Reference Model Based Learning in Expectation Formation: Experimental Evidence*

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Abstract

How do people form expectations about future prices in financial markets? One of the dominant learning rules that explains the forecasting behavior is the Adaptive Expectation Rule (ADA), which suggests that people adjust their predictions by adapting to the most recent prediction error at a *constant* weight. However, this rule also implies that they will continually learn and adapt until the prediction error is zero, which contradicts recent experimental evidence showing that people usually stop learning long before reaching zero prediction error. A more recent learning rule — Reference Model Based Learning (RMBL) — extends and generalizes ADA, hypothesizing that: i) People apply ADA but *dynamically* adjust the adaptive coefficient with regards to the auto-correlation of the prediction error in the most recent two periods; ii) Meanwhile, they also utilize a satisficing rule so that people would only adjust their adaptive coefficient when the prediction error is higher than their anticipation. This paper utilizes a rich set of experimental data with observations of 41,490 predictions from 801 subjects from the Learning-to-Forecast Experiments (LtFEs), i.e., the experiment that has been used to study expectation formation. Our results concludes that RMBL fits better than ADA in all the experiments.

Keywords: Reference-Model Based Learning; Learning-to-Forecast Experiments; Model-Free Reinforcement Learning; Adaptive Expectation Rule
JEL Codes: D83, D84, E37, E71

1 Introduction

How do people form expectation about future prices in financial market?

The rational expectations hypothesis (REH) has been the dominant paradigm in expectation formation since Muth (1961) and Lucas Jr (1972). It suggests that people's expectations are the objective mathematical expectations conditional upon the available information, assuming that they have all the information of the underlying market equilibrium; as well as the mental capacity to calculate this rational expectation forecasts. Later, bounded rationality literature argue that it is possible for agents to not have perfect knowledge of the underlying market equilibrium equations and not to possess perfect knowledge about the beliefs of all other agents in the market (Sonnemans et al., 2004). At the same time, it is also argued that people may not endowed with enough mental capacity to easily calculate the complicated market equilibrium.

In response, many other learning heuristics have been proposed in an attempt to understand how people actually form expectations when they do not know the law of motion of prices, as a replacement for REH. Adaptive expectation rule (ADA; Evans and Honkapohja, 2001) is one of the dominant heuristics that have gained attention by finding support in survey and experimental data.

ADA suggests that people adjust their prediction by adapting to the most recent prediction error at a constant weight[1], which also means they will perpetually adapt to the past prediction error until they reach no prediction error. But the recent experimental evidence suggest that subjects implement a stopping rule on their learning, i.e., employ a simple satisficing heuristic. Bao et al. (2022) designed an experiment to test whether subjects submit prediction on the price following least square learning (i.e., by estimating the coefficient in the principle of minimizing the sum of the squared prediction error). Specifically, they tasked the subjects with predicting asset prices in an experimental financial market through asking them to provide structural expectations instead of point predictions of price in a cobweb economy model. They find that even when they are endowed with the correct perceived law of motion, subjects still choose to employ a simple satisficing approach, instead of constant learning using the least square learning[2] which would lead them to the lowest prediction error.

A recent model called reference-model based learning (RMBL) proposed by Bossaerts

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[1]ADA hypothesizes that $p_t^e = p_{t-1}^e + \bar{G}(p_{t-1} - p_{t-1}^e)$. p^e denotes the prediction and p denotes the realized value, where $\bar{G}$ is a constant and ranges between 0 and 1.

[2]See discussion on how least square learning differs from ADA in footnote 4.

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(2018) incorporate this satisficing heuristic, and it has also been extended to model investors' behavior in asset pricing (Berrada et al., 2024). The RMBL model can be regarded as a generalized adaptive expectation, hypothesizing that people's prediction adapts to prediction error with respect to a reference model. In other words, it extends ADA in two important aspects as follows.

First, RMBL involves a *dynamic* weighted average of the previous prediction and the last observed price. Specifically, the weight increases (decreases) — implying a stronger (weaker) adaptation to the prediction error—when people under-adjust (over-adjust) the prediction error for two consecutive periods, leading to a positive (negative) correlation in the prediction error.

The learning approach where people adjust with regards to the correlation of the prediction error in the most recent two periods is similar as the sample autocorrelation (SAC) learning Hommes and Sorger (1998) proposed.[3] Simply put, SAC suggests that people adapt in the similar fashion to RMBL, just that people in SAC are with *perfect forecast* but their counterpart in RMBL are rather *myopic*. First, SAC user are adapting to the distance between most recent realized price and the long run average price, instead of the distance between the most recent realized price the most recent prediction as in RMBL. Second, SAC assumes subjects to update belief in a fashion that takes the full history of the autocorrelation of the prediction error into the consideration, while RMBL assumes that subjects only consider the autocorrelation of the price in the most recent two periods.[4]

Secondly, there is a stopping rule or satisficing rule (Simon, 1955) in RMBL—where people would only speed up learning by adjusting the weights when the squared prediction error is larger than a threshold. The threshold is primarily modeled as the error from a reference model. In a way, the reference model could be viewed as encapsulating the notion of ambition and aspirations, so that the learner will be satisfied if her realized prediction

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[3]The theory of SAC suggests that the prediction is the sum of the long-run average realized price, plus the product of the first-order autocorrelation coefficient of the prediction error and the deviation of the previous price from the long-run average. Given the price is a convergent sequence, the sign of the first-order autocorrelation coefficient is the same as the autocorrelation of the price. Mathematically, it hypothesizes that $p_t^e = \alpha + \beta(p_{t-1} - \alpha)$, where α is the long-run average price, $\alpha = \bar{p}$, with $\bar{p} = \lim_{T \to \infty} \frac{1}{T+1} \sum_{t=0}^{T} p_t$; for β is the first-order autocorrelation coefficient, $\text{sgn}(\beta^j) = \text{sgn}(\rho_j)$ when $(p_t)_{t=0}^{\infty}$ is a convergent sequence, where $\rho_j = \lim_{T \to \infty} \frac{c_{j,T}}{c_{0,T}}$, $j \geq 1$, and $c_{j,T} = \frac{1}{T+1} \sum_{t=0}^{T-j} (p_t - \bar{p})(p_{t+j} - \bar{p})$. p^e denotes the prediction and p denotes the realized value.

[4]But SAC is still less demanding for the subjects compared to adaptive learning (or known as least square learning). This is because least square learning hypothesizes that people update the belief in a way that minimizes the square error of the prediction error. Mathematically, α is the same as in SAC, but $\beta_t = \frac{\sum_{i=1}^{t-1}(p_i - \bar{p}_t^-)(p_i - \bar{p}_t^+)}{\sum_{i=1}^{t-1}(p_i - \bar{p}_t^-)^2}$ for $t \geq 2$, where $\bar{p}_t^- = \frac{1}{t} \sum_{i=0}^{t-1} p_i$, and $\bar{p}_t^+ = \frac{1}{t} \sum_{i=0}^{t} p_i$.

error is no larger than the prediction error from a reference model[5] (Bossaerts, 2018).

In this paper, we test whether RMBL explains the experimental data in the learning to forecast experiment (LtFE), an experimental setup primarily used to study subjects' expectation formation by incentivizing subjects to submit the best prediction for the next period's price. We utilize a rich set of experimental data with observations of 41,490 predictions from 801 subjects from the Learning-to-Forecast Experiment (LtFEs). The advantage of LtFEs — compared to other more common market like double auctions — is that it provides a clean elicitation on price prediction. Not only does the payoff function incentivize subjects to submit their best price predictions that are closer to the realized price, but the direct price belief in LtFEs also avoids the "testing joint hypothesis" problem that occurs in traditional markets where people submit quantity decisions. Specifically, it is possible for subjects to accurately predict the price but fail to translate the price information into optimal trading quantities.

The challenging aspect of testing whether subjects implement RMBL is that we lack knowledge of *which* reference model, and hence its resulting prediction error, subjects are comparing the realized prediction error with during the experiment. Specifically, although the reference model is defined as a Kalman filter (also known as ADA in economics) in Bossaerts (2018), we still do not know that specific constant weight subjects assign to the past prediction error when adjusting its prediction. In turn, we are unaware of the prediction error from the Kalman filter for each subject. In fact, when the latter work of Berrada et al. (2024) applies RMBL to asset pricing, there is nowhere to find the Kalman filter. Instead, the reference model is defined as the desired level of mean return and return volatility.

In a way, it suggests that the reference model can be *any* model. Therefore, the key point is not to find out the exact prediction error from one specific reference model; rather, it is that subjects stop adapting to the prediction error when the error is small enough — or smaller than a constant threshold. In turn, we refer to the prediction error anticipated from the reference model as the "maximum allowable error" in this paper. Note that the maximum allowable error of RMBL users must be greater than zero. This is because those with a maximum allowable error of zero cannot be categorized as an agent who is satisficing, but rather as an agent whose sole aim is to optimize.

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[5]In active inference, the reference model guide actions with the goal of minimizing inference complexity while maximizing prediction accuracy. In engineering, the reference model acts like a principal to control surprise, where the agent would compare the observable outcome from his action with the desired outcome from the reference model. As discussed later, reference model in RMBL is not a specific model and can be any model depending on the context.

We set up a simple regression equation to conduct a horse race test and determine whether the data fits more closely to RMBL against model-free reinforcement learning (i.e., RMBL without satisficing rule; henceforth "model-free RL") and ADA (RMBL with a constant gain factor). Our results show that RMBL can explain all the experiments in our dataset from at least one of either the discrete or continuous perspectives. Specifically, we find that in most of the experiments, the learning rate—the incidence where there is an increment of adaptive response with regards to positive correlation of the error term—increases when there is a larger absolute prediction error. In other words, this observation is in line with predictions from RMBL, as it suggests that in the LtFE: i) Each time a subject makes a forecast, they attempt to minimize prediction error by adjusting the coefficient on how they adapt to prediction errors occurring in that period, and ii) they satisfice, meaning they stop adjusting the coefficient as long as the prediction error in that period is tolerable. Furthermore, model-free RL could also provide explanations for 3 out of the total 18 experiments in either discrete or continuous analyses. By contrast, there is no evidence showing that ADA fits best among the three learning models in any of the experiments. Our result support the statement that decision-making based on minimizing surprise[6] relative to reference world has become a key method to ensure fast adaption in an ever-changing world (Berrada et al., 2024).

Overall, this paper makes two main contributions to the literature.

Firstly, to the best of our knowledge, this is the first paper that tests whether people follow RMBL in an economic experiment. All existing experimental evidence that partially supports the notion that subjects update their belief in a prediction task following RMBL comes from studies in neuroscience. For example, by studying the neurobiological mechanisms, d'Acremont and Bossaerts (2016; N=21) find that subjects exhibit a dynamic learning rate. In their experiment, subjects are tasked to guide a robot to track closely to a moving target. The results show that subjects reduce the learning rate below baseline when the autocorrelation of the prediction errors is estimated to be negative, while increasing it when the autocorrelation of the prediction errors is positive. Additionally, neuroscience work also find evidence that the insula is responsible for people's satisficing behavior. Specifically, Danckert et al. (2012; N=35) find that optimization but without satisficing typically emerges in left-hemisphere lesion patients, specifically those with lesions in the insula, but not in the healthy controls. While neural evidence has suggested that subjects implement RMBL, their sample size is small because data collection in neuroscience (e.g., using fMRI) is rather complicated. In contrast, our study utilizes rich observations of 41,490 predictions

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[6]Surprise is defined as the outcome prediction error that is larger than the reference model expected in Bossaerts (2018).

from 801 subjects in 18 economic experiments, where subjects are asked to play the role of financial forecasters, and their only task is to submit a point prediction on the price in the next period as accurately as possible. As a result, we find combined evidence that subjects' learning fits RMBL the best.

Secondly, we contribute to the literature testing learning rules in the financial market. Heuristics switching model (HSM; Anufriev and Hommes, 2012) has been proposed as a universal model for learning behavior in LtFEs, capable of explaining substantially different price dynamics across various groups in the asset pricing experiment of (Hommes et al., 2005). The concept behind HSM is that in each period, each subject selects from a menu of forecasting heuristics such as ADA, trend-following rules, or anchoring and adjusting rules. The notion is that subjects are more likely to switch to a specific rule when it performs better — in terms of generating smaller forecasting errors — in the recent past[7]. In contrast, the results from this paper suggest the possibility that subjects are universally implement an extended and generalized form of ADA.

