Is Doublet-Triplet Splitting Necessary?

Ilja Doršner[1, 2, *] and Shaikh Saad[3, †]

[1] *University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture in Split, Ruđera Boškovića 32, HR-21000 Split, Croatia*
[2] *J. Stefan Institute, Jamova 39, P. O. Box 3000, SI-1001 Ljubljana, Slovenia*
[3] *Department of Physics, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland*

We demonstrate that it is not necessary to substantially break the mass degeneracy between the Standard Model Higgs boson doublet and the corresponding scalar leptoquark, where these two fields comprise a single five-dimensional $SU(5)$ representation. More precisely, we show that the experimental data on partial proton decay lifetimes cannot place any meaningful bound on the mass of the scalar leptoquark, a.k.a. the color triplet, if one includes effects of higher-dimensional operators within the Georgi-Glashow model. We also point out that the complete suppression of proton decay via the scalar leptoquark mediation is compatible with the partial suppression of the gauge mediated two-body proton decay signatures. Finally, we derive a proper limit on the cutoff scale of the aforementioned higher-dimensional operators and outline how this scenario could be tested.

I. INTRODUCTION

One of the most prominent features of the the Georgi-Glashow model [1] is an ever-present need to introduce an enormous splitting between the masses of the two multiplets residing in the same five-dimensional scalar representation 5_H of the theory. One of these two multiplets is the Standard Model (SM) Higgs boson doublet D, whereas the other one is the color triplet T that generates proton decay. Since the Higgs boson is light while the proton decay mediating scalar leptoquark should be extremely heavy, one needs to strongly break mass degeneracy between them. This issue has been referred to in the literature as the (D)oublet-(T)riplet splitting problem.

We demonstrate that it is not necessary to accomplish the aforementioned mass split if one includes effects of the higher-dimensional operators within the Georgi-Glashow model. The inclusion of these operators allows one to completely suppress proton decay signatures via the scalar leptoquark mediation without affecting viability of the mechanism that generates masses of the SM charged fermions. We furthermore show that this intriguing possibility is also compatible with the partial suppression of the gauge mediated two-body proton decay signatures.

We introduce the most relevant details of the $SU(5)$ setup and the accompanying notation in Sec. II. The proton decay suppression mechanisms for both the scalar and gauge mediations are discussed in detail in Sec. III. We subsequently show how to establish an accurate upper bound on the cutoff scale associated with the higher-dimensional operators if one resorts to these suppression mechanisms in Sec. IV. Final remarks are presented in Sec. V and all our findings summarized in Sec. VI.

* E-mail:dorsner@fesb.hr
† E-mail:shaikh.saad@unibas.ch

II. SETUP

We consider the following $d=4$ and $d=5$ operators

$$\mathcal{L}_Y = 10_F^{\alpha ij}\Big\{Y_d^{\alpha\beta}\overline{5}^\beta_{Fi}5^*_{Hj} + \frac{1}{\Lambda}Y_1^{\alpha\beta}\overline{5}^\beta_{Fi}5^*_{Hk}24^k_{Hj} + \frac{1}{\Lambda}Y_2^{\alpha\beta}\overline{5}^\beta_{Fk}5^*_{Hi}24^k_{Hj}\Big\} + 10_F^{\alpha ij}10_F^{\beta kl}5^m_H\Big\{Y_u^{\alpha\beta}\epsilon_{ijklm} + \frac{1}{\Lambda}Y_3^{\alpha\beta}24^n_{Hm}\epsilon_{ijkln} + \frac{1}{\Lambda}Y_4^{\alpha\beta}24^n_{Hi}\epsilon_{jklmn}\Big\} + \text{h.c.}, \quad (1)$$

where Λ is a cutoff scale of the theory, $\alpha,\beta=1,2,3$ are family indices, $i,j,k,l,m,n=1,\ldots,5$ are $SU(5)$ indices, and Y_d, Y_1, Y_2, Y_u, Y_3, and Y_4 are Yukawa coupling matrices. Here, scalars comprise 5_H and 24_H, the SM fermions reside in $\overline{5}^\alpha_F$ and 10^α_F, while gauge fields are accommodated in 24_G [1].

The first bracket of Eq. (1) contains operators that generate masses of the charged leptons and down-type quarks, whereas the operators from the second bracket of Eq. (1) affect exclusively the up-type quark masses.

