

Spin caloric resistance of Dirac plasma in graphene Corbino device

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The thermal resistance of a spin-polarized hydrodynamic Dirac plasma in graphene is considered. A mechanism for the coupling of heat and spin flows is discussed, demonstrating that spin diffusion and spin thermocurrent modify viscous dissipation, leading to a significant enhancement of thermal resistance. Practical calculations are then presented for graphene devices in the Corbino geometry.

Thermoelectric (TE) and thermomagnetic (TM) transport effects in metals reflect particle-hole asymmetry and arise from the coupling of charge and heat currents [1]. A different class of thermomagnetolectric (TME) effects emerges when one considers coupling of charge and heat flow with the spin current [2, 3]. The distinction between TM and TME phenomena lies in the former being driven by the Lorentz force of the applied external magnetic field exerted on the charge flow, while the latter arises from the flow of spins driven by a magnetic field.

In metallic magnetic heterostructures, such as spin valves, spin-dependent thermoelectric effects can be understood through two main models. The first model involves two parallel spin-transport channels with distinct thermoelectric properties. Alternatively, these effects can be attributed to collective phenomena, influenced by spin waves, which are also present in insulating ferromagnets. For further references, refer to the collection of papers and reviews on spin caloritronics [4, 5]. The two-channel model, incorporating spin-dependent conductivities, can be derived from the kinetic equation. This model assumes different drift velocities for two spin subsystems and is particularly suitable in the weak interaction limit. Essentially, it extends the single-particle Drude-Sommerfeld theory to account for spin-polarized transport. Consequently, it adheres to Mott's law for the Seebeck coefficient and the Wiedemann-Franz law for spin-dependent thermal conductivity. In contrast, in the collision-dominated regime, the drift velocity is the same for all the spin components. The reason is that frequent collisions between electrons with different spins form a common drift of the electron system. This scenario can be realized in high-mobility and low-density semiconductor and semimetal devices, where strong correlations lead to the electron liquid reaching a hydrodynamic limit [6, 7]. In this situation, the coupling of spin and heat currents exhibits a distinct character, leading to the specific mechanism of thermal resistance explored in this paper.

The hydrodynamic behavior of the electron liquid has been demonstrated in a number of recent experiments, see reviews [8–11] on this topic and references therein. Of particular relevance to this work are strong thermoelectric anomalies observed in graphene devices tuned to the proximity of charge neutrality [12, 13]. Physically,

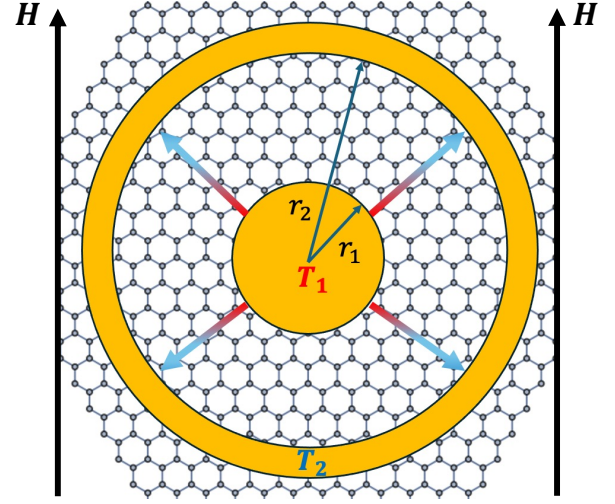


FIG. 1. A schematic for the graphene Corbino disk. Inner electrode has radius r_1 and is kept at temperature T_1 . The outer electrode has radius r_2 and temperature $T_2 < T_1$. The arrows pointing outwards from the center mark heat fluxes carried by electron fluid in graphene layer which is subjected to the in-plane magnetic field H .

these profound features, namely giant enhancement of the Lorenz ratio and violation of the Mott relation by inelastic scattering, stem from the decoupling of charge and heat flows.

