 4 (2019).
[66] M. M. Leite, EPL **137**, 34001 (2022).
[67] W. K. Theumann and M. A. Gusmão, Phys. Rev. B **31**, 379 (1985).
[68] C. Itzykson and J. M. Drouffe, *Statistical Field Theory: Vol. 1* (Cambridge Monographs on Mathematical Physics, 1989).
[69] J. Zinn-Justin, Nucl. Phys. B **367**, 105 (1991).
[70] N. Zerf, L. N. Mihaila, P. Marquard, I. F. Herbut, and M. M. Scherer, Phys. Rev. D **96**, 096010 (2017).
[71] Hans-Karl Janssen and U. C. Täuber, Ann. Phys. **315**, 147 (2005).
[72] U. C. Täuber, M. Howard, and B. P. Vollmayr-Lee, J. Phys. A Math. Theor. **38**, R79 (2005).