Analysis of $B_s^0 \to X(3872)[\psi(2S)]\pi^+\pi^-(K^+K^-)$ decays

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We have phenomenologically investigated the decays $B_s^0 \to X(3872)\pi^+\pi^-(K^+K^-)$ and $B_s^0 \to \psi(2S)\pi^+\pi^-(K^+K^-)$. In our analysis, the scalar meson $f_0(980)$ is formed through the final state interactions of coupled channels $\pi\pi$ and $K\bar{K}$. Our findings indicate that the $\pi^+\pi^-$ invariant mass distribution of the $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay can be accurately reproduced. Furthermore, we have explored the $\pi^+\pi^-(K^+K^-)$ invariant mass distribution of the $B_s^0 \to X(3872)\pi^+\pi^-(K^+K^-)$ decay, accounting for the different production mechanisms between $X(3872)$ and $\psi(2S)$, up to a global factor. It is found that the production rates for $X(3872)$ and $\psi(2S)$ are much different, which indicates that the structure of $X(3872)$ is more complicated than the $\psi(2S)$, which is a conventional $c\bar{c}$ state. Additionally, we have considered the contributions from $f_0(1500)$ to $\pi^+\pi^-$ and the ϕ meson to K^+K^- in our analysis. Utilizing the model parameters, we have calculated the branching fraction of $B_s^0 \to X(3872)K^+K^-$, and anticipate that the findings of our study can be experimentally tested in the future.

I. INTRODUCTION

The nonleptonic weak decays of bottom hadrons are widely acknowledged as a valuable means to elucidate the nature of certain enigmatic hadrons [1–6], especially these decays with charmonia in the final states [7, 8]. For example, it was found that the scalar meson $f_0(500)$ has a relatively bigger signal than $f_0(980)$ in the decay of $\bar{B}^0$ into $J/\psi\pi^+\pi^-$ [9]. While the decay of $B_s^0 \to J/\psi\pi^+\pi^-$ was measured by the LHCb collaboration [10], and a pronounced peak was found for the scalar meson $f_0(980)$ in the $\pi^+\pi^-$ invariant mass distributions. However, there was no appreciable signal for the scalar meson $f_0(500)$ [10]. This counter-intuitive result attracted experimental and theoretical attention. New measurements about the B and B_s decays have been performed by the Belle Collaboration [11], CDF Collaboration [12], D0 Collaboration [13], and LHCb Collaboration [14, 15].

The $B_s^0 \to J/\psi\pi^+\pi^-$ decay was studied in Ref. [6] based on the final state interaction of pseudo-scalar meson-pseudo-scalar meson provided by the chiral unitary approach, where the scalar mesons $f_0(500)$ and $f_0(980)$ were dynamically generated. The theoretical results are in agreement with the experimental data [10]. The approach of Ref. [6] was successfully extended to study other weak decays of B_s^0 and B mesons [16–21] (see also Ref. [1] for an extensive review). The $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay was firstly measured by the LHCb collaboration [22] and the $f_0(980)$ meson played an important role in the $\pi^+\pi^-$ invariant mass distributions. Recently, the $B_s^0 \to X(3872)\pi^+\pi^-$ decay was also firstly observed by the LHCb collaboration [23], where a large contribution from $B_s^0 \to X(3872)[f_0(980) \to \pi^+\pi^-]$ was found. Determining the $f_0(980)$ nature in the B_s^0 decays is possible. Indeed, it is interesting to investigate $f_0(980)$ in $B_s^0 \to X(3872)\pi^+\pi^-$, since it is the analogous decay compared with the decay of B_s^0 into $\psi(2S)\pi^+\pi^-$ within the assumption that $X(3872)$ can be generated by the hadronization of $c\bar{c}$ which is used to produce $\psi(2S)$ in the former case. The $B_s^0 \to X(3872)\pi^+\pi^-$ decay is also a useful platform to explore the exotic feature of $X(3872)$ [24–33]. Even if it was discovered about two decades ago [34–43], its nature is still unclear. For instance, molecular perspective is one common explanation for $X(3872)$ rather than a pure chamonium. As discussed in Ref. [33], one has investigated the decays of B meson into $X(3872)$ with a pseudoscalar or vector meson based on the molecular perspective of $X(3872)$ from the interaction of $D\bar{D}^* + c.c.$(charge conjugate). Following the analysis about the $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay, we will also study the $B_s^0 \to X(3872)\pi^+\pi^-$ decay.

It is natural to study the role of $f_0(980)$ in the K^+K^- invariant mass distribution of $B_s^0 \to \psi(2S)K^+K^-$ and $B_s^0 \to X(3872)K^+K^-$ decays using the chiral unitary approach, since $f_0(980)$ has strong coupling to the $K\bar{K}$ channel [44, 45]. Note that, within the chiral unitary approach [46–49], the production of $f_0(980)$ and $f_0(500)$ mesons in B^0

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and B_s^0 into J/ψ and a $\pi^+\pi$ or K^+K^- pair were investigated in Refs. [4, 6, 21]. To understand the new experimental data collected by the LHCb collaboration [23] and study the nature of $X(3872)$ and the scalar meson $f_0(980)$, in this work, we perform a coherent analysis of the $B_s^0 \to X(3872)\pi^+\pi^-(K^+K^-)$ and $B_s^0 \to \psi(2S)\pi^+\pi^-(K^+K^-)$ decays. In addition to the $f_0(980)$, we also consider the contribution from the scalar meson $f_0(1500)$, since its signal is clearly seen in the invariant $\pi^+\pi^-$ mass distributions [23, 50, 51].