The hydrodynamic equations in the presence of finite spin-polarization of an electron liquid induced by an in-plane magnetic field should be supplemented by the conservation law for spin. This is particularly relevant to graphene multilayers, as the spin relaxation time is known to be long, even at room temperature, in the range of nanoseconds, with corresponding spin diffusion lengths reaching $10 \mu\text{m}$ [14, 15]. In the linear response, the spin current consists of a spin diffusion current and a spin thermocurrent induced by temperature gradients. According to Onsager's reciprocity principle [16], the heat flux acquires an additional spin-dependent contribution related to the Peltier effect. These processes modify the viscous flow of the electron fluid, thus influencing dissipative friction and consequently strongly increasing thermal resistance.

To explore this physics, a particularly useful device ge-

ometry is a graphene Corbino membrane, as illustrated in Fig. 1. In this setup, the continuity of currents enforces the electronic flow profile to be spatially nonuniform, even at uniform particle and entropy density. This amplifies the interplay of dissipative viscous, spin-diffusion, and spin-thermal effects. It should be noted that charge and thermal transport measurements in gated graphene Corbino devices have been reported, including observations of magneto-thermal effects [17–19]. Additionally, powerful thermal and electrochemical potential imaging techniques can be implemented [20–23]. Thus far, theoretical works have focused on the orbital effects of the magnetic field in the magnetotransport [24–27], while hydrodynamic effects dominated by spin thermotransport remain unexplored, which motivates this work.

To this end, consider a two-dimensional electron system subjected to an in-plane magnetic field. In the regime where the rate of momentum-conserving electron-electron collisions exceeds the rates of momentum, energy, and spin relaxation, the corresponding macroscopic hydrodynamic equations of the electron fluid can be written as follows:

$$\partial_t n + \nabla \cdot \mathbf{j} = 0, \quad (1a)$$

$$\partial_t s + \nabla \cdot \mathbf{j}_s = \dot{s}, \quad (1b)$$

$$\partial_t \sigma + \nabla \cdot \mathbf{j}_\sigma = \dot{\sigma}, \quad (1c)$$

$$\partial_t \mathbf{p} + k\mathbf{u} + \nabla P + en\nabla\phi = \nabla \cdot \hat{\Sigma}. \quad (1d)$$

Here P is the pressure, $\mathbf{u}$ is the hydrodynamic velocity, k denotes the disorder-induced friction coefficient, and $\hat{\Sigma} \equiv \Sigma_{ij}$ denotes the viscous stress tensor [28]

$$\Sigma_{ij} = \eta(\partial_i u_j + \partial_j u_i) + (\zeta - \eta)\delta_{ij}\partial_k u_k, \quad (2)$$

with η and ζ being, respectively, shear (first) and bulk (second) viscosity of the electron liquid. The electric potential ϕ is related to the electron charge density en by the Poisson equation. The densities of particles, entropy, momentum, and spin are denoted by n , s , $\mathbf{p}$, and σ respectively. The corresponding current densities of particles, entropy, and spin are labeled respectively as $\mathbf{j}$, $\mathbf{j}_s$, and $\mathbf{j}_\sigma$. The local rate of entropy production due to electron-electron collisions is denoted by $\dot{s}$ in Eq. (1b). Likewise, $\dot{\sigma}$ in Eq. (1c) corresponds to the spin relaxation. The continuity equation for the spin current is written in analogy to the hydrodynamic theory of spin waves [29]. It should be noted that only spin component along the field appears in this framework as the other spin components are not conserved due to spin precession.