The remainder of this paper is organized as follows. Section 2 presents the theoretical models. Section 3 explains the data. Section 4 presents the procedure of our empirical strategy, with the testable hypotheses laid out in Section 5. Section 6 and 7 present the results of the study, and Section 8 concludes and limitations of our study, and offers avenues for future research.

2 Theory

We layout the theoretical models of adaptive expectation rule (ADA), reference-model based learning (RMBL), and model-free reinforcement learning (model-free RL) in this section, in a manner that focusing on highlight the key difference between the three models.[8]

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[7]According to Figure 2 in Bao et al. (2012), the results from HSM suggest that people are more inclined to use ADA more frequently in negative feedback markets, although its probability is still much smaller than that of contrarian expectations (CTR). Furthermore, they tend to prefer other learning rules such as strong trend extrapolation (TRE) in positive feedback markets. It's worth noting that both CTR and TRE are different heuristics compared to ADA, where they suggest that the prediction is the last price plus the last observed price change multiplied by a constant parameter γ (i.e., $p_{t+1}^e = p_t + \gamma(p_t - p_{t-1})$, CTR: $\gamma < 0$; TRE: $\gamma > 1$).

[8]One may note that we do not consider the popular Bayesian learning. This is because such learning requires subjects to have full knowledge of the distributions used to generate the target moves Berrada et al. (2024) In fact, when they do have the full knowledge of the distribution, Bayesian learning can also be approximated using model-based reinforcement learning. Because subjects typically do not have full knowledge of data generating process of the asset pricing in LtFEs or in most cases in the real life, we do not consider Bayesian learning or model-based RL in our study.

2.1 Adaptive Expectation Rule

The key to the adaptive expectation rule (ADA; Evans and Honkapohja, 2001)[9] is that people adjust their prediction for the next period by a constant fraction of the prediction error, based on their last period prediction. In a setting where the task is to predict the price, and with p^* denoting the price prediction and p denoting the realized price, ADA can be written as:

$$p_t^* = p_{t-1}^* + \bar{G}(p_{t-1} - p_{t-1}^*), \quad 0 < \bar{G} < 1 \tag{1}$$

Because $\bar{G}$ is a constant number that does not vary with time, when defining $\Delta G_{t+1,t} = G_{t+1} - G_t$, the hypothesis derived from ADA is hence:

$$\Delta G_{t+1,t} = \bar{G} - \bar{G} = 0$$

2.2 Reference Model Based Learning

Reference Model Based Learning (RMBL; Bossaerts, 2018) relaxes the constant $\bar{G}$, because it says an agent would speed up learning by adjusting G as long as the current prediction error is larger than the error anticipated from his reference model, or his maximum allowable error. In other words, $G \neq \bar{G}$, and it is possible that $G_t \neq G_{t+1}$.

Denote Z as the squared forecast error from the reference model (RM; e.g., a Kalman filter)[10], and e_t as the prediction error, where $e_t = p_t - p_t^*$, the agent in RMBL is essentially finding a G that minimizes the absolute difference between $(e_t)^2$ and Z:

$$\min_G \Omega_t^2 \quad \text{only if } \Omega_t = (e_t)^2 - Z > 0 \tag{2}$$

Solving the problem[11] by finding the first-order conditions for optimality, the RMBL can be written as:

$$p_t^* = p_{t-1}^* + G_t[p_{t-1} - p_{t-1}^*], \quad \text{where } 4\Omega_t e_t e_{t-1} = 0.$$

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[9]ADA is sometimes referred to as constant α Monte-Carlo methods (Sutton and Barto, 2018).

[10]As discussed in the introduction, Z may not be the prediction error from a specific reference model. In fact, subjects may appoint different reference models, and therefore result in different Z for different subjects. As a result, Z can instead be interpreted as a "maximum allowable error" one can tolerate before speeding up learning by adjusting the adaptive coefficient.

[11]For $\Omega_t = (e_t)^2 - Z$ and $e_t = p_t - p_{t-1}^* = p_t - p_{t-1}^* - Ge_{t-1}$, the first-order derivative is:

$$\frac{d\Omega_t^2}{dG} = \frac{d\Omega_t^2}{d\Omega_t} \cdot \frac{d\Omega_t}{de_t^2} \cdot \frac{de_t^2}{de_t} \cdot \frac{de_t}{dG} = 2\Omega_t \cdot 2e_t \cdot e_{t-1} = 4\Omega_t e_t e_{t-1}.$$

The hypothesis derived from RMBL is hence: When $\Omega_t < 0$, so that the realized forecast error is smaller than expected in RM, the agent would not need to adjust G, and hence $G_{t+1} = G_t$. Similarly, when $\Omega_t = 0$, so that the realized forecast error is the same as expected in RM, G is already set to be at optimality as $4\Omega_t e_t e_{t-1} = 0$, and hence $G_{t+1} = G_t$. In either case, the agent would remain at the current speed of learning and would not adjust G.

$$\Delta G_{t+1,t} = 0 \quad \text{when } \Omega_t \leq 0.$$

By contrast, when $\Omega_t > 0$, G needs to be adjusted and learning needs to be sped up, so that $e_t e_{t-1} = 0$ in the next period, in order to fulfill the optimality.

If $e_t e_{t-1} > 0$, it means that G_t is too timid, which results in an under-prediction followed by an under-prediction, or an over-prediction followed by an over-prediction. Either way, G_t needs to be set to be more aggressive in the next period so that $e_t e_{t-1} = 0$:

$$\Delta G_{t+1,t} > 0 \quad \text{when } \Omega_t \leq 0 \text{ and } e_t e_{t-1} > 0.$$

If $e_t e_{t-1} < 0$, it means that G_t is too aggressive, which results in an under-prediction followed by an over-prediction, or an over-prediction followed by an under-prediction. Either way, G_t needs to be set to be less aggressive in the next period so that $e_t e_{t-1} = 0$:

$$\Delta G_{t+1,t} < 0 \quad \text{when } \Omega_t \leq 0 \text{ and } e_t e_{t-1} < 0.$$

2.3   Model-Free Reinforcement Learning

Compared to RMBL user, the counterpart user that do not satisfice is called a user of model-free reinforcement learning (model-free RL; d'Acremont and Bossaerts, 2016). Specifically, model-free RL relies on autocorrelation of prediction errors to adapt the learning rate. It suggests that *as long as the prediction error is not zero*, the agent would speed up (slow down) adaption to learning rate when autocorrelation of the prediction error is positive (negative).

Practically, by setting the squared error from the reference model Z in RMBL as zero, one gets a learning function that is same as model-free RL:

$$\min_G \left(\Omega_t\right)^2 \quad \text{if } \Omega_t = (e_t)^2 \neq 0, \text{ given } Z = 0 \quad (3) \tag{3}$$

so that:

$$\Delta G_{t+1,t} = 0 \quad \text{when } \Omega_t = (e_t)^2 = 0$$

$$\Delta G_{t+1,t} > 0 \quad \text{when } \Omega_t = (e_t)^2 \neq 0 \text{ and } e_t e_{t-1} > 0$$

$$\Delta G_{t+1,t} < 0 \quad \text{when } \Omega_t = (e_t)^2 \neq 0 \text{ and } e_t e_{t-1} < 0$$

3 Data

To facilitate a clean test of whether subjects implement the learning rules, the experimental environment should enable subjects to be exposed to the history of realized prices and predictions. Meanwhile, they should also be incentivized to submit an accurate price rather than being incentivized to strategize.

3.1 Dataset

Learning-to-Forecast Experiments (LtFEs) fulfills the requirement as stated above. In our study, we utilize the prediction data and the realized value in LtFEs to calculate G in Equation 1 and find out which learning model subjects are using while making the predictions. This is in contrast to the analysis in LtFEs, which usually focuses on the data of the realized value instead of individual's prediction.

Our dataset contains five set LtFEs experimental data (Bao et al., 2012; 2013; 2017; Bao and Hommes, 2019; Bao et al., 2024) to test the predictions of the models. The related summary on the five studies can be found in Table A.1 - A.4.

We are aware that the history of LtFEs can be traced back to Marimon and Sunder (1993) and has been used as a setup to study whether people can learn the fundamental value of assets in various markets, so it is impossible to include all of them in the dataset. Nonetheless, the experimental results we include are generally recent and contain rich observations of 41,490 predictions from 801 subjects.

The next section describes the setting and typical results from LtFEs that are related to this study. They are largely adapted from Bao et al. (2021).

3.2 Learning to Forecast Experiment

A baseline LtFE is usually a market experiment containing 6-10 subjects in each market, spanning across 40-65 consecutive periods. In each market, the subjects play the role of

professional forecasters instead of trader, where their only task is to submit their expectation on the economic variable, most commonly price. They are incentivized to make their best forecasts on these economic variables, because their payoff function is either a quadradic loss function of their prediction error, or a function where prediction error is in the denominator.

The advantage of LtFEs compared to other more common market like double auctions that it provides the clean result on price prediction. Not only the payoff function incentives subjects to submit their best price prediction that are closer to the realized price. The direct price belief in LtFEs also avoid "testing joint hypothesis' problem that would happen in a traditional market where people submit quantity decision. Specifically, it is possible that subjects are able to predict the price accurately but fail to transform the price information to the optimal trading quantity.

A typical user interface can be found in Figure B.1. When making the prediction on the economic value, subjects have full access to the full history of prediction and realization of the economic value. They do not know the data generating process (DGP) of this economic value, just as most market participates do not know the DGP of stock price. However, they do know the value of the variables that determines the realized value, except for others' predictions. For example, the realized price of a risky asset depends on its dividend, the risk-free interest rate, and the aggregative expectation on the price of this risky asset. But subjects would not have information on other people's prediction, so that it prevents the possibility of subjects' strategic behavior. In other words, they are playing with "the market" because none of the subject has larger market power to affect the realized value, as the realized value is the function of the average prediction of all subjects in the market.

The LtFEs are designed to mainly look at whether people can learn and play the fundamental value of the asset, when do not start from it and do not have the knowledge about the specification of the DGP of the economy.

There are mainly two types of LtFEs: positive and negative feedback market. The asset markets are generally considered as positive feedback market because the realized market price is positively correlation of individual's price expectation. By contrast, a supply-driven cobweb market with a production lag is a typical negative feedback market. Specifically, a higher price expectation leads to an increase a production and hence a decreased realized market price.

The typical result from LtFEs is that there is usually a rapid convergence to the fundamental value of the asset when it is a negative feedback market, while a persistent bubbles and crashes if it is a positive feedback market. The recent experimental evidence suggest

that these typical results are robust to size of the experimental market, quantity and return prediction, as well as time horizon.

The example of a typical LtFE instruction Bao et al. (2024) can be found in Appendix C.

4 Methods

While LtFEs are almost suited to test the models, they do not contain information on Z, i.e., the threshold of maximum prediction error that the agent would endure before speeding up the learning by adjusting G. Meanwhile, it is highly possible that Z is heterogeneous among subjects, making it inaccurate to compare the speed of learning between the two samples divided by the ad-hoc assigned threshold. In response, we find the alternatives by using continuous and discrete analysis, which will be discussed in the following sections.

Another modification we make to the model is replacing squared prediction error with absolute prediction error. This is because the squared error can be very large (e.g., up to a maximum of 648,073 in Model 6), making it difficult to interpret the coefficient using squared prediction error.

4.1 Continuous Analysis

A continuous analysis allows us to observe whether the probability of adjusting G in the right direction — raise (reduce) G in the next period when the absolute prediction error in this and previous period are positively (negatively) correlated — increase when the absolute prediction error experienced in this period is larger.