This paper is organized as follows. In Sec. II, we present the theoretical formalism for the production of the scalar meson $f_0(980)$ in the B_s^0 decays into $\psi(2S)$ or $X(3872)$ and $\pi^+\pi^-$ or K^+K^-, together with a discussion about the scalar meson $f_0(1500)$ in the corresponding decays, while the contribution of the ϕ meson in the $B_s^0 \to X(3872)K^+K^-$ decay is also shown. In Sec. III, we show our theoretical numerical results and discussions, followed by a summary in the last section.

II. THEORETICAL FORMALISM

A. The $B_s^0 \to \psi(2S)[f_0(980), f_0(1500) \to \pi^+\pi^-]$ decay

The leading contributions to the decays of B_s^0 into $\psi(2S)$ plus a scalar meson is the Cabibbo favored $\bar{b} \to c\bar{c}\bar{s}$ process, therefore, the decay diagram of $B_s^0 \to \psi(2S)[f_0(980) \to \pi^+\pi^-]$, at the quark level, is shown in Fig. 1, which can be separated into two steps. The first step, namely the Cabbibo favored process, consists of the $\bar{b}$ decaying into a $\bar{c}$ quark and a W^+ boson followed by its decay into a c quark and a $\bar{s}$ quark. Then, in addition to the hadronization of $c\bar{c}$ to produce $\psi(2S)$, we need another $q\bar{q}$ ($\equiv u\bar{u} + d\bar{d} + s\bar{s}$) pair to generate the $\pi^+\pi^-$ in the final states from $s\bar{s}$.

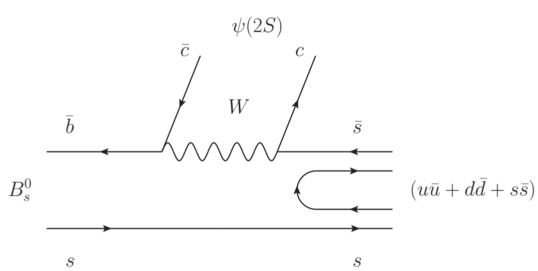


FIG. 1: Diagram for the decay of B_s^0 into $\psi(2S)$ (formed by the $c\bar{c}$ pair) and a primary $s\bar{s}$ pair, which hadronizes with an extra $(u\bar{u} + d\bar{d} + s\bar{s})$ pair from the vacuum.

Following Refs. [4, 6], the hadronization of $s\bar{s}$, in terms of pseudoscalar mesons, can be written as

$$s\bar{s}(u\bar{u} + d\bar{d} + s\bar{s}) \to K^+K^- + K^0\bar{K}^0 + \frac{2}{3}\eta\eta. \tag{1}$$

After the pseudoscalar meson-pseudoscalar meson pair is produced, final-state interactions between the mesons occur, where the $\pi^+\pi^-$ pair can be obtained in the final states. The scalar meson $f_0(980)$ is dynamically generated from the s-wave interaction of the pseudoscalar meson-pseudoscalar meson in coupled channels [52–54]. Hence, the decay amplitude for $B_s^0 \to \psi(2S)[f_0(980) \to \pi^+\pi^-]$ can be written as [6],

$$\begin{aligned}\mathcal{M}^{f_0(980)}_{B_s^0\to\psi(2S)\pi^+\pi^-} &= g_1\mathcal{M}_a = \frac{g_1 V_{cs}\boldsymbol{p}_{\psi(2S)}\cos\theta}{m_{B_s^0}} \times \\ &\Big(G_{K^+K^-}t_{K^+K^-\to\pi^+\pi^-} + G_{K^0\bar{K}^0}t_{K^0\bar{K}^0\to\pi^+\pi^-} \\ &+\frac{2}{3}\frac{1}{2}G_{\eta\eta}t_{\eta\eta\to\pi^+\pi^-}\Big),\end{aligned} \tag{2}$$

where $\boldsymbol{p}_{\psi(2S)}$ is the three momentum of $\psi(2S)$ in the center-mass system of B_s^0 and θ is an integration variable of final-state phase space. Note that for the $B_s^0 \to \psi(2S)[f_0(980) \to \pi^+\pi^-]$ decay, we shall need a p-wave interaction to match angular momentum conservation. We introduce a parameter g_1 to contain all dynamical factors, which is assumed to be real and positive in this work. The V_{cs} is one matrix element of the Cabbibo-Kobayashi-Maskawa matrix which is related to the Cabbibo angle [21]: $V_{cs} = \cos\theta_c = 0.97427$.

