# Belief Bias Identification 

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#### Abstract

This paper proposes a unified theoretical model to identify and test a comprehensive set of probabilistic updating biases within a single framework. The model achieves separate identification by focusing on the updating of belief distributions, rather than classic point-belief measurements. Testing the model in a laboratory experiment reveals significant heterogeneity at the individual level: All tested biases are present, and each participant exhibits at least one identifiable bias. Notably, motivated-belief biases (optimism and pessimism) and sequence-related biases (gambler's fallacy and hot hand fallacy) are identified as key drivers of biased inference. Moreover, at the population level, base rate neglect emerges as a persistent influence. This study contributes to the belief-updating literature by providing a methodological toolkit for researchers examining links between different conflicting biases, or exploring connections between updating biases and other behavioural phenomena.


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## 1 Introduction

Over the last decades, there have been substantial efforts to identify and categorize belief-updating biases, both within Economics and Psychology (Edwards, 1968; Tversky and Kahneman, 1974; Grether, 1980). Scholars have typically studied these biases separately. This has made it more challenging to tell if someone's updating behaviour can be attributed to one bias or another. Despite recent shifts towards a more comprehensive structure of these biases (Benjamin, 2019; Bordalo et al., 2023a,b), there is still a lack of a unifying testable model that can tell multiple conflicting biases apart.

The essential reason why such a model would be desirable is that the identification of belief-updating biases can be confounded when some potential biases are not taken into consideration. For example, imagine a person who consistently updates her beliefs "too much" in the face of new information. Traditional models are able to tell whether this behaviour is due to this person neglecting her prior beliefs (base rate neglect), or interpreting too much out of the information she receives (overinference). However, other potential biases could also explain her updating behaviour. If she has preferences over different outcomes, is she updating her beliefs too much because she generally overinterprets information, or because she cares about the outcome (optimism/pessimism)? Or may she instead be jumping to conclusions because the information she receives is confirming her previous beliefs (confirmation bias)? Could it also be that she has very little doubt about the outcome that she believes to be more likely to happen (overconfidence)? By excluding some of these biases from the model, we may wrongly attribute her behaviour to a different bias than the one(s) she is actually exhibiting. This implies that some of the biases which happen to be present in the literature may only seem to be relevant because we lack a more complete model. For instance, base-rate neglect might appear to be driving behaviour because, say, confirmation bias and overconfidence are unaccounted for.

Addressing confounded belief-updating biases is important for two related purposes. First, identifying these biases separately is relevant because the actions that people may take under different updating biases could be completely different even if they lead to the same posterior. For example, someone suffering from confirma-
tion bias may keep voting for the same political party after receiving social media information that aligns with her previous beliefs, while someone who simply neglects prior information might be more likely to change her mind if reminded of the actual base rate. Secondly, once confoundedness is taken care of, such a model would be able to shed light onto which specific biases are more common and which biases drive inference the most.

Integrating multiple belief biases into one testable model poses one important methodological challenge. Namely, that the way we usually measure beliefs may not suffice to separably identify a full range of potentially confounded biases. The vast majority of the literature measures point beliefs to study biased updating (Benjamin, 2019). This approach is simple and might be enough when the number of biases to take into consideration is small. But with the inclusion of more biases, the amount of flexibility we have to distinguish one updating pattern from another is reduced.

This paper constructs a theoretical model that allows to separably identify multiple conflicting belief-updating biases, using belief distributions ${ }^{1}$ to measure prior and posterior beliefs. One can understand belief distributions as an individual holding beliefs over multiple outcomes, or being uncertain about her own beliefs when there are only two states (in a similar sense to cognitive uncertainty (Enke and Graeber, 2023)). Either way, the central idea is that belief distributions give us more leeway to better distinguish updating biases.

The theoretical framework I propose adopts an approach where an agent observes a series of Bernoulli trials. The agent's prior beliefs are assumed to follow a beta distribution and are updated via Bayesian inference. Since the beta distribution is conjugate to the binomial distribution, the posterior of the Bayesian agent will also result in a beta distribution ${ }^{2}$. To integrate different biases, the model accounts for deviations from Bayesian updating, utilizing distorted likelihoods and priors. These distortions yield non-Bayesian posterior beta distributions, allowing for the identification of core biases such as over/under inference (Khaw et al., 2021; Augenblick et al.,

[^1]2021; Ba et al., 2022) or base rate neglect/over use (Benjamin et al., 2019; Bucher and Glimcher, 2022; Enke et al., 2023a). It also explores asymmetries in reacting to good and bad news in the observed data generating process, alongside preferencerelated biases (Eil and Rao, 2011; Zimmermann, 2020; Möbius et al., 2022). Further, it incorporates confirmation bias (Rabin and Schrag, 1999; Charness and Dave, 2017; Zhenxun, 2023b), sequence-related biases like the hot hand fallacy and gambler's fallacy (Rabin, 2002; Rabin and Vayanos, 2010), and confidence biases (Serra-Garcia and Gneezy, 2021; Huffman et al., 2022; Enke et al., 2023b) that affect the variance of the agent's posterior distribution, all within the same framework.

The model is tested with help of a novel laboratory experiment. Participants solved a series of tasks where they had to guess the percentage of red balls in a selected urn from a pool of 99 urns, each containing varying proportions of red and blue balls. Participants received information signals via ball draws from the selected urn, forming and reporting their belief beta distributions twice. Then they reported their updated beliefs after observing another sequence of independently drawn signals from the same urn.

The results of the study support the idea that the inclusion of a more complete set of biases mitigates the artificial presence of some biases in more reduced models. At the population level, both overinference and base rate neglect significantly affect updating behavior, when these are the only biases that the model accounts for. However, when the model encompasses a broader spectrum of biases, the impact of overinference vanishes, leaving base rate neglect as the sole remaining persistent bias.

Nonetheless, if people exhibit different biases, findings at the aggregate level might not be able to capture the importance of each bias, as many of them will be averaged out across individuals. In other words, due to the presence of individual heterogeneity ${ }^{3}$, the experiment arguably offers even more insightful findings when applied at the individual level. First, when only a reduced amount of biases is accounted for, the model is unable to identify biased inference in some subjects who would otherwise exhibit a clear bias. In fact, when the full array of biases is factored in, the model shows

[^2]that all individuals exhibit at least one specific bias; and more importantly, all tested biases are present to a certain extent. Finally, despite the presence of individual heterogeneity, motivated-belief biases (optimism and pessimism) and sequence-related biases (gambler's fallacy and hot hand fallacy) appear to be the main drivers of biased inference. On the other end, the presence of confirmatory biases (confirmation and disconfirmation bias) is scarce.

This paper does not only contribute to the literature of belief updating as a compass to gauge the intensity of various belief biases, but also aims to be a toolkit for researchers who identify belief-updating biases in settings where these could be potentially confounded. In this direction, there are three ways in which this paper contributes to the existing literature.

First, there has been quite some recent work trying to trace links between different updating biases. For example, Heger and Papageorge (2018) and Gneezy et al. (2023) study how wishful thinking (optimism) can affect overconfidence; while Charness and Dave (2017) and Zhenxun (2023b,a) try to separate behaviour stemming from motivated beliefs and unmotivated confirmation bias. In the absence of an overarching model that encompasses a wide array of biases, findings can be confounded and results about such links could therefore be biased. For example, bias A may appear to be linked to bias B , only because a third bias C is missing.

Second, many researchers search for belief-based explanations of behavioural phenomena, when conflicting biases come into play. For example, political polarization has been separately explained from the standpoint of overconfidence (Ortoleva and Snowberg, 2015), and confirmation bias (Del Vicario et al., 2017). In the financial literature, there is some debate as to whether the disposition effect could be caused by discrepant biases, such as motivated beliefs (Heinke et al., 2023), the gambler's fallacy (Jiao, 2017) or general underinference (Pitkäjärvi, 2022). More examples range from linking confirmation bias to several stylised facts in financial markets (Pouget et al., 2017); or confidence biases to poor investment performance (Ahmad and Shah, 2020) and biased memory (Huffman et al., 2022). As previously mentioned, utilizing an approach that is able to tell conflicting biases apart has the advantage to better pinpoint what biases are precisely underpinning certain behavioural regularities; or even let us know if some biases are simply an artifact of using a less complete model.

Third, the problem of conflicting biases also arises in the field of behavioural interventions. Boosting is a psychological technique aimed at de-biasing individuals when they suffer some kind of cognitive bias. One of the challenges mentioned in this area of research is the fact that these interventions normally target one bias at a time, when indeed multiple biases could be at stake (Kahneman et al., 2021). Once again, having a method to distinguish between different conflicting biases could come in handy in order to focus the intervention on those biases which end up being more common, or drive inference the most.