The vacuum expectation values (VEVs) of relevance are

$$\langle 24_H\rangle = v_{24}\mathrm{diag}\left(-1,-1,-1,3/2,3/2\right), \quad (2)$$

$$\langle 5_H\rangle = (0 \quad 0 \quad 0 \quad 0 \quad v_5/\sqrt{2})^T, \quad (3)$$

where $v_5 \approx 246\,\mathrm{GeV}$ correctly reproduces masses of the SM gauge boson fields $W^{\pm1}_\mu \in 24_G$ and $Z^0_\mu \in 24_G$, where the field superscripts denote electric charges in units of the positron charge.

Unification of the SM gauge couplings at scale M_{GUT}, if successful, stipulates that

$$M_{\mathrm{GUT}} \equiv M_X = M_Y = \sqrt{25/8}\,g_{\mathrm{GUT}}v_{24}, \quad (4)$$

where g_{GUT} is a gauge coupling constant at M_{GUT}, while M_X and M_Y are masses of the proton decay mediating gauge fields $X^{\pm4/3}_\mu \in 24_G$ and $Y^{\pm1/3}_\mu \in 24_G$, respectively.

$\mathcal{L}_Y$ yields the following charged fermion mass matrices

$$M_E = v_5\left\{\frac{1}{2}Y_d + \frac{3}{4}Y_1\epsilon - \frac{3}{4}Y_2\epsilon\right\}, \tag{5}$$

$$M_D = v_5\left\{\frac{1}{2}Y_d^T + \frac{3}{4}Y_1^T\epsilon + \frac{1}{2}Y_2^T\epsilon\right\}, \tag{6}$$

$$M_U = v_5\left\{\sqrt{2}\left(Y_u + Y_u^T\right) + \frac{3}{\sqrt{2}}\left(Y_3 + Y_3^T\right)\epsilon + \left(\frac{1}{2\sqrt{2}}Y_4 - \sqrt{2}Y_4^T\right)\epsilon\right\}, \tag{7}$$

where we introduce a dimensionless parameter $\epsilon = v_{24}/\Lambda$.

Mass matrices of the SM fermions are diagonalized via

$$E_c^T M_E E = M_E^{\rm diag}, \tag{8}$$
$$D_c^T M_D D = M_D^{\rm diag}, \tag{9}$$
$$U_c^T M_U U = M_U^{\rm diag}, \tag{10}$$
$$N^T M_N N = M_N^{\rm diag}, \tag{11}$$

where E_c, E, D_c, D, U_c, U, and N are 3×3 unitary matrices. M_N is a 3×3 mass matrix for neutrinos, where we explicitly assume neutrinos to be Majorana particles.

III. PROTON DECAY SUPPRESSION

The only scalar that mediates proton decay in this scenario is $T_i^{-1/3} \equiv T_i \in 5_H^i$, where $i = 1, 2, 3$. Its interactions with the SM fermions, as derived from Eq. (1), are (i) $u_{k,\alpha}^T C^{-1} e_\beta T_k^*$:

$$\left\{U^T\left[-\frac{Y_d}{\sqrt{2}} + \frac{Y_1}{\sqrt{2}}\epsilon + \frac{3}{2\sqrt{2}}Y_2\epsilon\right]E\right\}_{\alpha\beta}, \tag{12}$$

(ii) $\epsilon_{ijk} u_{i,\alpha}^{C,T} C^{-1} d_{j,\beta}^C T_k^*$:

$$\left\{U_c^\dagger\left[\frac{Y_d}{\sqrt{2}} - \frac{Y_1}{\sqrt{2}}\epsilon + \frac{Y_2}{\sqrt{2}}\epsilon\right]D_c^*\right\}_{\alpha\beta}, \tag{13}$$

(iii) $d_{k,\alpha}^T C^{-1} \nu_\beta T_k^*$:

$$\left\{D^T\left[\frac{Y_d}{\sqrt{2}} - \frac{Y_1}{\sqrt{2}}\epsilon - \frac{3}{2\sqrt{2}}Y_2\epsilon\right]N\right\}_{\alpha\beta}, \tag{14}$$