In Eq. (2), G_i is the loop function of two meson propagators

$$G_i(s) = \mathrm{i}\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\frac{1}{(P-q)^2 - m_1^2 + \mathrm{i}\varepsilon}\frac{1}{q^2 - m_2^2 + \mathrm{i}\varepsilon}, \tag{3}$$

where "i" represents the ith-channel, m_1, m_2, and q are the masses and four-momentum of one meson in this channel, respectively. P is the total momentum in this system, satisfying $s = P^2$. The three-momentum integral is carried out by precisely integrating the q^0 variable and applying a cutoff Λ of the order of 1 GeV, which is impacted by the number of channels. The element of the scattering matrix, t_{ij}, for the transition of channel i to j, is given by $t = (1 - VG)^{-1}V$. Now numbering the channels as 1 for $\pi^+\pi^-$, 2 for $\pi^0\pi^0$, 3 for K^+K^-, 4 for $K^0\bar{K}^0$, and 5 for $\eta\eta$, the V matrix can be used in the same form as [6]. It is worth noting whether or not considering the $\eta\eta$ channel does not affect the results much, as long as a reasonable cutoff Λ is used. See more details in Refs. [6, 44, 53]. We don't consider the $\eta\eta$ channel in this work and take $\Lambda = 903$ MeV. The loop function G and two-body scattering amplitude t depend on the invariant mass $M_{\pi\pi}$ of the $\pi^+\pi^-$ system.

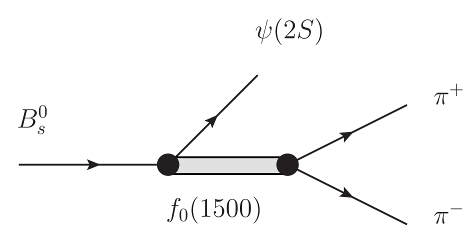


FIG. 2: Diagram for the decay of B_s^0 into $\psi(2S)$ and $\pi^+\pi^-$ through the resonance $f_0(1500)$.

In addition to the scalar meson $f_0(980)$, we consider the scalar meson, namely $f_0(1500)$ as shown in Fig. 2. It is treated in the amplitude as a Breit-Wigner (BW) propagator.

$$\mathcal{M}^{f_0(1500)}_{B_s^0\to\psi(2S)\pi^+\pi^-} = g_2\mathcal{M}_b = \frac{\mathrm{i}g_2 m_{f_0(1500)}\Gamma_{f_0(1500)}\boldsymbol{p}_{\psi(2S)}\cos\theta}{m_{B_s^0}\left(M^2_{\pi\pi} - m^2_{f_0(1500)} + \mathrm{i}m_{f_0(1500)}\Gamma_{f_0(1500)}\right)}, \quad (4)$$

where $m_{f_0(1500)}$ and $\Gamma_{f_0(1500)}$ are the mass and width of $f_0(1500)$. Here, g_2 is a free parameter, and we consider it real and positive. Furthermore, ongoing debates exist about the nature of $f_0(1500)$, and its mass and width are not well determined [55]. Hence, $m_{f_0(1500)}$ and $\Gamma_{f_0(1500)}$ will be fitted to the experimental data.

Then, the total decay amplitude for $B_s^0 \to \psi(2S)\pi^+\pi^-$ is written as,

$$\mathcal{M}_{B_s^0\to\psi(2S)\pi^+\pi^-} = g_1\mathcal{M}_a + g_2\mathcal{M}_b e^{\mathrm{i}\varphi}, \quad (5)$$

where φ is the relative phase between $\mathcal{M}_a$ and $\mathcal{M}_b$, and it is a free parameter. In fact, as discussed in Ref. [23], there are indeed contributions from the interference between $f_0(980)$ and $f_0(1500)$ to the $\pi^+\pi^-$ invariant mass spectrum of the $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay.

B. The mechanism of $B_s^0 \to X(3872)\pi^+\pi^-(K^+K^-)$

In contrast with the charmonium state $\psi(2S)$, the production of $X(3872)$ in the decay of $B_s^0 \to X(3872)\pi^+\pi^-$ may have a more involved mechanism because of the exotic nature of the $X(3872)$ state. Therefore, we should involve a different parameter g_1' [see Eq. (2)] for the $B_s^0 \to X(3872)\pi^+\pi^-$ decay: [1]

$$\mathcal{M}_{B_s^0\to X(3872)\pi^+\pi^-} = \frac{g_1' V_{cs}\boldsymbol{p}_{X(3872)}\cos\theta}{m_{B_s^0}} \times \left(G_{K^+K^-}t_{K^+K^-\to\pi^+\pi^-} + G_{K^0\bar{K}^0}t_{K^0\bar{K}^0\to\pi^+\pi^-}\right). \quad (6)$$

In other words, the mechanism for the production of $X(3872)$ is the same as that shown in Fig. 1 if we only consider the short-range contribution to the hadronization of $c\bar{c}$.

