Quantum deformed phantom dynamics in light of the generalized uncertainty principle

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Quantum gravity has been baffling the theoretical physicist for decades now: both for its mathematical obscurity and phenomenological testing. Nevertheless, the new era of precision cosmology presents a promising avenue to test the effects of quantum gravity.

In this study, we consider a bottom-up approach. Without resorting to any candidate quantum gravity, we invoke a generalized uncertainty principle (GUP) directly into the cosmological Hamiltonian for a universe sourced by a phantom scalar field with potential to study the early epoch of the evolution. This is followed by a systematic analysis of the dynamics, both qualitatively and quantitatively. Our qualitative analysis shows that the introduction of GUP significantly alters the existence of fixed points for the potential considered in this contribution. In addition, we confirm the existence of an inflationary epoch and analyze the behavior of relevant cosmological parameters with respect to the strength of GUP distortion.

CONTENTS

I. INTRODUCTION

The advent of Einstein's general theory of relativity, with its field equation, gave birth to many fields of research. Since gravity is the only dominant force at large distances, general relativity provides a viable mathematical framework to construct models of cosmology. Over a period of 100 years, General Relativity (GR) has seen profound successes. A few examples include the explanation of the perihelion precession of Mercury[1], the deflection of light rays when passing close by massive bodies [2], and the gravitational redshift of light [3].

In particular, to talk about cosmology, in the year 1929 the discovery of the Hubble's expansion law laid the foundation of modern cosmology. This observational evidence of uniform and isotropic expansion of the universe as incorporated by the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric globally gives rise to the standard model of cosmology (SM) when applied in GR. The FLRW metric is a maximally symmetric geometry of spacetime that supports the Copernican principle. One of the remarkable successes of the standard model of cosmology is the prediction of cosmic microwave background radiation (CMB).

Although successful, the SM was confronted with some serious drawbacks. To give an instance: the causal explanation for two otherwise spatially disconnected regions of space was lacking within the scope of the SM. Technically, these drawbacks of SM are summarized under the name of: horizon, flatness and entropy problem[4–8].

The inflationary paradigm proposed by A. Guth (1981) rescues the situation by providing a mechanism to solve the puzzles of the SM with the help of a nearly exponential expansion of the universe early on. The scalar field serves as a good candidate for an inflationary scenario.

The inflationary epoch not only rescues the standard model of cosmology but also predicts the formation of the large-scale structure of the universe. Although the universe looks almost homogeneous and isotropic at large scale[9], the tiny fluctuation of the order of 10^{-5} is observed in CMB radiation. This tininess of the scale allows us to employ perturbative theory, wherein the zeroth order, background, and spacetime is still FLRW and any in-

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homogeneity is given by the leading order correction. The physical reason for the perturbation of spacetime is the quantum fluctuation of matter content, which is inflaton in this situation. Of course, any perturbation of the matter field would induce a perturbation in the metric field, resulting in the clumping of energy and matter density, leading to the formation of the structure we see today. In the process, inflation expands the tiny causally connected quantum fluctuation into the super-Hubble mode, which re-enters the Hubble radius at a later epoch, giving us a causal mechanism for the large-scale structure we see today[10–12].

The background metric is still treated classically while quantizing the first-order correction in the linearized theory of gravity. However, in a true theory of quantum gravity, one would expect the quantum nature of the background metric to play a significant role, at the scale approaching to Planck region[13, 14].

This incomplete picture of the theory of physical cosmology is due to the continued lack of a consistent candidate for quantum gravity. In fact, this is one of the burning issues in modern theoretical physics. This is obscure because the current understanding of our nature is based on two mathematically incompatible frameworks; viz, general relativity (GR) and quantum mechanics (QM) discussed in [15–18].

In the literature, there exist different candidates based on different philosophical approaches to quantize gravity, each with its own advantages and issues. The two major streams of quantum gravity are–string theory and loop quantum gravity. While string theory is based on the unification of gravity with three other fundamental forces, loop quantum gravity (LQG) is the quantization of the Riemannian geometry of general relativity on its own right[19–22]. The LQG is background independent and non-perturbative. The technique of LQG, when applied to cosmological spacetime, gives rise to various models of quantum corrected cosmology, also called loop quantum cosmology (LQC)[23]. One of the striking features of LQC is the supplant of initial singularity by quantum bounce owing to the quantization of geometry. In lieu of its endeavor to empirically grasp the semi-classical physics near the Planck region, LQC faces serious criticism as it still remains an open problem to recover it directly from the full theory (LQG), in addition to its interpretational issue from a realist perspective.

Nevertheless, in view of the continued absence of a consistent theory of quantum gravity, radically different paths have been adopted. The generalized uncertainty principle is one such attempt that can generate quantum corrected dynamics when applied to cosmology to study the early universe. In this approach, we consider each classical point of space-time as a probability density associated with basis vectors with additional fluctuations in the geometry, giving rise to the extended generalized uncertainty principle(EGUP)[24, 25].

The departure point from classical mechanics to standard quantum mechanics is the Heisenberg's uncertainty principle (HUP), which states the incompatibility of position and momentum operators, reflecting the inherent imprecision of the measurement of one when the other is known precisely.

However, at scales approaching the Planck length, theories of quantum gravity suggest that the geometry of space-time cannot be measured below the Planck scale or below the minimal length. The observation of minimally measured length scale can be studied in different theories like string theory (minimal length scale is the string length itself)[26–31], minimally measured length in loop quantum gravity[32, 33], and in black holes[34].

This immediately implies that HUP is not applicable at the Planck scale as it puts no limit to precisely measure length provided momentum is undetermined. The inconsistency in Heisenberg's uncertainty principle indicates the need to modify the existing canonical Heisenberg uncertainty principle(HUP) by incorporating gravitational correction.

The consideration of quantum fluctuation in the space-time geometry leads to the Generalized uncertainty principle(GUP) which describes the limitation of measurement of position and momentum. The uncertainties of position and momentum depend on the fluctuation of spacetime. The greater the uncertainty in the geometry of space, the greater the uncertainty in the position and momentum of the particles[35, 36]. The notion that gravity might influence the uncertainty principle was first proposed by Mead [37]. Later, candidate theories of quantum gravity such as String Theory [38], Doubly Special Relativity (DSR) Theory, and Black Hole Physics[39] introduced modifications to the commutation relations between position and momentum, which are known as the Generalized Uncertainty Principle (GUP)[40, 41].

Keeping in view the current status of quantum cosmology, the GUP-modified cosmological dynamics require more attention than before to extract the low-energy domain of quantum gravity. In particular, in this article, we focus on the inflationary era due to GUP-corrected effective dynamics for phantom scalar fields.

Phantom inflation leads to a cosmological scenario of the Big Rip, where the universe undergoes a catastrophic expansion that leads to tearing apart all the bound structures, including planets, galaxies, stars, and even fundamental particles. However, investigating these consequences helps in understanding the possible fate of our universe. Lately, there has been significant attention to phantom cosmology, discussed in [42–45]

In this paper, in Sec.II A we review the formulation of GUP-corrected Hamiltonian by taking the Einstein-Hilbert action with a minimally coupled phantom scalar field with a positive cosmological constant. We obtain the GUP-corrected Friedmann equation, Raychaudhuri equation and the klien-Gordon equation. Later, this is extended to include the arbitrary potential in Sec.II B. The techniques of dynamical system analysis have been employed to extract qualitative information about the system in Sec.III. We limit ourselves to quadratic and exponential potentials. In Sec.IV 1, we study inflationary dynamics by calculating the Equation of State (EoS) parameter and slow climb parameters with plots for quadratic potential and exponential potential in Sec.IV 2.

II. GUP-MODIFIED BACKGROUND DYNAMICS

The standard model of cosmology is based on the "Copernican Principle" which says universe is homogeneous and isotropic on a large scale. This is encoded in the maximally symmetric FLRW (Friedmann-Lemaitre-Robertson-Walker) metric:

$$ds^2 = -N^2(t)dt^2 + a^2(t)\left[dr^2 + r^2 d\Omega^2\right], \tag{2.1}$$

Chronologically, GR was developed using the Lagrangian setup. Therefore, the dynamics are obtained as the Euler–Lagrange equation by varying the metric and the matter field of the Einstein-Hilbert action:

$$S_{EH} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}R + \mathcal{L}_m, \tag{2.2}$$

to give $G_{\mu\nu} = T_{\mu\nu}$. Where $G_{\mu\nu}$ is the Einstein Tensor and $T_{\mu\nu}$ is the energy momentum tensor. $\kappa \equiv \frac{1}{8\pi G}$ is set equal to one for the rest of the paper owing to natural units.

GR says that the dynamics of the geometry of the universe are dictated by the matter content of the universe, which in turn tells matter how to move. This implies that the dominant matter content of the universe determines the rate of expansion of the universe for the given epoch in question. An alternative school of thought proposes to modify the geometric sector of the Einstein-Hilbert action Eq.(2.2) giving rise to modified theories of gravity to produce the desired rate of expansion in a given epoch. The modified theories of gravity have been extensively employed to address both early- and late-time scenarios.[46–49].

In this paper, we focus on the former approach-modify the matter content to address the early epoch of the universe. To this effect, we adopt a phantom scalar field. This has been extensively studied in the context of the late-time era of evolution. The fact that the phantom field produces a phase of accelerated expansion of the universe makes it interesting to investigate its implication in inflationary dynamics.

One of our prime focus areas is to explore the tail-end dynamics of the universe where quantum gravity effects could still be important, though not dominant. This, in the domain of loop quantum cosmology, has been reported as *transition phase* from quantum to the classical universe in pre-inflationary dynamics[50, 51].

Having said that, this paper takes a radically different approach by directly invoking a generalized uncertainty principle into the Hamiltonian cosmological dynamics. This is an effective way of modeling the quantum corrected background evolution of the universe.

In this section and in what follows, we review the construction of the GUP-deformed background equation of motion for a phantom scalar field.

Let us begin with the Lagrangian for the phantom field with an arbitrary potential $V(\phi)$ [52],

$$\mathcal{L} = -3a\dot{a}^2 - a^3\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right). \tag{2.3}$$

The Hamiltonian is constructed from the Lagrangian with the help of the Legendre transformation $\mathcal{H} \equiv \dot{a}P_a + \dot{\phi}P_\phi - \mathcal{L}$, to give

$$\mathcal{H} = -\frac{P_a^2}{12a} - \frac{P_\phi^2}{2a^3} + a^3V(\phi). \tag{2.4}$$

Eq.(2.4) is the classical Hamiltonian that governs the background dynamics of a universe dominated by a phantom scalar field with the following symplectic structure.

$$\{a, P_a\} = 1, \quad \{\phi, P_\phi\} = 1, \tag{2.5}$$

where $P_a \equiv \frac{\delta\mathcal{L}}{\delta\dot{a}}$ and $P_\phi = \frac{\mathcal{L}}{\delta\dot{\phi}}$ are the conjugate momentum to a and ϕ respectively. Thus, the complete phase space consists of (a, P_a, ϕ, P_ϕ). While the symplectic algebra 2.5 represents the kinematical structure of a theory, the dynamical evolution is given by the Poisson flow of the phase space variables w.r.t. the Hamiltonian $\mathcal{H}$.

$$\begin{aligned}\dot{a} &= \{a, \mathcal{H}\}, \quad \dot{P}_a = \{P_a, \mathcal{H}\},\\ \dot{\phi} &= \{\phi, \mathcal{H}\}, \quad \dot{P}_\phi = \{P_\phi, \mathcal{H}\}.\end{aligned} \tag{2.6}$$

Using the above Eq.(2.6) the the Klien-Gordon and Raychaudhuri equations for the phantom scalar field are obtained as follows:

$$\ddot{\phi} + 3\dot{\phi}\frac{\dot{a}}{a} - \frac{dV(\phi)}{d\phi} = 0, \tag{2.7}$$

$$2\frac{\ddot{a}}{a^2} + \left(\frac{\dot{a}}{a}\right)^2 = \frac{\dot{\phi}^2}{2} + V(\phi). \tag{2.8}$$

The treatment thus far has been purely classical. However, reformulating the dynamics in terms of Poisson algebra is the point of departure from classical to quantum mechanics. Next, we will see how to incorporate quantum correction arising from GUP into the dynamics.