We run subject level fixed-effect logit regression, where each unit of observation is a decision made by a subject i at period t.

$$Y_{i,t+1,t} = \sum_{j=1}^{N} D_{ij}\alpha_i^c + \beta^c E_{i,t} + \gamma^c R_{i,t,t-1} + \delta^c(E_{i,t} \times R_{i,t,t-1}) + \epsilon_{i,t} \qquad (4)$$

The dependent variable $Y_{i,t}$ is binary – it equals to 1 if at period t, subject i increases G in period $t+1$, in other words $\Delta G_{i,t+1,t} > 0$, where $\Delta G_{i,t+1,t} = G_{i,t+1} - G_t$, $G_{i,t} = \frac{p_t^* - p_{t-1}^*}{p_{t-1} - p_{t-1}^*}$. p^* is the prediction and p is the realized price at the corresponding period; and equals to 0 if subject i reduces G in period $t+1$, so that $\Delta G_{i,t+1,t} < 0$.

α_i^c is the subject-level dummy, representing the subject level fixed effect in the logit regression. Specifically, $D_{ij} = 1$ when $i = j$.

$E_{i,t}$ denotes the absolute prediction error subject i incurs at period t, i.e., $|e_{i,t}|$, where $e_{i,t} = p_{i,t} - p_{i,t}^*$.

$R_{i,t,t-1}$ denotes the indicator variable. It equals to 1 if the prediction error in period t and $t-1$ subject i incurs is positively correlated, i.e., $Cov(e_{i,t}, e_{i,t-1}) = e_{i,t}e_{i,t-1} > 0$; but equals to 0 if it is smaller than 0, i.e., $Cov(e_{i,t}, e_{i,t-1}) < 0$.

4.2   Discrete Analysis

In discrete analysis, we split the sample using median of the prediction error for each subject in each treatment.

$$Y_{i,t+1,t} = \sum_{j=1}^{N} D_{ij}\alpha_i^d + \beta^d \text{SE}_{i,t} + \gamma^d R_{i,t,t-1} + \delta^d(\text{SE}_{i,t} \times R_{i,t,t-1}) + \epsilon_{i,t} \qquad (5)$$

Equation 5 is identical to Equation 4, except that we replace $E_{i,t}$ into $\text{SE}_{i,t}$.

$\text{SE}_{i,t}$ equals to 1 if at period t, $E_{i,t}$ is smaller than the median of E_i in a treatment and equals to 0 otherwise.

In turn, a $\delta^d < 0$ would suggest support for RMBL because it indicates that a subject is less likely to increase G with regards to a positively correlated prediction error when experiencing an absolute error that is smaller than his individual median.

One may argue that Z in the RMBL models is the threshold of the maximum allowable absolute prediction error. Therefore, in theory, it should be quite small, and one should use a smaller percentile, e.g., 10th percentile, instead of the 50th percentile. While it is theoretically sound, using a lower percentile as a cutoff creates the problem of unequal sample size between the larger-than-median sample and smaller-than-median sample. The smaller sample size in the smaller-than-median sample, when a lower percentile is set, will tend to favor the hypothesis of RMBL. Specifically, when subjects are less likely to increase G with regards to a positively correlated prediction error when the prediction error is smaller than the median, we expect to see a weaker coefficient $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ in the smaller-than-median.

Similarly, the choice of median over the mean value not only helps mitigate the outliers but also creates a balanced sample size that is critical to our analysis as discussed.

A discrete analysis can serve not only as a robustness check but also, as it will be discussed in Section 5.2, it provides clues about where Z may lie in each experiment and whether Z is unique or heterogeneous with respect to various experimental settings.

5   Testable Hypotheses

The testable hypotheses are summarized in Table 1, and they are validated in the rest of the section. A coefficient is said to be positive if it shows a positive sign, and with a significance level that is at least 5% level. By contrast, a coefficient with a significant that is larger than 5% level is said to be not statistically different from zero.

<div align="center">Table 1. Testable Hypotheses</div>

	Continuous	**Discrete**
RMBL	H1: $\delta^c > 0$ H2: $\gamma^c \geq 0$	H1: $\delta^d < 0$ H2: $\gamma^d > 0$ H3: $\gamma^d + \delta^d = 0$ if $Z_i = \text{Med}(E_{i,t})$; $\quad\ \gamma^d + \delta^d > 0$ if $Z_i < \text{Med}(E_{i,t})$;
Model-free RL	H1: $\delta^c = 0$ H2: $\gamma^c > 0$	H1: $\delta^d = 0$ H2: $\gamma^d > 0$
ADA	H1: $0 < \frac{d(p_{i,t}^* - p_{i,t-1}^*)}{d(p_{i,t-1} - p_{i,t-1}^*)} < 1$ H2: $\delta^c = 0$ H3: $\gamma^c = 0$	H1: $0 < \frac{d(p_{i,t}^* - p_{i,t-1}^*)}{d(p_{i,t-1} - p_{i,t-1}^*)} < 1$ H2: $\delta^d = 0$ H3: $\gamma^d = 0$

5.1   User of Reference Model Based Learning

In a continuous setting, we call the subjects in a market as RMBL users in period t — if he is more likely to increase (decrease) G at period $t+1$ with regards to a positively (negatively) correlated prediction error in period t and $t-1$, when he experiences a larger absolute period error in period t.

$$\frac{d^2 Y_{i,t+1,t}}{dR_{i,t,t-1}\, dE_{i,t}} = \delta^c > 0 \tag{6}$$

Further, because the satisficing rule says that $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = 0$ when $E_{i,t} \le Z_i$, but greater than 0 if otherwise. Therefore, we should see a positive correlation when aggregating all the sample.

$$\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = \gamma^c + \delta^c\, E_{i,t} \ge 0 \tag{7}$$

And hence, we conclude that the subjects in an experiment can be categorized as a RMBL using continuous setting if $\delta^c > 0$, $\gamma^c \ge 0$.

In the discrete setting, the agent is a RMBL user in period t if he is more likely to increase (decrease) G at period $t+1$ with regards to a positively (negatively) correlated prediction error in period t and $t-1$, when he experiences a larger-than-his-individual-median absolute prediction error in period t.

$$\frac{d^2 Y_{i,t+1,t}}{dR_{i,t,t-1}\, dSE_{i,t}} = \delta^d < 0 \tag{8}$$

Further, the satisficing rule suggests that $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ is larger when $SE_{i,t} = 0$:

$$\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = \gamma^d + \delta^d\, SE_{i,t} > 0 \quad \text{when } SE_{i,t} = 0 \tag{9}$$

$$\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = \gamma^d + \delta^d\, SE_{i,t} \ge 0 \quad \text{when } SE_{i,t} = 1 \tag{10}$$

We get $\gamma^d > 0$ and $\delta^d < 0$ from Equation 8 and 9.

By contrast, it is a bit trickier on the solution of Equation 10. Specifically, whether $\gamma^d + \delta^d$ is greater than or equal to 0 depends on the location of Z with regards to the median of the absolute prediction error.

When Z_i is at the median of $E_{i,t}$, then we get a clean result that $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = 0$ when $SE_{i,t} = 1$, so that $\gamma^d + \delta^d = 0$.

But if Z_i is much smaller than $E_{i,t}$, then we would still observe a positive correlation between $Y_{i,t}$ and $R_{i,t,t-1}$ when $SE_{i,t} = 1$, so that $\gamma^d + \delta^d > 0$.

5.2 User of Model-Free Reinforcement Learning

Model-free RL suggests that its user increases (decreases) G at period $t+1$ with regards to a positively (negatively) correlated prediction error in period t and $t-1$ at a constant rate even if he encounters different absolute prediction errors $E_{i,t}$ — so long as the absolute prediction error in period t is not zero, i.e., $E_{i,t} \ne 0$.

Our dataset as well as Equation 4 and 5 are already fit to test whether the user is model-free RL for two reasons, although the two equations do not seem to identify $E_{i,t} = 0$.

First, $E_{i,t} = 0$ is in theory a rare case due to the random error terms that are implemented in the Data Generating Process (DGP) in Learning-to-Forecast Experiments (LtFEs). In other words, even if subjects have learned the fundamental value of the asset and submit it as the prediction in each period, they would still encounter a nonzero absolute prediction error. The experimental results support this hypothesis, where $E_{i,t} = 0$ only accounts for 0.2

Second and more importantly, the incidence of $E_{i,t} = 0$ is already excluded from the observations when analyzing Equation 4 and 5. This is because $E_{i,t} = 0$ would result in a zero correlation between this and the last period observation, and hence a missing value of $R_{i,t,t-1}$.

Therefore, it is safe to say that the condition for a model-free RL user is just that of RMBL but removing the satisficing condition. Specifically,

In the continuous setting:
$$\frac{d^2 Y_{i,t+1,t}}{dR_{i,t,t-1} dE_{i,t}} = \delta^c = 0 \tag{11}$$

and
$$\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = \gamma^c + \delta^c E_{i,t} > 0 \quad \text{when } E_{i,t} \neq 0 \tag{12}$$

so that $\delta^c = 0$ and $\gamma^c > 0$ would be the condition for a model-free RL user.

In the discrete setting:
$$\frac{d^2 Y_{i,t+1,t}}{dR_{i,t,t-1} dSE_{i,t}} = \delta^d = 0 \tag{13}$$

and
$$\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = \gamma^d + \delta^d SE_{i,t} > 0, \quad \forall SE_{i,t} \text{ when } E_{i,t} \neq 0 \tag{14}$$

so that $\delta^d = 0$ and $\gamma^d > 0$ would be the condition for a model-free RL user.

5.3 User of Adaptive Expectation Rule

The sole condition for the users in an experiment to be classified as ADA is a constant G between 0 and 1, so that:
$$0 < G_{i,t} = \frac{p_{i,t}^* - p_{i,t-1}^*}{p_{i,t-1} - p_{i,t-1}^*} < 1 \tag{15}$$

In other words, an ADA user adjusts G at period $t+1$ neither with regards to the correlation of prediction error in period t and $t-1$, nor with regards to a change in the absolute prediction error in period t. It is therefore, equivalently to say that ADA would result in zero coefficients in our estimations:

In the continuous setting:

$$\frac{d^2 Y_{i,t+1,t}}{dR_{i,t,t-1} dE_{i,t}} = \delta^c = 0 \tag{16}$$

and

$$\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = \gamma^c + \delta^c E_{i,t} = 0, \quad \forall E_{i,t} \tag{17}$$

so that $\delta^c = 0$ and $\gamma^c = 0$ would be the condition for an ADA user.

In the discrete setting:

$$\frac{d^2 Y_{i,t+1,t}}{dR_{i,t,t-1} dSE_{i,t}} = \delta^d = 0 \tag{18}$$

and

$$\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}} = \gamma^d + \delta^d SE_{i,t} = 0, \quad \forall SE_{i,t} \tag{19}$$

so that $\delta^d = 0$ and $\gamma^d = 0$ would be the condition for an ADA user.

6   Main Results

This section tests the prediction from ADA, model-free RL, and RMBL, where the hypotheses that are used to distinguish which learning methods subjects in each experiment are using are tabulated in Table 1. The results are tabulated in Table A.5 - A.9. Note that in these tables, prediction error is denoted as PE, and absolute prediction error is denoted as $|PE|$.

6.1   Satisficing in Learning

Result 1: *All the experiments in our dataset show signs of satisficing in at least one of the discrete or continuous analyses. Meanwhile, model-free RL could also provide explanatory power for 3 out of the total of 18 experiments.*

Table A.5 and Table A.6 present our main logit regression analysis to test the hypotheses in Table 1. In general, all the experiments in our dataset can be concluded as RMBL users through either discrete or continuous analysis.

Subjects in most experiments can be classified as RMBL users by both continuous and discrete analysis because they all display $\delta^c > 0$, $\gamma^c \geq 0$ in continuous analysis, and $\delta^d < 0$, $\gamma^d > 0$ in discrete analysis, where they are statistically significant at least at the 5% level.

Specifically, we see a δ^c ranging from 0.01 to 1.42, indicating that when the absolute error term is one unit larger, the incidence where there is an increment of adaptive response with regards to positive correlation of the error term is 1.01 (i.e., $e^{0.01}$) to 4.14 times as likely to occur. Similarly, we see a δ^d ranging from -0.75 to -1.92, suggesting when the absolute error term is *above* the individual median, the incidence where there is an increment of adaptive response with regards to positive correlation of the error term is 2.11 to 6.82 times as likely to occur. Further, a $|\delta^d| > |\delta^c|$ is consistent with the average of the median error to be 4.74, as $|\delta^d| > |\delta^c|$ signals that the absolute value of the individual median error is mostly larger than one unit.