On the other hand, the contribution of $f_0(1500) \to \pi^+\pi^-$ in $B_s^0 \to X(3872)\pi^+\pi^-$ is different from that in the decay $B_s^0 \to \psi(2S)\pi^+\pi^-$. Referring to the masses of relevant particles in the Review of Particle Physics (RPP) [55], the phase space is tiny for the $B_s^0 \to X(3872)\pi^+\pi^-$ decay. The upper limit of the invariant mass of $M_{\pi\pi}$ is barely bigger than the mass of $f_0(1500)$, which means that the peak of $f_0(1500)$ in the $\pi^+\pi^-$ invariant mass distribution is seriously suppressed. Even if there is some contribution from $f_0(1500)$, it can be omitted in our mechanism. Thus, Eq. (6) is essentially the complete amplitude of the $B_s^0 \to X(3872)\pi^+\pi^-$ decay.

The K^+K^- pair can not only be directly produced by the hadronization of $s\bar{s}$ with $u\bar{u}$ from the vacuum in Fig. 1, but also be dynamically produced by the final-state interaction of $K\bar{K}$ in s-wave. According to the diagrams shown in Fig. 3, the decay amplitude of $B_s^0 \to X(3872)f_0(980) \to X(3872)K^+K^-$ is given by

$$\mathcal{M}_{B_s^0\to X(3872)K^+K^-} = \frac{g_1' V_{cs}\boldsymbol{p}_{X(3872)}\cos\theta}{m_{B_s^0}}\Big(1 + G_{K^+K^-}t_{K^+K^-\to K^+K^-} + G_{K^0\bar{K}^0}t_{K^0\bar{K}^0\to K^+K^-}\Big), \quad (7)$$

where we have used the same coupling constant g_1' as in Eq. (6) because of the similar mechanism and the same final state $X(3872)$.

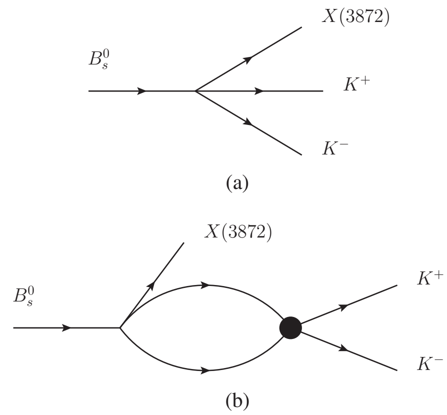


FIG. 3: Diagram for the decay of $B_s^0 \to X(3872)K^+K^-$ where K^+K^- is produced in s-wave. (a) is the tree diagram, and (b) is the rescattering.

On the other hand, we also consider the contribution of the ϕ meson to the $B_s^0 \to X(3872)K^+K^-$ decay. In this case, the K^+K^- is produced in p-wave. The decay amplitude is written as,

$$\mathcal{M}^{\phi}_{B_s^0\to X(3872)K^+K^-} = g_{BX\phi}g_{\phi KK}\varepsilon^{\mu\nu\rho\sigma} \times \epsilon^*_\mu(\boldsymbol{p}_X)p_{X\nu}q_\sigma\frac{\mathrm{i}(p_{K^+} - p_{K^-})_\rho}{q^2 - m_\phi^2 + \mathrm{i}m_\phi\Gamma_\phi}, \quad (8)$$

where $\epsilon^*_\mu(\boldsymbol{p}_X)$ and p_X are the polarization and four momentum of $X(3872)$. And $\epsilon^*_\nu(\boldsymbol{q})$, q, m_ϕ and Γ_ϕ are the polarization, four-momentum, mass, and width of the ϕ meson. Besides, $g_{BX\phi}$ and $g_{\phi KK}$ are the coupling parameters of the vertexes of $B_s^0X(3872)\phi$ and ϕKK. With the branching fractions of $\mathcal{B}[B_s^0 \to X(3872)\phi] = (1.1 \pm 0.4)\times 10^{-4}$ and $\mathcal{B}[\phi \to K^+K^-] = (49.1 \pm 0.5)\%$ from RPP [55], one can obtain that $g^2_{BX\phi} = (7.3 \pm 2.7)\times 10^{-22}\mathrm{MeV}^{-2}$ and $g^2_{\phi KK} = (20.0 \pm 0.2)$.

C. The mass distribution and partial decay width of $B_s^0 \to X(3872)[\psi(2S)]\pi^+\pi^-(K^+K^-)$

With these decay amplitudes obtained above, the $\pi^+\pi^-$ and K^+K^- invariant mass distributions of $B_s^0 \to$

[1] Note that the $\eta\eta$ channel is also neglected.

$X(3872)[\psi(2S)]\pi^+\pi^-(K^+K^-)$ decay can be easily obtained as follows

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}M_{\mathrm{inv}}} = \frac{1}{512\pi^5 m_{B_s^0}^2}\int \mathrm{d}\Omega\Omega^*|\boldsymbol{p}||\boldsymbol{p}^*||\mathcal{M}|^2, \quad (9)$$

where $(\boldsymbol{p}, \Omega)$ is the three momentum of $X(3872)$ or $\psi(2S)$ in the rest frame of B_s^0, while $(\boldsymbol{p}^*, \Omega^*)$ is the three momentum of one π (K) in the final $\pi^+\pi^-$ (K^+K^-) center of mass frame with invariant mass M_{inv}.