We proceed to determine the entropy production rate. For this purpose, we recall that the entropy density s is a

function of the conserved quantities, and its differential is given by the general thermodynamic relation [30]

$$ds = \frac{d\varepsilon}{T} - \frac{(\mu + e\phi)dn}{T} - \frac{\mu_\sigma d\sigma}{T} - \frac{\mathbf{u} \cdot d\mathbf{p}}{T}, \quad (3)$$

where μ and μ_σ are chemical potentials for particle and spin. Noting that all these thermodynamic variables correspond to conserved quantities, we express their time derivatives in terms of the divergences of the corresponding currents from Eq. (1). This yields the following expression for $\dot{S} = \int \dot{s} d\mathbf{r}$,

$$\begin{aligned} \dot{S} = \int \frac{1}{T} & [-\nabla \cdot \mathbf{j}_\varepsilon + (\mu + e\phi)\nabla \cdot \mathbf{j} + \mu_\sigma \nabla \cdot \mathbf{j}_\sigma \\ & + k\mathbf{u}^2 + n\mathbf{u} \cdot \nabla e\phi + \mathbf{u} \cdot \nabla \cdot \hat{\Pi}] d\mathbf{r}, \end{aligned} \quad (4)$$

where we used $\partial_t \varepsilon = -\nabla \cdot \mathbf{j}_\varepsilon$ for the energy current density $\mathbf{j}_\varepsilon$, and similarly for $\partial_t n$, $\partial_t \sigma$, and $\partial_t \mathbf{p}$ from Eqs. (1a)-(1c). The tensor $\hat{\Pi} \equiv \Pi_{ij} = P\delta_{ij} - \Sigma_{ij}$ denotes the local part of the momentum flux. Next, using the thermodynamic identity $\nabla P = n\nabla\mu + s\nabla T + \sigma\nabla\mu_\sigma$ and integrating by parts we get

$$\begin{aligned} T\dot{S} = \int & [-\mathbf{j}'_s \nabla T - \mathbf{j}' \nabla(\mu + e\phi) - \mathbf{j}'_\sigma \nabla \mu_\sigma \\ & + k\mathbf{u}^2 + \Sigma_{ij} \partial_j u_i] d\mathbf{r} \end{aligned} \quad (5)$$

where we introduced dissipative fluxes $\mathbf{j}'_s = \mathbf{j}_s - s\mathbf{u}$, $\mathbf{j}' = \mathbf{j} - n\mathbf{u}$, and $\mathbf{j}'_\sigma = \mathbf{j}_\sigma - \sigma\mathbf{u}$ that are proportional to gradients of the equilibrium parameters. The entropy flux is related to the energy flux as follows $\mathbf{j}_s = \frac{1}{T}[\mathbf{j}_\varepsilon - (\mu + e\phi)\mathbf{j} - \mu_\sigma \mathbf{j}_\sigma]$.

In a situation with a vanishing electromotive force, $e\mathcal{E} = -\nabla(\mu + e\phi)$, the second term in the above expression is absent since $\mathbf{j}' = 0$. Recalling the definition $\dot{S} = -\sum_i X_i \dot{x}_i$ from Ref. [30] and identifying $\dot{x}_i$ with the entropy current $\mathbf{j}'_s$ and the spin diffusion current $\mathbf{j}'_\sigma$ we conclude that the role of the corresponding thermodynamic forces is played by $\nabla T/T$ and $\nabla \mu_\sigma/T$ respectively. Introducing the kinetic coefficients following the standard convention $\dot{x}_i = -\gamma_{ij} X_j$ we write

$$\begin{pmatrix} \mathbf{j}'_s \\ \mathbf{j}'_\sigma \end{pmatrix} = -\frac{1}{T} \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} \begin{pmatrix} \nabla T \\ \nabla \mu_\sigma \end{pmatrix}. \quad (6)$$

We can express $\gamma_{11} = \kappa$ in terms of the thermal conductivity κ , and γ_{22} in terms of the spin diffusion D_σ as $\gamma_{22} = TD_\sigma$. Since the spin density changes sign with respect to time reversal symmetry and energy does not, the Onsager symmetry principle gives $\gamma_{12}(H) = -\gamma_{21}(-H)$. On the other hand both $\gamma_{12}(H)$ and $\gamma_{21}(H)$ are odd functions of the magnetic field H . Therefore, we conclude that $\gamma_{12}(H) = \gamma_{21}(H)$. Thus the matrix of kinetic coefficients is symmetric. In the following we denote $\gamma_{12} \equiv \gamma_\sigma$ and introduce

$$\hat{\Xi} \equiv \hat{\gamma}/T = \begin{pmatrix} \kappa/T & \gamma_\sigma/T \\ \gamma_\sigma/T & D_\sigma \end{pmatrix}. \quad (7)$$