The exceptions are Model 2 in discrete analysis and Models 14 and 15 in continuous analysis, where their coefficient indicates subjects in these experiments can also be classified as model-free RL users (i.e., $\delta^d = 0$ and $\gamma^d > 0$, or $\delta^c = 0$ and $\gamma^c > 0$).

As none of the models find a zero value δ and γ, we conclude that ADA does not explain as well as RMBL and model-free RL in any of the models.

Result 2: *The observation where RMBL explains well on the learning behaviour in all the experiments are robust in split-sample comparison and cross-study analysis.*

We further conduct the robustness check by splitting the above E_i median sample from below E_i median, and the results support the finding where RMBL explains well of the learning behavior.

The analyses are reported in Table A.7 and A.8. Specifically, in all of the models, we find a smaller and weaker coefficient of $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ in the below E_i median sample compared to that of the above E_i median sample. Specifically, the coefficient of $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ ranges from 0 to 2.24 when the absolute error is smaller than subject level median, suggesting that a positively correlated prediction error makes an increment in the adaptive response 1 to 9.4 times as likely to happen. By contrast, the coefficient of $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ ranges from 1.12 to 4.31 when the absolute error is larger than or equal to subject level median, suggesting that a positively correlated prediction error makes an increment in the adaptive response 3.06 to 74.44 times as likely to happen.

Interestingly, in the below E_i median, the coefficient of $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ in the positive feedback market is still statistically significant and larger than 0, while those in the negative feedback

market display a coefficient that is no longer statistically significant in 5 out of the 6 models. By contrast, in the above E_i median sample, all the coefficients of $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ are greater than 0 and with at least a 5% significance level. This observation show some signals that the learning speed $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ (that will be discussed in Section 7) is more chaotic in smaller-than-median PE sample in a negative feedback market.

Finally, we plot the coefficient of $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ against the mean E_i each of the models in Table A.7 and A.8. As illustrated by Figure 1, the coefficient is clustered at a lower level when its absolute prediction error is smaller than individual mean, compared to the sample where the absolute prediction error is greater than individual mean.
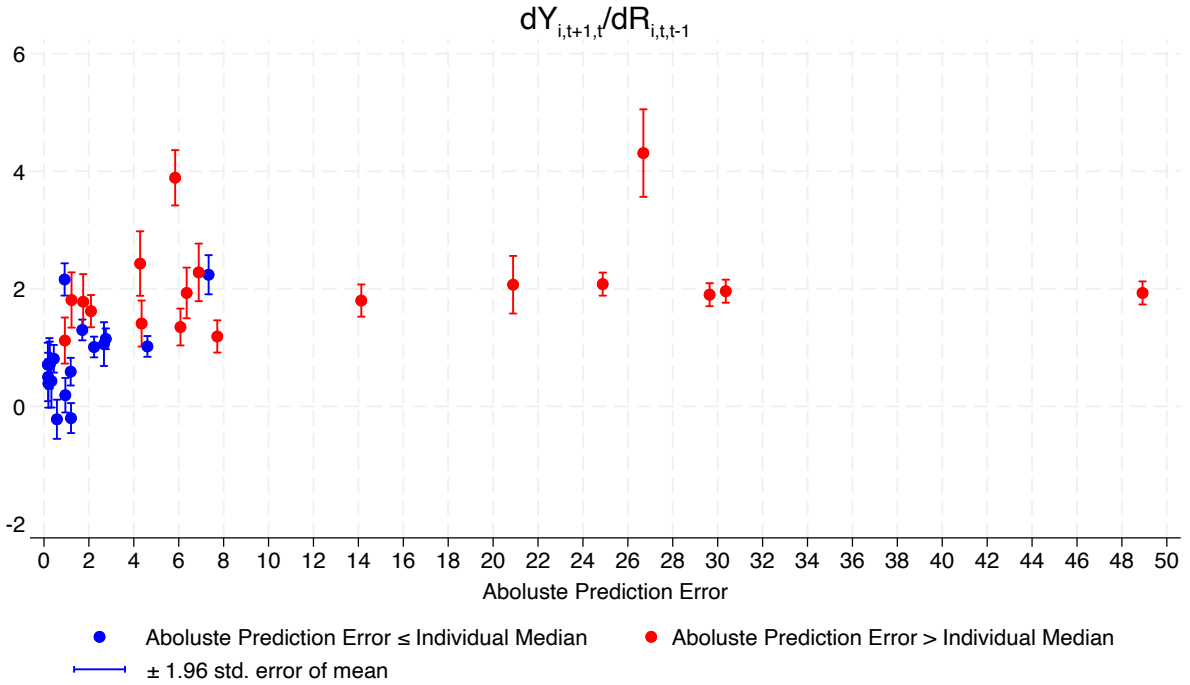
Figure 1. Coefficients of $\frac{dY_{(i,t+1,t)}}{dR_{(i,t,t-1)}}$ with regards to absolute prediction error in 18 experiments, separated by its absolute prediction error with regards to individual median. The detailed regression coefficients can be found in Table A.7 and A.8, or A.9.

6.2 Heterogeneity of Maximum Allowable Prediction Error

Result 3: *Z is not a universal constant and display heterogeneity across the experiment.*

Our theoretical model suggests that among the experiments where its user can be classified as RMBL user, Z_i is right at the median E_i when $\gamma^d + \delta^d = 0$, but much smaller

than median E_i when $\gamma^d + \delta^d > 0$. The estimates on the linear combinations of the two parameters for each model are reported in the last row of Table A.5 and A.6. We find that only in Models 14, 16, 17, and 18, Z_i is at the median E_i, and Z_i are much smaller than at the median E_i in the rest of the models.

To find out whether Z_i is a universal constant, we run a regression comparing the median E_i in the four models. The result is reported in Table 2. We find a significant difference in the median Z_i across the four, which are statistically significant at the 1% level.

Table 2. Comparison of Median E_i in Models where $\gamma^d + \delta^d = 0$

	Median E_i
	(1)
Model 16 (LtFE) = 1	0.67***
	(0.06)
Model 17 (LtFE + LtOE Both) = 1	2.06***
	(0.15)
Model 18 (LtFE + LtOE Either) = 1	1.50***
	(0.08)
Constant (Model 14 (REE = 41, $21 \leq t \leq 43$) = 0)	0.51***
	(0.03)
Observations	150
R-squared	0.70

Note: Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

7 Extended Investigation on Learning Speed

In this section, we repeat the analysis but replace the dependent variable into learning speed instead of learning rate, i.e., replace $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ with $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$. Note that neither RMBL, model-free RL, or ADA predicts anything on $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$. Therefore, this section only serves as an extended investigation beyond the main results. The results are tabulated in Table A.10 - A.13. As before and in these tables, prediction error is denoted as PE, and absolute prediction error is denoted as $|PE|$.

Result 4: *Learning speed does not provide insightful observation. In other words, subjects do not increase G to a larger extent in magnitude with regards to a positive correlation when error is larger. Instead, they are adjusting the magnitude of G at a chaotic manner.*

The previous results suggest that the learning rate (i.e., $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$, where $Y_{i,t+1,t}$ is binary, equaling 1 when G increases but equaling 0 when G decreases) is a positive function of the absolute prediction error that also exhibits satisficing. This observation is consistent with the RMBL prediction from Bossaerts (2018) and is laid out in Section 2 of this paper. However, it does not tell us whether subjects increase G to a larger extent in magnitude with regards to a positive correlation when the error is larger, which Bossaerts (2018) does not discuss.

In response, we repeat our analysis but replace the dependent variable into $\Delta G_{(i,t+1,t)}$, i.e.,

$$\Delta G_{i,t+1,t} = \sum_{j=1}^{N} D_{ij}\alpha_i^c + \beta^c E_{i,t} + \gamma^c R_{i,t,t-1} + \delta^c(E_{i,t} \times R_{i,t,t-1}) + \epsilon_{i,t} \tag{20}$$

and

$$\Delta G_{i,t+1,t} = \sum_{j=1}^{N} D_{ij}\alpha_i^d + \beta^d SE_{i,t} + \gamma^d R_{i,t,t-1} + \delta^d(SE_{i,t} \times R_{i,t,t-1}) + \epsilon_{i,t} \tag{21}$$

The criteria for classifying users in experiments as RMBL, RL, or ADA are the same as in Table 1, with the only difference being the interpretation of the results. Further, we refer $\frac{\Delta G_{(i,t+1,t)}}{dR_{(i,t,t-1)}}$ as the learning speed.

The result in Table A.10 and Table A.11 suggests that while subjects in more than half of the experiments can still be categorized as RMBL users from either a continuous or discrete perspective based on the assessment of the coefficients of γ and δ, there are more inconsistent results from discrete and continuous analysis compared to the results in the learning rate.

But instead of concluding that ADA and model-free RL could explain 40% of our experiments, these results should be interpreted with caution. If subjects truly adhere to the use of ADA or model-free RL models, the analyses using learning rates should yield the same result. Since analyses using learning rates suggest that subjects in all experiments are RMBL users, one can only interpret the results in Table A.10 and Table A.11 as subjects adjusting the learning speed in a chaotic manner with respect to the error. In other words, learning speed does not provide insightful observations as those from the learning rate.

The split-sample illustration in Figure B.2, along with the detailed split-sample coefficient of $\frac{\Delta G_{(i,t+1,t)}}{dR_{(i,t,t-1)}}$ in Table A.12 and Table A.13, is consistent with the doubts regarding the results. As shown in Figure B.2, it is very hard to observe a clean pattern since most coefficients in B.2 have a large standard error. Specifically, there is no clear cutoff on the coefficient of $\frac{\Delta G_{(i,t+1,t)}}{dR_{(i,t,t-1)}}$ that separates the sample into small and large absolute prediction errors. Instead, we even observe multiple red dots lying lower than the blue dots, indicating that the smaller absolute error would lead subjects to speed up learning to a larger extent. As a robustness check, we

plot another graph similar to Figure B.2 but only keep the sample where $\frac{dY_{(i,t+1,t)}}{dR_{(i,t,t-1)}} > 0$. The graph can be found in Figure B.3, and it shows similar chaotic pattern as in Figure B.2.

Nevertheless, except for one observation, all the other coefficients of $\frac{\Delta G_{(i,t+1,t)}}{dR_{(i,t,t-1)}}$ are greater than or equal to zero, indicating that subjects do increase G with regards to the positively correlated prediction error. And the only exception, on the other hand, is shown to be not significantly different from zero, as indicated in Table A.13.

Finally, one interesting observation from our learning speed analysis is that RMBL seems to fit better in positive feedback markets — indicating a consistent finding compared to the learning rate, at least from the continuous analysis. This observation is in line with our finding from learning rate. A possible explanation is that since the characteristic of a positive feedback market is that the price is more volatile and the prediction error is generally larger than the counterpart in negative feedback, subjects are more likely to exhibit satisficing and only willing to exert more cognitive effort to adjust G carefully when the prediction error is larger.

8 Concluding Remarks and Future Research

The existing literature finds the dominant explanatory power of ADA in people's forecasting behavior in the experimental financial market. RMBL (Bossaerts, 2018) can be considered a generalized ADA: it hypothesizes that people adjust how they adapt to past prediction errors with regards to the correlation of the error term; meanwhile, they exhibit satisficing behavior, where they would only do so when the most recent prediction error is larger than their maximum allowable threshold.

While there are some neural evidence suggesting that subjects implement RMBL in decision-making, their sample size is small as data collection using fMRI is rather complicated. In contrast, our study utilizes the rich observations of 41,490 predictions from 801 subjects in 18 economic experiments, where subjects are asked to play the role of financial forecaster, and their only task is to submit a point prediction on the price in the next period as accurately as possible.

We provide experimental evidence that RMBL, as a generalized ADA, best explains the learning behavior when human subjects are tasked as forecasters in a financial market. Specifically, we find that in most of the experiments, the learning rate — whether there is an increment of adaptive response with regards to the positive correlation of the error

term — increases as there is a larger prediction error. Our observation that people satisfice is consistent with the existing evidence in LtFE where subjects are found to implement stopping rules and choose to stop learning when the error is small, when their task is to provide structural estimates of the price in the price (Bao et al., 2022). Furthermore, our results suggest that model-free RL could also provide explanations for 3 out of the total 18 experiments in either discrete or continuous analyses. In contrast, there is no evidence showing that simple ADA fits best among the three learning models in any of the experiments.

