QSPR Analysis with Curvilinear Regression Modeling and Temperature-based Topological Indices

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Abstract

Establishing quantitative correlations between various molecular properties and chemical structures is of great technological importance for environmental and medical aspects. These approaches are referred to as Quantitative Structure-Property Relationships (QSPR), which relate the physicochemical or thermodynamic properties of compounds to their structures. The main goal of OSPR studies is to find a mathematical relationship between the property of interest and several molecular descriptors derived from the structure of the molecule. Topological indices are the molecular descriptors that characterize the formation of chemical compounds and predict certain physicochemical properties. In this study, the QSPR models are designed using certain temperature-based topological indices such as the sum connectivity temperature index, product connectivity temperature index, F-temperature index, and symmetric division temperature index to predict the thermodynamic properties, such as enthalpies of formation (ΔH_f^0 liquid), enthalpies of combustion (ΔH_C^0 liquid), and enthalpies of vaporization $(\Delta H_{vap}^0 \text{ gas})$ of monocarboxylic acids $(C_2H_4O_2 - C_{20}H_{40}O_2)$. The relationship analysis between thermodynamic properties and topological indices is done using linear, quadratic, and cubic equations of a curvilinear regression model. These regression models are then compared.

Keywords: Temperature of a vertex, sum connectivity temperature index, product connectivity temperature index, F-temperature index, symmetric division temperature index.

1 Introduction

The properties of a molecule are indeed closely tied to its structural characteristics and composition. This concept is fundamental to understanding how molecules behave and interact in various chemical reactions and physical processes. In this connection, graph theory has been successfully applied and some thermodynamic properties [1, 2, 6, 13, 17].

Chemical graph theory is a specialized field within mathematical chemistry that focuses on the study of molecules and chemical systems through the lens of graph theory. Graph theory provides a mathematical framework for analyzing the relationships between objects connected by edges, and in the context of chemical graph theory, these objects are atoms and the edges represent chemical bonds. Chemical graph theory has practical applications in fields such as drug discovery, materials science, computational chemistry, and chemical informatics. It provides a powerful approach to understanding the relationships between molecular structure and properties, which is crucial for designing new molecules with desired characteristics and predicting how molecules will behave under different conditions.

Chemical graph theory plays a crucial role in developing QSPR models. These models correlate graph-based molecular descriptors with various properties such as boiling points, melting points, solubility, etc. For more details, the reader can refer to [3, 7, 9, 10, 11, 12, 15]. Numerous studies have been made relating to QSPR models by using what are called topological indices (TI). The first topological index was the Wiener index, which was introduced by Harold Wiener in 1947. It was used to determine the physical properties of paraffin [17]. Since then, many topological indices have been defined and used in many applications.

A topological index can be classified according to the structural characteristics of the graph such as the degree of a vertex, the distance between vertices, the matching, and the spectrum. The best-known topological indices are the Wiener index which is based on distance, the Zagreb and the Randić indices which are based on degree, the Estrada index which is based on the spectrum of a graph, the Hosaya index based on the matching.

Shafiei in [14] designed the QSPR models using topological indices such as the connectivity index, Szeged index, Balaban index, and Harary number to predict the thermodynamic properties such as enthalpies of formation of liquid, enthalpies of combustion of liquid, enthalpies of vaporization, and enthalpies of sublimation of monocarboxylic acids ($C_2H_4O_2 - C_{20}H_{40}O_2$). Later in [5], Havare designed the QSPR models using topological indices such as the Gutman index, variance of degree index, product connectivity Banhatti index, and Sigma index to predict these thermodynamic properties of monocarboxylic acids.

Motivated by these, the structure-property relationship between certain temperature-based topological indices such as sum connectivity temperature index, product connectivity temperature index, F-temperature index, and symmetric division temperature index to the enthalpies of formation (ΔH_f^0 liquid), enthalpies of combustion (ΔH_C^0 liquid), and enthalpies of vaporization (ΔH_{vap}^0 gas) of monocarboxylic acids and their quantitative structure-property relationship are presented in this paper.