III. NUMERICAL RESULTS AND DISCUSSIONS

We calculate the invariant $\pi^+\pi^-$ mass distributions of the process $B_s^0 \to \psi(2S)\pi^+\pi^-$ with the above theoretical formalism. There are five free parameters to be obtained by fitting to the experimental data: g_1 in Eq. (2), g_2, $m_{f_0(1500)}$, and $\Gamma_{f_0(1500)}$ in Eq. (4), and φ in Eq. (5). Since our numerical results are $\mathrm{d}\Gamma/\mathrm{d}M_{\mathrm{inv}}$, and the experimental data are events as a function of the $\pi^+\pi^-$ invariant mass, there is a global factor C between our theoretical calculations and the experimental data. On the other hand, in the fitting to the experimental data, we use the following form:

$$\mathrm{data} = C\frac{\mathrm{d}\Gamma}{\mathrm{d}M_{\mathrm{inv}}} = \frac{Cg_2^2}{512\pi^5 m_{B_s^0}^2}\int \mathrm{d}\Omega\Omega^*|\boldsymbol{p}||\boldsymbol{p}^*| \times \left[(\frac{g_1}{g_2})^2|\mathcal{M}_a|^2 + |\mathcal{M}_b|^2 + \frac{2g_1}{g_2}\mathrm{Re}\Big(\mathcal{M}_a^*\mathcal{M}_b e^{\mathrm{i}\varphi}\Big)\right]. \quad (10)$$

Thus, we take Cg_2^2, g_1/g_2, $m_{f_0(1500)}$, $\Gamma_{f_0(1500)}$, and φ as free parameters. It should be noted that the global factor C can normalize the theoretical results to match the experimental mass distribution. And more importantly, the factor C is the same for the two processes B_s^0 into $\psi(2S)\pi^+\pi^-$ and $X(3872)\pi^+\pi^-$. In this way, the fitted parameters are listed in Table I. The obtained χ^2/d.o.f is 1.4, which is reasonably small.

TABLE I: The fitted parameters in this work.

Parameters	Fitting Results
Cg_2^2	$(2.77 \pm 0.35) \times 10^8$
g_1/g_2	0.68 ± 0.04
$\varphi(^\circ)$	-85.11 ± 8.65
$m_{f_0(1500)}$ (MeV)	1450.0 ± 6.8
$\Gamma_{f_0(1500)}$ (MeV)	164.4 ± 22.4
χ^2/d.o.f	1.4

The fitted results of the $\pi^+\pi^-$ invariant mass distributions of $B_s^0 \to \psi(2S)\pi^+\pi^-$ are shown in Fig. 4. One can see that thanks to the contributions from $f_0(980)$ and $f_0(1500)$, the experimental data can be well reproduced. In the calculations, the scalar meson $f_0(980)$ is produced in the final-state interaction of $K\bar{K}$ and $\pi\pi$ in coupled channels. The first higher peak can be described by only the $f_0(980)$ state. In contrast, the second small peak and the long tail between the two peaks can be reproduced by the $f_0(1500)$ and the interference between $f_0(980)$ and $f_0(1500)$. It is worth mentioning that the mass and width of $f_0(1500)$ state are mainly determined by the second peak, and the fitted results are different from the values quoted in the RPP [55].

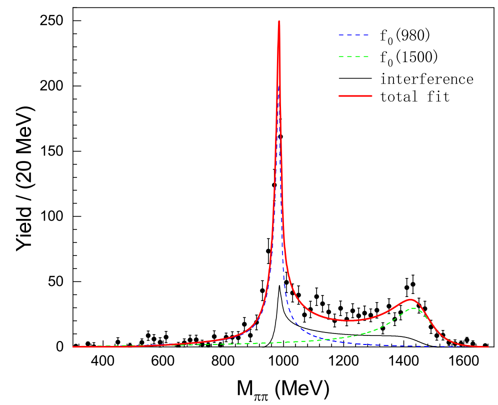


FIG. 4: Invariant mass distribution of $\pi^+\pi^-$ for the $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay, compared with the experimental data taken from Ref. [23]. The blue-dashed, green-dashed, and black-solid curves are the contributions from the $f_0(980)$, $f_0(1500)$, and their interference, respectively. The red-solid line is their total contribution.

With the fitted parameters and the branching ratio of $\mathcal{B}[B_s^0 \to \psi(2S)\pi^+\pi^-] = (6.9 \pm 1.2) \times 10^{-5}$ [55], we can extract the global factor C, which is $C = (8.28 \pm 1.44) \times 10^{17}$. Then, we can also get $g_2 = (1.83 \pm 0.20) \times 10^{-5}$ and $g_1 = (1.24 \pm 0.15) \times 10^{-5}$. If we take the same coupling constant g_1 for the $B_s^0 \to J/\psi\pi^+\pi^-$ decay, we obtain $\Gamma[B_s^0 \to J/\psi f_0(980) \to J/\psi\pi^+\pi^-] = (3.9 \pm 1.0) \times 10^{-14}$ MeV, which is in agreement with the value of $(5.4 \pm 0.6) \times 10^{-14}$ MeV quoted in the RPP [55]. This indicates that the coupling constants for producing charmonium states in the B_s^0 decays are universal.