While our paper establishes correlational findings indicating that all experiments in the dataset can be explained by RMBL, further studies are necessary for robustness checks.

The possible approach is to conduct a causal study. One way is to manipulate and vary the median prediction error, so that we can observe whether subjects accelerate learning when the prediction error exceeds the exogenously given threshold. Another potential way is to directly ask subjects about threshold information during the experiment while participants make predictions, or to inquire in the post-experiment questionnaire about the exact strategies they use for forecasting. Additionally, it is interesting to conduct a more comprehensive study like Sonnemans et al. (2004) that requires subjects to submit algorithms for price forecasts and delegates the calculation and implementation to the robots that subjects design.

Unfortunately, our results do not provide insights into the exact location of the maximum allowable error; they only suggest heterogeneity in these locations across experiments. One possibility is that instead of a maximum allowable error, subjects in the experiment implement a minimum allowable profit. We were unable to test this hypothesis because profit calculations vary across experiments. An interesting avenue for future research might be to design a tailored experiment and investigate whether the minimum allowable profit serves as the criterion subjects use to adjust their learning strategies.

Finally, our results suggest limited insights into learning speed, though learning speed is not within the predictions of any models. Nevertheless, one interesting observation from our learning speed analysis is that RMBL seems to fit better in positive feedback markets — indicating a consistent finding compared to learning rate, at least from the continuous analysis. It may be possible that subjects are more likely to exhibit satisficing and only exert more cognitive effort to adjust the learning speed carefully when the prediction error is large, and when the market is more volatile. We could not test this hypothesis because none of the subjects in our dataset complete both positive- and negative- feedback LtFE tasks, and hence raises concerns on the heterogeneity of cognitive load. Further research could robustly test this hypothesis by conducting a within-subject comparison on their learning speed between

a positive- and negative- feedback market, and therefore control for the cognitive ability in adjusting the adaptive coefficient with regards to the prediction error. Another approach is to deliberately lower the cognitive effort to adjust the adaption coefficient. For example, one could provide a calculator to all the subjects and observe whether learning speed are consistent with the findings in learning rate, i.e., whether learning speed increases as the prediction error is larger.

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Appendices

Appendix A   Additional Tables

Table A.1. Summary of the Dataset Used: Positive Feedback Market

Study / Abbrev	Description	Treatment	Summary Statistics		Model	Realized Price Dynamics
Bao et al. (2012), JEDC / LtFE in Positive and Negative Feedback Market	LtFEs investigate the converge behaviour in positive and feedback market. They find that negative feedback market converge quickly while positive feedback market do not and show underreaction to short run and overreaction in the long run. - Market size = 6 - # Subject = 48 in each treatment - Convergence to REE: ✗ - Within-subject design, from (1) to (2) to (3)	REE = 56, $1 \le t \le 20$	Var(Price): E($\mathcal{E}$): #Obs:	14.6 0.973 960	(1)	
		REE = 41, $21 \le t \le 43$	Var(Price): E($\mathcal{E}$): #Obs:	47.7 0.573 1104	(2)	
		REE = 62, $44 \le t \le 65$	Var(Price): E($\mathcal{E}$): #Obs:	67.1 0.744 1056	(3)	
Bao et al.(2024), JEBO / Theory of Mind (ToM)	LtFEs investigate whether market become more stable, resulting in lower volatility and fewer price bubbles when it is filled with people high theory of mind (ToM) capability, compared with the counterpart that filled with low ToM subjects. No significant differences are found. - Market size = 6 - # Subject = 96 in each treatment - # Obs = 4800 in each treatment - Convergence to REE: ✗ - Between- subject	High ToM	Var(Price): E($\mathcal{E}$):	9347.0 13.56	(4)	
		Medium High	Var(Price): E($\mathcal{E}$):	21963.2 16.25	(5)	
		Medium Low	Var(Price): E($\mathcal{E}$):	10444.8 16.04	(6)	
		Low ToM	Var(Price): E($\mathcal{E}$):	33306.2 26.80	(7)	

Note: In the column of realized price dynamics, y-axis denotes the average price while x-axis represents the period. There are 70 periods in Bao et al. (2012) while only 50 periods in Bao et al. (2024). In both studies, the dotted line are the fundamental value or rational expectation equilibrium (REE) of the price, while the solid lines are the realized market price (which is a function of all subjects prediction on the price). As the solid line is still far away from the dotted line, it is concluded that the price does not converge to REE at the end of the experiment. The quantitative approach of measuring whether the price converges using relative and absolute deviation can be found in respective original studies.

Table A.2. Summary of the Dataset Used: Positive Feedback Market, contd.

Study / Abbrev	Description	Treatment	Summary Statistics		Model	Realized Price Dynamics
Bao et al. (2017), EJ / LtFE vs. LtOE Positive	Compare the price dynamics and bubbles formation in asset across three treatments: (1) LtFE where subjects submit price only; (2) LtOE where subjects choose quantity to buy/sell; (3) perform both tasks, where payoff depends on price or quantity decision in equal probability. They find that bubble is larger in (2) and (3) compared to (1). - Exclude data in (2) because no price prediction - Market size = 6 - # Subject = 48 in each treatment - # Obs = 2400 in each treatment - Convergence to REE: ✗ - Between- subject	LtFE LtFE + LtOE Both	Var(Price): $E(\mathcal{E})$: Var(Price): $E(\mathcal{E})$:	71.3 1.267 1416.8 7.665	(8) (9)	

Left: LtFE in (1); Right: Mixed in (3)

Note: In the column of realized price dynamics, y-axis denotes the average price while x-axis represents the period. There are 50 periods of the game. The dotted black lines are the fundamental value or rational expectation equilibrium (REE) of the price or the quantity, while the solid lines are the realized market price or quantity (which is a function of all subjects prediction/decision on the price). As the solid line of price prediction in both graph are still far away from the dotted line, it is concluded that the price does not converge to REE at the end of the experiment. The quantitative approach of measuring whether the price converges using relative and absolute deviation can be found in respective original studies.

Table A.3. Summary of the Dataset Used: Positive Feedback Market, contd.

Study / Abbrev	Description	Treatment	Summary Statistics		Model	Realized Price Dynamics
Bao and Hommes (2019), JEDC / Speculator vs. Supplier in Housing Market	Housing market is a combination of positive feedback market (through speculative demand) and negative feedback market (through endogenous supply of housing). The study designs an experimental housing market and study how the strength of the negative feedback, the price elasticity of supply (PES), affect market stability. The result suggests that market stabilizes and price converge to REE only when there is strong PES where there is elastic housing supply (Treatment H: PES = 0.25), but fail to do so when there is no supplier (Treatment N: PES = 0) or when PES is low (Treatment L: PES = 0.1). - Market size = 6 in N, Market size = 9 in L and H - # Subject: Treatment N = 24; Treatment L = 45; Treatment H = 54 - # Obs: Treatment N = 1200; Treatment L = 2250; Treatment H= 2700 - Between-subject	No Supplier (N)	Var(Price): E($\mathcal{E}$): Converge to REE?	11004.0 11.78 ✗	(10)	
		Low PES (L)	Var(Price): E($\mathcal{E}$): Converge to REE?	265.0 17.01 ✗	(11)	
		High PES (H)	Var(Price): E($\mathcal{E}$): Converge to REE?	24.0 3.386 ✓	(12)	

Note: In the column of realized price dynamics, y-axis denotes the average price while x-axis represents the period. There are 50 periods in total. The black line are the fundamental value or rational expectation equilibrium (REE) of the price, while the blue line is the realized market price (which is a function of all subjects prediction on the price). As the solid line in N1 and H1 is still far away from the black line at the end of the experiment, while stick around the black line in H1, we conclude that only H1 converge to REE. The quantitative approach of measuring whether the price converges using relative and absolute deviation can be found in respective original studies.

Table A.4. Summary of the Dataset Used: Negative Feedback Market

Study / Abbrev	Description	Treatment	Summary Statistics		Model	Realized Price Dynamics
Bao et al. (2012), JEDC / LtFE in Positive and Negative Feedback Market	*Same as in Model 1 – 3:* LtFEs investigate the converge behaviour in positive and feedback market. They find that negative feedback market converge quickly while positive feedback market do not and show underreaction to short run and overreaction in the long run. - Market size = 6 - # Subject = 48 in each treatment - Convergence to REE: ✓ Within-subject design, from (1) to (2) to (3)	REE = 56, $1 \le t \le 20$	Var(Price): 3.5 E($\mathcal{E}$): 2.314 #Obs: 960		(13)	
		REE = 41, $21 \le t \le 43$	Var(Price): 11.7 E($\mathcal{E}$): 3.426 #Obs: 1104		(14)	
		REE = 62, $44 \le t \le 65$	Var(Price): 21.9 E($\mathcal{E}$): 3.591 #Obs: 1056		(15)	
Bao et al. (2013), EER / LtFE vs. LtOE Negative	Consider both forecasting (LtFE) and optimization decisions (LtOE) in a negative feedback market (i.e., experimental cobweb economy). The treatment include (1) LtFE: price forecasts only; (2) LtOE: quantity only; (3) LtFE + LtOE Both; (4) LtFE + LtOE Either, where they are paired in teams of 2, where one assigned with LtFE and another assigned with LtOE. All treatments converge to REE but at different speed. Performance is the best in (1) and worst in (3). - Exclude data in (2) because no price prediction - Market Size (i.e., number of subjects subject price prediction) in each treatment = 6 - # Valid Subject: LtFE: 24; LtFE+LtOE Both:42; LtFE + LtOE Either: 36 - # Obs: LtFE: 1200; LtFE+LtOE Both:2100; LtFE + LtOE Either: 1800 - Convergence to REE: ✓ - Between- subject	LtFE	Var(Price): 5.7 E($\mathcal{E}$): 2.465		(16)	
		LtFE + LtOE Both	Var(Price): 56.7 E($\mathcal{E}$): 4.463		(17)	
		LtFE + LtOE Either	Var(Price): 21.4 E($\mathcal{E}$): 3.517		(18)	

Note: In the column of realized price dynamics, y-axis denotes the average price while x-axis represents the period. There are 50 periods in total. The smooth lines are the fundamental value or rational expectation equilibrium (REE) of the price, while the dotted line is the realized market price (which is a function of all subjects prediction on the price). As the smooth line is close to the dotted line in all the market, we conclude that price converge to REE. The quantitative approach of measuring whether the price converges using relative and absolute deviation can be found in respective original studies.