2 Basic definitions

Let G = (V, E) be a simple connected graph with vertex set |V(G)| = n and edge set E(G). The number of edges incident to a vertex v is called the *degree* of the vertex v and is denoted by deg(v). Fajtolowicz [4] introduced the notion of the temperature of a vertex v as follows.

Definition 2.1 The *temperature* of a vertex $v \in V(G)$ is defined by $\mathcal{T}(v) = \frac{deg(v)}{n - deg(v)}$

Later in 2019, Kulli [8] introduced the concept of sum connectivity temperature index, product connectivity temperature index, F-temperature index, and symmetric division temperature index of a graph G as follows:

Sum connectivity temperature index:

$$S\mathcal{T}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\mathcal{T}(u) + \mathcal{T}(v)}}$$
(1)

Product connectivity temperature index:

$$P\mathcal{T}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\mathcal{T}(u) \times \mathcal{T}(v)}}$$
(2)

F-temperature index:

$$F\mathcal{T}(G) = \sum_{uv \in E(G)} \left(\mathcal{T}(u)^2 + \mathcal{T}(v)^2 \right)$$
(3)

Symmetric division temperature index:

$$SD\mathcal{T}(G) = \sum_{uv \in E(G)} \left(\frac{\mathcal{T}(u)}{\mathcal{T}(v)} + \frac{\mathcal{T}(v)}{\mathcal{T}(u)} \right)$$
(4)

3 Methods and techniques

The method used in this article includes finding the temperature of vertices, division of temperature of vertices, and partition of edges based on the temperature of end vertices. The temperature of vertices and the edge separation methods are used for the computation of temperature-based topological indices. The correlation coefficients are calculated using JMP statistical software. 2D graphs are drawn using JMP statistical software/Microsoft Excel.

4 2D molecular structures and computations of monocarboxylic acids

With the help of these four temperature-based topological indices, the molecular structures of 19 monocarboxylic acids are explored. There are three thermodynamic properties of monocarboxylic acids under observation. These properties are enthalpies of formation (ΔH_f^0 liquid), enthalpies of combustion (ΔH_C^0 liquid), and enthalpies of vaporization (ΔH_{vap}^0 gas). Thermodynamic properties of monocarboxylic acids as given in Table 1 are taken from [5, 14].

Let α =Enthalpies of formation of liquids; β =Enthalpies of combustion of liquids; and Γ =Enthalpies of vaporization. Then,

| Name of compounds | Formula | α | β | Г |
|--------------------|-------------------|--------|---------|-------|
| Acetic acid | $C_2H_4O_2$ | 483.50 | 875.16 | 46.3 |
| Propanoic acid | $C_3H_6O_2$ | 510.8 | 1527.3 | 50 |
| Butanoic acid | $C_4H_8O_2$ | 533.9 | 2183.5 | 54.9 |
| Pentanoic acid | $C_5 H_{10} O_2$ | 558.9 | 2837.8 | 58.2 |
| Hexanoic acid | $C_6 H_{12} O_2$ | 581.8 | 3494.3 | 63 |
| Heptanoic acid | $C_7 H_{14} O_2$ | 608.5 | 4146.9 | 64.8 |
| Octanoic acid | $C_8 H_{16} O_2$ | 634.8 | 4799.9 | 69.4 |
| Nonanoic acid | $C_9H_{18}O_2$ | 658 | 5456.1 | 72.3 |
| Decanoic acid | $C_{10}H_{20}O_2$ | 713.7 | 6079.3 | 76.6 |
| Undecanoic acid | $C_{11}H_{22}O_2$ | 736.2 | 6376.5 | 78.9 |
| Dodecanoic acid | $C_{12}H_{24}O_2$ | 775.1 | 7377 | 82.2 |
| Tridecanoic acid | $C_{13}H_{26}O_2$ | 807.2 | 8024.2 | 84.9 |
| Tetradecanoic acid | $C_{14}H_{28}O_2$ | 834.1 | 8676.7 | 87.7 |
| Pentadecanoic acid | $C_{15}H_{30}O_2$ | 862.4 | 9327.7 | 91.4 |
| Hexadecanoic acid | $C_{16}H_{32}O_2$ | 892.2 | 9977.2 | 94.5 |
| Heptadecanoic acid | $C_{17}H_{34}O_2$ | 924.4 | 10624.4 | 100.7 |
| Octadecanoic acid | $C_{18}H_{36}O_2$ | 947.2 | 11280.1 | 102.8 |
| Nonadecanoic acid | $C_{19}H_{38}O_2$ | 984.1 | 11923.4 | 105 |
| Eicosanoic acid | $C_{20}H_{40}O_2$ | 1012.6 | 12574.2 | 109.9 |