As a reminder, for temperatures $T < E_F$ in the Fermi liquid theory $\kappa \sim E_F^2/T$ [31] and $D_\sigma \sim A/T^2$ [32], modulo logarithmic renormalizations specific to the 2D case, see e.g. Ref. [33]. A detailed analytical calculations of these coefficients in the limit of weak interactions, $r_s = V/E_F \lesssim 1$, where V is the characteristic energy of Coulomb interaction and E_F is the bare Fermi energy, can be found in Ref. [34].

For compactness of further expressions, it is convenient to introduce the column-vector notations

$$\vec{s} = \begin{pmatrix} s \\ \sigma \end{pmatrix}, \quad \vec{X} = \begin{pmatrix} \nabla T \\ \nabla \mu_\sigma \end{pmatrix}. \quad (8)$$

Then it can be readily checked that the sum of the first and third term in the right hand side of Eq. (5) can be rewritten as $\vec{X}^\mathbb{T} \hat{\Xi} \vec{X}$, with the superscript $\mathbb{T}$ denoting the transposition. As a result,

$$T\dot{S} = \int \left[\Sigma_{ij} \partial_j u_i + \vec{X}^\mathbb{T} \hat{\Xi} \vec{X} + k u^2 \right] d\mathbf{r}, \quad (9)$$

where summation over the repeated indices is implicit. The current densities take the final form in explicit notations

$$\mathbf{j}_s = s\mathbf{u} - \frac{\kappa}{T} \nabla T - \frac{\gamma_\sigma}{T} \nabla \mu_\sigma, \quad (10a)$$

$$\mathbf{j}_\sigma = \sigma\mathbf{u} - \frac{\gamma_\sigma}{T} \nabla T - D_\sigma \nabla \mu_\sigma, \quad (10b)$$

where the first term in the right hand side represents the equilibrium components of the currents, while the remaining two terms in each expression represent the dissipative components. With the appropriate boundary conditions for a given device geometry, the system of equations Eqs. (1), (9), and (10) enable determination of the linear response transport coefficients.

We apply the above formalism to consider heat transport in a graphene flake shaped like a Corbino disk, see Fig. 1. The aspect ratio of the disk is defined by the ratio of its radii, $p = r_2/r_1 > 1$. In the linear response regime, a small heat current is induced in the device by a temperature difference $\Delta T = T_1 - T_2$ between the electrodes. The calculations presented below will focus on the regime of charge neutrality, where $n \rightarrow 0$. For simplicity, we also ignore the effect of bulk viscosity, which is known to vanish for both parabolic and linear spectra of quasiparticle excitations [35]. The latter corresponds to the case of graphene monolayer.

In a steady state, the continuity equation of entropy and spin currents can be presented in the form specific to the cylindrical geometry

$$\vec{x}u(r) - \hat{\Xi} \vec{X}(r) = \frac{\vec{I}}{2\pi r}, \quad \vec{I} = \begin{pmatrix} I_s \\ I_\sigma \end{pmatrix}, \quad (11)$$

where $u(r)$ is the radial hydrodynamic velocity and $\vec{X}(r)$ is the column-vector of thermodynamically conjugate

forces, with $r \in [r_1, r_2]$ being the radial coordinate. The force balance condition exerted on an element of the fluid takes the form of the Navier-Stokes equation, which can be found with the help of Eqs. (1d) and (2):

$$\eta \hat{\Delta} u(r) - k u(r) = \vec{x}^\mathbb{T} \vec{X}, \quad \hat{\Delta} = \frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} \right) - \frac{1}{r^2}. \quad (12)$$

Equations (11) and (12) determine the spatial dependence of the flow velocity, temperature gradients, and gradients of the spin chemical potential in the interior of the Corbino disk in terms of the entropy and spin currents $\vec{I}$. The crucial point to realize here is that even though thermal resistance is defined under the global condition of vanishing spin current, locally dissipative parts of these currents are nonzero due to finite $\vec{X}(r)$ in the bulk of the flow. This can be seen explicitly.