(iv) $\epsilon_{ijk} u_{i,\alpha}^T C^{-1} d_{j,\beta} T_k$:

$$\left\{U^T\left[-2\left(Y_u + Y_u^T\right) + 2\left(Y_3 + Y_3^T\right)\epsilon - \frac{1}{2}\left(Y_4 + Y_4^T\right)\epsilon\right]D\right\}_{\alpha\beta}, \tag{15}$$

(v) $u_{k,\alpha}^{C,T} C^{-1} e_\beta^C T_k$:

$$\left\{U_c^\dagger\left[2\left(Y_u + Y_u^T\right) - 2\left(Y_3 + Y_3^T\right)\epsilon + \left(3Y_4 - 2Y_4^T\right)\epsilon\right]E_c^*\right\}_{\alpha\beta}. \tag{16}$$

Since the linear combinations of Yukawa coupling matrices that enter the charged fermion masses M_E, M_D, and M_U differ from those that are featured in the interactions of the scalar leptoquark T with the SM fermions, one can suppress the latter without affecting viability of the former. For example, one can set to zero all quark-quark couplings of T, as given in Eqs. (13) and (15), by imposing the following three relations

$$Y_2\epsilon = Y_1\epsilon - Y_d \tag{17}$$
$$Y_u + Y_u^T = (Y_3 + Y_3^T)\epsilon \equiv Y_S, \tag{18}$$
$$Y_4 = -Y_4^T \equiv Y_A, \tag{19}$$

where the viability of Eq. (18) will be discussed in detail later on. These assumptions consequentially lead to

$$Y_d = \frac{4}{5v_5}E_c^* M_E^{\rm diag} E^\dagger, \tag{20}$$

$$Y_1 = \frac{4}{5\epsilon v_5}D^* M_D^{\rm diag} D_c^\dagger, \tag{21}$$

$$Y_S = \frac{\sqrt{2}}{10v_5}\left(U_c^* M_U^{\rm diag} U^\dagger + U^* M_U^{\rm diag} U_c^\dagger\right), \tag{22}$$

$$Y_A = \frac{\sqrt{2}}{5\epsilon v_5}\left(U_c^* M_U^{\rm diag} U^\dagger - U^* M_U^{\rm diag} U_c^\dagger\right). \tag{23}$$

Even though we still need to demonstrate that one can indeed cancel $d = 4$ contributions with $d = 5$ contributions in the up-type quark sector with perturbative Yukawa couplings, Eqs. (17) through (23) already imply that generation of viable charged fermion masses is compatible with complete suppression of scalar mediated proton decay signatures. Therefore, one of the main results of this *Letter* is that experimental data on partial proton decay lifetimes cannot provide any meaningful constraint on the scalar leptoquark mass if one includes effects of higher-dimensional operators. Our result is applicable to any $SU(5)$ setup that (i) uses a single 5_H and (ii) resorts to $d = 5$ operators to provide viable masses for the SM charged fermions.

The next question we want to address is whether the complete suppression of proton decay signatures via the scalar leptoquark mediation is compatible with the partial suppression of the gauge mediated proton decay.

To that end, we recall that the relevant $d = 6$ operators, which govern all eight two-body proton decay channels via exchanges of $X_\mu^{\pm4/3}$ and $Y_\mu^{\pm1/3}$ gauge bosons are [2, 3]

$$\mathcal{O}_I = \frac{g_{\rm GUT}^2}{2M_{\rm GUT}^2}c(e_\alpha^C, d_\beta)\epsilon_{ijk}\overline{u_i^C}\gamma^\mu u_j\overline{e_\alpha^C}\gamma_\mu d_{k\beta}, \tag{24}$$

$$\mathcal{O}_{II} = \frac{g_{\rm GUT}^2}{2M_{\rm GUT}^2}c(e_\alpha, d_\beta^C)\epsilon_{ijk}\overline{u_i^C}\gamma^\mu u_j\overline{d_{k\beta}^C}\gamma_\mu e_\alpha, \tag{25}$$

$$\mathcal{O}_{III} = \frac{g_{\rm GUT}^2}{2M_{\rm GUT}^2}c(\nu_\rho, d_\alpha, d_\beta^C)\epsilon_{ijk}\overline{u_i^C}\gamma^\mu d_{j\alpha}\overline{d_{k\beta}^C}\gamma_\mu\nu_\rho. \tag{26}$$