Table 1. The values of enthalpies of formation of liquid $(\Delta H_f^0 k J/mol)$, enthalpies of combustion of liquid $(\Delta H_C^0 k J/mol)$, and enthalpies of vaporization $(\Delta H_{vap}^0 k J/mol)$ of monocarboxylic acids at conditions, normally at 298.15 K, 1 atm.

5 Computation of temperature-based topological indices

Table 2 shows the temperature-based topological indices of 19 monocarboxylic acids calculated using the formulas (1-4).

| Formula | $S\mathcal{T}(G)$ | $P\mathcal{T}(G)$ | $F\mathcal{T}(G)$ | $SD\mathcal{T}(G)$ |
|-------------------|-------------------|-------------------|-------------------|--------------------|
| $C_2H_4O_2$ | 1.643167 | 3 | 27.3333333 | 27.3333333 |
| $C_3H_6O_2$ | 3.23569 | 6.715476 | 7.826389 | 18.069445 |
| $C_4H_8O_2$ | 4.837467 | 9.048627 | 4.12 | 17.8 |
| $C_5 H_{10} O_2$ | 6.585927 | 16.355579 | 2.730833 | 18.669444 |
| $C_6H_{12}O_2$ | 8.478948 | 22.649944 | 2.030113 | 19.993658 |
| $C_7H_{14}O_2$ | 10.97437 | 32.485281 | 1.421875 | 21.5 |
| $C_8H_{16}O_2$ | 12.671578 | 38.220202 | 1.338057 | 23.224868 |
| $C_9H_{18}O_2$ | 14.957559 | 47.500261 | 1.143533 | 24.985648 |
| $C_{10}H_{20}O_2$ | 17.700444 | 59.821403 | 0.998127 | 26.8 |
| $C_{11}H_{22}O_2$ | 19.879457 | 69.054894 | 0.885875 | 28.651768 |
| $C_{12}H_{24}O_2$ | 22.505505 | 81.330388 | 0.796447 | 30.530691 |
| $C_{13}H_{26}O_2$ | 25.235916 | 94.605041 | 0.723516 | 32.429945 |
| $C_{14}H_{28}O_2$ | 28.066954 | 106.017218 | 0.662893 | 34.344811 |
| $C_{15}H_{30}O_2$ | 30.995246 | 124.1525061 | 0.611696 | 36.271925 |
| $C_{16}H_{32}O_2$ | 34.017723 | 140.425548 | 0.567881 | 38.208823 |
| $C_{17}H_{34}O_2$ | 37.131587 | 157.698239 | 0.529953 | 39.581903 |
| $C_{18}H_{36}O_2$ | 40.334265 | 175.9706351 | 0.496798 | 42.105034 |
| $C_{19}H_{38}O_2$ | 43.62339 | 195.2427855 | 0.467565 | 44.061842 |
| $C_{20}H_{40}O_2$ | 46.99677 | 215.514726 | 0.441595 | 47.229573 |

Table 2. The values of temperature-based topological indices of monocarboxylic acids.