A. Phantom scalar field with cosmological constant

1. *Classical Dynamics*

In this section, we consider the Einstein–Hilbert action with a minimally coupled phantom scalar field and a positive cosmological constant.

$$S_{EH} = \int \sqrt{-g}\left[\frac{1}{2\kappa}(R-2\Lambda)+\frac{1}{2}\partial_\mu\phi\partial_\nu\phi\right]d^4x, \quad (2.9)$$

On the backdrop of a maximally symmetric spacetime $ds^2 = -N^2(t)dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2]$, where N(t) is the lapse function. Given the FLRW background, our action takes the following form:

$$S = V_0 \int dt\left[-\frac{3a\dot{a}^2}{N} - a^3\left(\frac{\dot{\phi}^2}{2N} + N\Lambda\right)\right]. \quad (2.10)$$

where V_0 is the fictitious volume introduced to facilitate our calculation in an otherwise non-compact FLRW spacetime. However, because it does not affect the dynamics, it can be set equal to 1 without loss of generality. Furthermore, for the rest of the paper we choose the natural unit $\kappa = 1$.

Thus, the expression of the Lagrangian from Eq.(2.10) is given by

$$\mathcal{L} = -\frac{3a\dot{a}^2}{N} - a^3\left(\frac{\dot{\phi}^2}{2N} + N\Lambda\right). \quad (2.11)$$

It is obvious that the Lagrangian, Eq.(2.11), is devoid of $\dot{N}(t)$. and hence there is no dynamics in the lapse function $N(t)$, $P_N \equiv \frac{\partial \mathcal{L}}{\partial \dot{N}} = 0$ being a constant of motion. Therefore, the dynamics of the system are completely contained in the equation of motion for (a, P_a, ϕ, P_ϕ) governed by the Hamiltonian:

$$\mathcal{H} = N\left[\frac{P_a^2}{12a} + \frac{P_\phi^2}{2a^3} - a^3\Lambda\right], \quad (2.12)$$

which is obtained from Eq.(2.11) through the Legendre transformation.

The Equation of motion can be obtained using Eq.(2.6) and by substituting the form of P_a and P_ϕ so obtained from the Lagrangian Eq.(2.11). The Raychaudhuri equation is

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \frac{\dot{\phi}^2}{2} + \Lambda. \quad (2.13)$$

The Friedmann equation is

$$3H^2 = -\frac{\dot{\phi}^2}{2} + \Lambda. \quad (2.14)$$

Furthermore, the Klein-Gordon (KG) equation is

$$\ddot{\phi} + 3H\dot{\phi} = 0. \quad (2.15)$$

The absence of the $\frac{dV}{d\phi}$ term in the KG equation above is because Λ acts as a constant potential.

2. GUP deformed dynamics

In this subsection, we review the inclusion of higher-order correction of the uncertainty principle in the cosmological Hamiltonian. To achieve this, we perform a canonical transformation of the phase space as follows:

$$x = \frac{a^{3/2}}{\mu}\sin(\mu\phi), \qquad y = \frac{a^{3/2}}{\mu}\cos(\mu\phi), \quad (2.16)$$

While preserving the dynamics. To see it explicitly, we note that differentiation of Eq.(2.16) gives

$$\dot{x} = \frac{3}{2}\frac{a^{1/2}\dot{a}}{\mu}\sin(\mu\phi) + (a^{3/2}\dot{\phi})\cos(\mu\phi), \quad (2.17)$$

$$\dot{y} = \frac{3}{2}\frac{a^{1/2}\dot{a}}{\mu}\cos(\mu\phi) - (a^{3/2}\dot{\phi})\sin(\mu\phi). \quad (2.18)$$

Now, multiplying both sides of Eq.(2.17) by $\sin(\mu\phi)$ and Eq.(2.18) by $\cos(\mu\phi)$ and adding them together gives

$$\dot{x}\sin(\mu\phi) + \dot{y}\cos(\mu\phi) = \frac{3}{2}\frac{a^{1/2}}{\mu}\dot{a}. \quad (2.19)$$

Next by squaring both sides of Eq.(2.19), followed by dividing the expression by $2N$ and considering $\mu = \sqrt{3/8}$ one obtains,

$$\frac{\dot{x}^2\sin^2(\mu\phi) + \dot{y}^2\cos^2(\mu\phi) + 2\dot{x}\dot{y}\sin(\mu\phi)\cos(\mu\phi)}{2N} = \frac{3a\dot{a}^2}{N}. \quad (2.20)$$

Instead, multiply both sides of Eq.(2.17) by $\cos(\mu\phi)$ and Eq.(2.18) by $\sin(\mu\phi)$ and subtracting

$$\dot{x}\cos(\mu\phi) - \dot{y}\sin(\mu\phi) = a^{3/2}\dot{\phi}. \quad (2.21)$$

This is followed by squaring Eq.(2.21) and dividing it by $2N$ to obtain

$$\frac{\dot{x}^2\cos^2(\mu\phi) + \dot{y}^2\sin^2(\mu\phi) - 2\dot{x}\dot{y}\cos(\mu\phi)\sin(\mu\phi)}{2N} = \frac{a^3\dot{\phi}^2}{2N}. \quad (2.22)$$

In addition, by considering the sum of the squares of the components in Eq.(2.16), we have:

$$x^2 + y^2 = \frac{a^3}{\mu^2}\sin^2(\mu\phi) + \frac{a^3}{\mu^2}\cos^2(\mu\phi)$$

$$\implies x^2 + y^2 = \frac{a^3}{\mu^2} = \frac{8a^3}{3}. \quad (2.23)$$

Now, from Eq.(2.23) we observe that the physical volume of the universe under study can be elegantly expressed as the radius of the circle, with $(0,0)$ as the center in the plane containing the configuration variable (x, y). Since the Friedmann equation is nothing but the fractional rate of change of volume, intuitively, one can speculate that the knowledge of the dynamics of the pair (x, y) suffices to predict the evolution of the universe.

Having set the stage in terms of Cartesian pair (x,y), we now return to the question of dynamics. Using Eq.(2.20) Eq.(2.21) and Eq.(2.23) we obtain the final form of the Lagrangian in terms of configuration coordinates $(x, y, \dot{x}, \dot{y})$ to be

$$\mathcal{L} = -\left[\frac{\dot{x}^2 + \dot{y}^2}{2N} + \frac{3}{8}(x^2 + y^2)\Lambda N\right]. \tag{2.24}$$

A quick look at the form of the Lagrangian suggests that dynamics is symmetric w.r.t the origin of the circle in the plane of (x, y).

Given this, it is straightforward to obtain the canonically transformed Hamiltonian using the Legendre transformation. Thus, the final form of the Hamiltonian looks like

$$\mathcal{H}_0 = N\left[\frac{P_x^2}{2} + \frac{P_y^2}{2} + \frac{\omega^2}{2}(x^2 + y^2)\right], \tag{2.25}$$

where $\omega^2 = -\frac{3}{4}\Lambda$. The utility of the canonical transformation of Eq.(2.16) is clear from the elegant expression of 2.25. It is straightforward to see that the dynamics of a universe with a phantom scalar and a positive cosmological constant can be expressed as a system of two decoupled simple harmonic oscillators in the new phase space.

However, the Eq.(2.25) is still classical, though expressed in a different guise. To write down the dynamics due to momentum deformation owing to the Generalized Uncertainty Principle (GUP), we introduce semi-classical canonical variables q_i and P_i and introduce GUP in the WDW equation in our cosmological model as followed by [53].

$$q_i = q_{0i}, \quad P_i = P_{0i}\left(1 - \beta\gamma P_0 + 2\gamma^2\frac{\beta^2 + 2\epsilon}{3}P_0^2\right), \tag{2.26}$$

where $P_0^2 = P_{0j}P_{0j}$, β, ϵ and γ are constants. we calculate the GUP distorted Hamiltonian up to the order of γ^2 [54] as:

$$\mathcal{H} = \mathcal{H}_0 - \beta\gamma(P_{0x}^2 + p_{0y}^2)^{3/2} + \gamma^2(P_{0x}^2 + p_{0y}^2)^2\left(\frac{\beta^2}{6} + \frac{2\epsilon}{3}\right) + \mathcal{O}(\gamma^3), \tag{2.27}$$

where $\mathcal{H}_0 = \frac{P_{0x}^2}{2} + \frac{P_{0y}^2}{2} + \frac{\omega^2}{2}(x^2 + y^2)$ is the unperturbed Hamiltonian before introducing GUP. From now on, the subscript 0 will be used to denote the unperturbed version to represent Eq.(2.25). For example, the unperturbed x, y are represented as (q_{0x}, q_{0y}) while the unperturbed pair (P_x, P_y) are to be recognized as (P_{0x}, P_{0y}).

In the final step, we re-express the GUP deformed Hamiltonian Eq.(2.27) in the cosmological phase-space variable. This is achieved by applying the inverse transformation to express Eq(2.27) in the cosmological variables, namely, the scale factor, the scalar field, and their corresponding conjugate momenta. Using Eq.(2.17) and Eq.(2.17), we obtain

$$P_{0x} = \dot{x} = \frac{3}{2}\frac{a^{1/2}\dot{a}}{\mu}\sin(\mu\phi) + (a^{3/2}\dot{\phi})\cos(\mu\phi), \tag{2.28}$$

$$P_{0y} = \dot{y} = \frac{3}{2}\frac{a^{1/2}\dot{a}}{\mu}\cos(\mu\phi) - (a^{3/2}\dot{\phi})\sin(\mu\phi). \tag{2.29}$$

Because there are no dynamics in the lapse function, without loss of generality $N(t) = 1$,

$$\frac{\partial\mathcal{L}}{\partial\dot{a}} = P_a = -6\dot{a}a,$$

and

$$\frac{\partial\mathcal{L}}{\partial\dot{\phi}} = P_\phi = -a^3\dot{\phi},$$

by substituting P_a and P_ϕ in Eq.(2.28) the equations becomes

$$P_{0x} = -\frac{P_a}{4a^{1/2}\mu}\sin(\mu\phi) - \frac{P_\phi}{a^{3/2}}\cos(\mu\phi), \tag{2.30}$$

$$P_{0y} = -\frac{P_a}{4a^{1/2}\mu}\cos(\mu\phi) + \frac{P_\phi}{a^{3/2}}\sin(\mu\phi). \tag{2.31}$$

Applying P_{0x} and P_{0y} in the momentum-deformed Hamiltonian due to GUP correction in Eq.(2.27) by taking $\epsilon = 1$,

$$\mathcal{H}_{GUP} = \left[\frac{\omega^2}{2}a^3 + \frac{P_a^2}{12a} + \frac{P_\phi^2}{2a^3} + 2\gamma^2\left(\frac{P_a^4}{108a^2} + \frac{P_\phi^4}{3a^6} + \frac{P_a^2P_\phi^2}{9a^4}\right)\right]. \tag{2.32}$$

This is the required GUP distorted Hamiltonian in the cosmological phase space dominated by a phantom scalar field with a positive cosmological constant up to second-order perturbation.