The paper continues as follows: Section 2 explains the theoretical framework, the updating setting and progressively introduces different biases into the model. Section 3 deals with the experimental design and presents the belief-measuring tool. In section 4, I bring the experimental data and the theory together and compare two regression models, which differ in the amount of biases they incorporate. Section 5 discusses the results of the experiment, and section 6 concludes with some final remarks.

## 2 A model of multiple belief biases

### 2.1 Theoretical framework

Let an agent observe a signal $S=\left\{s_{1}, \ldots, s_{n}\right\}$ consisting of the realization of $n$ independent and identically distributed Bernoulli trials. Let $p$ denote the probability of success of each trial $\left(s_{i}=1\right)$ and $1-p$ denote the probability of each failure $\left(s_{i}=0\right)$. For such data generating process (DGP), the likelihood function is the probability mass function of a binomial distribution with parameters $(n, p)$ :

$$
\begin{equation*}
L\left(p \mid s_{1} \ldots s_{n}\right)=\binom{n}{k} p^{k}(1-p)^{n-k} \tag{1}
\end{equation*}
$$

where $k=\sum_{i=1}^{n} s_{i}$ and $(n-k)$ are the number of successes and failures in the DGP respectively.

Let $\Omega=(0,1)$ be the set of possible values that $p$ may take ${ }^{4}$, and let a prior belief

[^3]$\pi(p)$ be beta distributed with prior parameters $\left(a_{0}, b_{0}\right)$. Namely,
\[

$$
\begin{equation*}
\pi\left(p \mid a_{0}, b_{0}\right)=\frac{1}{B\left(a_{0}, b_{0}\right)} p^{a_{0}-1}(1-p)^{b_{0}-1} \tag{2}
\end{equation*}
$$

\]

where $a_{0}, b_{0}>0$, and $B($.$) is the beta function.$
Given a prior $\pi(p)$ and a likelihood $L\left(p \mid s_{1} \ldots s_{n}\right)$ the agent forms a posterior. As the beta distribution is a conjugate prior of the binomial distribution, a Bayesian agent updates her beliefs such that her posterior distribution of $p, \pi\left(p \mid a_{n}, b_{n}\right)$, is also beta distributed with parameters $a_{n}, b_{n}$. This means that ${ }^{5}$ :

$$
\begin{equation*}
a_{n}=k+a_{0} \quad b_{n}=n-k+b_{0} \tag{3}
\end{equation*}
$$

In order to incorporate updating biases, let a non-Bayesian agent use a distorted likelihood and prior when she updates ${ }^{6}$. These distortions can be expressed as exponential deviations of the likelihood and prior. Namely, $\tilde{L}\left(p \mid s_{1} \ldots s_{n}\right)=\left(L\left(p \mid s_{1} \ldots s_{n}\right)\right)^{\gamma}$ would represent the distorted likelihood and $\tilde{\pi}(p)=(\pi(p))^{\delta}$ would represent a distorted prior of the non-Bayesian agent $(\gamma, \delta>0)$. The parameters $\gamma \neq 1$ and $\delta \neq 1$ indicate deviations from Bayesian updating due to distortions of the likelihood and prior respectively. With these modified functions such a non-Bayesian agent has a posterior beta distribution with parameters $\tilde{a}_{n}, \tilde{b}_{n}$ such that:

$$
\begin{gather*}
\tilde{a}_{n}=\gamma k+\delta\left(a_{0}-1\right)+1  \tag{4}\\
\tilde{b}_{n}=\gamma(n-k)+\delta\left(b_{0}-1\right)+1 \tag{5}
\end{gather*}
$$

state-space of an agent who forms beliefs over every possible realisation of the objective parameter $p \in(0,1)$. In the latter case, the agent would form beliefs over a binary state-space (whether the signal $s_{i}$ takes value 1 or 0 ), and the set $\Omega$ would, in this case, represent the subjective state-space ( $p$ ), which describes the set of possible states where the agent expresses uncertainty about her own beliefs. Both alternatives are contemplated in this model.
${ }^{5}$ See section A in the appendix for a proof; as well as a proof to derive eq.(4) and (5), which suffice to derive all other equations of posterior parameters in section 2 .
${ }^{6}$ These distortions are often seen (Benjamin, 2019) as part of an "as-if model." This means that the model does not take the stand that biased agents actually follow Bayes' Theorem with different likelihood and prior functions, but instead, that these distortions imply equivalent behaviour to agents interpreting too little, or too much, from information signals (or prior beliefs) when they update.

In particular $\gamma>1$ indicates "overinference" while $\gamma<1$ shows "underinference"; that is, believing that the information signal $S$ is more/less informative than it actually is. Similarly, parameter $\delta>1$ indicates "base rate overuse" and $\delta<1$, "base rate neglect"; which implies that the prior is more/less informative than a Bayesian agent perceives it to be. This bias structure conceptually resembles Grether-regressions ${ }^{7}$, as the same type of biases can be identified. The next section shows how equations (4) and (5) can be extended to incorporate more biases.

### 2.2 Introducing multiple belief biases

This section shows how an extended set of biases can be introduced into the model.

### 2.2.1 Asymmetries between successes and failures \& Preference-related biases

Equations (4) and (5) assume that deviations from the Bayesian agent in the likelihood are symmetric for successes and failures. That is, the agent over or under-reacts to "positive information" the same way she over or under-reacts to "negative" information. For a non-Bayesian agent this need not be the case. Consider instead that the agent weights successes and failures of the data generating process differently. Then her likelihood function would be $\tilde{L}\left(p \mid s_{1} \ldots s_{n}\right)=\binom{n}{k} p^{\alpha k}(1-p)^{\beta(n-k)}$. In turn, her posterior would be beta distributed with shape parameters $\tilde{a}_{n}, \tilde{b}_{n}$ such that:

$$
\begin{gather*}
\tilde{a}_{n}=\alpha k+\delta\left(a_{0}-1\right)+1  \tag{6}\\
\tilde{b}_{n}=\beta(n-k)+\delta\left(b_{0}-1\right)+1 \tag{7}
\end{gather*}
$$

where $\alpha, \beta>0$. And $\alpha \neq \beta$ indicates asymmetric reactions to successes and failures in the DGP. Equations (6) and (7) are especially interesting if the agent has preferences over the state-space. Suppose this is the case; and suppose further that preferences are expressed by a utility function, which is continuous and monotonically increasing over $p$. Then, every success $k$ would be informative of a higher realisation of $p \in(0,1)$, i.e.

[^4]a preferred state. Conversely, every failure $(n-k)$ is informative of a lower realisation of $p$. Therefore, successes and failures, can be interpreted as pieces of good and bad news respectively. This means that $\alpha>1$ (overreacting to positive information) or $\beta<1$ (underreacting to negative information) can be interpreted as optimism bias; while $\alpha<1$ or $\beta>1$ would indicate pessimism. Furthermore, $\alpha>\beta$ represents the good news effect, while $\alpha<\beta$ implies there is bad news effect. ${ }^{8}$

### 2.2.2 Confirmation Bias

Confirmation bias is modeled as a positive correlation between overreaction to information signal $S$, and how confirming the signal is. The degree of confirmation of the signal is unrelated to the agent holding preferences ${ }^{9}$ over the state-space. In particular, the degree of confirmation $c$ is expressed as the area, in the density function of the prior, comprised between the expected value of the prior $E(\pi(p))$ and the mean of the information signal $k / n$. Formally:

$$
\begin{equation*}
c=\left|\int_{(k \pm \varepsilon) / n}^{E(\pi(p))} \pi\left(p \mid a_{0}, b_{0}\right) d p\right| \tag{8}
\end{equation*}
$$

Equation (8) specifies a relative measure of confirmation ${ }^{10}$ (see Figure 1 for an example). The higher the value of $c$, the less confirming a signal will be. To incorporate over or under reaction to confirmation a separate term $\rho c$ is added to the distorted likelihood function. Equations (6) and (7) are modified as follows:

$$
\begin{gather*}
\tilde{a}_{n}=\alpha k+\rho c+\delta\left(a_{0}-1\right)+1  \tag{9}\\
\tilde{b}_{n}=\beta(n-k)+\rho c+\delta\left(b_{0}-1\right)+1 \tag{10}
\end{gather*}
$$

[^5]where $\rho<0$ indicates confirmation bias (as $c$ becomes smaller the perception of the number of successes or failures grows); and $\rho>0$ indicates disconfirmation bias.