6 Regression models

Regression analysis is a statistical method that shows the relationship between two or more variables.

To study the relationship between thermodynamic properties of monocarboxylic acids and the temperature-based topological indices, the following equations from [14] are used. Linear equation:

$$Y = a + b_1 X_1; \quad n, R^2, s, F$$

Quadratic equation:

$$Y = a + b_1 X_1 + b_2 X_1^2; \quad n, R^2, s, F$$

Cubic equation:

$$Y = a + b_1 X_1 + b_2 X_1^2 + b_3 X_1^3; \quad n, R^2, s, F$$

Here, Y is the dependent variable, a is the regression model, b_i $(1 \le i \le 3)$ are the coefficients for the individual descriptor, X_i $(1 \le i \le 3)$ are the independent variables, n is the number of samples used for building the regression equation, R^2 is the correlation coefficient, s is the standard error of deviation, and F is the calculated value of the F-ration test. Note that the quality of a QSPR model can be conveniently measured by the correlation coefficient (R^2) . A good QSPR model must have $R^2 > 0.99$. The observed values and model predictions must be compared to measure the predictive quality of the model. So, we deal with the RMSE (Root Mean Square Error) metric for the predictive power of the model. The best predictive model is the minimum error, i.e. the minimum RMSE.

Furthermore, R^2 and F parameters will be considered for the goodness of fit of the model. As given in [16], the best goodness of fit in models is selected by using the parameters either max (R^2) or max (F).

We now determine the linear, quadratics, and cubic curvilinear models of the sum connectivity temperature index, product connectivity temperature index, F-temperature index, and symmetric division temperature index for ΔH_f^0 , ΔH_C^0 , and ΔH_{vap}^0 .

The significance of bold numbers in each table denote highest correlation value.

6.1 Curvilinear regression models of $S\mathcal{T}(G)$, $P\mathcal{T}(G)$, $SD\mathcal{T}(G)$ for ΔH^0_f

Table 3. The curvilinear regressions models of $S\mathcal{T}(G)$ index for ΔH_f^0 .

| ΔH_f^0 | R^2 | F | RMSE |
|---|--------|----------|---------|
| $485.412 + 11.8001(S\mathcal{T}(G))$ | 0.9918 | 2056.329 | 15.8062 |
| $456.784 + 15.606(ST(G)) - 0.08110(ST(G)^2)$ | 0.9988 | 6658.513 | 6.2330 |
| $460.745 + 14.626(S\mathcal{T}(G)) - 0.0296(S\mathcal{T}(G)^2) - 0.000717(S\mathcal{T}(G)^3)$ | 0.9988 | 4449.708 | 6.2257 |

The analysis as mentioned in Table 3 indicates that the best goodness of fit among obtained curvilinear equations using $S\mathcal{T}(G)$ topological index for ΔH_f^0 are:

 $\Delta H_f^0 = 456.784 + 15.606(S\mathcal{T}(G)) - 0.08110(S\mathcal{T}(G)^2); \text{ and} \\ \Delta H_f^0 = 460.745 + 14.626(S\mathcal{T}(G)) - 0.0296(S\mathcal{T}(G)^2) - 0.000717(S\mathcal{T}(G)^3).$

| Table 4. | The cu | ırvilinear | regressions | models | of . | PT(| G | index | for | ΔH_{f}^{0} |) |
|----------|--------|------------|-------------|--------|------|-----|---|-------|-----|--------------------|---|
|----------|--------|------------|-------------|--------|------|-----|---|-------|-----|--------------------|---|

| ΔH_f^0 | R^2 | F | RMSE |
|--|--------|----------|--------|
| 532.892 + 2.465(PT(G)) | 0.9662 | 487.167 | 32.053 |
| $489.678 + 4.024(P\mathcal{T}(G)) - 0.0077(P\mathcal{T}(G)^2)$ | 0.9972 | 2950.84 | 9.355 |
| $479.340 + 4.754(P\mathcal{T}(G)) - 0.0168(P\mathcal{T}(G)^2) - 2.995e - 5(P\mathcal{T}(G)^3)$ | 0.9986 | 3753.734 | 6.777 |

The analysis as mentioned in Table 4 indicates that the best goodness of fit among obtained curvilinear equations using $P\mathcal{T}(G)$ topological index for ΔH_f^0 is:

 $\Delta H_f^0 = 479.340 + 4.754(P\mathcal{T}(G)) - 0.0168(P\mathcal{T}(G)^2) - 2.995e - 5(P\mathcal{T}(G)^3)$

Table 5. The curvilinear regressions models of $F\mathcal{T}(G)$ index for ΔH_f^0 .

| ΔH_f^0 | R^2 | F | RMSE |
|--|--------|---------|---------|
| $782.969-14.820(F\mathcal{T}(G))$ | 0.2913 | 6.9901 | 146.942 |
| $873.844-87.507(F\mathcal{T}(G))+2.692(F\mathcal{T}(G)^2)$ | 0.6220 | 13.1658 | 110.619 |
| $1013.011-271.1496(F\mathcal{T}(G))+34.177(F\mathcal{T}(G)^2)-0.9134(F\mathcal{T}(G)^3)$ | 0.8510 | 28.574 | 71.713 |

The analysis as mentioned in Table 5 indicates that the best goodness of fit among obtained curvilinear equations using $F\mathcal{T}(G)$ topological index for ΔH_f^0 is: $\Delta H_f^0 = 1013.011 - 271.1496(F\mathcal{T}(G)) + 34.177(F\mathcal{T}(G)^2) - 0.9134(F\mathcal{T}(G)^3).$

Table 6. The curvilinear regressions models of $SD\mathcal{T}(G)$ index for ΔH_f^0 .

| ΔH_f^0 | R^2 | F | RMSE |
|--|--------|----------|--------|
| $224.623+17.044(SD\mathcal{T}(G))$ | 0.8630 | 107.0981 | 64.607 |
| $276.589 + 13.4216(SD\mathcal{T}(G)) + 0.0578(SD\mathcal{T}(G)^2)$ | 0.8636 | 50.6906 | 66.429 |
| $653.886-26.121(SD\mathcal{T}(G))+1.3603(SD\mathcal{T}(G)^2)-0.0135(SD\mathcal{T}(G)^3)$ | 0.8658 | 32.275 | 68.059 |

The analysis as mentioned in Table 6 indicates that the best goodness of fit among obtained curvilinear equations using $SD\mathcal{T}(G)$ topological index for ΔH_f^0 is:

 $\Delta H_f^0 = 653.886 - 26.121(SD\mathcal{T}(G)) + 1.3603(SD\mathcal{T}(G)^2) - 0.0\dot{1}35(SD\mathcal{T}(G)^3).$

From the analysis as mentioned in Table 3 - Table 6, the topological indices $S\mathcal{T}(G)$ and $P\mathcal{T}(G)$ are the best suitable for predicting the ΔH_f^0 of monocarboxylic acids $(C_2H_4O_2 - C_{20}H_{40}O_2)$ since $R^2 > 0.99$.

6.2 Curvilinear regression models of $S\mathcal{T}(G)$, $P\mathcal{T}(G)$, $SD\mathcal{T}(G)$ for ΔH^0_C

Table 7. The curvilinear regressions models of $S\mathcal{T}(G)$ index for ΔH_C^0 .

| ΔH_C^0 | R^2 | F | RMSE |
|--|--------|----------|---------|
| $1258.293 + 253.772(S\mathcal{T}(G))$ | 0.9889 | 1526.730 | 394.502 |
| $512.470 + 352.946(S\mathcal{T}(G)) - 2.1129(S\mathcal{T}(G)^2)$ | 0.9992 | 1038.96 | 107.495 |
| $274.317 + 411.859(S\mathcal{T}(G)) - 5.2041(S\mathcal{T}(G)^2) + 0.0431(S\mathcal{T}(G)^3)$ | 0.9998 | 3037.92 | 51.341 |

The analysis as mentioned in Table 7 indicates that the best goodness of fit among obtained curvilinear equations using $S\mathcal{T}(G)$ topological index for ΔH_C^0 is: $\Delta H_C^0 = 274.317 + 411.859(S\mathcal{T}(G)) - 5.2041(S\mathcal{T}(G)^2) + 0.0431(S\mathcal{T}(G)^3).$

Table 8. The curvilinear regressions models of $P\mathcal{T}(G)$ index for ΔH_C^0 .

| ΔH_C^0 | R^2 | F | RMSE |
|---|--------|----------|----------|
| $2288.883 + 52.909(P\mathcal{T}(G))$ | 0.9594 | 402.091 | 757.149 |
| $1304.786 + 88.4016(PT(G)) - 0.1753(PT(G)^2)$ | 0.9941 | 1351.173 | 297.2921 |
| 920.142+115.588($PT(G)$)-0.5165($PT(G)^2$) +0.0010($PT(G)^3$) | 0.9982 | 2795.027 | 169.119 |

The analysis as mentioned in Table 8 indicates that the best goodness of fit among obtained curvilinear equations using $P\mathcal{T}(G)$ topological index for ΔH_C^0 is: $\Delta H_C^0 = 1304.786 + 88.4016(P\mathcal{T}(G)) - 0.1753(P\mathcal{T}(G)^2).$

Table 9. The curvilinear regressions models of $F\mathcal{T}(G)$ index for ΔH_C^0 .

| ΔH_C^0 | R^2 | F | RMSE |
|--|--------|---------|----------|
| $7702.569-334.266(F\mathcal{T}(G))$ | 0.3195 | 7.9835 | 3101.051 |
| 9679.411-1915.462 $(F\mathcal{T}(G))$ +58.562 $(F\mathcal{T}(G)^2)$ | 0.6569 | 15.3177 | 2269.749 |
| $12543.022-5694.203(F\mathcal{T}(G))+706.436(F\mathcal{T}(G)^2)-18.795(F\mathcal{T}(G)^3)$ | 0.8659 | 32.312 | 1465.029 |

The analysis as mentioned in Table 9 indicates that the best goodness of fit among obtained curvilinear equations using $F\mathcal{T}(G)$ topological index for ΔH_C^0 is: $\Delta H_C^0 = 12543.022 - 5694.203(F\mathcal{T}(G)) + 706.436(F\mathcal{T}(G)^2) - 18.795(F\mathcal{T}(G)^3).$

| Table 10. The c | curvilinear | regressions | models | of | SDT(| G | index f | or ΔH_C^0 | |
|-----------------|-------------|-------------|--------|----|------|---|---------|-------------------|--|
|-----------------|-------------|-------------|--------|----|------|---|---------|-------------------|--|

| ΔH_C^0 | R^2 | F | RMSE |
|--|--------|---------|----------|
| $-4241.663+362.973(SD\mathcal{T}(G))$ | 0.8437 | 91.831 | 1485.794 |
| $-3123.265 + 284.995(SD\mathcal{T}(G)) + 1.2453(SD\mathcal{T}(G)^2)$ | 0.8444 | 43.4386 | 1528.186 |
| $1084.461 - 156.0012(SDT(G)) + 15.7711(SDT(G)^2) - 0.1515(SDT(G)^3)$ | 0.8450 | 27.269 | 1575.346 |

The analysis as mentioned in Table 10 indicates that the best goodness of fit among obtained curvilinear equations using $SD\mathcal{T}(G)$ topological index for ΔH_C^0 is: $\Delta H_C^0 = 1084.461 - 156.0012(SD\mathcal{T}(G)) + 15.7711(SD\mathcal{T}(G)^2) - 0.1515(SD\mathcal{T}(G)^3).$

From the analysis as mentioned in Table 7 - Table 10, the topological indices $S\mathcal{T}(G)$ and $P\mathcal{T}(G)$ are the best suitable for predicting the ΔH_C^0 of monocarboxylic acids $(C_2H_4O_2 - C_{20}H_{40}O_2)$ since $R^2 > 0.99$.

6.3 Curvilinear regression models of $S\mathcal{T}(G)$, $P\mathcal{T}(G)$, $F\mathcal{T}(G)$, $SD\mathcal{T}(G)$ for ΔH^0_{vap}

Table 11. The curvilinear regressions models of $S\mathcal{T}(G)$ index for ΔH^0_{vap} .

| ΔH_{vap}^0 | R^2 | F | RMSE |
|---|--------|----------|--------|
| $49.7480 + 1.3369(S\mathcal{T}(G))$ | 0.9835 | 1015.590 | 2.5482 |
| $45.531 + 1.897(S\mathcal{T}(G)) - 0.0119(S\mathcal{T}(G)^2)$ | 0.9952 | 1681.431 | 1.4087 |
| $42.932 + 2.540(S\mathcal{T}(G)) - 0.0456(S\mathcal{T}(G)^2) + 0.000470(S\mathcal{T}(G)^3)$ | 0.9978 | 2317.041 | 0.9811 |

The analysis as mentioned in Table 11 indicates that the best goodness of fit among obtained curvilinear equations using $S\mathcal{T}(G)$ topological index for ΔH^0_{vap} is: $\Delta H^0_{vap} = 42.932 + 2.540(S\mathcal{T}(G)) - 0.0456(S\mathcal{T}(G)^2) + 0.000470(S\mathcal{T}(G)^3).$

Table 12. The curvilinear regressions models of $P\mathcal{T}(G)$ index for ΔH^0_{vap} .

| ΔH_{vap}^0 | R^2 | F | RMSE |
|--|--------|----------|--------|
| 55.201 + 0.278(PT(G)) | 0.9522 | 338.806 | 4.341 |
| $49.973 + 0.4670(PT(G)) - 0.00039(PT(G)^2)$ | 0.9872 | 621.4481 | 2.3079 |
| $47.306 + 0.655(P\mathcal{T}(G)) - 0.0032(P\mathcal{T}(G)^2) + 7.510(P\mathcal{T}(G)^3)$ | 0.9943 | 880.0532 | 1.5891 |

The analysis as mentioned in Table 12 indicates that the best goodness of fit among obtained curvilinear equations using $P\mathcal{T}(G)$ topological index for ΔH^0_{vap} is: $\Delta H^0_{vap} = 47.306 + 0.655(P\mathcal{T}(G)) - 0.0032(P\mathcal{T}(G)^2) + 7.510(P\mathcal{T}(G)^3).$

Table 13. The curvilinear regressions models of $F\mathcal{T}(G)$ index for ΔH^0_{van} .

| ΔH_{vap}^0 | R^2 | F | RMSE |
|--|--------|---------|---------|
| $83.876-1.822(F\mathcal{T}(G))$ | 0.3402 | 8.7681 | 16.1314 |
| 94.306-10.164 $(F\mathcal{T}(G))$ +0.308 $(F\mathcal{T}(G)^2)$ | 0.6767 | 16.7478 | 11.6393 |
| $108.734-29.203(F\mathcal{T}(G))+3.5733(F\mathcal{T}(G)^2)-0.09470(F\mathcal{T}(G)^3)$ | 0.8669 | 32.572 | 7.7128 |

The analysis as mentioned in Table 13 indicates that the best goodness of fit among obtained curvilinear equations using $F\mathcal{T}(G)$ topological index for ΔH^0_{vap} is: $\Delta H^0_{vap} = 108.734 - 29.203(F\mathcal{T}(G)) + 3.5733(F\mathcal{T}(G)^2) - 0.09470(F\mathcal{T}(G)^3).$

Table 14. The curvilinear regressions models of $SD\mathcal{T}(G)$ index for ΔH^0_{vap} .

| ΔH_{vap}^0 | R^2 | F | RMSE |
|--|--------|---------|----------|
| $21.241 + 1.896(SD\mathcal{T}(G))$ | 0.8255 | 80.4709 | 8.2942 |
| $28.6641 + 1.3792(SD\mathcal{T}(G)) + 0.00826(SD\mathcal{T}(G)^2)$ | 0.8266 | 38.1525 | 8.5231 |
| $36.2095 + 0.5884(SD\mathcal{T}(G)) + 0.03431(SD\mathcal{T}(G)^2) - 0.08027(SD\mathcal{T}(G)^3)$ | 0.8267 | 23.8565 | 80800973 |

The analysis as mentioned in Table 14 indicates that the best goodness of fit among obtained curvilinear equations using $SD\mathcal{T}(G)$ topological index for ΔH^0_{vap} is: $\Delta H^0_{vap} = 36.2095 + 0.5884(SD\mathcal{T}(G)) + 0.03431(SD\mathcal{T}(G)^2) - 0.08027(SD\mathcal{T}(G)^3).$

From the analysis as mentioned in Table 11 - Table 14, the topological indices $S\mathcal{T}(G)$ and $P\mathcal{T}(G)$ are the best suitable for predicting the ΔH_{vap}^0 of monocarboxylic acids $(C_2H_4O_2 - C_{20}H_{40}O_2)$ since $R^2 > 0.99$.

7 Plots of the cubic regression equation

The analysis as mentioned in Table 3 - Table 14 indicates that the cubic equation gives the best goodness of fit among three regression equations.

Figure 1 - Figure 3 shows the correlation of $S\mathcal{T}(G)$ and $P\mathcal{T}(G)$ with ΔH_f^0 , ΔH_C^0 , and ΔH_{vap}^0 using cubic regression equation.



Figure 1: (a) Cubic regression equation of ΔH_f^0 with $S\mathcal{T}(G)$. (b) Cubic regression equation of ΔH_f^0 with $P\mathcal{T}(G)$.



Figure 2: (a) Cubic regression equation of ΔH^0_C with $S\mathcal{T}(G)$ using normal probability plot. (b) Cubic regression equation of ΔH^0_C with $P\mathcal{T}(G)$.



Figure 3: (a) Cubic regression equation of ΔH_{vap}^0 with $S\mathcal{T}(G)$. (b) Cubic regression equation of ΔH_{vap}^0 with $P\mathcal{T}(G)$.

8 Conclusion

The temperature-based topological indices such as the sum connectivity temperature index, product connectivity temperature index, F-temperature index, and symmetric division temperature index of 19 monocarboxylic acids ($C_2H_4O_2 - C_{20}H_{40}O_2$) are calculated. Using these topological indices, curvilinear regression models are designed to predict certain thermodynamic properties such as enthalpies of formation (ΔH_f^0 liquid), enthalpies of combustion (ΔH_c^0 liquid), and enthalpies of vaporization (ΔH_{vap}^0 gas) of monocarboxylic acids. The most accurate results for the prediction of these thermodynamic properties can be calculated by using the sum connectivity temperature index and product connectivity temperature index. Furthermore, these thermodynamic properties also have a good correlation with the symmetric division temperature index, but the F-temperature index is not enough to make a good prediction of thermodynamic properties of monocarboxylic acids ($C_2H_4O_2 - C_{20}H_{40}O_2$). The optimum is the cubic equation form among curvilinear equations.

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