For this purpose, we use Eq. (11) to exclude $\vec{X}(r)$ and arrive at the equation for $u(r)$ in the form

$$\hat{\Delta} u - k u - \frac{u}{l^2} = -\frac{1}{2\pi r \eta} (\vec{x}^\mathbb{T} \hat{\Xi}^{-1} \vec{I}). \quad (13)$$

Here we introduced the characteristic length scale l defined by

$$l^{-2} = \frac{\vec{x}^\mathbb{T} \hat{\Xi}^{-1} \vec{x}}{\eta} = \frac{s^2 \left[D_\sigma - 2 \frac{\sigma \gamma_\sigma}{s T} + \frac{\sigma^2 \kappa}{s^2 T} \right]}{\eta \left[\frac{\kappa D_\sigma}{T} - \frac{\gamma_\sigma^2}{T^2} \right]}. \quad (14)$$

The general solution of Eq. (13) consists of the linear superposition of solutions to the homogeneous equation and a particular solution due to the right hand side. The former is given by a linear combination of modified Bessel functions of the first and second kinds. These functions describe deviations of the hydrodynamic flow from that in the bulk of the disk. Due to properties of Bessel functions they are exponentially localized near the inner and outer boundaries on the length scale of the order of l . For the graphene monolayer this length scale is of the order of thermal de Broglie length $l \sim l_T = v/T$, which in itself is of the order of electron mean free path l_{ee} . Therefore, these solutions contribute to the resistance of the contacts. Consideration of these effects goes beyond the limit of hydrodynamic theory. We are interested in the solution that describes the hydrodynamic mode in the bulk of the device. This solution can be easily found as a particular solution of the inhomogeneous equation, yielding the result

$$u(r) = \frac{f}{2\pi r} \frac{\vec{x}^\mathbb{T} \hat{\Xi}^{-1} \vec{I}}{\vec{x}^\mathbb{T} \hat{\Xi}^{-1} \vec{x}}, \quad f = \frac{1}{1 + k l^2 / \eta}. \quad (15)$$

The factor defined by f can be safely set to unity. Indeed, viscosity of the Dirac fluid is $\eta \sim (T/v)^2 \sim l_T^{-2}$ [36]. In addition, friction coefficient can be related to the spatial average of the local density fluctuations $\delta n(\mathbf{r})$ of the globally charge neutral system, namely $k \sim \frac{e^2}{\epsilon} \langle \delta n^2 \rangle$

[37], where ς is the intrinsic conductivity of graphene [38–40]. Therefore, $kl^2/\eta \sim \langle \delta n^2 \rangle l_T^4 \ll 1$. Based on this argument, we set $f \rightarrow 1$ in what follows. Knowledge of $u(r)$ enables us to compute dissipative friction term and also the viscous term in Eq. (9). For a purely radial flow, there are only two nonvanishing components of the stress tensor that contribute to the dissipated power. In cylindrical coordinates, these are

$$\Sigma_{rr} = 2\eta \frac{\partial u}{\partial r}, \quad \Sigma_{\phi\phi} = 2\eta \frac{u}{r}. \quad (16)$$

As the next step, we need to determine $\vec{X}(r)$, which governs contribution of the relative transport mode to the dissipation rate in Eq. (9). The required solution can be obtained by multiplying Eq. (11) by a row vector $\vec{z}^T = (\sigma, -s)$ from the left. The result is

$$\vec{X}(r) = -\frac{\vec{z}}{2\pi r} \frac{\vec{z}^T \vec{I}}{\vec{z}^T \hat{\Xi} \vec{z}}. \quad (17)$$