Figure 1: Example of confirmation measure for an updating problem with prior mean $E(\pi(p))=0.5$ and information signal of 8 successes and 2 failures $(k / n=8 / 10)$

### 2.2.3 Sequence-related biases and unrelated inference biases

Over(under)-reaction to information signals can also be driven by other reasons than the presence of preferences over the state-space. We consider two kind of alternative deviations: Sequence-related biases (i.e. hot hand fallacy and gambler's fallacy) and over(under)-inference to information signals when preference over the state-space are not at stake.

Sequence-related biases: Consider a partition of the signal space $S$. Namely, $S_{1}=\left\{s_{1} \ldots s_{m}\right\}$ and $S_{2}=\left\{s_{m+1} \ldots s_{n}\right\}$ where $n>m$. Suppose all of the $s_{i} \in S_{2}$ are either successes (i.e. $\sum_{n-(m+1)}^{n} s_{i}=n-m+1, s_{i} \in S$ ) or failures (i.e. $\sum_{n-(m+1)}^{n} s_{i}=0$, $s_{i} \in S$ ). In this context, the hot-hand fallacy is defined as inferring too much from
information signals after observing a the last $n-m+1$ consecutive successes (i.e. $\alpha>1$ ) or failures (i.e. $\beta>1$ ). Conversely, gambler's fallacy is defined as inference against the information signal after observing a the last $n-m+1$ consecutive successes (i.e. $\alpha<0$ ) or failures (i.e. $\beta<0$ ).

Inference biases: Consider an almost equivalent state-space set, $\Omega_{N P}=(0,1)$ over which the agent form beliefs but where, contrary to $\Omega$, the agent does not hold any preference. So any element $\hat{p}$ of the set $\Omega_{N P}$, will have the characteristic that $u(\hat{p})=0$. Because this is the only difference between sets, only $\alpha$ and $\beta$ coefficients in equations (9) and (10) differ between forming beliefs over $\Omega$ or $\Omega_{N P}$. Therefore, when the agent forms beliefs over $\Omega_{N P}, \alpha$ and $\beta$ coefficients can be interpreted as over(under) inference for successes and failures, independent of preferences over the state-space.

The biases in this subsection are only separably identified from one another when the agent faces multiple belief-elicitation decisions and observes different data generating processes coming from both $\Omega$ and $\Omega_{N P}$.

### 2.2.4 Under and overconfidence

Confidence biases (overconfidence and underconfidence) are those which are strictly related to the variance of the agent's posterior distribution in relation to the Bayesian variance. Overconfidence implies that the agent's overall distribution of posterior beliefs is more dispersed than that of a Bayesian agent. Conversely, underconfidence yields a posterior distribution which is more dispersed over the values of $p$ than the distribution a Bayesian agent would have. Formally:

$$
\begin{equation*}
V \tilde{a} r_{n}=\nu \times V a r_{n} \tag{11}
\end{equation*}
$$

where $V \tilde{a} r_{n}$ is the variance of the agent's posterior beta distribution, and $V a r_{n}$ is the Bayesian variance. In equation (11), $\nu>1$ indicates overconfidence, while $\nu<1$ indicates underconfidence.

## 3 Experimental Design

The experiment tests the theoretical framework of section 2 both at the individual and population level. The experiment was conducted at the Behavioural and Experimental Economics Laboratory (BEELab) at Maastricht University. 88 participants were recruited and, each of them had to make 30 belief-elicitation tasks. For each task, participants report their belief beta distributions twice, after observing two consecutive information signals. The average payment per participant was 15.9 euros. The experiment was pre-registered in October, 2023 at aspredicted.org. The full instructions of the experiment can be found in section $D$ of the appendix.

### 3.1 The belief-elicitation task

In each of the 30 tasks, participants observe a pool of 99 urns; each urn containing 100 balls. Each one of these urns contains a different distribution of red and blue balls. That is, Urn 1 has only one red ball and 99 blue, in Urn 2 there are only two red balls and 98 blue. This continues until Urn 99 where 99 balls are red and one is blue. Then, one of these urns is selected at random (i.e. selected from a uniform distribution), but the content of the urn is not revealed to the participants. Participants' task is to guess the percentage of red balls in the selected urn (see Figure 2).


Figure 2: Urn selection

In order to make their guesses, participants receive information signals by observing two sequences of balls, drawn with replacement. In the first sequence of draws,
either one, two or three balls are extracted at random from the selected urn. After seeing this sequence participants are asked to report their belief distribution ${ }^{11}$ (Figure 3a) about the percentage of red balls in the selected urn. This first report, is taken as a prior. Once this prior distribution is elicited, a second sequence of draws is drawn from the same selected urn and shown to the subjects. In this case, either three, five or seven balls are extracted at random from the urn. After observing this second information signal, participants are asked to report their (posterior) belief distribution ${ }^{12}$ (Figure 3b). Once this is done, a new urn from the pool of 99 urns is selected, with replacement, and this process is repeated thirty times. All of the subjects had an identical selection of urns and sequences of draws, but the order of the tasks was randomised between participants. All of the belief-elicitation tasks were incentiviced with a binarised scoring rule (more details in section 3.2 and appendix B)

To assess the role of motivated beliefs, fifteen of the thirty urns (placed at a random spots of the experiment) had an extra payment attached to it. I henceforth refer to these urns as dollar urns. In particular, when participants reported beliefs about these urns they receive a payment, in cents, equal to the (unknown) number of red balls in the selected urn. This means that participants should have a preference over the content of dollar urns.

### 3.2 Eliciting beta distributions

I use a novel way of eliciting belief distributions, which is especially convenient if the elicited belief distribution follows a specific functional form, as opposed to already existing methods (Crosetto and de Haan, 2022; Harrison et al., 2017).

In order to report their beliefs, participants were presented with a dynamic graphical interface that allows them to select their preferred beta distribution (See figures 3 a and 3 b ). By moving two sliders, each associated with a different question, participants were able to select a specific beta distribution. Those questions were:

[^6]1. What percentage of red balls do you expect the selected urn to have?
2. What is your uncertainty level about this percentage?

The first slider in figure, associated with question (1.), allows the subjects to manipulate the expected value of a beta distribution, while the second slider, associated with question (2.), allows the subjects to manipulate the standard deviation of such beta distribution ${ }^{13}$. Participants were shown a five-minute explanatory video on how to interpret the graph they selected and how to manipulate it. Subjects were specifically instructed to solve the task graphically, and they were also shown that the scale of the graph updates dynamically in order for the plot to be informative. Participants also had the option to enable the graph to update on a fixed scale if they desired to do so. Additionally, they answered related comprehension questions ${ }^{14}$ and could test the software before starting to answer the relevant belief-elicitation tasks.

In order to avoid the possibility that participants reported bimodal beta distributions, the maximum range of the second slider (standard deviation) depends on the value of the first slider (expected value). The possibility of reporting bimodal beliefs was excluded for a number of reasons: First, it is challenging to interpret what a bimodal beta distribution means in the context of this experiment, secondly introducing a bimodal distribution could impair the understanding of the graphical interface by the subjects; and thirdly, a bimodal distribution could never be the Bayesian answer to the task, so reporting a Bayesian distribution is an option which is never disallowed to the participants.

[^7]The lst sequence of draws from Urn $E$ is:
$\bigcirc$


What percentage of RED balls do you expect Urn $E$ to have?
What is your uncertainty level about the percentage you have just chosen?

(a) Figure 3a: Example of prior belief elicitation after observing a first sequence of draws from a random urn showing "red, red". This example shows a specific selection of a scaled beta distribution with an expected value of 56 and a standard deviation of 18.47

The 2nd sequence of draws from Urn $E$ is:
$\bigcirc 000$


What is your uncertainty level about the percentage you have just chosen?

$$
\begin{aligned}
& 7.44 \\
& \text { Scaling Option: } \\
& \text { Automatic Scaling } ~
\end{aligned}
$$

(b) Figure 3b: Example of posterior belief elicitation after observing a second sequence of draws showing "blue, red, blue, red, blue". The updated beta distribution is selected in this example shows an expected value of 51 and a standard deviation of 7.44

To incentivise truthful reporting, a binarised scoring rule was implemented in every task. Specifically, I follow a similar method to the one suggested by Schlag et al. (2015), who propose different scoring rules to incentivise every moment of a probability distribution (see appendix B to understand the specific scoring rule). At the same time, I closely follow Danz et al. (2022) in not explicitly disclosing the exact scoring rule that is implemented. Instead, participants were told that they should always truthfully report their guess for the percentage red of balls, and their uncertainty level in the selected urn (i.e. the mean and standard deviation of the beta distribution) so as to maximise their expected payoff.