Next, we turn to the $B_s^0 \to X(3872)\pi^+\pi^-$ decay. We firstly set $g_1' = g_1$. The resulting invariant mass $M_{\pi\pi}$ distribution of $B_s^0 \to X(3872)\pi^+\pi^-$ is shown as the black-dashed curve in Fig. 5. In this case, the obtained peak of $f_0(980)$ is too high compared with the available experimental data around 980 MeV. This indicates that the coupling of g_1' should differ from that of g_1. In another words, the production mechanism of $X(3872)$ and $\psi(2S)$ in the $B_s^0 \to X(3872)[\psi(2S)]\pi^+\pi^-$ decays are different. Indeed, the contributions from the long-distance $\bar{D}D^*$ scattering to the $X(3872)$ production in the B_s^0 decays are important [32, 33].

To get a good description of the experimental data on the $B_s^0 \to X(3872)\pi^+\pi^-$ decay, we modify the value of g_1' and enable the theoretical results to pass through the highest experimental point around $M_{\pi\pi} = 980$ MeV. We get $g_1' = 0.69g_1$, and the corresponding results are shown in

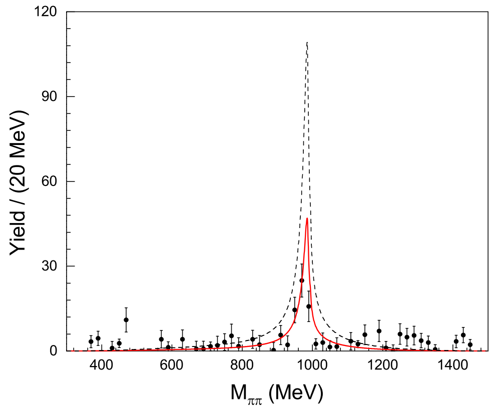


FIG. 5: Invariant mass distribution of $\pi^+\pi^-$ for the $B_s^0 \to X(3872)\pi^+\pi^-$ decay, compared with the experimental data [23]. The red-solid and black-dashed curves are obtained with different values for the production parameter g_1'.

Fig. 5 by the red curve. To explore more details about the difference between the production of $X(3872)$ and the charmonium states in the B_s^0 decays, we compare the modulus squared of their decay amplitudes, where the effect of the phase space is removed. For this purpose, we write,

$$|\mathcal{M}_{B_s^0\to Rf_0(980)}|^2 = \Gamma_{B_s^0\to Rf_0(980)}/|\boldsymbol{p}_R|, \tag{11}$$

where R represents the $X(3872)$, $\psi(2S)$, or J/ψ, respectively. For the partial decay width of $\Gamma_{B_s^0\to Rf_0(980)}$, we calculate them with the spectral function for the $\pi^+\pi^-$ distribution as follows [2, 21],

$$\Gamma_{B_s^0\to Rf_0(980)} = \frac{\int_{M_{\pi\pi}^{\min}}^{M_{\pi\pi}^{\max}} \frac{d\Gamma_{B_s^0\to R\pi^+\pi^-}}{dM_{\pi\pi}}/S(M_{\pi\pi}^2)dM_{\pi\pi}}{\int_{M_{\pi\pi}^{\min}}^{M_{\pi\pi}^{\max}} dM_{\pi\pi}}, \tag{12}$$

with the special function $S(M_{\pi\pi}^2)$ [2],

$$S(M_{\pi\pi}^2) = -\mathrm{Im}\frac{2m_{f_0(980)}/\pi}{M_{\pi\pi}^2 - m_{f_0(980)}^2 + im_{f_0(980)}\Gamma_{f_0(980)}}, \tag{13}$$

where we take $m_{f_0(980)} = 985$ MeV and $\Gamma_{f_0(980)} = 100$ MeV as quoted in the RPP [55], while $M_{\pi\pi}^{\max} = 1035$ MeV and $M_{\pi\pi}^{\min} = 935$ MeV.

On the other hand, we can also evaluate the modulus squared of decay amplitudes for $B_s^0 \to R\phi(\eta,\eta')$ by replacing $f_0(980)$ with ϕ, η or η', which contain an $s\bar{s}$ component. We define:

$$R_1 = \frac{|\mathcal{M}_{B_s^0\to X(3872)f_0(980)[\phi,\eta,\eta']}|^2}{|\mathcal{M}_{B_s^0\to J/\psi f_0(980)[\phi,\eta,\eta']}|^2}, \tag{14}$$

$$R_2 = \frac{|\mathcal{M}_{B_s^0\to \psi(2S)f_0(980)[\phi,\eta,\eta']}|^2}{|\mathcal{M}_{B_s^0\to J/\psi f_0(980)[\phi,\eta,\eta']}|^2}. \tag{15}$$

[2] Here, we use a Breit-Wigner form for the $f_0(980)$, and it will not change our main conclusion if we worked in the dynamically generated picture.