Using Eq.(2.6)we can obtain the final Raychaudhuri equation

$$2\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2=\frac{\dot{\phi}^2}{2}-\frac{\omega^2}{2}+\gamma^2\left(16a^3H^4+\frac{4a^3\dot{\phi}^4}{3}+\frac{32a^3H^2\dot{\phi}^2}{3}\right). \tag{2.33}$$

The final Friedmann equation is

$$3H^2=-\left[\frac{\dot{\phi}^2}{2}+\frac{\omega^2}{2}\right]-2\gamma^2\left[12H^4a^3+\frac{a^3\dot{\phi}^4}{3}+4a^3H^2\dot{\phi}^2\right]. \tag{2.34}$$

and the KG equation is

$$\ddot{\phi}+3\dot{\phi}H=0. \tag{2.35}$$

We notice that the Raychaudhuri Eq.(2.33) and the Friedmann Eq.(2.34) directly incorporate quantum corrections, whereas the Klein-Gordon Eq.(2.35), there is no explicit dependence on quantum correction. Any quantum deformation entering the KG Eq.(2.35) arises only implicitly through the Hubble parameter. This situation is similar to the models of loop quantum cosmology [57–62].

3. *GUP corrected Friedmann equation with cosmological constant*

In this subsection we simplify the Friedmann Eq.(2.34) Defining a new parameter for the perturbation term $2\gamma^2\equiv\alpha$, the Eq.(2.34) can be written as a quadratic equation in terms of $\tilde{H}=H^2$ as

$$3\tilde{H}^2+\left(\dot{\phi}^2+\frac{3}{4\alpha a^3}\right)\tilde{H}-C_0=0, \tag{2.36}$$

where $C\equiv-\left(\frac{\dot{\phi}^2}{2}+\frac{\omega^2}{2}+\frac{\alpha a^3\dot{\phi}^4}{3}\right)$ and $C_0\equiv\frac{C}{4\alpha a^3}$. Solving the quadratic equation gives us:

$$\tilde{H}=\frac{-\left(\dot{\phi}^2+\frac{3}{4\alpha a^3}\right)\pm\sqrt{\left(\dot{\phi}^2+\frac{3}{4\alpha a^3}\right)^2+12C_0}}{6}. \tag{2.37}$$

Since $\tilde{H}=H^2$, the right-hand side of the Eq.(2.37) must be greater than zero. This implies

$$\sqrt{\left(\dot{\phi}^2+\frac{3}{4\alpha a^3}\right)^2+12C_0}>\left(\dot{\phi}^2+\frac{3}{4\alpha a^3}\right). \tag{2.38}$$

this is equivalent to

$$C_0>0, \tag{2.39}$$

or

$$\frac{\dot{\phi}^2}{2}+\frac{\alpha a^3\dot{\phi}^4}{3}<\frac{3\Lambda}{8}. \tag{2.40}$$

Rewriting Eq.(2.37)

$$H^2=-\frac{\dot{\phi}^2}{6}-\frac{1}{8\alpha a^3}+\left(\frac{\dot{\phi}^2}{6}+\frac{1}{8\alpha a^3}\right)\sqrt{1+\frac{12C_0}{\left(\dot{\phi}^2+\frac{3}{4\alpha a^3}\right)^2}}, \tag{2.41}$$

and applying the conditions Eq.(2.38)

$$\sqrt{1+\frac{12C_0}{M^2}}>1, \tag{2.42}$$

where $M\equiv\dot{\phi}^2+\frac{3}{4\alpha a^3}$. Applying binomial expansion of the Eq.(2.41) we get,

$$H^2=-\frac{\dot{\phi}^2}{6}-\frac{1}{8\alpha a^3}+\left(\frac{\dot{\phi}^2}{6}+\frac{1}{8\alpha a^3}\right)\left[1+\frac{1}{2}\left(\frac{12C_0}{M^2}\right)-\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2!}\right)\left(\frac{12C_0}{M^2}\right)^2+\mathcal{O}(3)\right]. \tag{2.43}$$

Considering term up to the first order in $12C_0/M^2$,

$$H^2=\frac{\left(\frac{-\dot{\phi}^2}{2}+\frac{3\Lambda}{8}-\frac{\alpha a^3\dot{\phi}^4}{3}\right)}{4\alpha a^3\dot{\phi}^2+3}. \tag{2.44}$$

Eq.(2.44) is the required Friedmann equation with GUP modification for a phantom scalar field with a positive cosmological constant.

B. Phantom scalar field with an arbitrary potential

Now, we construct the GUP-modified Friedmann equation for arbitrary potential by applying the change in variables shown in the previous section II A 2

$$x = \frac{a^{\frac{3}{2}}}{\mu}\sin(\mu\phi), \qquad y = \frac{a^{\frac{3}{2}}}{\mu}\cos(\mu\phi). \tag{2.45}$$

The Lagrangian Eq.(2.3) can be written in terms of $(x,y,\dot{x},\dot{y})$ by applying the procedure prescribed in the section II A 2 as

$$\mathcal{L} = -\left[\frac{\dot{x}^2+\dot{y}^2}{2} + \frac{3}{8}(x^2+y^2)V(\phi)\right]. \tag{2.46}$$

In addition, the unperturbed Hamiltonian can be obtained by the Legendre transformation of the Eq.(2.46)

$$H_0 = \frac{P_x^2}{2} + \frac{P_y^2}{2} - \frac{3}{8}(x^2+y^2)V(\phi). \tag{2.47}$$

Introducing GUP, as given in section II A 2 to the unperturbed Hamiltonian Eq.(2.47) generates

$$\mathcal{H}_{GUP} = \frac{P_a^2}{12a} + \frac{P_\phi^2}{2a^3} - a^3V(\phi) + 2\gamma^2\left(\frac{P_a^4}{108a^2} + \frac{P_\phi^4}{3a^6} + \frac{P_a^2P_\phi^2}{9a^4}\right). \tag{2.48}$$

From the above GUP corrected Hamiltonian Eq.(2.48) one can easily obtain the Raychaudhuri equation as:

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \frac{\dot{\phi}^2}{2} + V(\phi) + \gamma^2\left(16a^3H^4 + \frac{4a^3\dot{\phi}^4}{3} + \frac{32a^3H^2\dot{\phi}^2}{3}\right). \tag{2.49}$$

Following the same procedure given in section II A 3, after some algebraic manipulation, the Friedmann equation for arbitrary potential can be derived as

$$H^2 = \frac{\left(-\frac{\dot{\phi}^2}{2} + V(\phi) - \frac{\alpha a^3\dot{\phi}^4}{3}\right)}{4\alpha a^3\dot{\phi}^2+3}, \tag{2.50}$$

and the KG equations remain the same for any arbitrary potential as

$$\ddot{\phi} + 3\dot{\phi}H - \frac{dV(\phi)}{d\phi} = 0. \tag{2.51}$$

We note that this feature is consistent with the loop quantum cosmology scenario in which the KG equation does not receive any explicit quantum correction. Any quantum correction occurs only through the Hubble parameter; thus, the classical form of the KG equation holds for any given potential in the GUP corrected regime.

In standard cosmology, the factor $\frac{\alpha a^3\dot{\phi}^4}{3}$ is absent in the numerator of the Eq.(2.50), retaining only the energy density term. The additional term arises solely from the quantum corrections due to GUP. In addition to this, the Freidemann Eq.(2.50) reveals the possibility of singularity resolution. The occurrence of non-singular bounce in phantom models has already been studied in [63–67]. The possibility of a non-singular bounce is further enhanced by the presence of a negative quantum corrected term, $-\frac{\alpha a^3\dot{\phi}^4}{3}$, in the Friedmann Eq.(2.50). Though $\dot{a}=0$ is a necessary condition for the occurrence of a bounce, the solution must meet $\ddot{a}>0$ at the turning point of the bounce for the contracting universe to reverse its trajectory and begin expanding again. This condition is satisfied by the Raychaudhuri Eq.(2.49) as the right-hand side of the equation all the terms are positive due to even powers for the non-zero arbitrary values of $V(\phi)$ and $\dot{\phi}$.

III. DYNAMICAL SYSTEM ANALYSIS

A nonlinear system is generally difficult to predict. However, the method of dynamical system analysis serves as a powerful tool for extracting qualitative information from nonlinear systems. The Einstein's equation when applied to FLRW spacetime, becomes a set of coupled second-order differential equations. However, with a suitable choice of variables, they can be transformed to first-order autonomous differential equations. The powerful technique of dynamical system analysis (DSA) can be widely employed to analyze the background dynamics of cosmological models.

The approach of DSA is qualitative in nature. It predicts the overall dynamics of the of the system in question without actually solving it. The most widely adopted method is linear stability analysis. In this method the system's dynamics is cast as a set of first-order autonomous differential equations, and the fixed points are defined as the points where the vector flow of the dynamical variables vanishes. The precise nature of the fixed points is obtained by examining the behavior of the leading order perturbation around the fixed points. Mathematically, the signs of the eigenvalues of the Jacobian matrix evaluated at the fixed points indicate the nature of the fixed points. For an extensive review of DSA, we refer the readers to [68–71].

Phase portraits, on the other hand, are visual representations of the trajectories of a dynamic system. It provides insights into the qualitative behavior of the system, pictorially. This is achieved by drawing a tangent at each point given by the flow vectors of the autonomous differential equations. When applied to cosmology, it offers an intuitive understanding of the fate of the Universe even without solving the equations.

In the subsequent subsections, we construct the autonomous equations and henceforth perform dynamic system analysis for background cosmology with GUP modification.

We know that the Einstein's equation are second order in nature. However, to perform a dynamical system analysis, they must be transformed into a set of first-order differential equations. A widely practiced method is to begin by normalizing the Friedmann equation by the square of the Hubble parameter to make each term dimensionless. In the process, it brings all the components contributing to the Hubble rate on equal footing. The next step is to write the equation of motion for each independent dimensionless dynamical variable obtained with the help of the Raychaudhuri and Klien-Gordon equation. The final set of equations for each variable, when expressed entirely in terms of newly defined dimensionless variables, constitutes the required autonomous system.

A. For $V(\phi) = V_0\phi^2$

In this subsection, we study qualitative dynamics (DSA) for a universe dominated by a phantom scalar field with quadratic potential considering GUP correction.

Following the above-mentioned recipe, we perform dynamic system analysis by constructing autonomous equations for the chosen potential. This will allow us to perform fixed point analysis and study the behavior of phase portraits. From the eq.(2.50), the expansion normalized Friedmann equation can be written as

$$4\alpha a^3\dot{\phi}^2 + 3 = -\frac{\dot{\phi}^2}{2H^2} + \frac{V(\phi)}{H^2} - \frac{2\alpha a^3\dot{\phi}^2}{3}\frac{\dot{\phi}^2}{2H^2}. \tag{3.1}$$

A suitable choice of dimensionless dynamical also called, Einstein-Nordstrom, variable is

$$x \equiv \sqrt{\frac{\dot{\phi}^2}{6H^2}}, \quad y \equiv \sqrt{\frac{V_0\phi^2}{3H^2}}, \quad z \equiv \sqrt{\alpha a^3\dot{\phi}^2}. \tag{3.2}$$

Expressing in terms of dynamic variables, the expansion-normalized Friedmann equation is

$$x^2 + y^2 - \frac{2}{3}z^2x^2 - \frac{4}{3}z^2 = 1. \tag{3.3}$$

In addition to this from eq.(2.8) the KG equation is

$$\ddot{\phi} + 3\dot{\phi}H - 2V_0\phi = 0. \tag{3.4}$$

From eq.(2.49) we can obtain,

$$\frac{\dot{H}}{H^2} = \frac{3}{2}(x^2 + y^2 - 1) + \frac{(2 + 6x^4 + 8x^2)(-1 - x^2 + y^2)}{(2x^4 + 4x^2)}. \tag{3.5}$$

With this, the ground has been prepared to construct the autonomous differential equation for EN variables. However, it should be noted that the EN variables alone fail to close the autonomous system for power law potential. For example, a new dynamic variable depending on ϕ appears in the form of $\lambda \equiv -\frac{V_{,\phi}}{V}$ with its independent equation of motion.