## 4 Baseline and Complete model regressions

This section presents the main analyses derived directly from the theoretical framework of section 2. In Section 4.1, the baseline model regression is outlined, introducing only a limited set of biases. Following that, Section 4.2 details the complete model (equations (14),(15) and (16)), encompassing all biases of the model. The decision to present these two models separately aims to facilitate comparison, investigating whether significant biases in the baseline model persist when accounting for multiple belief biases. Both models are executed and compared at both population and individual levels. In Section 4.3, a method is introduced to compare the relative importance of biases proven significant at the individual level.

### 4.1 Baseline model regressions

The baseline model, closely resembles equations (4) and (5). It tests for under(over)inference and base rate neglect(overuse):

$$
\begin{gather*}
\tilde{a}_{n}-1=\gamma_{s} k+\delta_{s}\left(a_{0}-1\right)+\varepsilon_{a}  \tag{12}\\
\tilde{b}_{n}-1=\gamma_{f}(n-k)+\delta_{f}\left(b_{0}-1\right)+\varepsilon_{b} \tag{13}
\end{gather*}
$$

In equations (12) and (13), successes and failures (variables $k$ and $(n-k)$ ), represent the number of red and blue balls observed in the second sequence of draws provided to participants. Variables $a_{0}$ and $b_{0}$ are the parameters of the beta distribution elicited by subjects after the first sequence of draws, while $\tilde{a_{n}}$ and $\tilde{b_{n}}$ are the parameters of the beta distribution elicited by subjects after observing the second sequence of draws. $\varepsilon_{a}$ and $\varepsilon_{b}$ are the error terms.

The parameter interpretation of equations (12) and (13) is akin to equations (9) and (10). Parameters $\gamma$ and $\delta$ similarly indicate under/over inference and base rate neglect/overuse respectively. However, equations (12) and (13) differ by acknowledging that these biases $\left(\gamma_{s}, \gamma_{f}\right)$ and $\left(\delta_{s}, \delta_{f}\right)$ may vary between successes and failures (i.e. realisations of red and blue balls). The presence of those biases is tested by comparing whether the estimated $\gamma$ and $\delta$ parameters are significantly different from their

Bayesian values (i.e. $\gamma_{s}=\gamma_{f}=1$ and $\delta_{s}=\delta_{f}=1$ ).
Note that equations (12) and (13) do not have an intercept. This has a very straightforward theoretical justification. If all the independent variables in those equations take value 0 (i.e. $k=0, n=0, a_{0}=1, b_{0}=1$ ), both $\tilde{a}_{n}$ and $\tilde{b}_{n}$ take a value of 1 . This means that when one starts updating from a uniform prior distribution ( $a_{0}=1, b_{0}=1$ ), and observes no information signals whatsoever $(k=0, n=0)$, one must not update, i.e. remain at such uniform prior after updating ( $\tilde{a}_{n}=1, \tilde{b}_{n}=1$ ). Having an intercept different than zero would imply that agents update in the absence of any kind of information.

### 4.2 Complete model regressions

In order to test for the presence of all the biases described in section 2.2, I run a slightly modified version of equations (9), (10) and (11). Namely:

$$
\begin{gather*}
\tilde{a}_{n}-1=\left(\alpha_{0}+\alpha_{\text {Pref }} I_{\text {Pref }}+\alpha_{\text {Seq }} I_{\text {Seq }_{s}}\right) k+\rho_{s} c+\delta_{s}\left(a_{0}-1\right)+\varepsilon_{a}  \tag{14}\\
\tilde{b}_{n}-1=\left(\beta_{0}+\beta_{\text {Pref }} I_{\text {Pref }}+\beta_{\text {Seq }} I_{\text {Seqf }}\right)(n-k)+\rho_{f} c+\delta_{f}\left(b_{0}-1\right)+\varepsilon_{b}  \tag{15}\\
V \tilde{a} r_{n}=\eta+\nu \times V a r_{n}+\varepsilon_{v} \tag{16}
\end{gather*}
$$

Variables in equations (14) and (15) are identical to the baseline model with the only exception of variable $c$, the relative measure of confirmation, as described in section 2.2.2. Equation (16) is almost identical to equation (11) in the interpretation of its variables and the $\nu$ parameter indicating over or underconfidence ( $\nu>1$ vs $\nu<1$ ). The inclusion of an intercept $\eta$ and an error term $\varepsilon_{v}$ are the only differences ${ }^{15}$.

Parameters $\rho$ and $\delta$ in equations (14) and (15) indicate confirmation/disconfirmation bias, and base rate neglect/overuse respectively. As in the baseline case, equations (14) and (15) differ by acknowledging that these parameters may differ between successes and failures ( $\rho_{s}$ vs $\rho_{f}$ ) and ( $\delta_{s}$ vs $\delta_{f}$ ). However, the notable difference between

[^8]equations (14), (15) and (9),(10) lies in accommodating the biases of section 2.2.3. This involves the inclusion of three dummy variables - IPref, ISeq $q_{s}$, and $I S e q_{f}-$ interacting with the number of successes (in (14)) or failures (in (15)). IPref equals 1 when the subject faces a decision with preferences over the state-space (that is, when beliefs about a dollar urn where reported) and 0 otherwise. $I S e q_{s}\left(I S e q_{f}\right)$ equals 1 when the last three balls observed in the second sequence are red (blue). By doing so, one can distinguish motivated beliefs (optimism and pessimism), sequence-related biases (gambler's fallacy and hot hand fallacy) and unrelated over/under inference. The presence of those biases is tested by comparing whether the estimated parameters are significantly different from their Bayesian values. Table 1 summarizes the baseline and complete models, and specifically shows the ranges of values of the different parameters, which correspond to each bias.

| Baseline Model <br> Eq (12) (13) | $\begin{aligned} & \text { Complete Model } \\ & \text { Eq (14)(15)(16) } \\ & \hline \end{aligned}$ |
| :---: | :---: |
| $\gamma_{s}$ or $\gamma_{f}>1$ Overinference $\gamma_{s}$ or $\gamma_{f}<1$ Underinference $\delta_{s}$ or $\delta_{f}>1$ Base Rate Overuse $\delta_{s}$ or $\delta_{f}<1$ Base Rate Neglect | ```\(\alpha_{0}\) or \(\beta_{0}>1\) Overinference \(\alpha_{0}\) or \(\beta_{0}<1\) Underinference \(\alpha_{0}+\alpha_{\text {Pref }}>1\) or \(\beta_{0}+\beta_{\text {Pref }}<1\) Optimism \(\alpha_{0}+\alpha_{\text {Pref }}<1\) or \(\beta_{0}+\beta_{\text {Pref }}>1\) Pessimism \(\alpha_{\text {Pref }}>\beta_{\text {Pref }}\) Good News Effect \(\alpha_{\text {Pref }}<\beta_{\text {Pref }}\) Bad News Effect \(\alpha_{0}+\alpha_{S e q}>1\) or \(\beta_{0}+\beta_{S e q}>1\) Hot Hand Fallacy \(\alpha_{0}+\alpha_{S e q}<0\) or \(\beta_{0}+\beta_{S e q}<0\) Gambler's Fallacy \(\rho_{s}\) or \(\rho_{f}<0\) Confirmation Bias \(\rho_{s}\) or \(\rho_{f}>0\) Disconfirmation Bias \(\delta_{s}\) or \(\delta_{f}>1\) Base Rate Overuse \(\delta_{s}\) or \(\delta_{f}<1\) Base Rate Neglect \(\nu>1\) Overconfidence \(\nu<1\) Underconfidence``` |

Table 1: Summary of biases for Baseline and Complete Model

### 4.3 The effect of each individual bias

In order to facilitate the comprehension and enhance the comparability of biases in Table 1 for the individual analysis, I construct a measure that is informative of the relative importance of each bias. This measure identifies the specific effect of each bias in the relative changes of mean and variance for each subject. In order to achieve this, I analyze the distortions to the Bayesian expected value and Bayesian variance caused by exhibiting a particular kind of bias. The idea is to compare these Bayesian values with a bias-specific measure of how the expected value and variance would look like if the subject exhibited only the specific bias we are interested in. I call these measures Bias-specific expected value and Bias-specific variance. The Bayesian expected value and variance of the posterior beta distribution are given by:

$$
\begin{align*}
E_{n} & \equiv E\left(p \mid a_{n}, b_{n}\right)=\frac{a_{n}}{a_{n}+b_{n}}  \tag{17}\\
\operatorname{Var}_{n} & \equiv \operatorname{Var}\left(p \mid a_{n}, b_{n}\right)=\frac{a_{n} b_{n}}{\left(a_{n}+b_{n}\right)^{2}\left(a_{n}+b_{n}+1\right)} \tag{18}
\end{align*}
$$