These obtained numerical results for R_1 and R_2 are listed in Table II. In the calculations, we take the two-body decay branching fractions from the RPP [55] except $\mathcal{B}[B_s^0 \to X(3872)\eta]$ and $\mathcal{B}[B_s^0 \to X(3872)\eta']$. While for the decays of $B_s^0 \to X(3872)\eta$ and $B_s^0 \to X(3872)\eta'$, there are still no experimental measurements, and thus we rely on the results obtained in Ref. [32] with the subtraction parameter $\alpha = -1.91$ (see more details in that reference). These values are listed in Table III. On the other hand, Ref. [33] also gives the results of $\mathcal{B}[B_s^0 \to X(3872)\eta]$ and $\mathcal{B}[B_s^0 \to X(3872)\eta']$ based on the molecular picture of $X(3872)$.

TABLE II: Ratios of the two-body decays of $B_s^0 \to X(3872)[\psi(2S)]f_0(980)[\phi,\eta,\eta']$ to the $B_s^0 \to J/\psi f_0(980)[\phi,\eta,\eta']$.

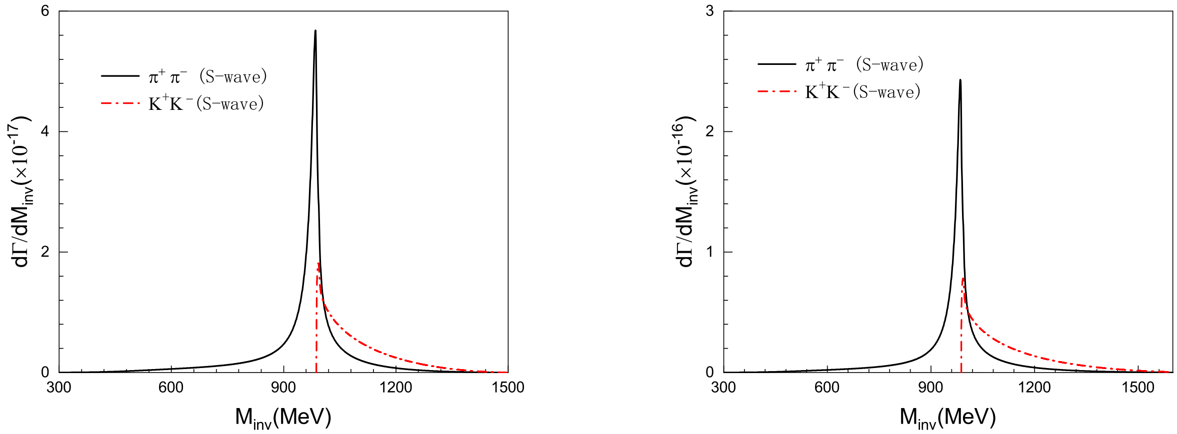
	R_1	R_2
$f_0(980)$	0.16 ± 0.09	0.51 ± 0.28
ϕ	0.18 ± 0.07	0.71 ± 0.06
η	0.05 ± 0.03	1.07 ± 0.35
η'	0.08 ± 0.04	0.54 ± 0.16

TABLE III: Branching fractions of B_s^0 decaying into $X(3872)[\psi(2S), J/\psi]$ and $\phi[\eta,\eta']$.

Decay modes	Branching fractions ($\times 10^{-4}$)
$X(3872)\phi$	1.1 ± 0.4
$X(3872)\eta$	0.15 ± 0.07
$X(3872)\eta'$	0.17 ± 0.08
$\psi(2S)\phi$	5.2 ± 0.4
$\psi(2S)\eta$	3.3 ± 0.9
$\psi(2S)\eta'$	1.29 ± 0.35
$J/\psi\phi$	10.4 ± 0.4
$J/\psi\eta$	4.0 ± 0.7
$J/\psi\eta'$	3.3 ± 0.4

Table II shows that the results for R_2 are close to one since both $\psi(2S)$ and J/ψ are charmonium states. Furthermore, the obtained ratios of R_1 are much smaller than that of R_2, which indicates that the $X(3872)$ state is not pure charmonium.

Finally, we consider the $B_s^0 \to X(3872)[\psi(2S)]K^+K^-$ process. The theoretical results for the invariant K^+K^- mass distributions are shown in Fig. 6, where the numerical results for the invariant $\pi^+\pi^-$ mass distributions of the $B_s^0 \to X(3872)[\psi(2S)]\pi^+\pi^-$ decays are also shown. The production rate of the $X(3872)$ is almost an order of magnitude smaller than that of the production of $\psi(2S)$ for both $\pi^+\pi^-$ and K^+K^- final states. The final-state interactions of $\pi^+\pi^-$ and K^+K^- occur in s-wave, where only the $f_0(980)$ contribution is considered. It is expected that future experimental measurements can test these calculations.



(a) (b)

FIG. 6: The $\pi^+\pi^-$ and K^+K^- invariant mass distribution of B_s^0 decay with the final state (a) $X(3872)$ and (b) $\psi(2S)$.