The physical phase space for a power law system is always represented by a positive y half-cylinder stretching from $\lambda = 0$ to $+\infty$ due to symmetry[72–75], which means that the phase space is not compact. To make phase space compact, we choose a new dynamic variable u, which is

$$u = \frac{\lambda}{\lambda + 1}.$$

This transformation makes our phase space compact with the range $0 \leq u \leq 1$. finally we can write our system of equations in terms of u.

The autonomous set of dynamical equations for quadratic potential with GUP correction read as

$$f(x,y) \equiv \frac{dx}{dN} = -3(1-u)x - u\left(\sqrt{6}y^2\right) - (1-u)x\left\{\frac{3}{2}(x^2+y^2-1) + \frac{(2+6x^4+8x^2)(-1-x^2+y^2)}{(2x^4+4x^2)}\right\}, \quad (3.6)$$

$$g(x,y) \equiv \frac{dy}{dN} = \sqrt{6}yxu - y(1-u)\left\{\frac{3}{2}(x^2+y^2-1) + \frac{(2+6x^4+8x^2)(-1-x^2+y^2)}{(2x^4+4x^2)}\right\}, \quad (3.7)$$

$$h(x,y) \equiv \frac{du}{dN} = -\sqrt{6}(\Gamma-1)(1-u)xz^2, \quad (3.8)$$

where $\Gamma = \frac{VV_{,\phi\phi}}{V_{,\phi}^2}$. The complete cosmological dynamics for the universe with quadratic potential and GUP correction are contained in the three equations 3.7, 3.8 and 3.8.

1. Fixed point analysis

In this subsection, we perform a thorough analysis of the fixed points of the cosmological system dictated by a phantom scalar field with quadratic potential in a GUP-modified scenario. Later, we compare the results with those without GUP.

Fixed points are obtained by simultaneously solving the autonomous system to zero. This physically means that the system becomes stationary at that point. In our present case, this is given by the set of all points for which autonomous equations(3.7-3.8) simultaneously equal to zero . We apply linear stability analysis to comment on the nature of the fixed points. The fixed points along with their behavior, as obtained, are tabulated in III A 1. To compare our results with the original dynamics, we turn off the quantum perturbation by setting $\alpha = 0$ in Eqs.(3.7-3.8). The results are summarized in Table (III A 1).

Comparison of the tables III A 1 and III A 1 reveals the dynamics without GUP are richer than the dynamics after GUP modification. This is clear as III A 1 depicting the dynamics without GUP contains a greater number of fixed points than in III A 1 with GUP. However, introducing distortion due to GUP does not change the fixed points as the eigenvalues in both tables contain zero. The theory of linear stability analysis has no say on the properties of such fixed points. One requires higher order analysis. However, in this paper, we confine ourselves to linear stability analysis.

TABLE I. **Fixed points and the stability analysis for the Potential $V(\phi) = V_0\phi^2$ without GUP**

	x	y	u	E_1	E_2	E_3	Stability
A	0	0	c	$-3(1-c)/2$	$3(1-c)/2$	0	saddle point
B	0	0	0	$-3/2$	$3/2$	0	saddle point
C	0	0	1	0	0	0	neutral point
D	c	0	1	$2\sqrt{6}c$	$-\sqrt{6}c$	0	saddle point
E	0	-1	0	-3	-3	0	stable point
F	0	1	0	-3	-3	0	stable point

TABLE II. **Fixed points and the stability analysis for the Potential $V(\phi) = V_0\phi^2$** with GUP

	x	y	u	E_1	E_2	E_3	Stability
A	$+c$(other than 0)	0	1	0	$2\sqrt{6}c$	$-\left(\frac{2\sqrt{6}c^3+\sqrt{6}c^5}{c^2(2+c^2)}\right)$	saddle point
B	$-c$(other than 0)	0	1	0	$-2\sqrt{6}c$	$\left(\frac{2\sqrt{6}c^3+\sqrt{6}c^5}{c^2(2+c^2)}\right)$	saddle point

B. Exponential Potential

In this subsection, we consider a potential of the form

$$V(\phi) = V_0 e^{-k\phi}, \quad (3.9)$$

to study the GUP modified background dynamics. EN variables are defined as

$$x \equiv \sqrt{\frac{\dot{\phi}^2}{6H^2}}, \quad y \equiv \sqrt{\frac{V_0 e^{-k\phi}}{3H^2}}, \quad z \equiv \sqrt{\alpha a^3 \dot{\phi}^2}. \quad (3.10)$$

The expansion-normalized Friedmann equation in the form of EN variables is

$$x^2 + y^2 - \frac{2}{3}z^2x^2 - \frac{4}{3}z^2 = 1, \tag{3.11}$$

and from eq.(2.8) the Klein Gordon equation reads as

$$\ddot{\phi} + 3\dot{\phi}H + V_0ke^{-k\phi} = 0. \tag{3.12}$$

Furthermore, the Raychaudhuri equation (2.49) in terms of EN variables is

$$\begin{aligned}\frac{\dot{H}}{H^2} &= \frac{3}{2}(x^2+y^2-1)\\ &\quad+\frac{(2+6x^4+8x^2)(-1-x^2+y^2)}{(2x^4+4x^2)}.\end{aligned} \tag{3.13}$$

Because our potential is exponential, we can get $\Gamma = \frac{VV_{,\phi\phi}}{V^2_{,\phi}}$ equal to 1. While constructing the autonomous equation, we obtain a factor of $Q \equiv -\frac{V_{,\phi}}{V} = k$, which is equal to a constant k. With these, we can write the autonomous differential equations as:

$$\tilde{f}(x,y) = \frac{dx}{dN} = -3x - \sqrt{\frac{3}{2}}Qy^2 - x\left\{\frac{3}{2}(x^2+y^2-1) + \frac{(2+6x^4+8x^2)(-1-x^2+y^2)}{(2x^4+4x^2)}\right\}, \tag{3.14}$$

$$\tilde{g}(x,y) = \frac{dy}{dN} = -\sqrt{\frac{3}{2}}Qyx - y\left\{\frac{3}{2}(x^2+y^2-1) + \frac{(2+6x^4+8x^2)(-1-x^2+y^2)}{(2x^4+4x^2)}\right\}. \tag{3.15}$$

The complete dynamics of the system with exponential potential and GUP corrections are contained in the two equations 3.15 and 3.15. The original dynamics without quantum fluctuations can be easily obtained as a limiting case by setting $\alpha = 0$.

1. *Fixed point analysis*

In this subsection, we perform dynamical system analysis for the exponential potential both with and without GUP. Because of the exponential potential, the system can be completely explained by two dynamical variables x and y. The fixed points of the system with the eigenvalues of the Jacobian are summarized in the table III B 1 and III B 1, respectively, for GUP and without GUP.

TABLE III. **Fixed points and corresponding eigenvalues of the Potential** $V(\phi) = V_0e^{-k\phi}$ without GUP

	x	y	E_1	E_2	Stability
A	$-Q/\sqrt{6}$	$\sqrt{(Q^2+6)/6}$	$-(6+Q^2)/2$	$-(3+Q^2)$	stable point
B	$-Q/\sqrt{6}$	$-\sqrt{(Q^2+6)/6}$	$-(6+Q^2)/2$	$-(3+Q^2)$	stable point
C	0	0	$-3/2$	$3/2$	saddle point

TABLE IV. **Fixed points and the corresponding eigenvalues of the Potential** $V(\phi) = V_0e^{-k\phi}$

	x	y	E_1	E_2	Stability
A	$-\frac{Q}{\sqrt{6}}$	$\sqrt{Q^2+6/6}$	$\frac{-72Q^2-18Q^4-Q^6}{2Q^2(12+Q^2))}$	$-\frac{72+84Q^2+21Q^4+Q^6}{Q^2(12+Q^2))}$	stable point
B	$-\frac{Q}{\sqrt{6}}$	$-\sqrt{Q^2+6/6}$	$\frac{-72Q^2-18Q^4-Q^6}{2Q^2(12+Q^2))}$	$-\frac{72+84Q^2+21Q^4+Q^6}{Q^2(12+Q^2))}$	stable point

The rigorous fixed point analysis shows that the introduction of GUP correction completely alters the fixed points and hence their stability in the case of exponential potential. It is found that the dynamics without GUP have two stable fixed points and one saddle point, whereas the GUP-modified dynamics for exponential potential leave us with two stable fixed points.

2. *Phase portrait*

However, from the physical perspective, only the upper half of the plane, $y \geq 0$, is of meaning. This is because $y \leq 0$ is not physically achievable for a positive potential. Thus, we discard the set of the points, including the fixed points, with $y \leq 0$ as not being physical. The dynamic variable x is proportional to the velocity. In both cases, with and without GUP,

The phase portraits of the system are presented in Fig.(1) for $Q = 1$. the saddle point $C = (0, 0)$ disappears upon the introduction of the GUP distortion. However, the physically relevant point $A = (-0.40, 1.08)$ remains there even after quantum correction. Physically, the stable fixed point $A = (-0.40, 1.08)$ implies that at a later time, the scalar field settles down to a negative velocity value with a positive field value.

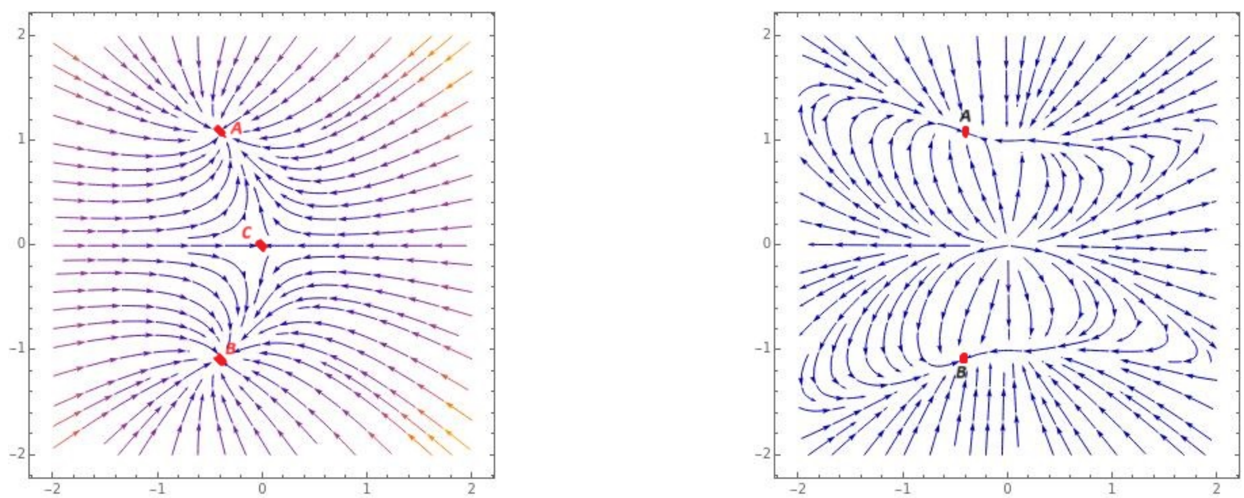


FIG. 1. Phase portrait for potential $V(\phi) = V_0 e^{-k\phi}$ for value of $Q = 1$. *Left* without GUP fluctuation where the stable fixed points are at **A**=(-0.40,1.08), **B**=(-0.40,-1.08) and a saddle fixed point at **C**=(0,0) and *Right* with GUP where the stable fixed point are **A**=(-0.40,1.08), **B**=(-0.40,-1.08)

IV. GUP-MODIFIED INFLATIONARY SCENARIO

In this section, we present inflationary dynamics under the influence of the Generalized Uncertainty Principle (GUP) for quadratic and exponential potentials. To understand the change in the behavior of the background dynamics of our cosmology in the presence of GUP distortion, we need to study the behavior of cosmologically relevant parameters like; of scale factor a(t), scalar field $\phi(t)$, Equation of State (EoS) (w_{eff}) and slow climb parameters ϵ and η with and without GUP deformation [76]. In what follows, we do this case by case for each potential considered in this article.