The bias-specific expected value $E_{\text {Bias }}$ and bias-specific variance $\operatorname{Var}_{\text {Bias }}{ }^{16}$ are given by:

$$
\begin{align*}
E_{\text {Bias }} & =\frac{a_{\text {bias }}}{a_{\text {bias }}+b_{\text {bias }}}  \tag{19}\\
\text { Var }_{\text {Bias }} & =\frac{a_{\text {bias }} b_{\text {bias }}}{\left(a_{\text {bias }}+b_{\text {bias }}\right)^{2}\left(a_{\text {bias }}+b_{\text {bias }}+1\right)} \tag{20}
\end{align*}
$$

where $a_{\text {bias }}$ and $b_{\text {bias }}$ are the hypothetical parameters of a posterior beta distribution if they were distorted by only one specific bias ${ }^{17}$. For example, suppose we want to

[^9]evaluate the impact of overinference. Suppose a particular subject exhibits overinference only in successes: That is $\alpha_{0}>1$. Then, $a_{\text {bias }}=a_{\text {Overinfernce }}=\alpha_{0} k+a_{0}$ and $b_{\text {bias }}=b_{n}$. Equations (19) and (20) for every other bias are calculated analogously.

To capture the relative importance of each bias in inference, I compare the distance between the Bayesian and distorted expected value and variance. Namely:

$$
\begin{align*}
\Delta E_{\text {Bias }} & =\left|E_{n}-E_{\text {Bias }}\right|  \tag{21}\\
\Delta \text { Var }_{\text {Bias }} & =\mid \text { Var }_{n}-V_{\text {Bias }} \mid \tag{22}
\end{align*}
$$

Equations (21) and (22) allow us to quantify the discrepancy between the Bayesian expected value and variance, and the bias-specific measures. These discrepancies serve as indicative measures of the influence of each bias on the overall inference process. Subsequently, by assessing these differences across subjects, we can assess the relative impact of each individual bias.

## 5 Results

### 5.1 Population-level analysis

Table (2) compares the baseline model (equations (12) and (13)) with the complete model (equations (14) (15) (16)) at the population level. Columns (1) and (2) of the table refer to the baseline model, while columns (3), (4) and (5) correspond to the complete model. All regressions in table (2) are run with clustered standard errors by participant. Importantly, the significance of each coefficient does not necessarily show whether an estimate is significantly different from 0 , but whether it is significantly different from the corresponding Bayesian value for a given coefficient (see Table 1).

Table 2: Baseline and Complete models at the population level. Significance is with respect to Bayesian values. Clustered standard errors

|  | Dependent variable: |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | a posterior <br> (1) | b post <br> (2) | a post <br> (3) | b post <br> (4) | Variance post (5) |
| Successes | $\begin{gathered} 37.339^{* *} \\ (11.66) \end{gathered}$ |  | $\begin{aligned} & 43.678 \\ & (36.34) \end{aligned}$ |  |  |
| a prior | $\begin{gathered} 0.017^{* * *} \\ (0.02) \end{gathered}$ |  | $\begin{gathered} 0.016^{* * *} \\ (0.057) \end{gathered}$ |  |  |
| Failures |  | $\begin{gathered} 76.199^{* *} \\ (31.96) \end{gathered}$ |  | $\begin{gathered} 67.102 \\ (40.072) \end{gathered}$ |  |
| $b$ prior |  | $\begin{gathered} -0.0002^{* * *} \\ (0.01) \end{gathered}$ |  | $\begin{gathered} -0.001^{* * *} \\ (0.019) \end{gathered}$ |  |
| Success : preference |  |  | $\begin{gathered} -30.889 \\ (27.86) \end{gathered}$ |  |  |
| Success: Seq pos |  |  | $\begin{aligned} & 14.726 \\ & (32.75) \end{aligned}$ |  |  |
| Failures : preference |  |  |  | $\begin{gathered} -27.602 \\ (71.12) \end{gathered}$ |  |
| Failures: Seq ${ }_{\text {neg }}$ |  |  |  | $\begin{gathered} 102.488 \\ (99.13) \end{gathered}$ |  |
| Confirmation |  |  | $\begin{gathered} 0.001 \\ (0.001) \end{gathered}$ | $\begin{gathered} -0.096 \\ (0.09) \end{gathered}$ |  |
| Bayesian variance |  |  |  |  | $\begin{gathered} 0.983 \\ (0.076) \end{gathered}$ |
| Constant |  |  |  |  | $\begin{aligned} & 0.002^{* *} \\ & (0.001) \end{aligned}$ |
| Observations | 2,640 | 2,640 | 2,636 | 2,636 | 2,640 |
| $\mathrm{R}^{2}$ | 0.002 | 0.003 | 0.002 | 0.004 | 0.289 |
| Adjusted $\mathrm{R}^{2}$ | 0.001 | 0.002 | 0.00000 | 0.002 | 0.288 |
| Note: |  | <0.1; ${ }^{* *} \mathrm{p}<0$ | ; ${ }^{* * *} \mathrm{p}<0$ | from the | yesian estimate |

In the baseline model, there are two significant biases at the population level: Overinference and base rate neglect. We see overinference both in successes $\left(\hat{\gamma}_{s}>1\right.$; $\left.\hat{\gamma}_{s}=37.339\right)$ and failures $\left(\hat{\gamma}_{f}>1 ; \hat{\gamma}_{f}=76.119\right)$. This can be interpreted as subjects, on average, interpreting each red ball they observe as 37 red balls; and each blue ball they see as 76 blue balls, when they form their beliefs. And we see base rate neglect (also in successes ( $\hat{\delta}_{s}<1 ; \hat{\delta}_{s}=0.017$ ) and failures $\left.\hat{\delta}_{f}<1 ; \hat{\delta}_{f}=-0.0002\right)$ ). This can be interpreted as subjects ignoring their priors (number of red/blue balls they expected to see) when they form their beliefs.

Despite the significance of overinference and base rate neglect at the population level in the baseline model, the effect of overinference seems to vanish when all of the biases of the model are controlled for (Columns (3), (4) and (5)), and base rate neglect is the only remaining significant bias. The complete model also minimizes the AIC and BIC as show in Table 3 (Appendix C). This precisely shows that the lack of inclusion of a wider array of biases within the same model, makes some biases look artificially significant ${ }^{18}$.

However, neither the baseline nor the complete model seem to be explanatory of subject's posterior belief formation at the population level. This can be seen by the extremely low goodness of fit ( $R^{2}$ ) of Columns (1) to (4) in Table 2. Yet, this is not the result of subjects being completely noisy, but rather a consequence of individual heterogeneity. That is, subjects are distinctly biased, but they exhibit widely different biases. These distinct biases average out at the population level, but clearly present at the individual level. Table 4 (Appendix C) compares the $R^{2}$ of all the equations at the population with the mean of all $R^{2} \mathrm{~s}$ in these same equations after running the model at the individual level. As one can see in the first and second column of the table, the goodness of fit from the population to the individual model shows a substantial improvement ${ }^{19}$. This is also the case for both the AIC and BIC criteria (columns (3) and (4) of table 3), which tremendously diminish when compared to running the models at the population level, with the only exception of the variance equation (equation (16)), which performs better at the population level. This is

[^10]evidence pointing to the fact that there is considerable individual heterogeneity in the data, and therefore, the model would benefit from an individual analysis.

### 5.2 Individual-level analysis

Figures 4 a and 4 b provide an overview of the biases detected at the individual level within the context of the two models: the baseline and the complete model. In Figure 4a, the bar chart depicts the occurrences of statistically significant biases at the $5 \%$ level within the baseline model. The color-coded representation distinguishes biases: red means significance solely for successes, blue for failures, and green for significance in both successes and failures ${ }^{20}$. Notably, overinference and base rate overuse emerge prominently. However, Figure 4b, illustrating individual biases in the complete model, shows a more complete picture. Here, a broader array of biases is accounted for, with the hot hand effect emerging as the most commonly exhibited bias among the subjects. Further comparing these models yields intriguing insights. First, at the descriptive level, it becomes evident that individual heterogeneity persists across both models. This observation underscores the varying proportions with which biases manifest among individuals: All tested biases were present among subjects to a certain extent. That is, there is no bias which fully vanishes after incorporating the complete set of biases. Notably, the prevalence of the hot hand fallacy contrasts sharply with the sparsity of confirmatory biases or overconfidence among subjects. But secondly, the complete model makes another important revelation: every subject exhibits at least one form of bias when the full spectrum of biases is considered.