The dimensionless coefficients of interest read

$$c(e_\alpha^C, d_\beta) = \left(U_c^\dagger U\right)_{11}\left(E_c^\dagger D\right)_{\alpha\beta} + (U_c^\dagger D)_{1\beta}(E_c^\dagger U)_{\alpha1}, \tag{27}$$

$$c(e_\alpha, d^C_\beta) = \left(U_c^\dagger U\right)_{11} \left(D_c^\dagger E\right)_{\beta\alpha}, \tag{28}$$

$$c(\nu_\rho, d_\alpha, d^C_\beta) = \left(U_c^\dagger D\right)_{1\alpha} (D_c^\dagger N)_{\beta\rho}. \tag{29}$$

Although proton decay via gauge boson exchange cannot be completely rotated away in $SU(5)$ [4], it is possible to significantly suppress coefficients in Eqs. (27), (28), and (29) [5]. This, for example, can be achieved with the followings set of conditions [5]

$$\left(U_c^\dagger D\right)_{1\alpha} = 0, \tag{30}$$
$$\left(E_c^\dagger D\right)_{1\alpha} = \left(E_c^\dagger D\right)_{\alpha 1} = 0, \tag{31}$$
$$\left(D_c^\dagger E\right)_{1\alpha} = \left(D_c^\dagger E\right)_{\alpha 1} = 0, \tag{32}$$

where $\alpha = 1, 2$. Namely, since Eq. (30) implies, via $U^\dagger D = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}) V_{\mathrm{CKM}} \mathrm{diag}(e^{i\phi_4}, e^{i\phi_5}, 1)$, that $|(U_c^\dagger U)_{11}| = |(V_{\mathrm{CKM}})_{13}|$, one finds that Eqs. (30), (31), and (32) facilitate the following levels of suppression

$$\max|c(e^C_\alpha, d_\beta)| = 2 \ \rightarrow\ \max|c(e^C_\alpha, d_\beta)| = |(V_{\mathrm{CKM}})_{13}|,$$
$$\max|c(e_\alpha, d^C_\beta)| = 1 \ \rightarrow\ \max|c(e_\alpha, d^C_\beta)| = |(V_{\mathrm{CKM}})_{13}|,$$
$$\max|c(\nu_\rho, d_\alpha, d^C_\beta)| = 1 \ \rightarrow\ \max|c(\nu_\rho, d_\alpha, d^C_\beta)| = 0.$$

Here, V_{CKM} represents the Cabibbo-Kobayashi-Maskawa mixing matrix with $|(V_{\mathrm{CKM}})_{13}| \approx 4 \times 10^{-3}$, while ϕ_1, ϕ_2, ϕ_3, ϕ_4, and ϕ_5 are arbitrary phases.

Even though the matrix elements of Y_d, Y_1, Y_S, and Y_A, as given in Eqs. (20), (21), (22), and (23), respectively, become somewhat more constrained after implementation of Eqs. (30), (31), and (32), it is still possible to completely suppress scalar mediated proton decay signatures. Again, Eq. (30) establishes relation between unitary matrices U_c and U via V_{CKM}, whereas Eqs. (31) and (32) connect unitary transformations in the charged lepton and down-type quark sectors. For example, Eq. (31) relates E_c and D via

$$E_c = D \begin{pmatrix} 0 & 0 & e^{i\xi_1} \\ 0 & e^{i\xi_2} & 0 \\ e^{i\xi_3} & 0 & 0 \end{pmatrix}, \tag{33}$$

where ξ_1, ξ_2, and ξ_3 are arbitrary phases. Eq. (32) introduces an analogous relationship between D_c and E.

IV. CUTOFF SCALE LIMIT

Let us now derive an accurate upper bound on the cutoff scale Λ of the setup that addresses charged fermion masses via Eq. (1) if one furthermore implements Eqs. (30), (31), and (32) to partially suppress gauge mediated proton decay signatures. This result improves the findings of Ref. [6], where the need to use $d = 5$ operators to generate experimentally observed mismatch between the masses of charged leptons and down-type quarks and conditions of Eqs. (31) and (32) were exploited to find the associated upper bound on the cutoff scale. We, on the other hand, find that the most stringent limit on Λ originates from implementation of Eq. (30) and the fact that the leading term in the up-type quark mass matrix M_U is symmetric in the flavor space, as evident from Eq. (7).