Finally, we introduce spin caloric resistance matrix $\hat{\mathcal{R}}$ that can be determined by equating the Joule heat,

$$\mathcal{P} = \vec{\mathcal{I}}^T \hat{\mathcal{R}} \vec{\mathcal{I}}, \quad \vec{\mathcal{I}}^T = (I_q, I_\sigma), \quad (18)$$

to the rate of energy dissipation in the bulk flow as defined by Eq. (9). Here $I_q = TI_s$ is the heat current. The thermal resistance R_{th} can be found by setting the spin current to zero $I_\sigma \rightarrow 0$. The computed dissipation in Eq. (18) is then simply $\mathcal{P} = \mathcal{R}_{11} I_q^2$. On the other hand, employing general thermodynamic relations, one finds the entropy production rate $\dot{S} = I_q \Delta T / T^2 = R_{\text{th}} I_q^2 / T^2$. As a consequence, from the condition $\mathcal{P} = T\dot{S}$ we determine that

$$R_{\text{th}} = T\mathcal{R}_{11} = \frac{1}{T I_s^2} (T\dot{S})_{I_\sigma \rightarrow 0}. \quad (19)$$

We insert Eqs. (15), (16), and (17) into Eq. (9), set $I_\sigma \rightarrow 0$, compute each of the radial integrals individually, and bring the resulting expression into Eq. (19) that will cancel I_s^2 in the denominator. The resulting formula can be written as a sum of respective contributions

$$R_{\text{th}} = R_\Sigma + R_\Xi + R_k. \quad (20)$$

The first term describes viscous dissipation modified by the spin diffusion and spin thermocurrent, which reads

$$R_\Sigma = \frac{\eta(p^2 - 1)}{\pi T s^2 r_2^2} \frac{(D_\sigma - \frac{\sigma\gamma_\sigma}{sT})^2}{(D_\sigma + \frac{\sigma}{s}\beta_\sigma)^2}, \quad (21a)$$

with the notation $\beta_\sigma = (\sigma\kappa/sT) - (2\gamma_\sigma/T)$. The second term captures contribution of the relative mode and is expressed solely in terms of the spin-dependent intrinsic dissipative coefficients of matrix $\hat{\Xi}$

$$R_\Xi = \frac{1}{2\pi T} \frac{(\sigma/s)^2 \ln p}{D_\sigma + \frac{\sigma}{s}\beta_\sigma}. \quad (21b)$$

The third term describes an extra contribution due to disorder-induced friction

$$R_k = \frac{k \ln p}{2\pi T s^2} \frac{(D_\sigma - \frac{\sigma\gamma_\sigma}{sT})^2}{(D_\sigma + \frac{\sigma}{s}\beta_\sigma)^2}. \quad (21c)$$

Eqs. (20) and (21) represent the main results of this work. Next we analyze several limiting cases.

In pristine systems, $k \rightarrow 0$, and without magnetic field, $\sigma \rightarrow 0$, the result simplifies to $R_{\text{th}} = \eta(p^2 - 1)/\pi T s^2 r_2^2$. For a graphene monolayer $\eta \sim s \sim (T/v)^2$, so that $R_{\text{th}} \propto 1/T^3$ [41].

The leading field dependent correction to R_{th} comes from R_Ξ . At low magnetic fields, we write the spin density in the form $\sigma = \chi H$, where χ denotes the spin susceptibility. As a result, for the relative thermal magnetoresistance, one obtains

$$\Delta R_{\text{th}}(H) \equiv \frac{R_{\text{th}}(H) - R_{\text{th}}(0)}{R_{\text{th}}(0)} \approx \frac{p^2 \ln p}{2(p^2 - 1)} \frac{r_1^2 (\chi H)^2}{\eta D_\sigma}, \quad (22)$$

which is controlled by the spin diffusion coefficient. We conclude that the spin caloric effect enables the extraction of D_σ and, thus, the spin conductance via the corresponding Einstein relation, in the transport regime dominated by electron collisions. This approach could offer valuable complementary experimental tools in addition to earlier methods based on spin Coulomb drag measurements [42].

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