While the "No Bias" column in Figure 4a shows that 15 individuals ( $17 \%$ of the sample) could not be categorised, this column is empty in Figure 4b. This implies that the shift from the baseline to the complete model is able to identify biased individuals, who would otherwise be categorised as either Bayesian or "too noisy" ${ }^{21}$.

[^11]
(a) Figure 4a: Number of times a specific bias is found to be significant ( $p<0.05$ ) in the baseline model at the individual level. Overinference and base rate overuse are the most common biases. 15 individuals ( $17 \%$ ) exhibit no bias or are too noisy to be distinguished from a Bayesian agent.

(b) Figure 4b: Number of times a specific bias is found to be significant ( $p<0.05$ ) in the complete model at the individual level. There is significant individual heterogeneity. All subjects exhibit at least one type of bias. The hot hand fallacy is found to be the most common bias.

Despite these analyses showing the prevalence of different biases at the individual level, it is also paramount to also consider how much each of these biases drives inference in more tangible and common units. Thus, I apply the methodology outlined in section 4.3, to asses the relative importance of each bias.

Figures 5a and 5b provide an examination of biases within the complete model, focusing on the expected value deviations from the Bayesian framework for each specific bias. While both figures provide insights into how important these deviations are, they take different approaches. Figure 5a presents the expected value deviations for each bias, irrespective of their frequency of occurrence. On the other hand, Figure $5 b$ adjusts for frequency by weighting these deviations with the number of times a bias was found to be statistically significant.

## Effects on Expected Value and Variance


(a) Figure 5a: Average expected-value deviations from each individual bias.

(b) Figure 5b: Expected-value deviations adjusted by the amount of times a bias was found to be significant.

In Figure 5a, two biases, gambler's fallacy and optimism, stand out with prominent expected value deviations, emphasizing their substantial impact on inference. In contrast, confirmation and disconfirmation bias exhibit less pronounced deviations. Remarkably, even after correcting for frequency in Figure 5b, gambler's fallacy and optimism retain their prominence, suggesting their enduring influence on biased inference.

Beyond these observations, Figure 5a and 5b also reveal interesting patterns. The biases exerting the most substantial influence on expected value deviations cluster into two distinct categories: Motivated beliefs (optimism and pessimism), and biases associated with updating against the information signal, including gambler's fallacy. Additionally, there is a notable observation regarding the hot hand fallacy: While identified as the most common bias in Figure 4b, its impact on expected value devi-
ations appears relatively modest in Figure 5a. However, when it comes to its effect on variance, the hot hand fallacy demonstrates a notably strong influence, as can be seen in Figures 6a and 6b.

Figure 6 a and 6 b are analogous to 5 a and 5 b , but represent variance deviations with respect to the Bayesian framework instead. While the relative contribution of each bias appears to be quite homogeneous (Figure 6a), once we correct for the frequency such biases (figure 6b), the hot hand fallacy shows its prominence, while optimism and pessimism come second and third respectively. This underlines the importance of motivated-belief biases in overall inference: Both in expected-value and variance deviations.


Putting it all together, this subsection emphasises the importance of incorporating a more complete spectrum of biases into account. When doing so, all biases are present at the individual level and there is no single subject who does not exhibit some kind of bias. It also showcases a key interplay between bias frequency and inference. While,
the hot hand fallacy is the most common bias, it is mainly so through its effect on variance. That is, overreaction to a consecutive set of signals in the same direction does not seem to distort the perception of how likely the outcome is as much as distorting how certain individuals are. However, optimism and pessimism seem to be driving inference in both dimensions. Overall, in spite of the presence of individual heterogeneity, two categories of biases (sequence-related biases and motivated-belief biases) clearly stand out above the rest.

## 6 Concluding remarks

In conclusion, this paper addresses the challenges in identifying and testing multiple belief-updating biases simultaneously. This is important because in the absence of such a model, many of these biases could be potentially confounded. The theoretical model introduces a novel approach by considering belief distributions over a continuous state space, allowing for a complete identification of multiple conflicting belief biases. The corresponding laboratory experiment applies this model, using a novel technique to elicit belief distributions.

The population-level analysis shows that, despite the prevalence of overinference and base rate neglect in the baseline model, only base rate neglect remains significant in the complete model, emphasizing the importance of accounting for a broader spectrum of biases. Individual-level analysis uncovers significant heterogeneity, with every participant exhibiting at least one identifiable bias. Further exploration into biases' impact on expected value and variance exposes motivated-belief biases (optimism and pessimism) and sequence-related biases (gambler's and hot hand fallacies) as key drivers of biased inference. Thus, capturing a wide array of biases provides a much more subtle understanding of belief updating dynamics. Overall, addressing confoundedness in belief-updating biases is key to researchers interested in understanding links between different biases, or delving deeper into the causes of various behavioural phenomena (political polarization, investing performance, disposition effect...) when conflicting updating biases are at stake.

I would like to conclude by pointing to some related future research avenues. While this paper takes detailed care in separating behaviour stemming from different
biases, it is important to note that it approaches biases as systematic departures from rational behaviour. That is, it does not dig into the cognitive mechanisms or channels by which a certain bias might be exhibited. In this direction Bordalo et al. (2023a), look at how some biases become more prominent when salient features of the information become more relevant. At the same time, Bordalo et al. (2023b) also emphasise the importance of memory and recall to explain well-known cognitive biases. Further exploring more of these cognitive mechanisms in a setting where biases can be de-confounded, could deliver a complete picture of what updating biases matter the most, and pinpoint the primitive traits that underlie biased behaviour.

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## Appendix to Belief Bias Identification

## A Conjugate analyses of the binomial distribution

This section shows the derivation of equations of posterior parameters in section 2 . I present two cases: The Bayesian case (eq. (3)), and introducing the biases of the baseline model (eq.(4) and (5)). The rest of the proofs for the remaining biases are analogous to these cases. Once the distorted likelihood and prior are given, the proof follows the exact same steps.

## A. 1 The Bayesian case

Statement 1. Given likelihood equation (1) and prior equation (2) of section 2.1., a Bayesian agent updates her beliefs such that her posterior distribution of $p: \pi\left(p \mid a_{n}, a_{n}\right)$ is beta distributed with parameters $a_{n}, b_{n}$ such that:

$$
a_{n}=k+a_{0} \quad b_{n}=n-k+b_{0}
$$

Proof. Let us apply Bayes' Theorem given likelihood eq.(1) and prior eq.(2). This yields:

$$
\begin{aligned}
\pi\left(p \mid s_{1} \ldots s_{n}, a_{0}, b_{0}\right) & =\frac{L\left(p \mid s_{1} \ldots s_{n}\right) \pi\left(p \mid a_{0}, b_{0}\right)}{\int_{p=0}^{1}\left(L\left(p \mid s_{1} \ldots s_{n}\right) \pi\left(p \mid a_{0}, b_{0}\right)\right) d p} \\
& =\frac{\binom{n}{k} p^{k+a_{0}-1}(1-p)^{n-k+b_{0}-1} / B\left(a_{0}, b_{0}\right)}{\int_{p=0}^{1}\left(\binom{n}{k} p^{k+a_{0}-1}(1-p)^{n-k+b_{0}-1} / B\left(a_{0}, b_{0}\right)\right) d p} \\
& =\frac{p^{k+a_{0}-1}(1-p)^{n-k+b_{0}-1}}{B\left(a_{0}+k, b_{0}+n-k\right)}
\end{aligned}
$$

Which is itself the probability density function of a beta distribution with parameters $\left(a_{0}+k, b_{0}+n-k\right)$. Therefore, the posterior Bayesian distribution is beta distributed with parameters $\left(a_{n}, b_{n}\right)$ as defined in Statement 1.