Since a symmetric form of the up-type quark mass matrix would imply $|(U_c^\dagger U)_{11}| = 1$, whereas Eq. (30) yields $|(U_c^\dagger U)_{11}| = |(V_{\mathrm{CKM}})_{13}|$, one needs to have a substantial skew-symmetric component in M_U. To quantify this, we decompose M_U into a sum of a symmetric part S and a skew-symmetric part A via

$$M_U = U_c^* M_U^{\mathrm{diag}} U^\dagger = \frac{v_5}{\sqrt{2}} (S + A). \tag{34}$$

For our purposes it is sufficient to take $M_U^{\mathrm{diag}} = \mathrm{diag}(0, 0, m_t) = \mathrm{diag}(0, 0, y_t v_5/\sqrt{2})$, where m_t and y_t are the top quark mass and Yukawa coupling at M_{GUT} scale, respectively. This decomposition allows us to find a lower limit on the largest possible entry in A, i.e., $\max|(A)_{\alpha\beta}|$, in units of y_t. Since the elements of A can only originate from the very last operator in Eq. (7), our approach should yield an accurate lower limit on parameter $\epsilon \equiv v_{24}/\Lambda$ if we insist on perturbativity of Yukawa couplings. Coincidentally, decomposition of Eq. (34) is compatible with our assumptions of Eqs. (18) and (19). This, in turn, will enable us to explicitly demonstrate that one can cancel $d = 4$ contributions with $d = 5$ contributions with perturbative couplings in the up-type quark sector.

To perform our numerical analysis we first note that Eq. (30) implies that

$$U_c = U \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}) V_{\mathrm{CKM}} \mathrm{diag}(e^{i\phi_4}, e^{i\phi_5}, 1) X^\dagger,$$

where

$$X = \begin{pmatrix} 0 & 0 & e^{i\eta_4} \\ e^{i(\eta_1/2+\eta_2+\eta_3)} c_\theta & e^{i(\eta_1/2+\eta_2-\eta_3)} s_\theta & 0 \\ -e^{i(\eta_1/2-\eta_2+\eta_3)} s_\theta & e^{i(\eta_1/2-\eta_2-\eta_3)} c_\theta & 0 \end{pmatrix}. \tag{35}$$

Here, $c_\theta \equiv \cos\theta$ and $s_\theta \equiv \sin\theta$, where θ is a free parameter, while η_1, η_2, η_3, and η_4 are four arbitrary phases. We can, furthermore, represent unitary matrix U via

$$U = \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}) U_{23} U_{13} U_{12} \mathrm{diag}(e^{i\beta_4}, e^{i\beta_5}, 1),$$

where

$$U_{12} = \begin{pmatrix} c_{u_{12}} & s_{u_{12}} & 0 \\ -s_{u_{12}} & c_{u_{12}} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{36}$$

$$U_{13} = \begin{pmatrix} c_{u_{13}} & 0 & s_{u_{13}} e^{-i\beta_6} \\ 0 & 1 & 0 \\ -s_{u_{13}} e^{i\beta_6} & 0 & c_{u_{13}} \end{pmatrix}, \tag{37}$$

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{u_{23}} & s_{u_{23}} \\ 0 & -s_{u_{23}} & c_{u_{23}} \end{pmatrix}. \tag{38}$$

To evaluate $U_c^* M_U^{\mathrm{diag}} U^\dagger$ of Eq. (34) and thus determine a lower limit on the largest entries in both A and S, one

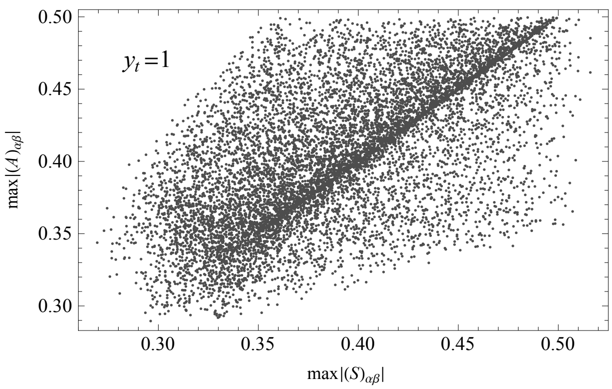

Figure 1. A plot of $\max|(A)_{\alpha\beta}|$ vs. $\max|(S)_{\alpha\beta}|$ for 10^4 sets of randomly chosen values of relevant parameters. For demonstration purpose we set $y_t = 1$.