Table A.5. $\dfrac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ in Positive Feedback Market

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Positive Feedback Markets*			Theory of Mind (ToM) / Bao et al (2024), JEBO				LtFE vs. LtOE Positive / Bao et al. (2017), EJ		Speculator vs. Supplier in Housing Market / Bao and Hommes (2019), JEDC		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	High-ToM	Medium High	Medium Low	Low-ToM	LtFE	LtFE+LtOE Both	No Supplier	Low PES	High PES
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Continuous Analysis												
Positively Correlated PE × |PE|, δ^c	1.42***	0.76***	0.93***	0.02***	0.01***	0.01***	0.01***	0.55***	0.15***	0.04***	0.16***	0.33***
	(0.38)	(0.25)	(0.26)	(0.00)	(0.00)	(0.00)	(0.00)	(0.11)	(0.03)	(0.01)	(0.03)	(0.06)
Positively Correlated |PE|, γ^c	0.40*	0.55***	0.65***	1.34***	1.37***	1.47***	1.30***	0.62***	0.72***	1.09***	1.25***	2.05***
	(0.21)	(0.17)	(0.20)	(0.07)	(0.07)	(0.07)	(0.07)	(0.14)	(0.12)	(0.18)	(0.22)	(0.15)
|PE|, β^c	-0.89***	-0.24*	-0.58***	-0.01***	-0.00**	-0.00***	-0.00***	-0.27***	-0.14***	-0.01	-0.12***	-0.14**
	(0.33)	(0.14)	(0.21)	(0.00)	(0.00)	(0.00)	(0.00)	(0.08)	(0.03)	(0.01)	(0.03)	(0.06)
Observations	852	1,053	978	4,558	4,576	4,572	4,551	2,246	2,269	1,138	2,142	2,513
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54
Classification	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL
Panel B: Discrete Analysis												
Positively Correlated PE × |PE|, δ^d	-1.21***	-0.39	-1.10***	-1.11***	-0.78***	-0.66***	-0.91***	-0.75***	-1.24***	-1.04***	-1.92***	-1.77***
	(0.30)	(0.26)	(0.29)	(0.13)	(0.13)	(0.13)	(0.13)	(0.18)	(0.18)	(0.30)	(0.39)	(0.27)
Positively Correlated |PE|, γ^d	1.70***	1.08***	1.80***	2.10***	1.93***	1.95***	1.96***	1.59***	1.81***	2.09***	4.16***	3.92***
	(0.23)	(0.19)	(0.22)	(0.10)	(0.10)	(0.10)	(0.10)	(0.13)	(0.14)	(0.24)	(0.36)	(0.23)
Small |PE|, β^d	0.61***	-0.16	0.46**	0.52***	0.25**	0.28***	0.38***	0.21	0.70***	0.61**	1.18***	0.84***
	(0.22)	(0.20)	(0.22)	(0.11)	(0.10)	(0.10)	(0.10)	(0.13)	(0.13)	(0.26)	(0.38)	(0.24)
Observations	852	1,053	978	4,558	4,576	4,572	4,551	2,246	2,269	1,138	2,142	2,513
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54
Classification	RMBL	Model-free RL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL
Test: $\gamma^d + \delta^d$	0.491**		0.692***	0.989***	1.145***	1.295***	1.045***	0.840***	0.568***	1.054***	2.240***	2.144***
	(0.2)		(0.19)	(0.09)	(0.09)	(0.09)	(0.09)	(0.12)	(0.12)	(0.19)	(0.17)	(0.14)
E (Median of ε_i)	0.391		0.522	5.168	6.201	3.771	12.11	0.919	2.432	5.905	13.90	2.142

Note: Logit estimates fit for panel data with subject level fixed effect (except for Model 9 in Panel A where subject level fixed effect model cannot converge, so that a random effect model is implemented). *** p<0.01, ** p<0.05, * p<0.1.

Table A.6. $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ in Negative Feedback Market

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Negative Feedback Markets*			LtFE vs. LtOE Negative / Bao et al. (2013), EER		
Treatment	REE = 56, $1 \le t \le 20$	REE = 41, $21 \le t \le 43$	REE = 62, $44 \le t \le 65$	LtFE	LtFE+LtOE Both	LtFE+LtOE Either
Model	(13)	(14)	(15)	(16)	(17)	(18)
Panel A: Continuous Analysis						
Positively Correlated PE × |PE|, δ^c	0.38***	0.04*	0.02	0.47***	0.14***	0.20***
	(0.10)	(0.02)	(0.02)	(0.09)	(0.02)	(0.04)
Positively Correlated |PE|, γ^c	0.75***	1.10***	1.33***	-0.21	-0.05	0.20
	(0.20)	(0.16)	(0.17)	(0.17)	(0.13)	(0.14)
|PE|, β^c	-0.16***	0.01	-0.01	-0.12***	-0.04***	-0.05***
	(0.05)	(0.01)	(0.01)	(0.03)	(0.01)	(0.02)
Observations	791	918	826	1,087	1,846	1,537
Number of Subject	48	48	48	24	42	36
Classification	**RMBL**	**Model-free RL**	**Model-free RL**	**RMBL**	**RMBL**	**RMBL**
Panel B: Discrete Analysis						
Positively Correlated PE × |PE|, δ^d	-1.84***	-1.51***	-1.41***	-1.58***	-1.37***	-1.19***
	(0.32)	(0.29)	(0.31)	(0.26)	(0.19)	(0.21)
Positively Correlated |PE|, γ^d	2.28***	1.91***	2.11***	1.39***	1.16***	1.36***
	(0.25)	(0.21)	(0.22)	(0.19)	(0.14)	(0.15)
Small |PE|, β^d	1.10***	0.56***	0.51***	0.79***	0.64***	0.54***
	(0.20)	(0.19)	(0.20)	(0.16)	(0.13)	(0.14)
Observations	791	918	826	1,087	1,846	1,537
Number of Subject	48	48	48	24	42	36
Classification	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**	**RMBL**
Test: $\gamma^d + \delta^d$	0.433**	0.401*	0.694***	-0.196	-0.214	0.170
	(0.22)	(0.2)	(0.22)	(0.17)	(0.13)	(0.15)
E (Median of $\mathcal{E}_i$)	0.782	**0.508**	0.489	**1.182**	**2.568**	**2.006**

Note: Logit estimates fit for panel data with subject level fixed effect *** p<0.01, ** p<0.05, * p<0.1.

Table A.7. $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ in Positive Feedback Market: Split Sample

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Positive Feedback Markets*			Theory of Mind (ToM) / Bao et al (2024), JEBO				LtFE vs. LtOE Positive / Bao et al. (2017), EJ		Speculator vs. Supplier in Housing Market / Bao and Hommes (2019), JEDC		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	High-ToM	Medium High ToM	Medium Low ToM	Low-ToM	LtFE	LtFE+LtOE Both	No Supplier	Low PES	High PES
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Error Smaller than Subject-Level Median (Small Error = 1)												
Positively Correlated PE	0.50**	0.71***	0.71***	1.01***	1.15***	1.30***	1.02***	0.81***	0.59***	1.06***	2.24***	2.16***
	(0.21)	(0.19)	(0.20)	(0.09)	(0.09)	(0.09)	(0.09)	(0.12)	(0.12)	(0.19)	(0.17)	(0.14)
Observations	433	486	473	2,264	2,275	2,308	2,266	1,152	1,150	573	1,058	1,221
Number of Subject	48	48	47	96	96	96	96	48	48	24	45	54
Panel B: Error Larger than or Equal to Subject-Level Median (Small Error = 0)												
Positively Correlated PE	1.78***	1.12***	1.81***	2.08***	1.90***	1.96***	1.93***	1.62***	1.80***	2.07***	4.31***	3.89***
	(0.24)	(0.20)	(0.24)	(0.10)	(0.10)	(0.10)	(0.10)	(0.14)	(0.14)	(0.25)	(0.38)	(0.24)
Observations	419	567	501	2,294	2,301	2,264	2,285	1,094	1,119	565	1,084	1,292
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54

Note: Logit estimates fit for panel data with subject level fixed effect. *** p<0.01, ** p<0.05, * p<0.1.

Table A.8. $\frac{dY_{i,t+1,t}}{dR_{i,t,t-1}}$ in Negative Feedback Market: Split Sample

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Negative Feedback Markets*			LtFE vs. LtOE Negative / Bao et al. (2013), EER		
Treatment	REE = 56, $1 \leq t \leq 20$	REE = 41, $21 \leq t \leq 43$	REE = 62, $44 \leq t \leq 65$	LtFE	LtFE+LtOE Both	LtFE+LtOE Either
	(13)	(14)	(15)	(16)	(17)	(18)
Panel A: Error Smaller than Subject-Level Median (Small Error = 1)						
Positively Correlated	0.43*	0.39*	0.73***	-0.22	-0.20	0.19
PE	(0.23)	(0.21)	(0.22)	(0.17)	(0.13)	(0.15)
Observations	376	383	358	553	902	745
Number of Subject	43	41	41	24	42	36
Panel B: Error Larger than or Equal to Subject-Level Median (Small Error = 0)						
Positively Correlated	2.43***	1.93***	2.28***	1.41***	1.19***	1.35***
PE	(0.28)	(0.22)	(0.25)	(0.20)	(0.14)	(0.16)
Observations	401	532	457	534	944	792
Number of Subject	47	48	48	24	42	36

Note: Logit estimates fit for panel data with subject level fixed effect. *** p<0.01, ** p<0.05, * p<0.1.

Table A.9. Coefficients in Plots

Column	Sample	Figure 1			Figure 2			Figure B.2		
		Coef	SE	|PE|	Coef	SE	|PE|	Coef	SE	|PE|
1	below median	0.5	0.21	0.184	1.83	1.66	0.184	12.75	2.67	0.21
1	equal and above median	0.71	0.19	0.172	4.46	2.07	0.172	25.8	2.95	0.203
2	below median	0.71	0.2	0.258	9.56	4.9	0.258	46.01	9.58	0.305
2	equal and above median	1.01	0.09	2.23	10.18	4.32	2.23	47.29	4.86	2.778
3	below median	1.15	0.09	2.753	11.82	3.11	2.753	54.79	7.17	3.184
3	equal and above median	1.3	0.09	1.706	9.49	5.73	1.706	53.42	5.33	1.953
4	below median	1.02	0.09	4.606	8.45	5.72	4.606	49.78	5.61	6.024
4	equal and above median	0.81	0.12	0.438	0.33	1.72	0.438	8.23	0.81	0.46
5	below median	0.59	0.12	1.19	10.7	4.56	1.19	33.4	7.35	1.368
5	equal and above median	1.06	0.19	2.671	31.68	21.1	2.671	66.71	40.74	3.067
6	below median	2.24	0.17	7.331	7.56	1.81	7.331	11.49	2.22	7.982
6	equal and above median	2.16	0.14	0.92	5.4	0.97	0.92	9.93	1.17	1.143
7	below median	0.43	0.23	0.331	1.79	0.83	0.331	7.73	1.42	0.524
7	equal and above median	0.39	0.21	0.197	-16.65	14.24	0.197	4.93	0.75	0.503
8	below median	0.73	0.22	0.241	2.16	0.92	0.241	8.31	1.76	0.415
8	equal and above median	-0.22	0.17	0.576	1.95	2.78	0.576	12.11	5.79	0.736
9	below median	-0.2	0.13	1.204	0.14	0.62	1.204	6.61	1.15	1.594
9	equal and above median	0.19	0.15	0.952	0.01	0.63	0.952	5.77	0.85	1.263
10	below median	1.78	0.24	1.746	5.15	1.28	1.746	8.21	1.59	1.038
10	equal and above median	1.12	0.2	0.93	4.57	1.73	0.93	14.07	3.12	1.109
11	below median	1.81	0.24	1.226	5.42	1.96	1.226	12.35	1.63	1.319
11	equal and above median	2.08	0.1	24.88	17.9	5.37	24.88	30.66	8.78	29.14
12	below median	1.9	0.1	29.64	14.99	2.46	29.64	27.52	3.42	33.67
12	equal and above median	1.96	0.1	30.36	17.95	3.65	30.36	35.17	5.67	34.35
13	below median	1.93	0.1	48.92	15.27	7.15	48.92	31.25	11.67	53.52
13	equal and above median	1.62	0.14	2.093	5.46	3.31	2.093	10.16	5.54	1.994
14	below median	1.8	0.14	14.13	17.45	5.79	14.13	29.11	7.97	19.6
14	equal and above median	2.07	0.25	20.89	26.63	17.88	20.89	47.65	33.39	22.91
15	below median	4.31	0.38	26.69	5.15	0.63	26.69	5.91	0.65	28.71
15	equal and above median	3.89	0.24	5.841	5.22	0.52	5.841	6.27	0.61	5.986
16	below median	2.43	0.28	4.282	2.75	0.7	4.282	4.25	0.85	3.736
16	equal and above median	1.93	0.22	6.354	27	23.14	6.354	38.5	29.92	7.386
17	below median	2.28	0.25	6.891	4.1	1.11	6.891	7.25	2.12	8.827
17	equal and above median	1.41	0.2	4.348	2.41	0.78	4.348	4.44	1.22	4.491
18	below median	1.19	0.14	7.717	1.41	0.48	7.717	4.18	0.88	7.76
18	equal and above median	1.35	0.16	6.078	1.83	0.46	6.078	3.84	0.58	6.297

Table A.10. $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Positive Feedback Market