1. *Quadratic potential, $V(\phi) = \mu^2\phi^2$*

During cosmic inflation, the behavior of a phantom field differs from that of a normal scalar field. While a normal scalar field undergoes a slow roll along its potential, a phantom field exhibits a slow climb along its potential. This distinction is evident both mathematically and graphically, as demonstrated in [77] for power-law potentials.
After introducing GUP corrections to the Friedmann equations, the slow climb parameters defined as $\epsilon \equiv -\frac{\dot{H}}{H^2}$, $\eta \equiv \frac{V"}{3H^2}$ and $\delta \equiv \eta - \epsilon$ are expressed as follows:

$$\epsilon = \frac{3}{2}\left(\frac{\dot{\phi}^2}{6H^2} + \frac{V_0\phi^2}{3H^2} - 1\right) + 4\alpha a^3 H^2 + \frac{\alpha a^3 \dot{\phi}^4}{3H^2} + \frac{8}{3}\alpha a^3 \dot{\phi}^2. \tag{4.1}$$

For $V(\phi) = V_0\phi^2$, we have

$$\eta = \frac{2V_0}{3H^2}. \tag{4.2}$$

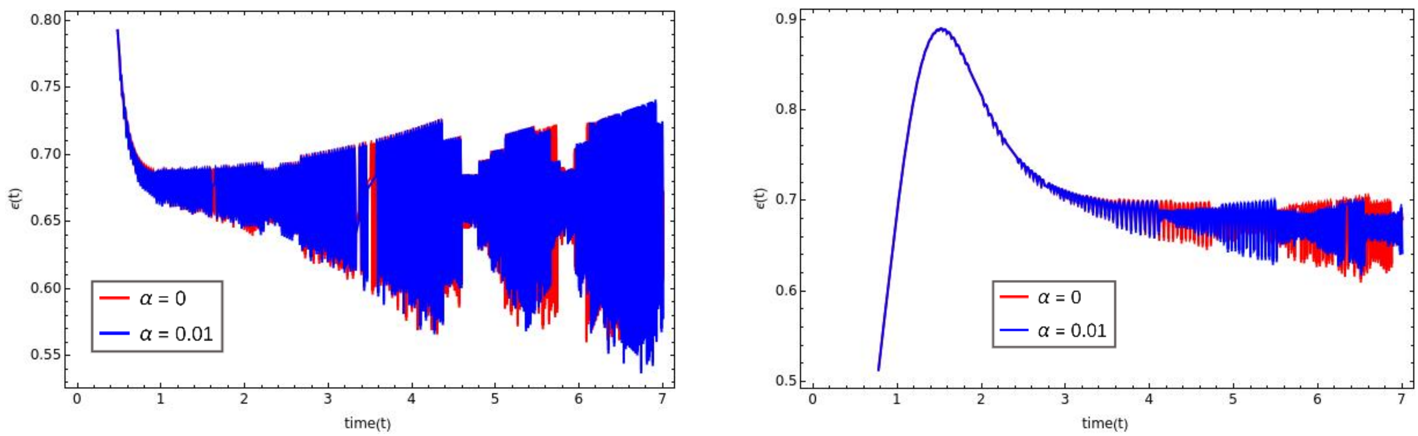


FIG. 2. *Left panel*: Plot of $\epsilon(t)$ vs time(t) for the initial condition as $a(0) = 1$, $\dot{a}(0) = 1$ and $\dot{\phi}(0) = 0.1$ for three different values of α where, $\alpha = 0$ shows no GUP fluctuation. *Right panel*: For initial condition as $a(0) = 4$, $\dot{a}(0) = 4$ and $\dot{\phi}(0) = 0.1$.

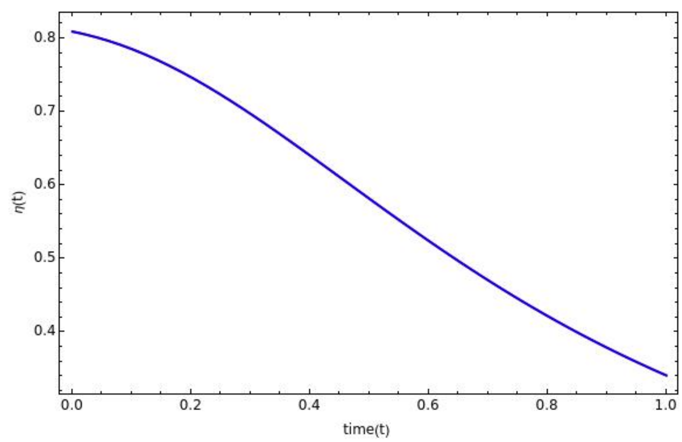


FIG. 3. $\eta(t)$ vs t for the square potential.

In the cosmological context, EoS is a useful parameter to understand and classify the acceleration and reacceleration phases of our universe. EoS is the relationship between energy density and pressure. For $w = 0$ corresponds to non-relativistic matter such as cold dark matter (CDM) or non-relativistic baryonic matter and $0 < w < 1/3$ refers to radiation dominated. For $w = -1$,$-1 < w < -1/3$ and $w < -1$ refer to the cosmological constant, Quintessence, and Phantom eras, respectively.
The Raychaudhuri equation, in the standard case, can be written in terms of EoS as follows:

$$\frac{\ddot{a}}{a} = -(1+3w)\frac{\rho}{6}, \tag{4.3}$$

the GUP modified Raychaudhuri Eq.(2.49) looks like

$$\frac{\ddot{a}}{a} = -\left[1 + 3\left(\frac{\frac{-\dot{\phi}^2}{2} - V(\phi) - \frac{2\alpha a^3\dot{\phi}^4}{3} - \frac{4\alpha a^3\dot{\phi}^2 V(\phi)}{3}}{-\frac{\dot{\phi}^2}{2} + V(\phi) - \frac{\alpha a^3\dot{\phi}^4}{3}}\right)\right]\frac{\rho}{6}, \tag{4.4}$$

comparing Eq.(4.4) with Eq.(2.49) we get

$$w = \frac{\frac{-\dot{\phi}^2}{2} - V(\phi) - \frac{2\alpha a^3\dot{\phi}^4}{3} - \frac{4\alpha a^3\dot{\phi}^2 V(\phi)}{3}}{-\frac{\dot{\phi}^2}{2} + V(\phi) - \frac{\alpha a^3\dot{\phi}^4}{3}}. \tag{4.5}$$

For the quadratic potential, the EoS parameter is

$$w = \frac{\frac{-\dot{\phi}^2}{2} - V_0\phi^2 - \frac{2\alpha a^3\dot{\phi}^4}{3} - \frac{4\alpha a^3\dot{\phi}^2 V_0\phi^2}{3}}{-\frac{\dot{\phi}^2}{2} + V_0\phi^2 - \frac{\alpha a^3\dot{\phi}^4}{3}}. \tag{4.6}$$

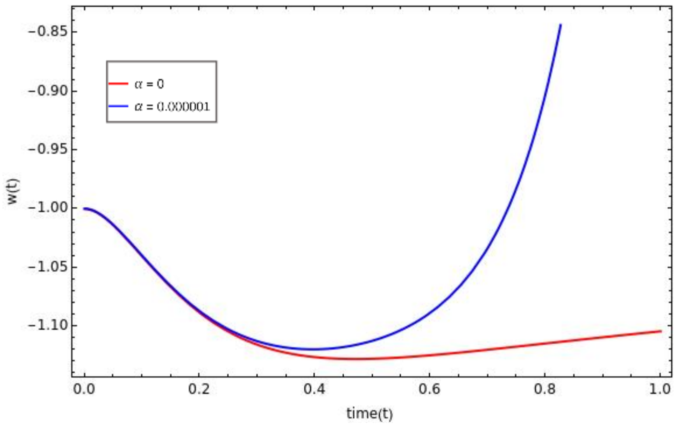
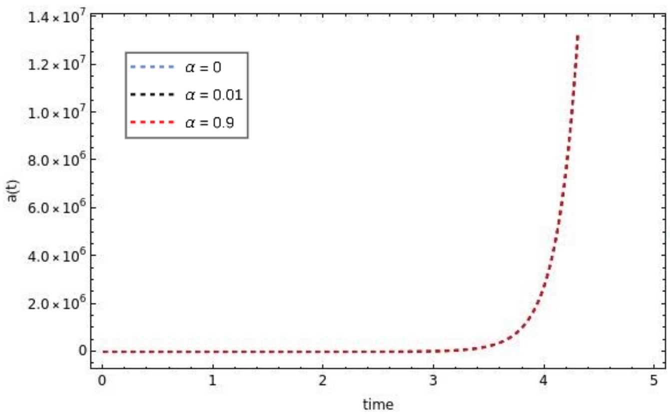


FIG. 4. Comparison of EoS vs time for potential $V = \mu^2\phi^2$ for different values of α.



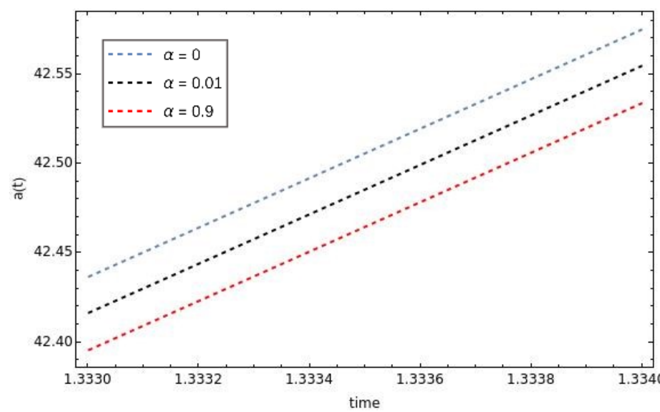
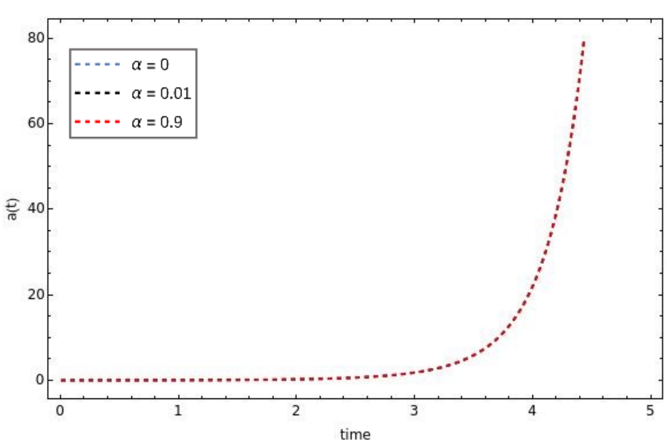


FIG. 5. *Left panel :* Plot of $a(t)$ along the Y-axis vs time(t) along the X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = \mu^2\phi^2$ potential with initial condition as $a(0) = 1$ and $\phi(0) = 1$ and $\dot{\phi}(0) = 0.1$. *Right panel :* Magnified view.