needs to vary angle θ in X and angles u_{12}, u_{13}, and u_{23} in U. It also seems that one would need to vary phases $\eta_1, \ldots, \eta_4$, $\phi_1, \ldots, \phi_5$, and $\beta_1, \ldots, \beta_6$. However, it turns out that η_4 drops out due to the form of M_U^{diag}, while η_1, η_2, and η_3 can be absorbed through redefinition of phases ϕ_4 and ϕ_5. Also, any changes in β_1, β_2, and β_3 equally affect elements $(S+A)_{\alpha\beta}$ and $(S+A)_{\beta\alpha}$. This makes β_1, β_2, and β_3 completely irrelevant for our analysis. Finally, phases β_4 and β_5 that appear on the right-hand side of M_U^{diag} in Eq. (34) drop out from considerations, while β_4 and β_5 that appear on the left-hand side of M_U^{diag} can be absorbed in ϕ_1 and ϕ_2, respectively. After all is said and done, one only needs to vary four angles and five phases. These phases are ϕ_1, ϕ_2, ϕ_4, ϕ_5, and β_6, where ϕ_3 is also irrelevant as it can be interpreted as an overall phase in Eq. (34), upon redefinition of phases ϕ_1 and ϕ_2.

We have accordingly performed a scan over 10^6 configurations of the randomly chosen value sets of these nine parameters, i.e., θ, u_{12}, u_{13}, u_{23}, ϕ_1, ϕ_2, ϕ_4, ϕ_5, and β_6, to find that $\max|(A)_{\alpha\beta}| \in (0.288 y_t, 0.500 y_t)$ and $\max|(S)_{\alpha\beta}| \in (0.261 y_t, 0.521 y_t)$. Moreover, $\max|(A)_{\alpha\beta}|$ and $\max|(S)_{\alpha\beta}|$ tend to be correlated. Namely, if the largest entry in A is on the larger side, so is the largest entry of matrix S. To that end we present in Fig. 1 a plot of $\max|(A)_{\alpha\beta}|$ vs. $\max|(S)_{\alpha\beta}|$ for 10^4 randomly chosen sets of values of four angles and five phases of relevance.

To find an upper bound on Λ we first observe that the relevant limits are $\max|(A)_{\alpha\beta}| > 0.288 y_t$ and $\max|(S)_{\alpha\beta}| > 0.261 y_t$. Also, the assumption that Y_4 is skew-symmetric, is consistent with the search for an upper limit on Λ as it yields the smallest possible maximum for values of elements in Y_4 matrix. We can thus write, if we take $Y_4 = -Y_4^T$ and use Eqs. (7) and (34) to find both the symmetric and skew-symmetric contributions in M_U, that $S = 2(Y_u + Y_u^T) + 3(Y_3 + Y_3^T)\epsilon$ and $A \equiv 5/2 Y_4 \epsilon$. The latter identity, then, leads to the following inequality

$$\frac{5}{2}\max|(Y_4)_{\alpha\beta}|\epsilon \equiv \frac{5}{2}\max|(Y_4)_{\alpha\beta}|\frac{v_{24}}{\Lambda} > 0.288 y_t. \quad (39)$$

To proceed, one would need to establish what the viable values of v_{24} and y_t, to be used as input in Eq. (39), are. We accordingly observe that $v_{24} = M_{\mathrm{GUT}}\sqrt{(2\alpha_{\mathrm{GUT}}^{-1})/25\pi}$, where $\alpha_{\mathrm{GUT}} = g_{\mathrm{GUT}}^2/(4\pi)$. Also, a model [6–8] that (i) uses perturbative couplings to generate neutrino masses, (ii) relies on operators of Eq. (1) to generate charged fermion masses, and (iii) implements Eqs. (30), (31), and (32) to suppress gauge mediated proton decay signatures, yields $\alpha_{\mathrm{GUT}}^{-1} = 37.1$ and $y_t = 0.42$, where α_{GUT} is evaluated via the two-loop level gauge coupling unification analysis, whereas y_t, at M_{GUT}, is obtained through the one-loop level renormalisation group equation running. If these values for α_{GUT} and y_t are inserted into Eq. (39) we find that the upper limit on Λ for a model of Ref. [7] reads $\Lambda < 57 M_{\mathrm{GUT}}$, where we assume that $5/2\max|(Y_4)_{\alpha\beta}| = \sqrt{4\pi}$. This bound on Λ, as inferred from Eq. (39), is another key finding of this *Letter*. It is, as already advocated, much more stringent than the $\Lambda \lesssim 900 M_{\mathrm{GUT}}$ bound reported in Ref. [6].