## A. 2 Biases of the baseline model: Inference and base-rate biases

Statement 2. Given likelihood $\tilde{L}\left(p \mid s_{1} \ldots s_{n}\right)=\left(L\left(p \mid s_{1} \ldots s_{n}\right)\right)^{\gamma}$ and prior $\tilde{\pi}(p)=(\pi(p))^{\delta}$ of section 2.1., where $\gamma, \delta$ indicate deviations from Bayesian updating, a non-Bayesian agent follows a posterior beta distribution with parameters $\tilde{a}_{n}, \tilde{b}_{n}$ such that:

$$
\begin{gathered}
\tilde{a}_{n}=\gamma k+\delta\left(a_{0}-1\right)+1 \\
\tilde{b}_{n}=\gamma(n-k)+\delta\left(b_{0}-1\right)+1
\end{gathered}
$$

Proof. Applying Bayes theorem yields:

$$
\begin{aligned}
\tilde{\pi}\left(p \mid s_{1} \ldots s_{n}, a_{0}, b_{0}\right) & =\frac{\left(L\left(p \mid s_{1} \ldots s_{n}\right)\right)^{\gamma}(\pi(p))^{\delta}}{\int_{p=0}^{1}\left(\left(L\left(p \mid s_{1} \ldots s_{n}\right)\right)^{\gamma}(\pi(p))^{\delta}\right) d p} \\
& =\frac{\left[\binom{n}{k}\right]^{\gamma} p^{\gamma k}(1-p)^{\gamma(n-k)}\left(1 / B\left(a_{0}, b_{0}\right)\right)^{\delta} p^{\delta\left(a_{0}-1\right.}(1-p)^{\delta\left(b_{0}-1\right)}}{\int_{p=0}^{1}\left(\left[\binom{n}{k}\right]^{\gamma} p^{\gamma k}(1-p)^{\gamma(n-k)}\left(1 / B\left(a_{0}, b_{0}\right)\right)^{\delta} p^{\delta\left(a_{0}-1\right.}(1-p)^{\delta\left(b_{0}-1\right)}\right) d p} \\
& =\frac{p^{\gamma k+\delta\left(a_{0}-1\right)}(1-p)^{\gamma(n-k)+\delta\left(b_{0}-1\right)}}{\int_{p=0}^{1}\left(p^{\gamma k+\delta\left(a_{0}-1\right)}(1-p)^{\gamma(n-k)+\delta\left(b_{0}-1\right)}\right) d p} \\
& =\frac{p^{\gamma k+\delta\left(a_{0}-1\right)}(1-p)^{\gamma(n-k)+\delta\left(b_{0}-1\right)}}{B\left(\gamma k+\delta\left(a_{0}-1\right)+1, \gamma(n-k)+\delta\left(b_{0}-1\right)+1\right)}
\end{aligned}
$$

This is the probability density function of a beta distribution with parameters $\left(\gamma k+\delta\left(a_{0}-1\right)+1, \gamma(n-k)+\delta\left(b_{0}-1\right)+1\right)$. Therefore, the posterior distribution of an agent that exhibits inference bias $\gamma$, and base-rate bias $(\delta)$ is beta distributed with parameters $\left(\tilde{a_{n}}, \tilde{b_{n}}\right)$ as defined in Statement 2.

## $B$ Incentivising mean and variance of beta distributions

The scoring rules follow Schlag et al. (2015) with a slight modification. In particular, random realisations of the Bayesian posterior distributions are taken as the random draws.

## B. 1 Incentivising the mean

Let $\tilde{m}$ be the reported mean of the agent's posterior beta distribution $\pi\left(p \mid \tilde{a}_{n}, \tilde{b}_{n}\right)$, and let $d$ be a random draw of the Bayesian posterior beta distribution $\pi\left(p \mid a_{n}, b_{n}\right)$. Then the Quadratic Scoring Rule is given by $g_{Q S R}(\tilde{m}, d)=-(\tilde{m}-d)^{2}$. Let $A, B$ be the boundaries of the state-space $\Omega$, and $M$ be any arbitrary amount of money ${ }^{22}$. Then, the randomised quadratic scoring rule is given by the following lottery:

$$
\tilde{g}_{Q S R}(\tilde{m}, d)=l\left(M, 0 ; 1+\frac{g_{Q S R}(\tilde{m}, d)}{(B-A)^{2}}\right)
$$

## B. 2 Incentivising the variance

Let $\tilde{v}$ be the the reported variance of the agent's posterior beta distribution. In order to elicit the variance consider two random draws of the agent's posterior beta distribution $\pi\left(p \mid \tilde{a}_{n}, \tilde{b}_{n}\right)$. Namely, $d_{1}$ and $d_{2}$. Then the variance scoring rule is given by $g_{v}\left(\tilde{v}, d_{1}, d_{2}\right)=-\left(\tilde{v}-\frac{1}{2}\left(d_{1}-d_{2}\right)^{2}\right)^{2}$. Applying randomisation, the randomised variance scoring rule yields:

$$
\tilde{g}_{v}\left(\tilde{v}, d_{1}, d_{2}\right)=l\left(M, 0 ; \frac{g_{v}+\frac{1}{4}(B-A)^{4}}{\frac{1}{4}(B-A)^{4}}\right)
$$

[^12]
## C Extra tables and figures

## C. 1 Information criteria and $R^{2}$ comparisons

| Model | AIC(pop.) | BIC(pop.) | AIC(ind.) | BIC(ind.) |
| :--- | :---: | :---: | :---: | :---: |
| Baseline model |  |  |  |  |
| Eq. (12) | 42454.31 | 42466.07 | 270.41 | 274.61 |
| Eq. (13) | 43636.76 | 43648.52 | 265.84 | 270.05 |
| Complete model |  |  |  |  |
| Eq. (14) | 42398.72 | 42428.1 | 267.99 | 276.39 |
| Eq. (15) | 43577.64 | 43607.03 | 254.45 | 262.84 |
| Eq. (16) | -22181.66 | -22169.9 | -187.21 | -183.01 |

Table 3: Information Criteria Comparison at population and individual level

| Model | $R^{2}$ Population | Mean $R^{2}$ Ind. | Mean adj. $R^{2}$ Ind. |
| :--- | :---: | :---: | :---: |
| Baseline model |  |  |  |
| Eq. (12) | 0.002 | 0.564 | 0.533 |
| Eq. (13) | 0.003 | 0.533 | 0.501 |
| Complete model |  |  |  |
| Eq. (14) | 0.002 | 0.761 | 0.713 |
| Eq. (15) | 0.003 | 0.812 | 0.774 |
| Eq. (16) | 0.289 | 0.139 | 0.108 |

Table 4: $R^{2}$ Comparison for Baseline and Complete model at population vs individual level

## C. 2 Individual bias frequency at a $1 \%$ significance level

As the individual-level analysis in section 5.2 tests a considerable number of null hypotheses (eleven per subject), one may wonder whether some of these results are driven by a high rate of false positives. To address this concern, this section replicates Figures 4a and 4 b for a $1 \%$ significance level, lowering the rate of potential false positives.

(a) Figure 7a: Number of times a specific (b) Figure 7b: Number of times a specific bias is found to be significant ( $p<0.01$ ) in bias is found to be significant ( $p<0.01$ ) in the baseline model at the individual level. the complete model at the individual level.

As one can see from figures 7a and 7b, the three main results from section 5.2 still hold. Namely, i) all biases are exhibited to a certain extent in the complete model, ii) transitioning from the baseline to the complete model shows that all subjects exhibit some kind of bias (empty "No Bias" column), and iii) the hot hand effect stands out as the most-frequently exhibited bias.

In fact, the second result seems to be stronger in this condition, with the vast majority of subjects appearing to be "too noisy" in the baseline model, but showing some type of bias in the complete model. However, the frequency of significant motivated-belief biases (optimism/pessimism) does diminish under $p<0.01$.

## D Experiment instructions

In this study, you will be asked to complete $\mathbf{3 0}$ guessing tasks. For each guessing task you have to make 2 related guesses. At the beginning of each guessing task, there is always a pool of 99 URNS, each containing 100 BALLS. Some balls in the urns are red, and some are blue. Each one of these urns contains a different percentage of red balls. For example, in Urn 1 there is only one red ball and 99 blue ( $1 \%$ of the balls are red), in Urn 2 there are only two red balls and 98 blue ( $2 \%$ of the balls are red). This continues until Urn 99 where 99 balls are red and one is blue. (See picture below).


For each guessing task, out of these 99 urns, one of them (say Urn X) has been selected at random. Each urn has the same chances of being selected from the pool. That is, you do not know how many of the balls are red and how many of them are blue in the selected urn. All combinations are possible. (See picture below).

URN X SELECTED:


Your task is to guess what the percentage of red balls in the selected urn is. In each one of the 30 guessing tasks, this process will be repeated with a new urn.