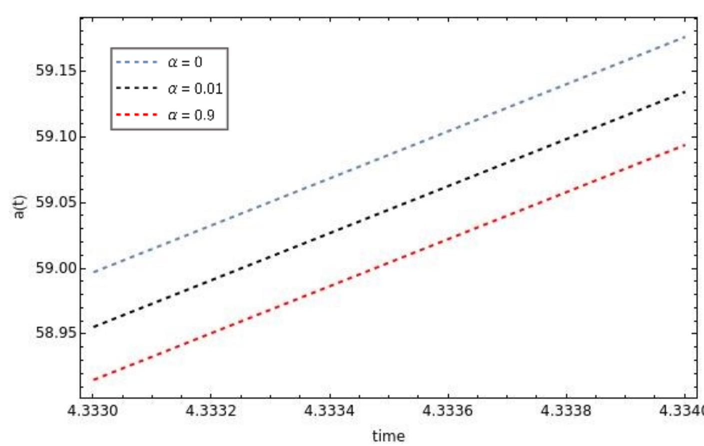


FIG. 6. *Left panel :* Plot of $a(t)$ along the Y-axis vs time(t) along the X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = \mu^2\phi^2$ potential with initial condition as $a(0) = 0.1$ and $\phi(0) = 0.1$. *Right panel :* Magnified view.

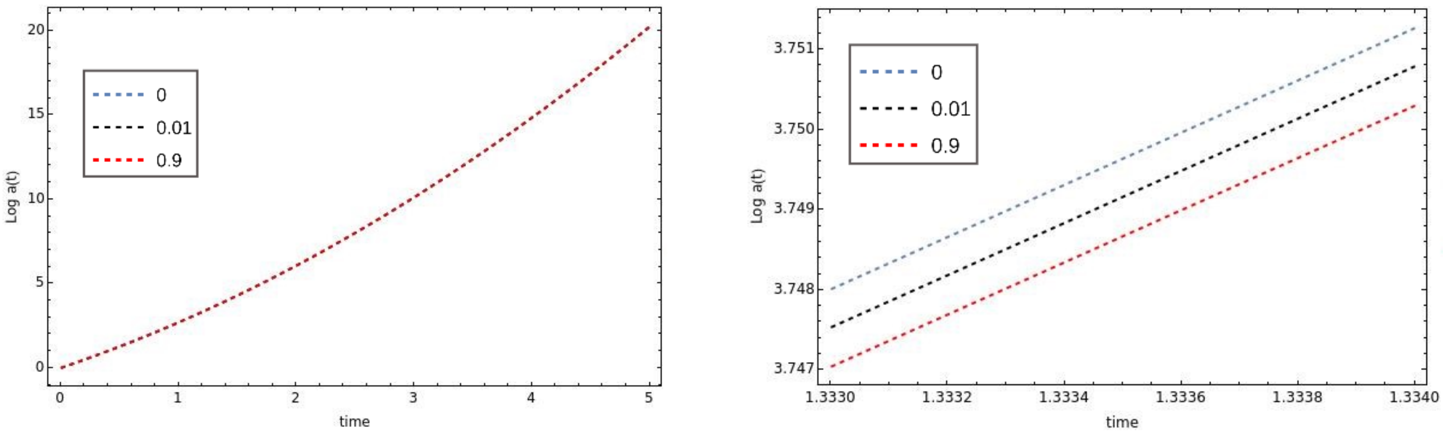


FIG. 7. *Left panel :* Plot of logarithm of $a(t)$ along the Y-axis vs time(t) along the X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = \mu^2\phi^2$ potential with initial condition as $a(0) = 1$ and $\phi(0) = 1$. *Right panel :* Magnified view.

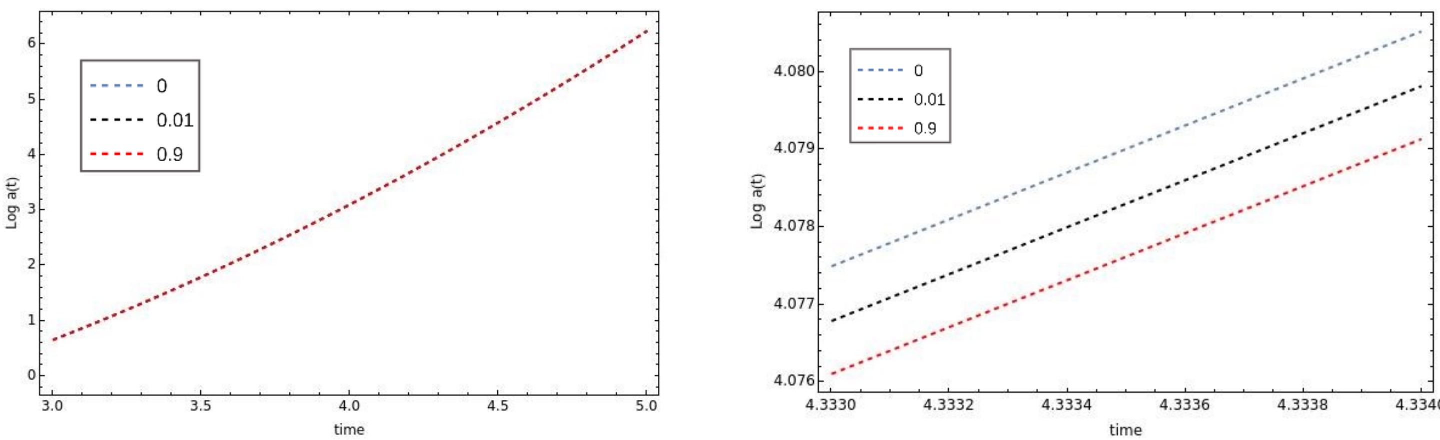


FIG. 8. *Left panel :* Plot of logarithm of $a(t)$ along the Y-axis vs time(t) along the X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = \mu^2\phi^2$ potential with initial condition as $a(0) = 0.1$ and $\phi(0) = 0.1$. *Right panel :* Magnified view.

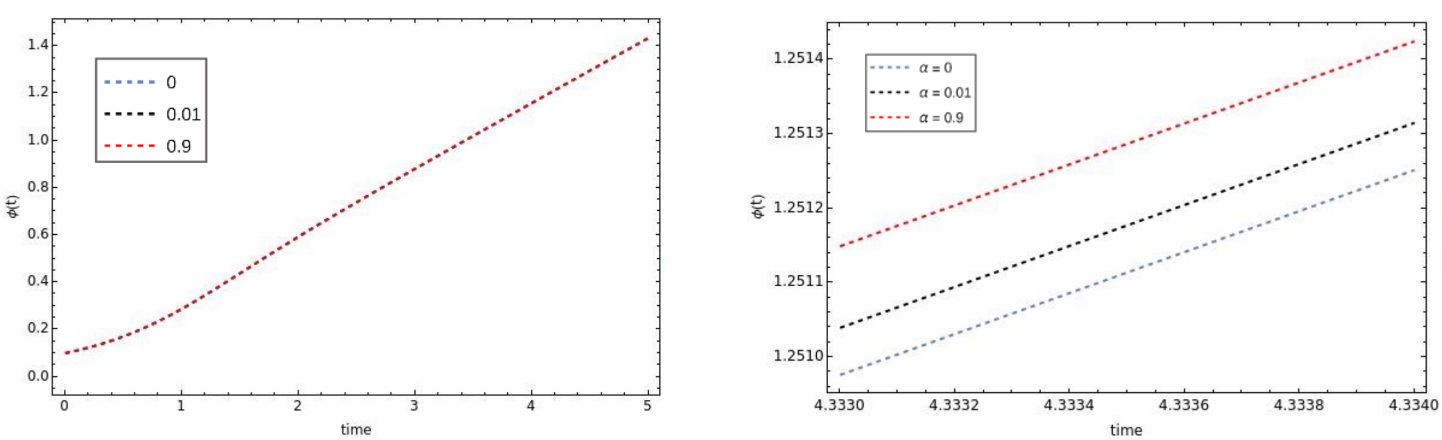


FIG. 9. *Left panel :* Plot of $\phi(t)$ along the Y-axis vs time(t) along the X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = \mu^2\phi^2$ potential with initial condition as $a(0) = 1$ and $\phi(0) = 1$. *Right panel :* Magnified view.

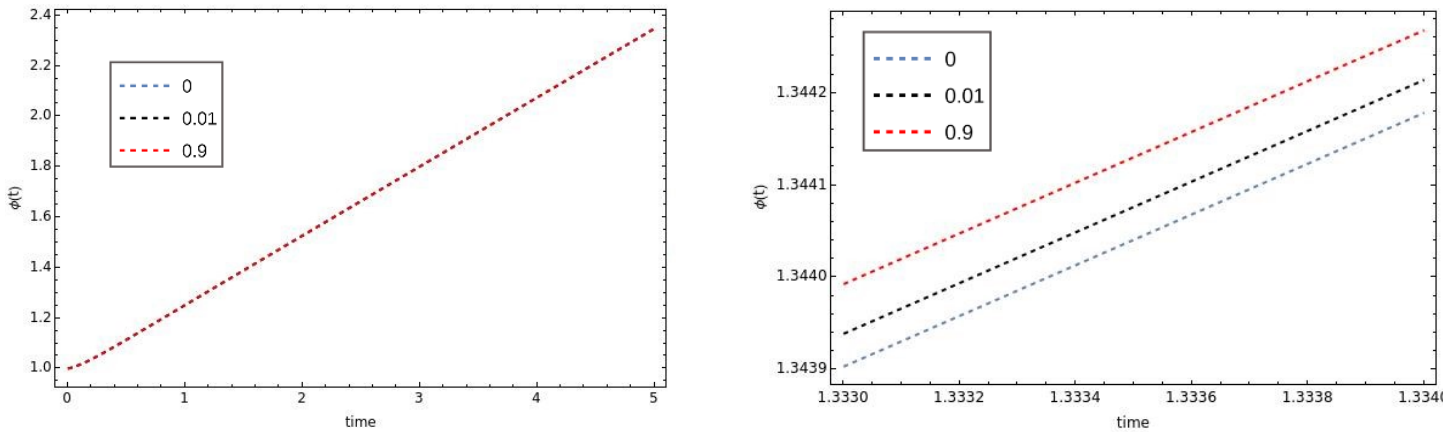


FIG. 10. *Left panel :* Plot of $\phi(t)$ along the Y-axis vs time(t) along the X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = \mu^2\phi^2$ potential with initial condition as $a(0) = 0.1$ and $\phi(0) = 0.1$. *Right panel :* Magnified view.

Fig.(5) represents dynamics of scale factor with time, illustrating a nearly exponential rise of the scale factor, indicating inflation. However, the effect of GUP is not readily discernible from the left side of Fig.(5) To observe this effect, it is necessary to magnify the range presented on the right side of Fig.(5). The blue line represents the original scale factor without GUP modification. As we increase the strength of (α), the value of the original scale factor becomes nonlinearly distorted. In addition, we present the behavior of the scale factor for different initial conditions in Fig.(6).
The behavior of ϕ w.r.t cosmic time t for different initial conditions is shown in Fig.(9) and Fig.(10). A magnified view of the effect of GUP deformation for different values of α is provided on the right-hand side of the figures. On the left side of the figures, the scalar field starts from a very low value and then increases linearly upward for different initial conditions. As we can observe from the graph of ϕ vs t, the inclusion of a higher value of GUP-strength, α, in the dynamics causes the evolution to distort upwards from the original dynamics without GUP.
The Fig.(4) depicts the behavior of the EoS parameter with the introduction of a small strength of GUP distortion. We observe that the EoS for the phantom remains always less than -1 in the absence of GUP fluctuations due to the negative pressure term. Even in this case of GUP-modified dynamics, the behavior of the EoS remains the same for most of the evolution. Only at the later phase EoS increases w.r.t the unperturbed dynamics.
In Fig.(2),Fig.(3) we present slow climb parameter $|\epsilon|$ and $|\eta|$ as a function of time with different initial conditions and we clearly observe that value of $|\eta|$,$|\epsilon| << 1$ indicate inflation.