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Positive Feedback Markets*			Theory of Mind (ToM) / Bao et al (2024), JEBO				LtFE vs. LtOE Positive / Bao et al. (2017), EJ		Speculator vs. Supplier in Housing Market / Bao and Hommes (2019), JEDC		
Treatment	REE = 56, $1 \le t \le 20$	REE = 41, $21 \le t \le 43$	REE = 62, $44 \le t \le 65$	High-ToM	Medium High	Medium Low	Low-ToM	LtFE	LtFE+LtOE Both	No Supplier	Low PES	High PES
Model	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Continuous Analysis												
Positively Correlated PE × |PE|, δ^c	7.86**	3.25***	3.29	1.16**	0.15***	0.59***	0.17**	26.77**	0.66***	3.43***	-0.14	0.14
	(3.65)	(0.63)	(2.34)	(0.49)	(0.03)	(0.20)	(0.07)	(12.86)	(0.22)	(0.30)	(0.13)	(0.14)
Positively Correlated |PE|, γ^c	-0.68	2.93**	5.23	-2.80	10.71***	4.87	6.01**	-25.87*	8.60***	-11.62	8.69***	5.14***
	(1.85)	(1.37)	(3.65)	(6.80)	(2.30)	(3.99)	(2.67)	(13.94)	(3.08)	(10.26)	(2.64)	(0.74)
|PE|, β^c	-4.85	-1.41**	-1.85	-1.01**	-0.04*	-0.15*	-0.04	-26.53**	-0.32	0.22	0.10	-0.12
	(3.22)	(0.61)	(1.95)	(0.50)	(0.02)	(0.08)	(0.04)	(13.10)	(0.22)	(0.24)	(0.12)	(0.12)
Observations	894	1,077	994	4,598	4,590	4,594	4,590	2,283	2,302	1,152	2,160	2,586
R-squared	0.06	0.03	0.01	0.09	0.01	0.04	0.00	0.27	0.30	0.43	0.03	0.05
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54
Classification	RMBL	RMBL	ADA	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	RMBL	Model-free RL	Model-free RL
Panel B: Discrete Analysis												
Positively Correlated PE × |PE|, δ^d	-3.32*	0.07	4.30	-10.72	-4.08	-11.65	-10.63	-5.84	-4.27	9.90	2.36	0.03
	(1.75)	(2.08)	(6.06)	(9.25)	(3.46)	(8.64)	(14.69)	(6.03)	(3.69)	(7.61)	(1.82)	(1.07)
Positively Correlated |PE|, γ^d	5.16***	4.70**	5.12*	19.72***	15.17***	20.01***	15.97**	5.25	16.71***	21.47	5.42***	5.34***
	(1.13)	(1.81)	(2.56)	(5.86)	(2.43)	(4.18)	(7.44)	(3.36)	(5.33)	(13.36)	(0.69)	(0.53)
Small |PE|, β^d	2.17	-0.09	-2.85	9.83	2.16	5.33	8.67	5.10	6.74	-20.46	-2.09	-0.43
	(1.84)	(2.11)	(4.77)	(8.71)	(3.63)	(7.11)	(14.33)	(5.97)	(6.12)	(16.15)	(1.65)	(0.72)
Observations	894	1,077	994	4,598	4,590	4,594	4,590	2,283	2,302	1,152	2,160	2,586
R-squared	0.02	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.01	0.01	0.03	0.05
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54
Classification	Model-free RL	Model-free RL	ADA	Model-free RL	Model-free RL	Model-free RL	Model-free RL	ADA	Model-free RL	ADA	Model-free RL	Model-free RL

Note: Subject level fixed effects OLS model with cluster-robust standard error for panels nested within subject level. *** p<0.01, ** p<0.05, * p<0.1.

Table A.11. $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Negative Feedback Market

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Negative Feedback Markets*			LtFE vs. LtOE Negative / Bao et al. (2013), EER		
Treatment	REE = 56, $1 \le t \le 20$	REE = 41, $21 \le t \le 43$	REE = 62, $44 \le t \le 65$	LtFE	LtFE+LtOE Both	LtFE+LtOE Either
Model	(13)	(14)	(15)	(16)	(17)	(18)
Panel A: Continuous Analysis						
Positively Correlated PE × |PE|, δ^c	-0.01	9.00	0.10	1.34	0.15**	0.19
	(0.13)	(7.09)	(0.16)	(1.06)	(0.07)	(0.15)
Positively Correlated |PE|, γ^c	2.21***	-24.57	3.16**	0.18	0.18	0.27
	(0.79)	(21.01)	(1.26)	(2.78)	(0.44)	(0.63)
|PE|, β^c	-0.07*	-0.33	-0.14	0.07	-0.12	-0.06
	(0.04)	(0.22)	(0.16)	(0.23)	(0.10)	(0.04)
Observations	2,586	910	1,088	998	1,150	2,012
R-squared	0.05	0.02	0.06	0.02	0.01	0.00
Number of Subject	54	48	48	48	24	42
Classification	Model-free RL	ADA	Model-free RL	ADA	RMBL	ADA
Panel B: Discrete Analysis						
Positively Correlated PE × |PE|, δ^d	-0.92	-41.25	-1.45	-0.44	-1.42**	-1.85**
	(0.62)	(33.50)	(0.96)	(3.06)	(0.55)	(0.71)
Positively Correlated |PE|, γ^d	2.74***	23.05	3.95***	2.61***	1.49***	1.78***
	(0.63)	(18.25)	(1.12)	(0.73)	(0.50)	(0.42)
Small |PE, β^d	2.10***	3.89	0.80	-1.24	1.39**	1.27**
	(0.71)	(2.44)	(1.12)	(2.96)	(0.57)	(0.56)
Observations	910	1,088	998	1,150	2,012	1,726
R-squared	0.04	0.01	0.02	0.00	0.00	0.01
Number of Subject	48	48	48	24	42	36
Classification	Model-free RL	ADA	Model-free RL	Model-free RL	RMBL	RMBL

Note: Subject level fixed effects OLS model with cluster-robust standard error for panels nested within subject level. *** p<0.01, ** p<0.05, * p<0.1.

Table A.12. $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Positive Feedback Market: Split Sample

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Positive Feedback Markets*			Theory of Mind (ToM) / Bao et al (2024), JEBO				LtFE vs. LtOE Positive / Bao et al. (2017), EJ		Speculator vs. Supplier in Housing Market / Bao and Hommes (2019), JEDC		
Treatment	REE = 56, $1 \le t \le 20$	REE = 41, $21 \le t \le 43$	REE = 62, $44 \le t \le 65$	High-ToM	Medium High ToM	Medium Low ToM	Low-ToM	LtFE	LtFE+LtOE Both	No Supplier	Low PES	High PES
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A: Error Smaller than Subject-Level Median (Small Error = 1)												
Positively Correlated PE	1.83	4.46**	9.56*	10.18**	11.82***	9.49	8.45	0.33	10.70**	31.68	7.56***	5.40***
	(1.66)	(2.07)	(4.90)	(4.32)	(3.11)	(5.73)	(5.72)	(1.72)	(4.56)	(21.10)	(1.81)	(0.97)
Observations	459	502	490	2,290	2,281	2,318	2,275	1,174	1,173	583	1,069	1,283
R-squared	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.04
Number of Subject	48	48	48	96	96	96	96	48	48	24	45	54
Panel B: Error Larger than or Equal to Subject-Level Median (Small Error = 0)												
Positively Correlated PE	5.15***	4.57**	5.42***	17.90***	14.99***	17.95***	15.27**	5.46	17.45***	26.63	5.15***	5.22***
	(1.28)	(1.73)	(1.96)	(5.37)	(2.46)	(3.65)	(7.15)	(3.31)	(5.79)	(17.88)	(0.63)	(0.52)
Observations	435	575	504	2,308	2,309	2,276	2,315	1,109	1,129	569	1,091	1,303
R-squared	0.05	0.02	0.02	0.01	0.03	0.01	0.00	0.00	0.01	0.00	0.02	0.07
Number of Subject	435	575	504	2,308	2,309	2,276	2,315	1,109	1,129	569	1,091	1,303

Note: Subject level fixed effects OLS model with cluster-robust standard error for panels nested within subject level. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table A.13. $\frac{d\Delta G_{i,t+1,t}}{dR_{i,t,t-1}}$ in Negative Feedback Market: Split Sample

Study and Description	LtFE in Positive and Negative Feedback Market / Bao et al. (2012), JEDC, *Negative Feedback Markets*			LtFE vs. LtOE Negative / Bao et al. (2013), EER		
Treatment	REE = 56, $1 \le t \le 20$	REE = 41, $21 \le t \le 43$	REE = 62, $44 \le t \le 65$	LtFE	LtFE+LtOE Both	LtFE+LtOE Either
	(13)	(14)	(15)	(16)	(17)	(18)
Panel A: Error Smaller than Subject-Level Median (Small Error = 1)						
Positively Correlated PE	1.79**	-16.65	2.16**	1.95	0.14	0.01
	(0.83)	(14.24)	(0.92)	(2.78)	(0.62)	(0.63)
Observations	474	509	486	590	1,023	872
R-squared	0.02	0.00	0.01	0.00	0.00	0.00
Number of Subject	48	48	48	24	42	36
Panel B: Error Larger than or Equal to Subject-Level Median (Small Error = 0)						
Positively Correlated PE	2.75***	27.00	4.10***	2.41***	1.41***	1.83***
	(0.70)	(23.14)	(1.11)	(0.78)	(0.48)	(0.46)
Observations	436	579	512	560	989	854
R-squared	0.05	0.01	0.03	0.02	0.01	0.02
Number of Subject	48	48	48	24	42	36

Note: Subject level fixed effects OLS model with cluster-robust standard error for panels nested within subject level. *** p<0.01, ** p<0.05, * p<0.1.