Bear in mind: Whenever a new urn is selected, it is always drawn from the same pool of 99 urns (WITH REPLACEMENT). This means that the same urn can be chosen either once or multiple times. The urns have letters (or combinations of letters) on top of them. This is just to highlight the fact that when a new letter(or combination of letters) is on top of the urn, it means that a new urn has been drawn.

## The guessing task

This section will explain how each one of the 30 guessing tasks works. Initially, as explained above, the computer has randomly selected, with equal probability, ONE out of the 99 urns. Remember that initially, you know nothing about the content of this urn. Once a given urn is selected, you will be given some information about the urn to help you make your guess. First, you will see a sequence of balls which have been randomly drawn from the urn. Each ball in the selected urn has the same chances of being drawn, and each of these draws is done WITH REPLACEMENT. This means that after a ball has been drawn and taken out of the urn, it is immediately replaced with one of the same colour. The urn will always have the same 100 balls. In order to help you make your guesses, two sequences of draws will be made from each urn.

- 1st sequence of draws: Either one, two or three balls from the urn are selected at first. After this selection, you will have to make your first guess. For your first guess you will answer two questions:

1. What percentage of red balls do you expect the selected urn to have?
2. What is your uncertainty level about this percentage?

## Example: Guessing Task A


$\mathbf{1}^{\text {st }}$ sequence of draws: 0 O $\longrightarrow$ Make your first guess for urn $A$

- 2nd sequence of draws: The second draw follows very similar rules to the first one. In this case three, five or seven balls from THE SAME URN are selected with replacement. After this selection, you will have to make your second guess. Once again, for your second guess you will answer the same two questions:

1. What percentage of red balls do you expect the selected urn to have?
2. What is your uncertainty level about this percentage?
```
2nd sequence of draws: ० ००००\longrightarrow Make your second guess for urn A
```

In order to further help you with your guesses a dynamic graph of your choice will be provided. Please watch the following video (next screen) to understand how this works.

Click here to see the explanatory video
[Page Break]

## Your Payment

You can earn up to $\in 29,70$ in this experiment. In particular, your payment is broken down as follows:

- You will receive $€ 5$ for taking the time to complete this experiment.
- You will receive up to €10 for your responses related to the guessing tasks.

How much of this amount (€10) you receive depends on the actual percentage of red balls in the selected urn. You can get money for EVERY SINGLE ONE of your guesses.

The payment rule we use, is optimized so that in order to maximize your expected payoff, you should ALWAYS give your best estimate of the percentage of red balls in the selected urn. In the same manner, the payment rule we use, is also optimized so that in order to maximize your expected payoff, you should ALWAYS give your best estimate of your uncertainty level.

- You can receive up to € 14,70 as extra payment.

This extra payment is divided across the 30 guessing tasks. Whether or not a guessing task has an extra payment attached depends on whether a dollar urn has come up. (See picture below. Urn A is in this example, a dollar urn). In particular, a dollar urn will come up randomly in 15 of the 30 guessing tasks.

If a dollar urn comes up, you will receive as many cents as red balls the selected dollar urn has. For example, if the selected dollar urn has 50 red balls you will get 50 cents.

At the end of the experiment, you will be informed about the number of red balls in each urn and your total payment. If you want to know more about the details of the payment rule, you can let me know after the experiment or write an email to p.gonzalezfernandez@maastrichtuniversity.nl

## Get Familiar with the Tools

Before you answer the comprehension questions you have the chance to get familiar with the guesses, the payment and the sliders with a trial guessing task.
[Page Break]

## THIS IS A TRIAL

## URN X



From the pool of 99 urns, Urn $\mathbf{X}$ has been selected at random. The 1st sequence of draws from Urn $X$ is:



What percentage of RED balls do you expect Urn X to have?
$44 \%$
What is your uncertainty level about the percentage you have just chosen?


Scaling Option:
Automatic Scaling $\vee$

## THIS IS A TRIAL

## URN X



The 2nd sequence of draws from Urn $X$ is:



What percentage of RED balls do you expect Urn $X$ to have?
$57 \%$

What is your uncertainty level about the percentage you have just chosen?


Scaling Option:
Automatic Scaling $\vee$


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[^1]:    ${ }^{1}$ While Jiao et al. (2020) utilize the idea of using belief distributions to simultaneously incorporate belief biases their model is unsuitable for probabilistic biases (relies on normally distributed beliefs), incorporates a reduced range of biases and is not tested in an experimental setting.
    ${ }^{2}$ This updating setting is comparable to the one employed in the ambiguity aversion literature (see Abdellaoui et al. (2021) as an example) or cognitive uncertainty (Enke and Graeber, 2023).

[^2]:    ${ }^{3}$ The idea of individual heterogeneity being a key determinant of biased belief updating has been recently discussed by Khaw et al. (2021) and Alós-Ferrer and Garagnani (2023); supporting the idea that while average reports might look almost Bayesian or noisy at the population level, an individual analysis reveals that there are different but systematic deviations among individuals.

[^3]:    ${ }^{4}$ The state-space set $\Omega$ can have two different interpretations. One could either consider an agent who is not uncertain about her own beliefs, or one who is indeed uncertain about her own beliefs (à la cognitive uncertainty (Enke and Graeber, 2023)). In the former case, the set $\Omega$ would specify the

[^4]:    ${ }^{7}$ That is, binary-state regressions where inference biases and base-rate biases are identified.

[^5]:    ${ }^{8}$ Asymmetries between successes and failures can also be present at the level of the prior. These distinctions are introduced in section 4, but left out of the main theoretical discussion. This is without loss of generality as the resulting equations are identical to equations (9) and (10) except for having separate $\rho$ and $\delta$ parameters for successes and failures. (see section 4.2)
    ${ }^{9} \mathrm{~A}$ discussion about the distinction between motivated and unmotivated confirmation bias can be found in Zhenxun (2023a)
    ${ }^{10}$ In equation (8), the signal mean $k / n$ is replaced by $(k \pm \varepsilon) / n$. This is because $p$ is not strictly defined at $p=0$ and $p=1$. Thus, one must take $(k+\varepsilon) / n$ as the inferior limit if $k=0$; and $(k-\varepsilon) / n$ if $k=n$.

[^6]:    ${ }^{11}$ see section 3.2 for details about the elicitation of beta distributions
    ${ }^{12}$ Both in the prior and posterior elicitations, participants need to update from a given default beta distribution. Before eliciting their prior, the default beta distribution they see is a uniform distribution (as this is the exogenously implemented "prior of the prior"). Accordingly, the default beta distribution they update before eliciting their posterior is their own prior report.

[^7]:    ${ }^{13}$ The probability density function of any beta distribution can also be parameterised by its expected value and variance.
    ${ }^{14}$ In order to participate in the experiment subjects must have answered 3 out of 5 comprehension questions correctly. If any comprehension question was answered incorrectly, participants had a second chance to answer every question correctly

[^8]:    ${ }^{15}$ Similarly to the baseline model, equations (14) and (15) do not have an intercept for the very same reason as outlined in section 4.1.

[^9]:    ${ }^{16}$ In the case of confidence biases, $V a r_{\text {Bias }}$ is already given by $\tilde{V a} r_{n}$, and $E_{\text {Bias }}$ would be unaffected.
    ${ }^{17}$ Because we are looking at the effect of specific biases for each subject, it is theoretically possible that the effect of the bias in isolation is so strong that it makes $a_{\text {bias }}$ or $b_{\text {bias }}$ negative. This would make $E_{\text {Bias }}$ and $V a r_{\text {Bias }}$ uninterpretable. Therefore, it is assumed that the maximum effect that a bias in isolation can have, is such that the associated parameter $a_{\text {bias }}$ or $b_{\text {bias }}$ is equal to $0+\varepsilon$. It is worth nothing that these cases were very rare, as it would require strongly updating against the information signal.

[^10]:    ${ }^{18}$ Additionally, comparing the adjusted $R^{2}$ column for the individual baseline and complete models in Table 4 further emphasises this point.
    ${ }^{19}$ With the only exception of the variance equation (Eq. (16)).

[^11]:    ${ }^{20}$ The column "against" classifies those individuals who update against the information signals.
    ${ }^{21}$ Because the individual-level analysis tests many null hypotheses (eleven per subject), one may wonder whether some of these results are driven by a high rate of false positives. To address this concern, section C. 2 of the appendix replicates Figures 4a and 4b for a $1 \%$ significance level.

[^12]:    ${ }^{22}$ In the experiment $A=1, B=99$ and $M=25 / 3$ cents for each report (that is a total maximum of 10 euros)