2. *Exponential potential,* $V(\phi) = V_0 e^{-k\phi}$

In this subsection, we study the GUP-modified background dynamics for exponential potential. We determine the slow roll parameters and equation of state and subsequently present them for different initial conditions.
The slow climb parameters are

$$\epsilon = \frac{3}{2}\left(\frac{\dot{\phi}^2}{6H^2} + \frac{V_0 e^{-k\phi}}{3H^2} - 1\right) + 4\alpha a^3 H^2 + \frac{\alpha a^3 \dot{\phi}^4}{3H^2} + \frac{8}{3}\alpha a^3 \dot{\phi}^2, \tag{4.7}$$

and

$$\eta = \frac{V_0 k^2 e^{-k\phi}}{3H^2}. \tag{4.8}$$

The effective equation of state (EoS) for $V(\phi) = V_0 e^{-k\phi}$ is

$$w = \frac{\frac{-\dot{\phi}^2}{2} - V_0 e^{-k\phi} - \frac{2\alpha a^3 \dot{\phi}^4}{3} - \frac{4\alpha a^3 \dot{\phi}^2 e^{-k\phi}}{3}}{-\frac{\dot{\phi}^2}{2} + V_0 e^{-k\phi} - \frac{\alpha a^3 \dot{\phi}^4}{3}}. \tag{4.9}$$

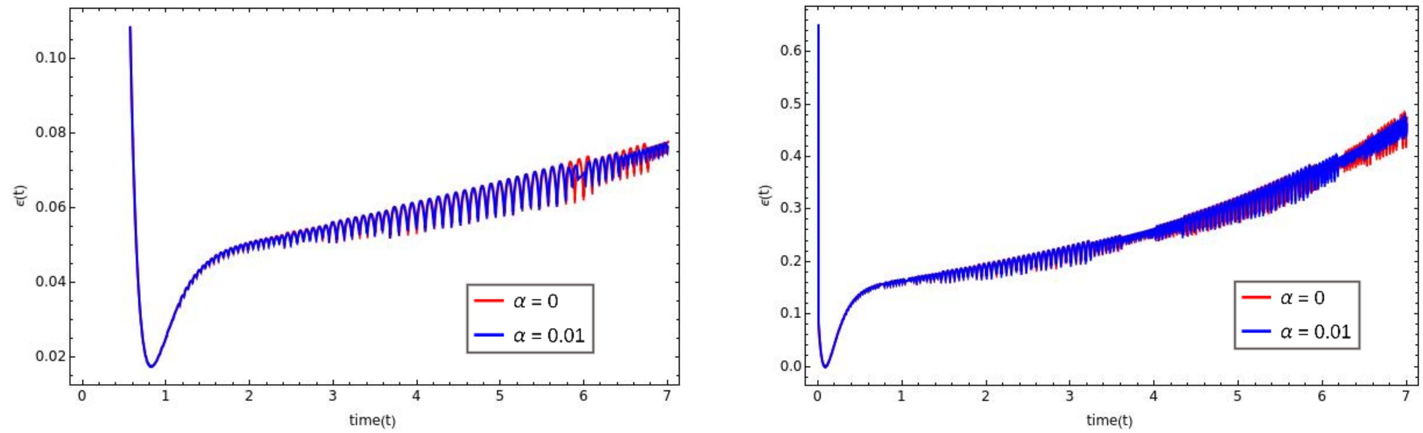


FIG. 11. *Left panel*: For $V(\phi) = V_0 e^{-k\phi}$ with initial condition as $a(0) = 1$, $\dot{a}(0) = 1$ and $\dot{\phi}(0) = 0.1$ plot of $\epsilon(t)$ along Y-axis vs time(t) along X-axis for three different values of α where, $\alpha = 0$ with no GUP fluctuation. *Right panel*: For initial condition as $a(0) = 0.1$, $\dot{a}(0) = 0.1$ and $\dot{\phi}(0) = 0.1$.

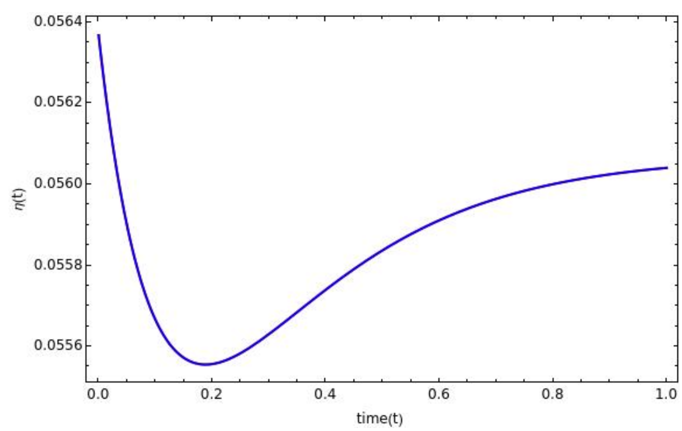


FIG. 12. $\eta(t)$ vs t for exponential potential.

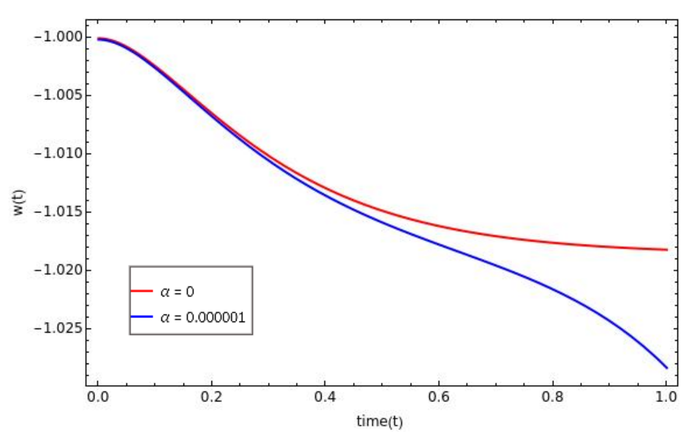


FIG. 13. Comparison of EoS vs time graph for potential $V = e^{-k\phi}$ for different values of α.

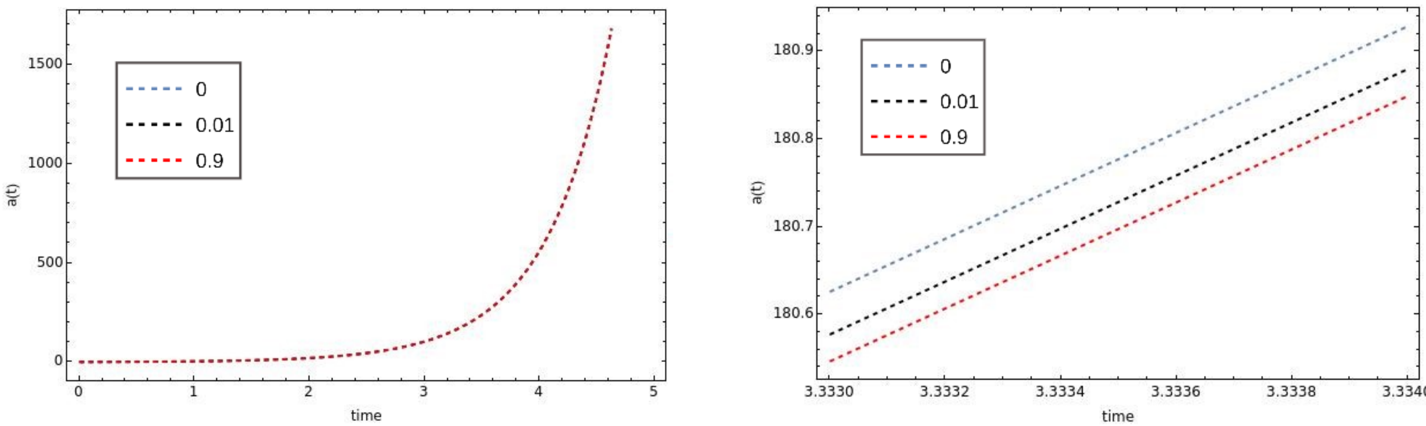


FIG. 14. *Left panel :* Plot of $a(t)$ along Y-axis vs time(t) along X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = e^{-k\phi}$ potential with initial condition as $a(0) = 1$ and $\phi(0) = 1$. *Right panel :* Magnified view.

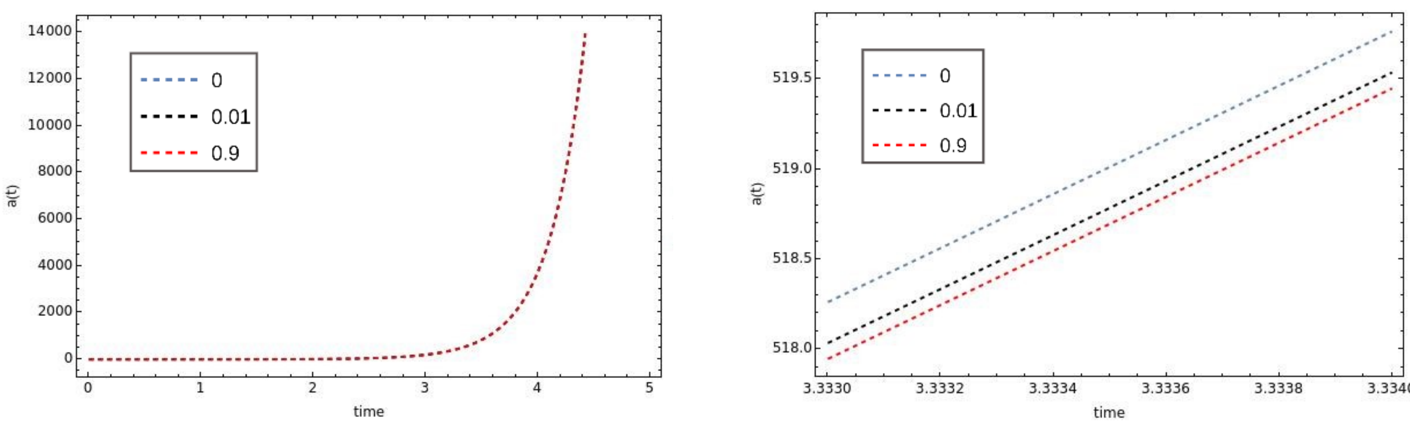


FIG. 15. *Left panel :* Plot of $a(t)$ along Y-axis vs time(t) along X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = e^{-k\phi}$ potential with initial condition as $a(0) = 0.1$ and $\phi(0) = 0.1$. *Right panel :* Magnified view.

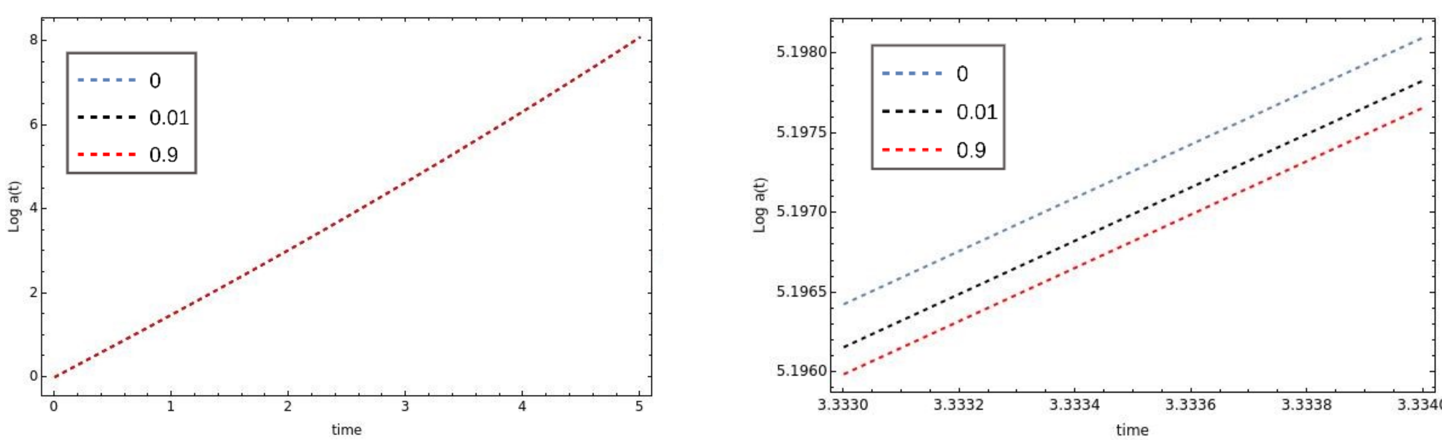


FIG. 16. *Left panel :* Plot of logarithm of $a(t)$ along Y-axis vs time(t) along X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = e^{-k\phi}$ potential with initial condition as $a(0) = 1$ and $\phi(0) = 1$. *Right panel :* Magnified view.

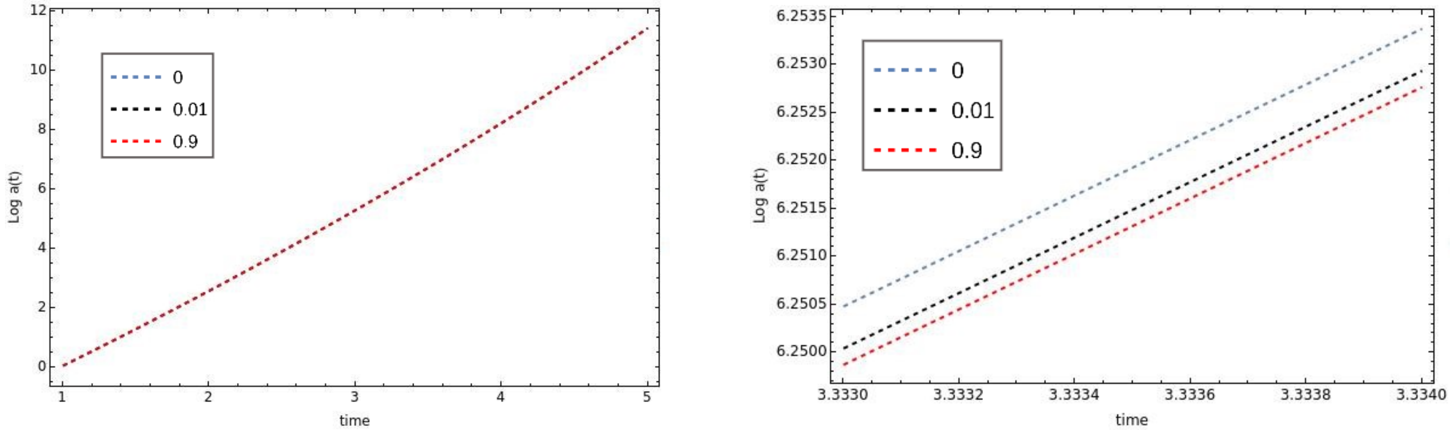


FIG. 17. *Left panel :* Plot of logarithm of $a(t)$ along Y-axis vs time(t) along X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = e^{-k\phi}$ potential with initial condition as $a(0) = 0.1$ and $\phi(0) = 0.1$. *Right panel :* Magnified view.

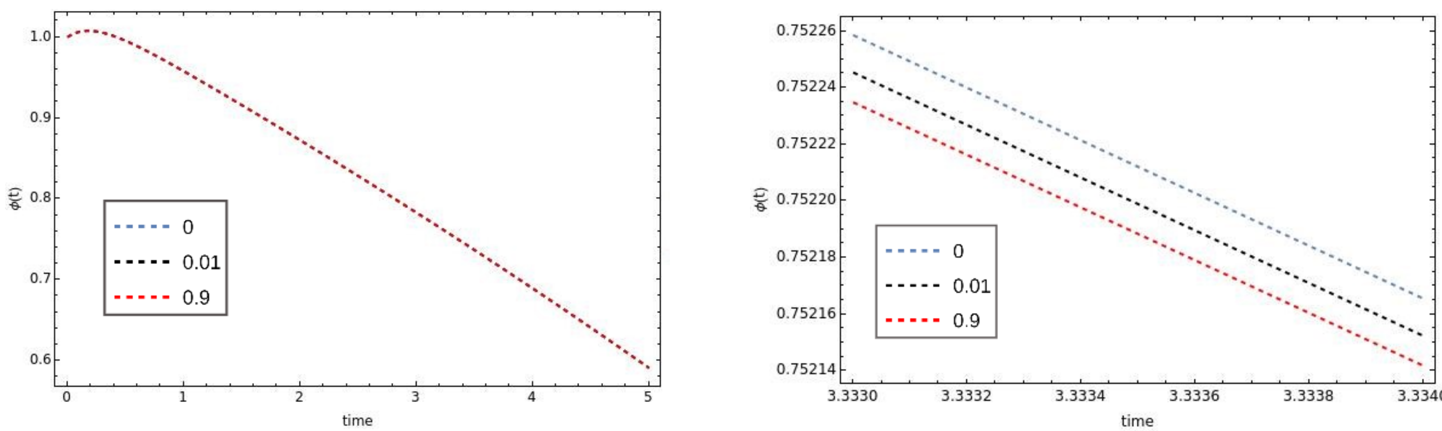


FIG. 18. *Left panel :* Plot of $\phi(t)$ along Y-axis vs time(t) along X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = e^{-k\phi}$ potential with initial condition as $a(0) = 1$ and $\phi(0) = 1$. *Right panel :* Magnified view.

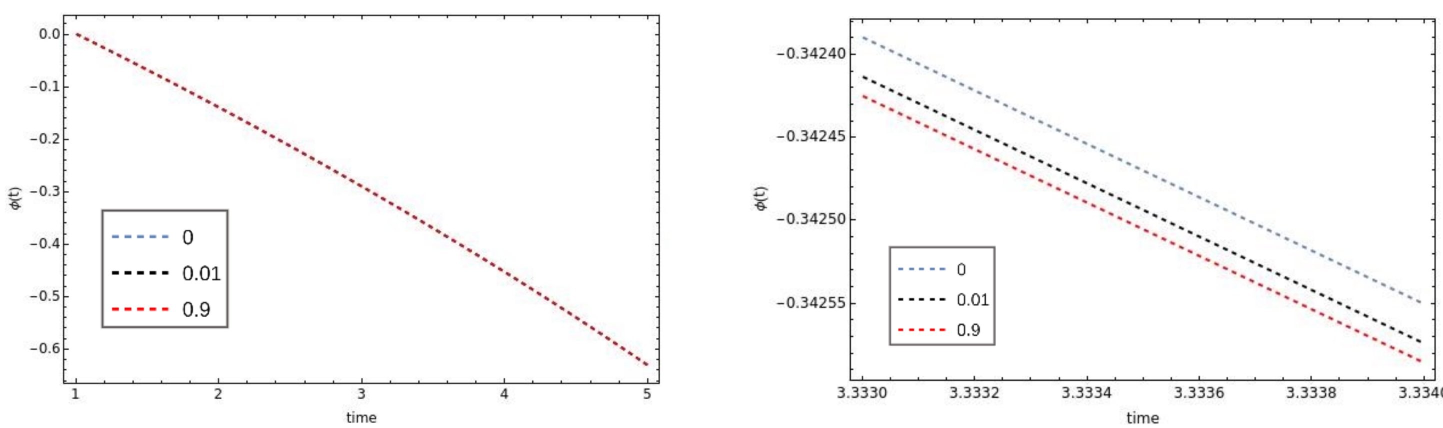


FIG. 19. *Left panel :* Plot of $\phi(t)$ along Y-axis vs time(t) along X-axis for three values of $\alpha = 0, 0.01, 0.9$ respectively in $V = e^{-k\phi}$ potential with initial condition as $a(0) = 0.1$ and $\phi(0) = 0.1$. *Right panel :* Magnified view.

We analyze the effect of GUP modification on the background dynamics for the exponential potential from

Fig.(14),(15),(16) and (17). The figures depict the line representing the small strength of α, $\alpha = 0.01$ is closer to the original dynamics. However, as we increase the strength of the GUP modification, the dynamics deviate further from the original dynamics.
The behavior of the EoS parameter in the phantom field with and without the effect of GUP is represented in Fig.(13). The EoS starts with a value of -1, indicating proximity to the Cosmological constant era, and transitions toward the phantom-dominated era for both GUP and without GUP dynamics.
The behavior of ϕ and t for exponential potentials with different initial conditions are shown in Fig.(18) and Fig.(19). In addition, a magnified view of the GUP deformation for different values of α is shown on the right-hand side of the Fig.(18) and Fig.(19). On the left side of the scalar field, it starts from the highest value and then decreases linearly downward for different initial conditions. We observe from the graph of ϕ vs t that the inclusion of a comparable level of GUP in the dynamics causes the line to distort downward from the original blue line, which represents the dynamics without GUP. Furthermore, the plots of $|\epsilon|$ and $|\eta|$ in Fig.(11) and Fig.(12) indicates inflation in the case of exponential potential.

V. CONCLUSION

In this paper, we comprehensively review the construction of the GUP-corrected effective Hamiltonian from the scratch, ie. Einstein-Hilbert action. To this effect, we consider a minimally coupled phantom scalar field with the cosmological constant as the toy model. Following this, we perform the same exercise with arbitrary potentials. In particular, we focus on introducing momentum deformation to our dynamics due to the generalized uncertainty principle. Having derived the effective Hamiltonian, we obtain all the background equations of motion in terms of the Raychaudhuri, Friedmann, and Klien-Gordon equations. Interestingly, we show that the Klien-Gordon equation is free from any explicit quantum correction due to GUP. On the other hand, the Raychaudhuri and Friedmann equation receives quantum correction explicitly. This situation is consistent with the cosmological models constructed using the framework of loop quantum gravity.

The system of equations obtained is highly nonlinear. This demands qualitative analysis using the tools of dynamical system analysis to extract information about the system before actually solving it.

We achieve this by performing a detailed dynamical system analysis using the tools of linear stability analysis. We observe that the introduction of GUP affects the local behavior of the system, although the overall dynamics remain similar by and large. This is confirmed from Fig.(1) and from table(I,II,IV,III). In the case of quadratic potential, we observe from table(I,II) that after introducing GUP distortion to the dynamics, certain fixed points disappear from the scenario. In the case of exponential potential, table(IV,III) indicates that after introducing GUP corrections, only one saddle point disappeared. Consequently, this leaves us with two stable points in the GUP modified scenario.

As our final goal, we return to the question of the cosmological implications of the considered model in Sec.(IV). We discuss inflationary scenarios after GUP correction through plots of scale factor and the logarithm of scale factor for different sets of initial conditions in Fig.(5,6,7,8)for square potential and in Fig.(14,15,16,17) for exponential potential. We observe nearly exponential expansion for both potentials, indicating inflation. Finally, we calculated the slow climb parameters for both potentials and plotted them in Fig.(2,3,11,12) which clearly shows inflation because of the values $|\eta|$,$|\epsilon| << 1$. This confirms the inflationary scenario in this study.

Furthermore, we calculated the GUP-induced EoS parameter and plotted it in Fig.(4,13) starting nearly from -1 and then decreasing to more negative values for phantom scenarios. To be precise in the case of quadratic potential, when we incorporate the GUP correction term, the graph of the Equation of Sate (EoS) parameter rapidly approaches the value -1 compared to the case without GUP corrections. We also observe a deviation from the original dynamics.

Our analysis is general. In the future, we propose to extend our analysis to more viable models of inflationary scenarios. Also, the effects of GUP correction have not been studied in the context of linearized gravity yet. We leave it as our future project.

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