It is interesting to compare the branching fractions through the integral of invariant mass $M_{\pi\pi}$ and M_{KK}. The results are given by

$$\frac{\mathcal{B}[B_s^0 \to X(3872)(f_0(980) \to K^+K^-)]}{\mathcal{B}[B_s^0 \to X(3872)(f_0(980) \to \pi^+\pi^-)]} = 0.5 \pm 0.3, \quad (16)$$

$$\frac{\mathcal{B}[B_s^0 \to \psi(2S)(f_0(980) \to K^+K^-)]}{\mathcal{B}[B_s^0 \to \psi(2S)(f_0(980) \to \pi^+\pi^-)]} = 0.6 \pm 0.3, \quad (17)$$

which shows that the branching fraction obtained from the integral over invariant mass M_{KK} is of the same order of magnitude as that for $M_{\pi\pi}$ while the strength of K^+K^- invariant mass distribution below the peak of $f_0(980)$ is much smaller than that for $\pi^+\pi^-$.

Moreover, it is easy to get the branching fraction from the measurements of Ref. [56],

$$\mathcal{B}[B_s^0 \to X(3872)(K^+K^-)_{\text{non}-\phi}] = (8.6 \pm 3.5) \times 10^{-5}. \quad (18)$$

Then, one can also get the following ratio,

$$\frac{\mathcal{B}[B_s^0 \to X(3872)(f_0(980) \to K^+K^-)]}{\mathcal{B}[B_s^0 \to X(3872)(K^+K^-)_{\text{non}-\phi}]} = 0.06 \pm 0.02, \quad (19)$$

which means that the s-wave K^+K^- contribution from $f_0(980)$ is extremely small compared with other non-ϕ contributions.

For the contribution of the ϕ meson to the $B_s^0 \to X(3872)K^+K^-$, with the above obtained couplings of $g^2_{BX\phi}$ and $g^2_{\phi KK}$, we get the branching fraction

$$\mathcal{B}[B_s^0 \to X(3872)(\phi \to K^+K^-)] = (8.3 \pm 3.0) \times 10^{-5}, \quad (20)$$

which is consistent with the following result from the narrow width approximation within the uncertainty,

$$\begin{aligned}\mathcal{B}[B_s^0 \to X(3872)(\phi \to K^+K^-)] &= \mathcal{B}[B_s^0 \to X(3872)\phi] \\ \times \mathcal{B}[\phi \to K^+K^-] &= (5.4 \pm 2.0) \times 10^{-5}, \end{aligned} \quad (21)$$

where we have used $\mathcal{B}[\phi \to K^+K^-] = (49.1 \pm 0.5)\%$ from the RPP [55].

IV. SUMMARY

We have investigated the decays of B_s^0 into $\psi(2S)\pi^+\pi^-$ and $X(3872)\pi^+\pi^-$ and performed a χ^2-fit to the $\pi^+\pi^-$ invariant mass distributions based on the experimental data from the LHCb collaboration. Taking the dominant Cabibbo favored weak decay mechanism of B_s^0, we firstly get $\psi(2S)$ or $X(3872)$ and an $s\bar{s}$ pair. Second, after the hadronization of $s\bar{s}$, we get $\pi^+\pi^-$ and K^+K^- in the final state, and this interaction is mediated by the scalar meson $f_0(980)$. In addition, the contribution from the scalar meson $f_0(1500)$ is also considered for the $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay. It is found that the recent LHCb experimental measurements on the $\pi^+\pi^-$ invariant mass distributions of $B_s^0 \to \psi(2S)\pi^+\pi^-$ decay can be well reproduced.

Within the same picture, we also studied the $B_s^0 \to X(3872)\pi^+\pi^-$ decay. We find that, to reproduce the experimental data, one needs a different production coupling parameter for $X(3872)$, which indicates that the production of $X(3872)$ is not the same as the production of the charmonium state $\psi(2S)$. Moreover, we have compared the modulus squared of amplitudes of B_s^0 decays into $X(3872)$ or $\psi(2S)$ and one light meson, namely $f_0(980)$, ϕ, η, and η'. The results indicate that the production amplitudes of $X(3872)$ in B_s^0 decays are different from that of one charmonium in the same B_s^0 decays. This may indicate that the $X(3872)$ is not a pure charmonium state.

The $\pi^+\pi^-$ and K^+K^- invariant mass distributions for the processes $B_s^0 \to \psi(2S)[X(3872)]\pi^+\pi^-$ and $B_s^0 \to \psi(2S)[X(3872)]K^+K^-$ are calculated, where we have naturally considered the K^+K^- final state from $f_0(980)$ for the decays of B_s^0 into $\psi(2S)K^+K^-$ and $X(3872)K^+K^-$ in the coupled channel approach, and compared with the $\pi^+\pi^-$ final state in the same situation. On the one hand, it is found that the peak strength of $f_0(980)$ in $m_{\pi\pi}$ is higher than that in m_{KK} for the production of $X(3872)$ or $\psi(2S)$ in the B_s^0 decays. On the other hand, we realize that $\mathcal{B}[B_s^0 \to X(3872)\pi^+\pi^-]$ is bigger than $\mathcal{B}[B_s^0 \to X(3872)K^+K^-]$ while both are of the same order of magnitude. The above result does not change with the substitution of $\psi(2S)$ for $X(3872)$. The results here shed light on the fact that the low-lying scalar meson $f_0(980)$ is formed from the interaction of pseudoscalar meson and pseudoscalar meson and that $X(3872)$ is indeed not a pure charmonium state.

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