It is now trivial to show that it is possible to cancel $d = 4$ contributions with $d = 5$ terms in the up-type quark sector, as required by Eq. (18), if one is to completely suppress scalar mediated proton decay signatures. Namely, if we insert conditions of Eqs. (18) and (19) into Eq. (7) and implement numerical analysis of the decomposition of Eq. (34), we find that

$$5\max|(Y_3 + Y_3^T)_{\alpha\beta}|\epsilon > 0.261 y_t. \quad (40)$$

Since the right-hand side of Eq. (39) is larger than the one in Eq. (40), we see that one can simultaneously accomplish complete suppression of the scalar mediated proton decay and partial suppression of the gauge mediated proton decay, where the correct upper bound on the cutoff scale Λ in Eq. (39) originates from Eqs. (7) and (30).

Let us, as a qualitative example, assume that $\epsilon = 0.05$. This, for $\alpha_{\mathrm{GUT}}^{-1} = 37.1$ and $y_t = 0.42$, implies that $\Lambda = 19.4 M_{\mathrm{GUT}}$, $5\max|(Y_3 + Y_3^T)_{\alpha\beta}| > 2.2$, $5\max|(Y_4)_{\alpha\beta}|/2 > 2.4$, and $2\max|(Y_u + Y_u^T)_{\alpha\beta}| > 0.88$ if one is to implement suppression of both scalar and gauge mediations of proton decay.

If one only wants to suppress scalar mediated proton decay, one can completely drop a skew-symmetric contribution towards M_U in Eq. (7). This could be accomplished by setting, for example, $Y_4 = 0$. In this scenario, $M_U = M_U^T$ implies $U_c \equiv U$. We can thus resort to the decomposition of Eq. (34), drop A, set $U_c \equiv U$, and vary three angles in U, together with three phases β_4, β_5, and β_6, to establish correct upper limit on the cutoff scale Λ. This procedure, combined with the condition of Eq. (18), yields

$$5\max|(Y_3 + Y_3^T)_{\alpha\beta}|\epsilon > 0.334 y_t. \quad (41)$$

It is clear that if one drops a skew-symmetric contribution to M_U, one would obtain somewhat more stringent limit on the cutoff scale Λ. It is thus preferable to have the $d = 5$ skew-symmetric contribution towards M_U or even obligatory if one is to suppress proton decay signatures through gauge boson mediation. Amazingly, though, it is entirely possible to completely suppress proton decay through scalar mediation even when the up-type quark mass matrix is symmetric as long as Λ satisfies Eq. (41).

V. FINAL REMARKS

Final comments are in order.

Our derivation of the limit on Λ is accurate up to the corrections of the order of m_c/m_t, where m_c is the charm quark mass. Moreover, the values of elements in $V_{\rm CKM}$ used to generate bounds in Eqs. (39), (40), and (41) are sourced from the Particle Data Group [9], although the only $V_{\rm CKM}$ entry that matters for our analysis is the Cabibbo angle.

We have explicitly demonstrated how to suppress the quark-quark couplings of the scalar leptoquark T in Sec. III. We could have alternatively opted to suppress the quark-lepton couplings of Eqs. (12), (14), and (16) via $Y_2\epsilon = \frac{2}{3}(Y_d - Y_1\epsilon)$ and $\left(Y_3 + Y_3^T\right)\epsilon = \left(Y_u + Y_u^T\right) + \left(\frac{3}{2}Y_4 - Y_4^T\right)\epsilon$ in order to eliminate proton decay through the color triplet mediation. Note, however, that we cannot simultaneously suppress both sets of the scalar leptoquark couplings. This distinguishes our proposal from an approach advocated in Ref. [10] that insists on complete decoupling of the color triplet from the SM fermions, thus resulting, as specified in Ref. [11], in existence of long-lived colored state(s).

If the color triplet $T \in 5_H$ is indeed light, it would form a complete $SU(5)$ multiplet with the SM Higgs boson doublet $D \in 5_H$ as far as the analysis of the running of the gauge coupling constants is concerned. Since the SM Higgs boson doublet is always beneficial for unification and aids the largeness of $M_{\rm GUT}$, the lightness of T would consequentially lower maximal possible value of $M_{\rm GUT}$, thus making the proton decay signatures through the gauge boson mediation more accessible.

Suppression of the scalar mediation of proton decay signatures somewhat constrains Yukawa couplings of leptoquark T to the SM fermions. (See, for example, Eqs. (20), (21), (22), and (23).) If one, furthermore, suppresses proton decay through gauge mediation, one would also introduce strong correlation between pairs of unitary matrices $(U,\, U_c)$, $(D,\, E_c)$, and $(E,\, D_c)$. Consequently, the detection of the scalar leptoquark T at current or future colliders would enable the verification of consistency between lepton-quark-leptoquark or quark-quark-leptoquark interactions and the observed final states.

VI. CONCLUSIONS

We demonstrate that without additional assumptions about the flavor structure of the theory, the experimental data on partial proton decay lifetimes cannot meaningfully constrain the mass of the scalar leptoquark if one includes effects of higher-dimensional operators. Our result is applicable to any $SU(5)$ setup that (i) uses a single 5_H, where the aforementioned leptoquark resides, and (ii) resorts to the use of $d = 5$ operators to provide viable masses for the SM charged fermions. This undermines the underlying premise behind the need to implement the doublet-triplet mass splitting in this class of models.

We also show that the complete suppression of proton decay via the scalar leptoquark mediation is compatible with the partial suppression of the gauge mediated proton decay signatures.

Finally, we show how to derive an accurate upper bound on the cutoff scale of the $d = 5$ operators in the fermion sector if one implements partial suppression of the two-body proton decay signatures due to the gauge boson exchanges, complete suppression of the scalar mediated proton decay, or both.

ACKNOWLEDGMENTS

I.D. acknowledges the financial support from the Slovenian Research Agency (research core funding No. P1-0035).

[1] H. Georgi and S. L. Glashow, "Unity of All Elementary Particle Forces," *Phys. Rev. Lett.* **32** (1974) 438–441.

[2] P. Fileviez Perez, "Fermion mixings versus d = 6 proton decay," *Phys. Lett.* **B595** (2004) 476–483, `arXiv:hep-ph/0403286 [hep-ph]`.

[3] P. Nath and P. Fileviez Perez, "Proton stability in grand unified theories, in strings and in branes," *Phys. Rept.* **441** (2007) 191–317, `arXiv:hep-ph/0601023`.

[4] S. Nandi, A. Stern, and E. C. G. Sudarshan, "Can proton decay be rotated away?," *Phys. Lett. B* **113** (1982) 165–169.

[5] I. Dorsner and P. Fileviez Perez, "How long could we live?," *Phys. Lett. B* **625** (2005) 88–95, `arXiv:hep-ph/0410198`.

[6] I. Dorsner, P. Fileviez Perez, and G. Rodrigo, "Fermion masses and the UV cutoff of the minimal realistic SU(5)," *Phys. Rev. D* **75** (2007) 125007, `arXiv:hep-ph/0607208`.

[7] I. Dorsner and P. Fileviez Perez, "Unification without supersymmetry: Neutrino mass, proton decay and light leptoquarks," *Nucl. Phys.* **B723** (2005) 53–76, `arXiv:hep-ph/0504276 [hep-ph]`.

[8] I. Dorsner, P. Fileviez Perez, and R. Gonzalez Felipe, "Phenomenological and cosmological aspects of a minimal GUT scenario," *Nucl. Phys. B* **747** (2006) 312–327, `arXiv:hep-ph/0512068`.

[9] **Particle Data Group** Collaboration, R. L. Workman *et al.*, "Review of Particle Physics," *PTEP* **2022** (2022)

083C01.

[10] G. R. Dvali, "Can 'doublet - triplet splitting' problem be solved without doublet - triplet splitting?," *Phys. Lett. B* **287** (1992) 101–108.

[11] G. R. Dvali, "Light color triplet Higgs is compatible with proton stability: An Alternative approach to the doublet - triplet splitting problem," *Phys. Lett. B* **372** (1996) 113–120, `arXiv:hep-ph/9511237`.