Appendix B Additional Figures
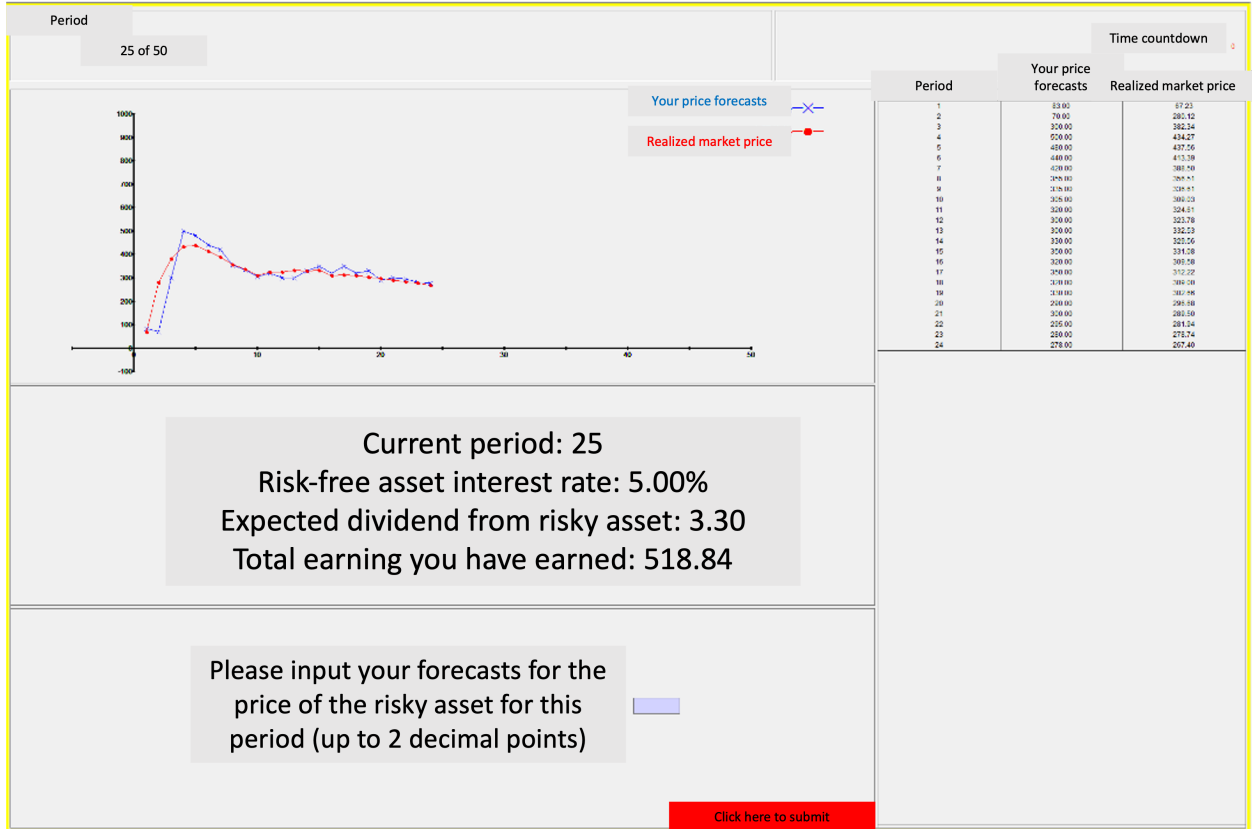

Figure B.1. Computer interface in Bao et al. (2024). DGP or the realized price of this asset that is unknown to the subject is: $p(t) = \frac{1}{1+r}(\bar{p}^e(t) + d) + e_t$, where $r = 0.05$, $d = 3.3$, $e_t \sim \mathcal{N}(0, 1)$, and $\bar{p}^e$ stands for the average prediction by all the subjects in the market. The fundamental value of the asset can be determined by equating $p(t) = \bar{p}^e(t)$, so that $p^* = 66$.
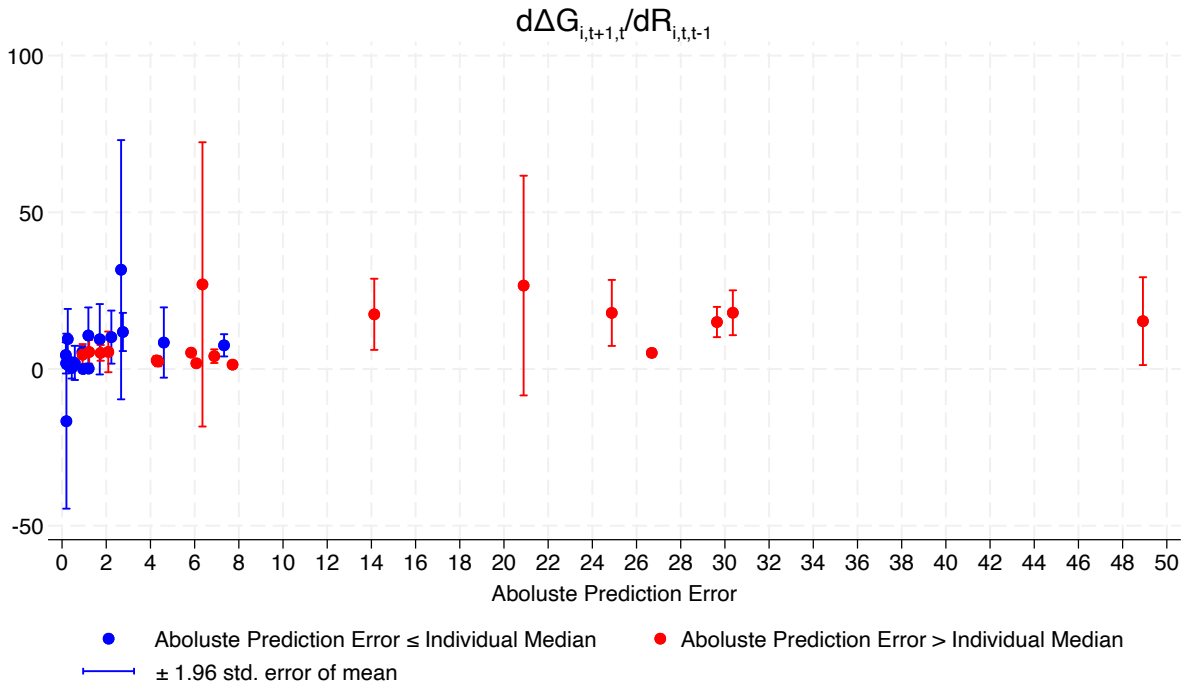
Figure B.2. Coefficient of $d\Delta G_{i,t+1,t}/dR_{i,t,t-1}$ with respect to absolute prediction error in 18 experiments, separated by its absolute prediction error with respect to individual median. The detailed regression coefficients can be found in Tables A.12 and A.13
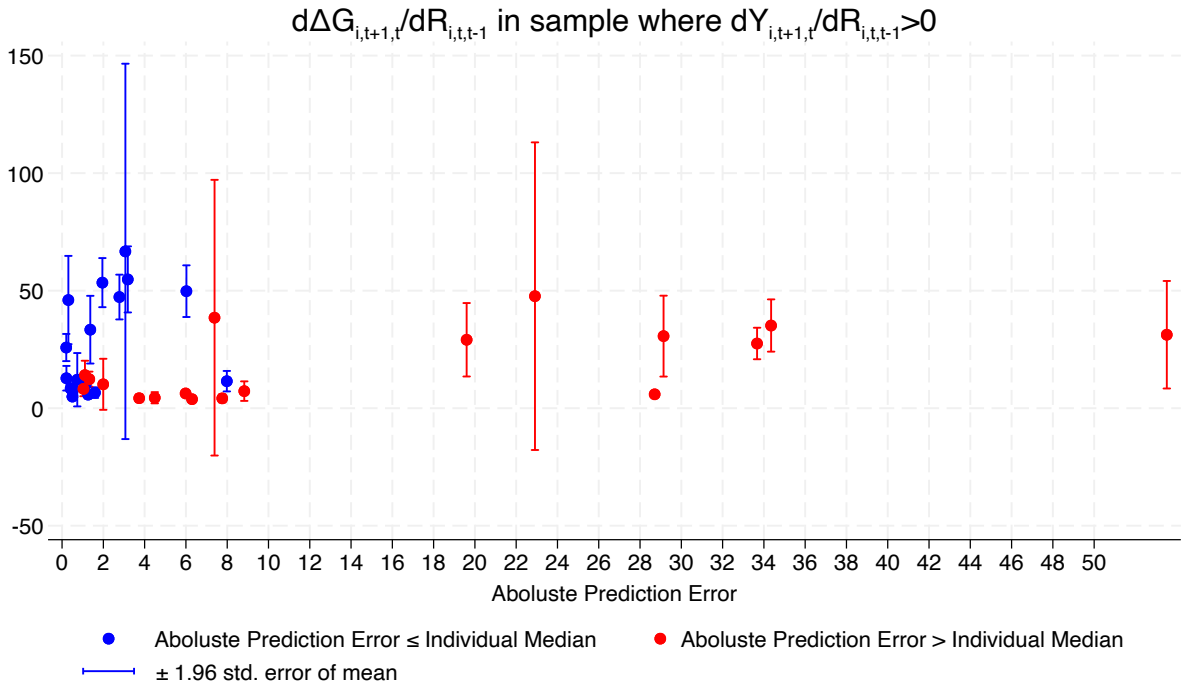
Figure B.3. Coefficients of $d\Delta G_{i,t+1,t}/dR_{i,t,t-1}$ with respect to absolute prediction error in the sample where $dY_{i,t+1,t}/dR_{i,t,t-1} > 0$ in 18 experiments, separated by its absolute prediction error with respect to individual median.

Appendix C   Experimental Instruction for a Typical LtFE

The following instructions are taken from Bao et al. (2024).

Welcome to our experiment! You are participating in an experiment with a real monetary reward. There are 6 participants in each market. In other words, your payoff from the experiment depends on your decision and the decisions of the other 5 participants in your market. The experiment consists of 50 periods. When the experiment ends, we will pay you according to the total number of points you earned. The exchange rate is 80 points = 1 RMB.

The experiment is anonymous. You do not know the identity of the other participants, nor do they know yours. You are not allowed to communicate with others during the experiment, and please place your cell phone in the place assigned by the experimenter.

The following is a detailed description of the experimental setup. Please read it carefully and listen to the explanation by the experimenter. If you have any questions, please feel free to ask.

Your role in the experiment is a financial advisor to an investment fund that wants to optimally invest a large amount of money. The fund is a major participant in the market of some risky assets. The experiment consists of 50 periods. Before the beginning of each period, you must predict the asset price of the risky asset for the investment fund. Based on your prediction, the fund will decide the unit of the risky asset to purchase or sell. The investment fund has two investment options for the limited fund: a risk-free investment (e.g., government bond), with an interest rate of 5%; and a risky investment (e.g., stock), where the value of the dividend of the stock is 3.3 points. According to finance theory, the fundamental value of the risky asset is positively correlated with its dividend and negatively correlated with the interest rate of the risk-free asset.

Note that your prediction is the only determinant of the fund's purchasing/selling decision. The more accurate you predict, the more money the fund will earn. Accordingly, your earnings in each period in the experiment solely depend on your forecasting accuracy in each period. At the end of the experiment, we will pay you according to the total points you earn in 50 periods.

Participants need to complete a test before the beginning of the experiment. Please answer the questions carefully.

1. The determination of the asset price

The stock price is determined by the following mechanism: when the total demand for risky asset in the market is larger than the total supply, or when the total assets firms want to purchase is larger than the total assets firms want to sell, the price will increase. Conversely, if the total demand for risky asset in the market is smaller than the total supply, the price will decrease. This rule is generally consistent with the reality.

There are some large investment funds in the market, where each of the investment funds is advised by a financial advisor played by a participant in the experiment (like you). Generally speaking, the funds will buy more of the risky asset if the financial advisor forecasts that the price of the risky asset will increase, whereas they will sell more assets if the financial advisor forecasts that the price of the risky asset will decrease. The total demand and total supply of the asset are determined by the total purchasing/selling decisions of these large investment funds in the market.

2. Your task in the experiment

Your only task in the experiment is to forecast the market price in each period. At the beginning of the experiment, you need to submit your forecast for the price in the first period, where the forecast should range between 0 to 100. After all participants have submitted their forecasts for the first period, the investment fund played by the computer will make the decisions on purchasing/selling quantities based on each participant's predictions. After that, the experimental program will determine the asset price in the current period using the total purchasing and selling quantities and reveal it to everyone. Based on your forecasting error, your earnings (in points) for period 1 will be calculated.

Next, you need to submit your forecasts for the price in the second period. After all participants have submitted their predictions for the second period, the market price in the second period will be calculated based on all the predictions and their corresponding trading decisions. This process continues for 50 periods. In each period, the available information comprises the previous market prices, your previous predictions, and your previous earnings. **Specifically, the experimental procedure in each period is as follows:**

In general, at period t ($t \geq 2$), participants need to **predict the asset price in the current period t at the beginning of period t.** When forecasting the price, the following information will be disclosed on the user interface: **the interest rate of risk-free asset γ, expected dividend of risky asset y, participant's previous predictions up to $t-1$, previous prices up to period $t-1$, previous total earnings up to period $t-1$.**

Participants only need to fill up **the price forecast for the current period** in the cor-

responding experimental program. After collecting the price forecasts from all participants, the program will disclose the market price in period t, P_t, and the earnings in point that is calculated based on the predicting error between the price forecasts $P_{h,t}^e$ and market price P_t, **at the end of period** t. In other words, **the actual asset price that was predicted at the beginning of each period will be disclosed at the end of each period.**

Simply put, when $t \geq 2$ such as at the beginning of period 5, participants will need to predict the market price in period 5. After all participants submit their forecasts on the price for period 5, the actual market price for the asset in period 5 will be disclosed at the end of period 5. Next is to predict the price in period 6 at the beginning of period 6, where the actual market price for period 6 will be derived using the forecasting price and its corresponding demand/supply equation. And so on.

Specifically, when $t = 1$, participants only need to submit the price forecasts, where no market price and earnings (in points) will be disclosed. The price forecasts for period 1 need to range between 0 and 100.

Note that 60 seconds is given to you for forecasting in each period. Please submit your price forecasts before the end of the countdown. Except for the first period, where the price forecasts need to range between 0 to 100, all price forecasts from period 2 could range between 0 to 1000. All predictions could have an accuracy up to 2 decimal points.

3. Your payoff

Earnings in each period will depend only on the forecasting accuracy in the corresponding period. The more accurate you predict the asset price in each period, the higher your aggregate earnings will be. In other words, as your prediction error increases, or as the difference between the actual stock price and your price forecasts increases, your payment decreases. When your forecast equals the stock price, you get 100 points. When your prediction error is greater than 7, you get 0 points. Hence, your earnings in each period are:

$$\text{earning} = \max\left\{100 - \frac{100}{49} \times (\text{prediction error})^2, 0\right\}$$

The earnings with regards to the prediction error is plotted as follows: