Review of Automaton Learning Algorithms with Polynomial Complexity - Completely Solved Examples

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1 Introduction

Automaton learning is a domain in which the target system aka System Under Learning (SUL) is inferred by the automaton learning algorithm in the form of an automaton, by synthesizing a finite number of inputs and their corresponding outputs. Automaton learning makes use of a Minimally Adequate Teacher (MAT). The learner learns the SUL by posing membership queries to the MAT.

In the early stages, theoretical automaton learning has successful realworld applications. It goes beyond formal verification and allows to infer behavioral model of black-box systems. The improvements in tool support and raising competition focuses on the practicality of automaton learning and encouraging Learning-based Testing (LBT) techniques, which shows the growing interest in the field of research.

Learning-based testing [10] is an emerging paradigm of software testing. It is a heuristic iterative approach which is useful to automate specification based black box testing [9][7]. The LBT framework consists of a system under test (SUT), a formal specification for SUT and a learned model of SUT. A learning base testing algorithm works by executing test case inputs on the SUT. Most of the learning algorithms learn in the limit to yield a minimal approximation of the target DFA. This concept of learning in the limit for target DFA was f i rst introduced by E.M.Gold in 1967 [5]. In his paper, he showed that with the help of some inference or learning algorithm, a regular language corresponding to some target DFA can be guessed by a f i nite number of wrong hypothesis. With respect to learning type, there are three kinds of learning algorithms: *complete* learning algorithms, *incremental* learning algorithms and *sequential* learning algorithms.

In complete learning algorithms, initially, the system under learning (SUL) is completely learned by giving different inputs and receiving their corresponding outputs. After complete learning of the system, a hypothesis *H* is

generated [1]. While in incremental learning algorithms, system is learned in incremental form [13]. Initially, a small part (increment) of the system is learned by giving the inputs and receiving corresponding outputs after that, these input/output pairs are synthesized into a hypothesis H_i by the learning algorithm in an incremental fashion. The next step it learns a new increment (small part of the system) and rebuilds the hypothesis H_{i+1} which contains the previously and newly learned information. This process of learning can potentially continue till the complete learning of the system. On the other hand, sequential learning algorithm similarly works as an incremental learning algorithm except that it does not reuse the previously learned information from the hypothesis H_{s-1} to construct the new hypothesis H_s [8].

In automaton learning, there is a *learner* (learning algorithm) and an *adequate teacher* [2]. The Learner is a learning algorithm which learns the regular set from queries and counter-examples. The Learner asks queries to the adequate teacher. The adequate teacher answers the questions from the Learner, about the unknown regular set. It answers two types of questions: First type is a *membership query*, consisting of a string $t \in \Sigma^*$. The adequate teacher answers as *yes* or *no* depending on whether string *t* is a member of the unknown set or not. The second type of question is a *conjecture*, consisting of a description of a regular set *S*; the answer is *yes* if *S* is equal to the unknown language and is a string *t* in the symmetric difference of *S* and the unknown language otherwise. In the second case, the string *t* is called a counter-example because it serves to show that the conjectured set *S* is incorrect.

The concept of the adequate teacher was first introduced by Dana Angluin, in ID algorithm [1]. After that this concept was used by other researchers. As they found that if there is an adequate teacher, the complexity of automaton learning is polynomial whereas, in the absence of the adequate teacher automaton learning is an NP-hard problem [10].

2 Automaton Learning Algorithms

We have briefly studied some automaton learning algorithms with the help of examples, which are specifically relevant to table data structure and tree data structure. The learning algorithms having table data structure are L* proposed by Dana Angluin in 1987 [2], ID proposed by Dana Angluin in 1981 [1], DLIQ and BDLIQ proposed by Farah Haneef and Muddassar Azam Sindhu in 2022 and 2023 respectively in [15, 16], IID proposed by R. Parekh, C. Nichitiu and V. Honavar in 1998 [13], IDS proposed by K. Meinke and Muddassar Azam Sindhu in 2010 [11], IDLIQ proposed by Farah Haneef and Muddassar Azam Sindhu in [17] and IKL proposed by K. Meinke and Muddassar Azam Sindhu in 2011 [12]. The learning algorithms having tree data structure are RPNI proposed by Oncina and Garcia in 1992 [3] and RPNII proposed by Pierre Dupont in 1996 [4].

2.1 L* Algorithm

The L* is a complete learning algorithm proposed by Dana Angluin in 1987 [2]. It learns a regular set by asking membership and equivalence queries. A membership query tells whether a string α is a member of the language of DFA A or not, $\alpha \in L(A)$? An equivalence query determines whether a hypothesis DFA is a correct representation of regular set or not i.e. L(H) = L(A)? L* asks membership queries and organizes this information in form of a table consisting of a tuple (S, E, T) which is called Observation Table *OT*. Where $s_1, s_2, ..., s_n$ are row labels belonging to *S* and $e_1, e_2, ..., e_n$ are column labels belonging to *E*. Whereas *T* is a transition function (($S \cup S \cdot \Sigma$) X E). A simple observation table (*OT*) is shown in Table 2.1.1. This table has two parts. Upper part consists of set $S = \{s_1, s_2\}$ and lower part consists of concatenation of *S* and Σ having s_1 . a_1 and s_2 . a_2 elements, where $s_1, s_2 \in S$ and $a_1, a_2 \in \Sigma$ and Σ is a f i nite set of alphabets.

Function row(s) is a f i nite function which represents the tuple of entries in the observation table *OT* corresponding to row labeled as *s*.

Т	e_1	e ₂
S 1		
S 2		
<i>s</i> ₁ . <i>a</i> ₁		
S2.02		

Table 2.1.1: Observation Table (OT)

This table should meet two basic properties before asking equivalence queries to make conjecture. These properties are *closure* and *consistency*. The *OT* is called closed if and only if, for each string *t* in lower part of the table i.e. *S*. Σ , there exist an *s* in upper part of the table i.e. *S* such that row(t) = row(s). If *OT* is not closed then rows of the observation table are extended as *S* with prefixes of *S*. For *OT* to be consistent, it is necessary that if any two rows of upper part of *OT* are same as row(s1) = row(s2) then for all $a \in \Sigma$, $row(s_1, a) = row(s_2, a)$. If the observation table is closed but not consistent then column of observation table is extended with a symbol *a* where $a \in \Sigma$.

When *OT* is closed and consistent, a conjecture can be constructed. Distinct *rows(S)* show the different states (states described by concatenation of elements of set *E*) and column *T* represents the strings *t* which are used to show the transitions from one state to another. The initial state is the *row*(λ) and the final states are the entries in the table under the column λ with value = 1. We start from the initial state q_0 and check all transitions of input alphabet, existing in Column *T* as $\delta(row(s), a) = row(s. a)$. After reading all input symbols from each state, a conjecture is represented in the form of a table. Rows represent the states as q_0, q_1, \ldots, q_n and columns represent input symbols as 0, 1.

If this conjecture *H* is language equivalent to the target DFA, *A*, L(H) = L(A) then the adequate teacher answers the equivalence query as Yes otherwise, it answer as No and gives the counter example either from L(H) - L(A) or L(A) - L(H), which is then accommodated in *OT* in the form of extension of rows. This process continues until *OT* becomes consistent and closed.

Example

An example run of the L* algorithm is given below:

Unknown Regular Set: U = Even number of 0's except the empty string. Fixed known Finite Alphabets: $\Sigma = \{0, 1\}$

The initial observation table is shown in Table 2.1.2. It shows that $S = \{\lambda\}$ and $E = \{\lambda\}$. The corresponding table entries are called as: λ . $\lambda = \lambda$ which is not an accepting string so value = 0 is inserted in the corresponding

cell. $0.\lambda = 0$ which shows odd number of 0's which are not accepted so value = 0, and $1.\lambda = 1$ which shows zero number of 0's therefore the corresponding cell value = 0.

<i>T</i> ₁	λ
λ	0
0	0
1	0

Table 2.1.2: Initial Observation Table

As in upper part of λ column there is no cell having value 1 therefore this table shows that there is no final state in the initial hypothesis DFA. Initial *OT* is closed as *row* (0) = *row* (λ) and *row* (1) = *row* (λ). This *OT* is also consistent as no two rows in *S* are same. Now L* makes a conjecture shows that if we read 0 or 1 from *state*0 then it will remain to the same state as itself. Conjecture *H*₁ and respective automaton is shown in Fig.2.1.1: (a) and (b)

δ	0	1
a_0	a_0	a_0

Fig.2.1.1(a): Conjecture H_1

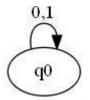


Fig.2.1.1(b): Automaton H₁

Above conjecture H_1 shows that there is no final state in it which makes its language inequivalent to the target DFA, *A*. Let H_1 gives counter-example as 00 which is an accepting string on target DFA but above automaton does not accept it. Therefore, we extend the rows of initial $OT T_1$ as 0, 00 in *S* and 01, 001, 000 in *S*. Σ . The extended observation table T_2 is shown below as Table 2.1.3.

<i>T</i> ₂	λ
λ	0
0	0
00	1
1	0
01	0
001	1
000	0

Table 2.1.3: Observation Table T_2

Table 2.1.3 shows that λ . $\lambda = \lambda$ so value = 0, $0.\lambda = 0$ so value = 0, $00.\lambda = 00$ which has even number of 0's so it is an accepting string and the value against it will be 1. Similarly, $1.\lambda = 1$ so value = 0, $01.\lambda = 01$ so value = 0, $000.\lambda = 000$ so value = 0, $001.\lambda = 001$ having two 0's so this string is also accepted and value against it will be 1. T_2 has $S = \{\lambda, 0, 00\}$ and $E = \{\lambda\}$

Observation table T_2 is closed as for every successor state in $S.\Sigma$, there is a row in S as row(s) = row(t) not consistent as $row(\lambda) = row(0)$ but row(0) is not equivalent to row(00). Therefore, we have to extend set E with 0. Observation table T_3 is shown below as Table 4 having $S = \{\lambda, 0, 00\}$ and $E = \{\lambda, 0\}$

<i>T</i> ₃	λ	0
λ	0	0
0	0	1
00	1	0
1	0	0
01	0	1
001	1	0
000		

Table 2.1.4: Observation Table *T*₃

In Table T_3 we have called the column 0 as: $\lambda .0 = 0$ as number of 0's is 1 so value=0, 0.0 = 00 as number of zeros are two so corresponding value = 1, 00.0 = 000 here number of zeros are three so value = 0 and similarly other values of column 0's are calculated for the lower part of the table.

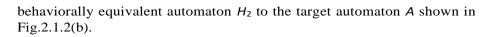
 T_3 is closed as well as consistent as $row(\lambda) = row(00)$ and row(1) = row(001) and row(0) = row(000). So, L* makes a conjecture H_2 which is shown in Fig.2.1.2(a). Rows against column λ and 0 shows the states of automaton as 00 = q_0 , 01 = q_2 , 10 = q_2 . whereas transitions are as follows.

If we read strings from column T_3 as 0 from the *state00* then we reach the *state01*. If we read 1 from *state00* will reach the *state00* (self-loop). If we read the string 0 from the *state01* we reach the *state10*. If we read the string 1 from the *state01* we reach the *state01*, if we read the string 0 from the *state01* (self-loop). Similarly, if we read the string 0 from the *state10*, we reach the *state01* and if we read the string 1 from the *state10*, we reach the *state10*.

δ	0	1
q_0	q_1	q_0
q_1	q 2	q_1
q ₂	q_1	q ₂

Fig.2.1.2(a): Conjecture H_2

As H_2 accepts all the strings having even number of 0's except the null string so the adequate teacher answers as Yes. L* terminates and gives a minimal



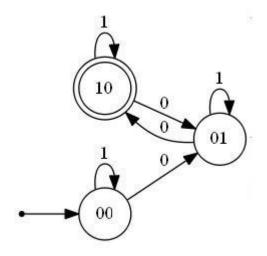


Fig.2.1.2(b): Automaton H_2

2.2 ID Algorithm

The ID algorithm is a complete learning algorithm proposed by Dana Angluin in 1981 [1]. It asks membership queries from the adequate teacher to learn the regular set. It uses the concept of live states and dead state. A state $q_1 \in Q$ is called a live state if there exists a string $\sigma_1, \sigma_2, \ldots, \sigma_n \in \Sigma^*$ such that $\delta^* (q_0, \sigma_1, \sigma_2, \ldots, \sigma_n) = q_i$ and $q_i \in F$ where *F* is a final state. The set of all live states is called live complete set denoted *P* and states which not live are called dead states. In isomorphic automata, there is only one dead state, d_0 . The set having live states as well as dead state is, denoted as P' that is $P = P \cup \{d_0\}$. The ID algorithm partitions the set T into blocks of accepting and non-accepting

strings, for this purpose it uses the concept of distinguishing strings *V*. *T* is a set having all live states as well as their concatenation with input alphabet β such as $T = P' \cup \{f(\alpha, \beta) \mid (\alpha, \beta) \in P \times \Sigma\}$ where $\alpha \in P'$ and $\beta \in \Sigma$.

To f i n d the blocks of accepting and nonaccepting states, the ID constructs a table. The first row of table shows the number of iterations, through which the set *T* is partitioned into accepting and nonaccepting blocks. The second row of table shows the set of distinguishing strings v_1, v_2, \ldots, v_n where $v_1, v_2, \ldots, v_n \in V$. First column of table shows the elements of the set *T* with transition function *E* where $E_i(\alpha) = \{v_j | v_j \in V, 0 \le j \le i, \alpha v_j \in L(A) ?\}$ and L(A) is the language of target DFA.

i	0	1	2
Vi	λ	а	b
$E(d_0)$	φ	φ	φ
$E(\lambda)$			
E(a)			
E(b)			
E(aa)			
E(ab)			
E(ba)			
E(bb)			

Table 2.2.1: Structure of Table

The purpose of distinguishing strings is to identify states, having same behavior For some particular string, $\alpha \in \Sigma^*$ but have different behavior for a suffix $\sigma \in \Sigma$.

Table 2.2.1 shows that two iterations have been completed to reach final partition of blocks and the second row shows that $V = \{\lambda, \alpha, b\}$. From third row we can see that any transition from dead state is always dead, $E_i(d_0) = \varphi$. In first iteration E_0 when $v_0 = \lambda$, $E(d_0) = \varphi$ and $E_0(\alpha) = \lambda$ when $\alpha \in L(A)$. Otherwise $E_0(\alpha) = \varphi$.

When rst iteration becomes complete, ID searches for a pair, such that $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) /= E_i(f(\beta, \sigma))$ whereas $\alpha, \beta \in P$ and $\sigma \in \Sigma$. This expression shows that if the ID algorithm nds a pair from set P' which shows the same behavior i.e. either both $E_i(\alpha)$ and $E_i(\beta)$ lie in the accepting block or both lie in rejecting block and when we concatenate α and β with some alphabet σ from the input set Σ then their behavior changes i.e. one lie in accepting block and other lie in rejecting block. This can give a potential distinguishing string. Then ID chooses some string $\gamma \in E_i(f(\alpha, \sigma)) \oplus E_i(f(\beta, \sigma))$ and a new distinguishing string is defined as $\sigma\gamma$. The ID performs next iteration i + 1 to further split the blocks, by reading distinguishing string $\sigma\gamma \in \Sigma$ from all elements of $E_i(\alpha)$. For this, it asks membership queries as $\alpha v_{i+1} \in L(A)$?, if the adequate teacher answers as Yes then $E_i(\alpha)$ becomes $E_{i-1}(\alpha)$.

If the ID algorithm f i nds no such pair that is $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma))$ /= $E_i(f(\beta, \sigma))$ then it constructs the hypothesis DFA, *H* which is isomorphic to the target DFA, *A*.

In hypothesis DFA, H, $E_i(\alpha)$ represents the states where $\alpha \in T$. $E(\lambda)$ is initial state and $E_i(\alpha)$ where $\alpha \in T$ and $\lambda \in E_i(\alpha)$ are f i nal states. The transition relation δ is constructed as $E_i(\alpha) = \varphi$ then self-loop to that state otherwise $\delta(E_i(\alpha), \sigma) = E_i(f(\alpha, \sigma))$.

Example

An example run of the ID algorithm is given below:

Target *DFA*: *A*= Consecutive even number of a's and all b's. $(b^*(aa)^*b^*)$ Input alphabet: $\Sigma = \{a, b\}$

 $P_0 = \{\lambda, a\}$ and $P_0' = \{d_0, \lambda, a\}$ and T_0' becomes as $T_0' = \{d_0, \lambda, a, b, aa, ab\}$

i	0
Vi	λ
$E(d_0)$	φ
Ε(λ)	$\{\lambda\}$
E(a)	φ
E(b)	$\{\lambda\}$
E(aa)	$\{\lambda\}$
E(ab)	φ

Table 2.2.2: Initial Table

Table 2.2.2 shows that distinguishing string set $V = \{\lambda\}$. The ID algorithm asks membership queries for all strings belong to *T* as $av_{i+1} \in L(A)$. The adequate teacher answers as Yes for $E(\lambda)$, E(b), E(aa) as these strings lead to the accepting states so $E_i(\alpha)$ becomes $\{\lambda\}$ and for all others, those are not leading to accepting states, adequate teacher answers as No so they set to φ .

Table 2.2.2 shows that $E(d_0) = E(a)$ but $E(d_0, \alpha) \neq E(a, a)$ therefore we can take a distinguishing string $\sigma \gamma$ as a'' in next iteration. The extended table is given in Table 2.2.3

i	0	1
Vi	λ	а
$E(d_0)$	φ	φ
Ε(λ)	$\{\lambda\}$	$\{\lambda\}$
E(a)	φ	{ a }
E(b)	$\{\lambda\}$	$\{\lambda\}$
E(aa)	$\{\lambda\}$	$\{\lambda\}$
E(ab)	φ	φ

Table 2.2.3: For Distinguishing String a

Table 2.2.3 shows that distinguishing string set $V = \{\lambda, \alpha\}$. The ID algorithm asks membership queries for all strings belong to *T* as $\alpha v_{i+1} \in L(A)$. The adequate teacher answers as Yes for $E(\lambda)$, $E(\alpha)$, E(b), $E(\alpha\alpha)$ as these strings lead to the accepting states so $E_i(\alpha)$ becomes $E_{i-1}(\alpha) \cup \{v_i\}$ and for all others, those are not leading to accepting states, adequate teacher answers as No so they set to φ .

Table 2.2.3 shows that $E(\lambda) = E(a)$ but $E(\lambda, b)$ not equal to E(a, b) therefore we can take a distinguishing string $\sigma \gamma$ as b'' in next iteration. The extended table is given in Table 2.2.4.

i	0	1	2
Vi	λ	а	b
$E(d_0)$	φ	φ	φ
Ε(λ)	$\{\lambda\}$	$\{\lambda\}$	$\{\lambda, b\}$
E(a)	φ	{ <i>a</i> }	{ a }
E(b)	$\{\lambda\}$	$\{\lambda\}$	$\{\lambda, b\}$
E(aa)	$\{\lambda\}$	$\{\lambda\}$	$\{\lambda, b\}$
E(ab)	φ	φ	φ

Table 2.2.4: For Distinguishing String b

Table 2.2.4 shows that distinguishing string set $V = \{\lambda, \alpha, b\}$. The ID algorithm asks membership queries for all strings belong to T as $\alpha v_{i+1} \in L(A)$. The adequate teacher answers as Yes for $E(\lambda)$, $E(\alpha)$, E(b), $E(\alpha\alpha)$ as these strings lead to the accepting states so $E_i(\alpha)$ becomes $E_{i-1}(\alpha) \cup \{v_i\}$ and for all others, those are not leading to accepting states, adequate teacher answers as No so they set to φ .

Table 2.2.4 shows that there is no pair such that $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) /= E_i(f(\beta, \sigma))$ so blocks will not be further partitioned. The hypothesis DFA, *H* is given below in Figure 2.2.1.

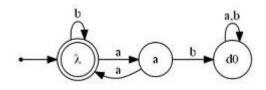


Figure 2.2.1: Hypothesis H

As we can see that this automaton is behaviorally equivalent to the target DFA, A i.e. L(H) = L(A) therefore the ID algorithm terminates.

2.3 IID Algorithm

The IID is an incremental extension of the ID algorithm [13]. It works similar to the ID algorithm except that it does not require the availability of live complete set at the start of inference procedure. The IID incrementally builds the live state set and its corresponding automata to provided labeled examples, those are taken as input by this algorithm. A labeled example is a pair (α , *label*(α)) where $\alpha \in \Sigma^*$ and *label*(α) shows that whether an equivalence query α is accepted or rejected by the adequate teacher. If $\alpha \in L(A)$ then it is called a positive example and if $\alpha \notin L(A)$ then it is called a negative example. Initial hypothesis DFA, H_0 consists of only one state that is the dead state. When the first positive example is seen then H_0 is updated and after that for each additional labeled example (α , *label*(α)), it is determined that whether it is consistent with our previous hypothesis DFA or we have to update it according to a new labeled example.

Like ID algorithm, IID has a set of live states *P* that is initially empty. *P'* is $P' = P \cup \{d_0\}$ where d_0 is a dead state. *T'* is a set having all live states as well as their concatenation with input alphabets \mathcal{B} such that $T' = P' \cup \{f(\alpha, \beta) \mid (\alpha, \beta) \in P' \times \Sigma\}$ whereas $\alpha \in P'$ and $\mathcal{B} \in \Sigma$. IID algorithm splits the set *T* into blocks of accepting and nonaccepting states and for this, it uses the concept of distinguishing strings, denoted by *V* like ID algorithm. The purpose of distinguishing strings is to identify states, having same behavior for some particular string, $\alpha \in \Sigma^*$ but have different behavior for a suffix $\sigma \in \Sigma$. Like the ID algorithm, to nd the blocks of accepting and nonaccepting states, the IID also constructs a table. The f i rst row of table shows the number

of iterations, through which the set *T* is partitioned into accepting and nonaccepting blocks. The second row of table shows the set of distinguishing strings $v_1, v_2, ..., v_n$ where $v_1, v_2, ..., v_n \in V$. First column of table shows the elements of the set *T* with transition function *E* where $E_i(\alpha) = \{v_j | v_j \in V, 0 \le j \le i, \alpha v_j \in L(A)\}$ and L(A) is the language of target DFA.

When the f i rst positive example arrives, the IID algorithm constructs the sets P, T and corresponding table as in the ID algorithm. In the f i rst iteration, the

function E_0 when $v_0 = \lambda$, $E_0(d_0) = \varphi$ and $E_0(\alpha) = \lambda$ when $\alpha \in L(A)$, otherwise $E_0(\alpha) = \varphi$. After that IID searches for a pair such that $E_i(\alpha) = E_i(\beta)$ but $E_i(\beta)$ $f(\alpha, \sigma)$ /= $E_i(f(\beta, \sigma))$ whereas $\alpha, \beta \in P$ and $\sigma \in \Sigma$. This expression shows that if IID algorithm f i nds a pair from set P' which shows the same behavior i.e. either both $E_i(\alpha)$ and $E_i(\beta)$ lie in the accepting block or both lie in rejecting block and when we concatenate α and β with some alphabet σ from the input set Σ then their behavior may change i.e. one lie in accepting block and other lie in rejecting block. This can give a potential distinguishing string. Then IID non-deterministically chooses some string $\gamma \in E_i(f(\alpha, \sigma)) \oplus E_i(f(\theta, \sigma))$ and the new distinguishing string is defined as σy . IID performs next iteration *i* + 1 to further split the blocks, by reading distinguishing string $\sigma \gamma \in \Sigma$ from all elements of $Ei(\alpha)$. For this it asks membership queries as $\alpha v_{i+1} \in L(A)$, if adequate teacher answers as Yes then $E_i(\alpha)$ becomes $E_{i-1}(\alpha) \cup \{v_i\}$, and if it replies as No then $E_i(\alpha)$ is set to $E_{i-1}(\alpha)$. If IID finds no such pair that is $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) = E_i(f(\beta, \sigma))$ then it construct the hypothesis DFA, H_m . If H_m becomes behaviorally equal to the target DFA, A then IID terminates otherwise, it waits for another positive example and above process repeats until hypothesis DFA, H becomes equivalent to the target DFA, A.

Example

An example run of the IID algorithm is given below:

Target *DFA*: A= Consecutive even number of a's and all b's. ($b^*(aa)^*b^*$) Input alphabet: $\Sigma = \{a, b\}$

Initial null automata H_0 is given below in Figure 2.3.1.

 $P_0 = \{ \}$ and $P_0' = \{ d_0 \}$ and T_0' becomes as $T_0' = \{ d_0, \lambda, a, b \}$

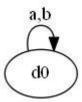


Figure 2.3.1: Null Hypothesis

Suppose the rst labeled example is (a, -), as it is negative example and is consistent with H_0 so H_0 does not change.

Suppose next labeled example is (b, +) as it is a first positive example so set $P_1 = \{\lambda, b\}, P_1 = \{d_0, \lambda, b\}$ and $T_1 = \{d_0, \lambda, a, b, ba, bb\}$ For k = 0

i	0
vi	λ
E(do)	φ
$E(\lambda)$	$\{\lambda\}$
E(a)	φ
E(b)	$\{\lambda\}$
E(ba)	φ
E(bb)	$\{\lambda\}$

Table 2.3.1: For Distinguishing String λ

Table 2.3.1 shows that distinguishing string set $V = \{\lambda\}$. The IID algorithm asks membership queries for all strings belong to *T* as $av_{i+1} \in L(A)$. The adequate teacher answers as Yes for $E(\lambda)$, E(b), E(bb) as these strings lead to the accepting states so $E_i(\alpha)$ becomes $\{\lambda\}$ and for all others, those are not leading to accepting states, adequate teacher replies as No so they set to φ .

Table 2.3.1 shows that there is no pair such that $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) / = E_i(f(\beta, \sigma))$ so blocks will not be further partitioned. The hypothesis DFA, H_1 for this iteration is given below in Figure 2.3.2.

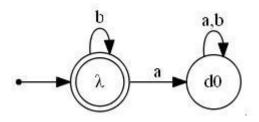


Figure 2.3.2: Hypothesis H_1

As automaton H_1 represented in Figure 2.3.2 is not equal to the target DFA, A therefore the IID algorithm learns the target DFA with more labeled example.

Suppose next input labeled example is (*aa*, +)

Now $P_2 = \{\lambda, a, b, aa\}$

 $P_{2}' = \{ d_{0}, \lambda, a, b, aa \}$

 $T_{1}^{\prime} = \{ d_{0}, \lambda, a, b, aa, bb, ab, ba, aaa, aab \}$

k = 1

i	0	1
Vi	λ	а
$E(d_0)$	φ	φ
<i>Ε</i> (λ)	$\{\lambda\}$	$\{\lambda\}$
E(a)	φ	{ <i>a</i> }
E(b)	$\{\lambda\}$	$\{\lambda\}$
E(aa)	$\{\lambda\}$	$\{\lambda\}$
E(ab)	φ	φ
E(ba)	φ	{ <i>a</i> }
E(bb)	$\{\lambda\}$	$\{\lambda\}$
E(aaa)	φ	{ a }
E(aab)	$\{\lambda\}$	$\{\lambda\}$

Table 2.3.2: For Distinguishing String *a*

In Table 2.3.2, the column λ shows that as $E(d_0) = \varphi = E(a)$ but $E(d_0.a) \neq E(a, a)$ so the IID algorithm partitions the accepting and nonaccepting blocks by using distinguishing string $\sigma \gamma = a$ shown in column a of Table 2.3.2.

Table 2.3.2 shows that distinguishing string set $V = \{\lambda, \alpha\}$. The IID asks membership queries for all strings belong to *T* as $\alpha v_{i+1} \in L(A)$. The adequate teacher answers as Yes for $E(\alpha)$, $E(b\alpha)$ and $E(\alpha\alpha\alpha)$ as these strings lead to the accepting states so $E_i(\alpha)$ becomes $E_{i-1}(\alpha) \cup \{v_i\}$ and for all others, those are not leading to accepting states, adequate teacher replies as No so they set to $E_{i-1}(\alpha)$.

Table 2.3.2 shows that there is no pair such that $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) /= E_i (f(\beta, \sigma))$ so blocks will not be further partitioned. Hypothesis DFA, H_2 is given below in Figure 2.3.3.

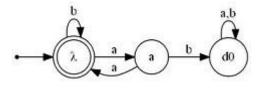


Figure 2.3.3: Hypothesis H₂

As above hypothesis DFA, H_2 is behaviorally equivalent to the target DFA, A i.e. $L(H_2) = L(A)$ therefore the IID algorithm terminates.

2.4 IDS Algorithm

The IDS algorithm is also an incremental extension of ID algorithm [11]. Like IID algorithm, it also does not require the availability of live complete set at the start of inference procedure. IDS incrementally build the live state set and its corresponding automata regarding to provide input labeled examples. A labeled example consists of two parts such as $(\alpha, label(\alpha))$ whereas $\alpha \in \Sigma^*$ and $label(\alpha)$ shows that whether an equivalence query α is accepted or rejected by the adequate teacher. A $label(\alpha)$ is valued as accepted if $\alpha \in L(A)$ and $label(\alpha)$ is valued as rejected if $\alpha \notin L(A)$. If $\alpha \in L(A)$ then it is called positive example and if $\alpha \notin L(A)$ then it is called negative example. Initial hypothesis *DFA*, H_0 consists of only one state (initial state) and all its input alphabet transitions

 $\beta \in \Sigma$. When a first labeled example (either positive or negative) arrives, then H_0 is updated and after that for each additional labeled example (α , *label*(α)), it is determined that whether it is consistent with our previous hypothesis *DFA* or we have to update it according to new labeled example.

Like IID algorithm [13], the IDS algorithm has also a set *P* that is initially as $P = \{\lambda\}$ and $P = P \cup \{d_0\}$, where d_0 is a dead state. *T* is a set having all states

as well as their concatenation with input alphabet β such that $T = P \cup \{f(\alpha, \beta) \mid (\alpha, \beta) \in P \times \Sigma\}$ where $\alpha \in P$ and $\beta \in \Sigma$ (for prefix closed). The IDS

algorithm partitions the set T into the blocks of accepting and nonaccepting states and for this, it uses the concept of distinguishing strings V like the ID and IID algorithms. The purpose of distinguishing strings is to identify a string, having same behavior for some particular string, $\alpha \in \Sigma^*$ but have different behavior for a suffix $\sigma \in \Sigma$.

Like ID and IID algorithms, to f i nd the blocks of accepting and nonaccepting states, the IDS algorithm also constructs a table. The first row of table shows

the number of iterations, through which set *T* is partitioned into accepting and nonaccepting blocks. The second row of table shows the set of distinguishing strings $v_1, v_2, ..., v_n$ where $v_1, v_2, ..., v_n \in V$. First column of table shows the elements of the set *T* with transition function *E* where $E_i(\alpha) = \{v_j | v_j \in V, 0 \le j \le i, \alpha v_j \in L(A)\}$ and L(A) is the language of target DFA, *A*.

When IDS receives a first labeled example, it constructs the set P, P, T and corresponding table like in ID algorithm. In first iteration E_0 when $v_0 = \lambda$, $E(d_0) = \varphi$ and $E_0(\alpha) = \lambda$ when $\alpha \in L(A)$. Otherwise $E_0(\alpha) = \varphi$. After that IDS searches for a pair such that $E_i(\alpha) = E_i(\beta)$ but E_i ($f(\alpha, \sigma)$) $\neq E_i$ ($f(\beta, \sigma)$) whereas $\alpha, \beta \in P$ and $\sigma \in \Sigma$. This expression shows that if IDS

finds a pair from set P which shows the same behavior i.e. either both $E_i(\alpha)$ and $E_i(\beta)$ lie in accepting block or both lie in rejecting block and when we concatenate $E_i(\alpha)$ and $E_i(\beta)$ with some alphabet σ from the input set Σ then their behavior changes i.e. one lie in accepting block and other lie in rejecting block. Then the IDS algorithm chooses some string $\gamma \in E_i(f(\alpha, \sigma)) \bigoplus E_i(f(\beta, \sigma))$ and a new distinguishing string is defined as $\sigma\gamma$. IDS perform next iteration i + 1to further split the blocks, by reading distinguishing string $\sigma\gamma \in \Sigma$ from all elements of $E_i(\alpha)$. For this, it asks membership queries as $\alpha v_i + 1$

 $\in L(A)$? if the adequate teacher answers as Yes then $E_i(\alpha)$ becomes $E_{i-1}(\alpha) \cup \{v_i\}$, if the adequate teacher answers as No then $E_i(\alpha)$ is set to $E_{i-1}(\alpha)$. If the IDS f i nds no such pair that is $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) /= E_i(f(\beta, \sigma))$ then it constructs the hypothesis DFA H_m . If H_m is equivalent to the target DFA, A then the IDS algorithm stops its execution otherwise it waits for another labeled example. Above process repeats until hypothesis DFA, H becomes equivalent to the target DFA, A.

This algorithm has two versions. One is prefix closed like L*, ID, IID and other one is prefix free. The main difference between these two is; in prefix free version, set P contains the strings gained from the labeled examples without their prefixes whereas, in prefix closed version, set P contains the strings gained from labeled examples as well as their prefixes.

Example

Prefix Closed

An example run of prefix closed version of IDS algorithm is given below: Input alphabet: $\Sigma = \{a, b\}$

Target DFA: A= Consecutive even number of a's and all b's. $(b^*(aa)^*b^*)$

Initially $P_0 = \{\lambda\}, P'_0 = \{d_0, \lambda\}$

 $T_0' = \{ do, \lambda, a, b \}$

Initial null automata H_0 is given below in Figure 2.4.1.

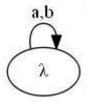


Figure 2.4.1: Null Automaton H₀

Suppose the first labeled example is (a, -), as this example shows that transition *a* from initial state λ leads to the dead state so it is not consistent with H_0 . Here $P_1 = \{\lambda, a\}, P' = \{d_0, \lambda, a\}$ and $T' = \{d_0, \lambda, a, b, aa, ab\}$ so Table

2.4.1 is given below:

i	0			
Vi	λ			
$E(d_0)$	φ			
14				

$E(\lambda)$	$\{\lambda\}$
E(a)	φ
E(b)	$\{\lambda\}$
E(aa)	φ
E(ab)	φ

Table 2.4.1: For Labeled Example (a, -)

Table 2.4.1 shows that distinguishing string set $V = \{\lambda\}$ and IDS asks membership queries for all strings belong to T as $av_{i+1} \in L(A)$. The adequate teacher answers Yes, for $E(\lambda)$ and E(b) as these strings lead to the accepting states so $E_0(\alpha)$ becomes $\{\lambda\}$ and for all others, those are not leading to accepting states, adequate teacher replies No so they are set to φ .

Table 2.4.1 shows that there is no pair such that $Ei(\alpha) = Ei(\beta)$ but $Ei(f(\alpha, \sigma)) /= Ei(f(\beta, \sigma))$ so blocks will not be further partitioned. Hypothesis *DFA*, *H*₁ is given below in Figure 2.4.2.

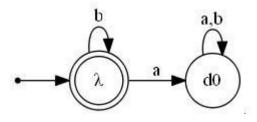


Figure 2.4.2: Hypothesis H_1

Suppose next labeled example is (ab, -) so $P_2 = \{\lambda, a, ab\}$ and $P_2' = \{d_0, \lambda, a, ab\}$ and T'_2 becomes as $T'_2 = \{do, \lambda, a, b, aa, ab, aba, abb\}$

The corresponding table for this iteration is given below:

i	0
vi	Λ
E(do)	Φ
$E(\lambda)$	$\{\lambda\}$
E(a)	Φ
E(b)	$\{\lambda\}$
E(aa)	Φ
E(ab)	Φ
E(aba)	Φ
E(abb)	Φ

Table 2.4.2: For Labeled Example (ab, -)

Table 2.4.2 shows that distinguishing string set $V = \{\lambda\}$ and IDS asks membership queries for all strings belong to T as $av_{i+1} \in L(A)$. The adequate teacher answers Yes for $E(\lambda)$ and E(b) as these strings lead to the accepting states so $E_0(\alpha)$ becomes $\{\lambda\}$ and for all others, those are not leading to accepting states, adequate teacher replies No so they are set to φ .

Table 2.4.2 shows that there is no pair such that $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha), \beta)$ σ)) /= $E_i(f(\theta, \sigma))$ so blocks will not be further refined. Hypothesis DFA, H_2 is given below in Figure 2.4.3

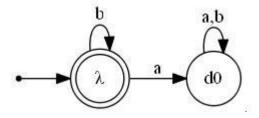


Figure 2.4.3: Hypothesis H₂

Suppose next labeled example is (b,+) so $P_3 = \{\lambda, a, b, ab\}$ and $P'_3 = \{d_0, \lambda, a, b, ab\}$ and T' becomes as $T' = \{do, \lambda, a, b, aa, ab, ba, bb, aba, abb\}$ The corresponding table for the current iteration is given below:

i	0
vi	λ
E(do)	φ
$E(\lambda)$	$\{\lambda\}$
E(a)	φ
E(b)	$\{\lambda\}$
E(aa)	φ
E(ab)	φ
E(ba)	φ
E(bb)	$\{\lambda\}$
E(aba)	φ
E(abb)	φ

Table 2.4.3: For Labeled Example (b, +)

Table 2.4.3 shows that distinguishing string set $V = \{\lambda\}$ and IDS asks membership queries for all strings belong to T as $\alpha v_{i+1} \in L(A)$. The adequate teacher answers Yes for $E(\lambda)$, E(b) and E(bb) as these strings lead to the accepting states so $E_0(\alpha)$ becomes $\{\lambda\}$ and for all others, those are not leading to accepting states, adequate teacher replies No so they are set to φ .

Table 2.4.3 shows that there is no pair such that $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) /= E_i(f(\beta, \sigma))$ so blocks will not be further partitioned. Hypothesis *DFA*, *H*₃ is given below in Figure 2.4.4.

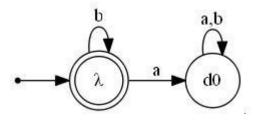


Figure 2.4.4: Hypothesis H_3

Suppose next labeled example is (aa, +)Now $P_4 = \{\lambda, a, b, aa, ab\}$ $P_4' = \{do, \lambda, a, b, aa, ab\}$ $T'_4 = \{do, \lambda, a, b, aa, bb, ab, ba, aaa, aab, aba, abb\}$ therefore Table 2.4.4 is given below:

i	0	1
vi	λ	а
E(do)	φ	φ
<i>Ε</i> (λ)	$\{\lambda\}$	$\{\lambda\}$
E(a)	φ	{ <i>a</i> }
E(b)	$\{\lambda\}$	$\{\lambda\}$
E(aa)	$\{\lambda\}$	$\{\lambda\}$
E(ab)	φ	φ
E(ba)	φ	{ <i>a</i> }
E(bb)	$\{\lambda\}$	$\{\lambda\}$
E(aaa)	φ	{ <i>a</i> }
E(aab)	$\{\lambda\}$	$\{\lambda\}$
E(aba)	φ	φ
E(abb)	φ	φ

Table 2.4.4: For Labeled Example (aa, +)

In Table 2.4.4, the column λ shows that as $E(d_0) = \varphi = E(a)$ but $E(d_0.a) \neq E(a.a)$ hence the IDS algorithm partitions the accepting and nonaccepting blocks by using distinguishing string $\sigma \gamma = a$ shown in column a of Table 2.4.4.

Table 2.4.4 shows that distinguishing string set $V = \{\lambda, \alpha\}$. The IDS asks membership queries for all strings belong to T as $av_{i+1} \in L(A)$. The adequate teacher answers Yes for $E(\alpha)$, $E(b\alpha)$ and $E(\alpha\alpha\alpha)$ as these strings lead to the accepting states so $E_i(\alpha)$ becomes $E_{i-1}(\alpha) \cup \{v_i\}$ and for all others, those are not leading to accepting states, adequate teacher replies No so they are set to $E_{i-1}(\alpha)$.

Table 2.4.4 shows that there is no pair such that $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) / = E_i(f(\beta, \sigma))$ so blocks will not be further partitioned. Hypothesis DFA, H_4 is given below in Figure 2.4.5.

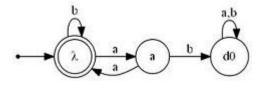


Figure 2.4.5: Hypothesis H₄

As the above hypothesis *DFA*, H_4 is behaviorally equivalent to the target DFA, *A* i.e. $L(H_4) = L(A)$ therefore the IDS algorithm terminates.

Prefix Free

An example run of prefix free version of IDS algorithm is given below:

Target *DFA*: A= Consecutive even number of a's and all b's. ($b^*(aa)^*b^*$) Input alphabet: $\Sigma = \{a, b\}$

Initially $P_0 = \{\lambda\}, P'_0 = \{d_0, \lambda\}$ $T'_0 = \{d_0, \lambda, a, b\}$

Initial null automata H_0 is given below in Figure 2.4.6.

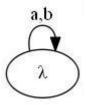


Figure 2.4.6: Null Hypothesis H₀

Suppose the first labeled example is (ab, -) therefore $P_{1} = \{\lambda, ab\}, P'_{1} = \{d_{0}, \lambda, ab\}$ and $T'_{1} = \{d_{0}, \lambda, a, b, ab, aba, abb\}$ so

corresponding Table 2.4.5 is given below:

i	0
vi	λ
$E(d_0)$	φ
$E(\lambda)$	$\{\lambda\}$
E(a)	φ
E(b)	$\{\lambda\}$
E(ab)	φ
E(aba)	φ
E(abb)	φ

Table 2.4.5: For Labeled Example (*ab*, -)

Table 2.4.5 shows that distinguishing string set $V = \{\lambda\}$ and IDS asks membership queries for all strings belong to *T* as $av_{i+1} \in L(A)$. The adequate teacher answers Yes for $E(\lambda)$ and E(b) as these strings lead to the accepting states so $E_0(\alpha)$ becomes $\{\lambda\}$ and for all others, those are not leading to accepting states, adequate teacher replies No so they are set to φ .

Table 2.4.5 shows that there is no pair such that $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) /= E_i(f(\beta, \sigma))$ so blocks will not be further partitioned. Hypothesis *DFA*, H_1 is given below in Figure 2.4.7.

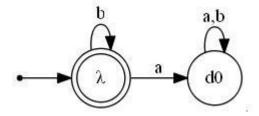


Figure 2.4.7: Hypothesis H₁

Suppose next labeled example is (b, +) so $P_2 = \{\lambda, b, ab\}$ and $P_2' = \{d_0, \lambda, b, ab\}$ and T_2' becomes as $T_2' = \{d_0, \lambda, a, b, ab, ba, bb, aba, abb\}$

The corresponding table for the current iteration is given below:

i	0
vi	λ
E(do)	φ
$E(\lambda)$	$\{\lambda\}$
E(a)	φ
E(b)	$\{\lambda\}$
E(ab)	φ
E(ba)	φ
E(bb)	$\{\lambda\}$
E(aba)	φ
E(abb)	φ

Table 2.4.6: For Labeled Example (*b*, +)

Table 2.4.6 shows that distinguishing string set $V = \{\lambda\}$ and IDS asks membership queries for all strings belong to *T* as $av_{i+1} \in L(A)$. The adequate teacher answers Yes for $E(\lambda)$, E(b) and E(bb) as these strings lead to the accepting states so $E_0(\alpha)$ becomes $\{\lambda\}$ and for all others, those are not leading to accepting states, adequate teacher replies No so they are set to φ .

Table 2.4.6 shows that there is no pair such that $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) /= E_i(f(\beta, \sigma))$ so blocks will not be further partitioned. Hypothesis DFA, H_2 is given below in Figure 2.4.8.

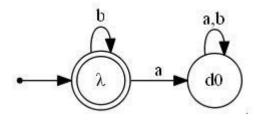


Figure 2.4.8: Hypothesis H₂

Suppose next labeled example is (aa, +)Now $P_3 = \{\lambda, b, aa, ab\}$ $P_3' = \{d_0, \lambda, b, aa, ab\}$ $T'_3 = \{d_0, \lambda, a, b, aa, bb, ab, ba, aaa, aab, aba, abb\}$ therefore Table 2.4.7 is given below:

i	0	1
vi	λ	а
E(do)	φ	φ
Ε(λ)	$\{\lambda\}$	$\{\lambda\}$
E(a)	φ	{ <i>a</i> }
E(b)	$\{\lambda\}$	$\{\lambda\}$
E(aa)	$\{\lambda\}$	$\{\lambda\}$
E(ab)	φ	φ
E(ba)	φ	{ <i>a</i> }
E(bb)	$\{\lambda\}$	$\{\lambda\}$
E(aaa)	φ	{ <i>a</i> }
E(aab)	$\{\lambda\}$	$\{\lambda\}$
E(aba)	φ	φ
E(abb)	φ	φ

Table 2.4.7: For Labeled Example (aa, +)

In Table 2.4.7, the column λ shows that as $E(a) = \varphi = E(ab)$ but $E(a, a) \neq E(ab, a)$ so the IDS algorithm partitions the accepting and nonaccepting blocks by using distinguishing string $\sigma \gamma = a$ shown in column a of table 2.4.7.

Table 2.4.7 shows that distinguishing string set $V = \{\lambda, \alpha\}$. The IDS asks membership queries for all strings belong to *T* as $\alpha v_{i+1} \in L(A)$. The adequate teacher answers Yes for $E(\alpha)$, $E(b\alpha)$ and $E(\alpha\alpha\alpha)$ as these strings lead to the accepting states so $E_i(\alpha)$ becomes $E_{i-1}(\alpha) \cup \{v_i\}$ and for all others, those are not leading to accepting states, adequate teacher replies No so they are set to $E_{i-1}(\alpha)$.

Table 2.4.7 shows that there is no pair such that $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) /= E_i (f(\beta, \sigma))$ so blocks will not be further partitioned. Hypothesis DFA, H_3 is given below in Figure 2.4.9.

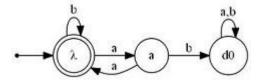


Figure 2.4.9: Hypothesis H₃

As above hypothesis DFA, H_3 is behaviorally equivalent to the target DFA, A i.e. $L(H_3) = L(A)$ therefore the IDS algorithm terminates.

2.5 IKL Algorithm

IKL is an incremental extension of the ID algorithm like the IDS algorithm but the major difference is, the IID algorithm is one bit whereas the IKL is a multibit extension [12]. The IKL learns deterministic Kripke structures multi-bit moore machine having k-bit outputs. The IKL algorithm uses two basic ideas. One is bit-slicing i.e. converting k-bits Kripke structure into k 1-bit Kripke structures having 1-bit output which is given below in Figure 2.51. Second concept used is of *partition refinement* which is similar to the consistency maintanence of the ID algorithm but difference is, the IKL algorithm uses the concept of lazy partition refinement.

Like IDS algorithm, the IKL algorithm has also a set P that is initially as P_0 $= \{\varepsilon\}$ and $P'_0 = P_0 \cup \{d_0\}$, where d_0 is a dead state. T' is a set having all states as well as their concatenation with input alphabet $\boldsymbol{\beta}$ such that $T_k = T_{k-1} \cup \boldsymbol{P}'$ $\cup \{(\alpha, \beta) \mid (\alpha \in P_k - P_{k-1}, \beta \in \Sigma\}$ for prex closure. The IKL algorithm partitions the set T' into the blocks of accepting and nonaccepting states and for this, it uses the concept of distinguishing strings V like the ID and IID algorithms. The purpose of distinguishing strings is to identify a state, having same behavior for some particular string, $\alpha \in \Sigma^*$ but have different behavior for a suffix $\sigma \in \Sigma$. Like ID and IDS algorithms, to find the blocks of accepting and nonaccepting states, the IDS algorithm also constructs a table. The rst row of table shows the number of iterations, through which set T' is partitioned into accepting and nonaccepting blocks. The second row of table shows the set of distinguishing strings v_1, v_2, \ldots, v_n where $v_1, v_2, \ldots, v_n \in V$. First column of table shows the elements of the set T' with transition function *E* where $E^c(\alpha) = \{v_i \mid v_i \in V, 0 \le i \le i, \alpha v_i \in L(A)\}$? and L(A) is the language of target DFA, A.

In the IKL algorithm, the target automaton A is initially converted into k 1-bit automata i.e. B_1 , B_2 ... B_n by bit slicing. After that all 1-bit automata are incrementally learned and then the IKL algorithm finds the product of all 1-bit automata B_1 , B_2 ... B_n to convert all 1-bit Kripke structures into k-bit target automata A.

The IKL constructs the set P, P, T and corresponding tables for all 1 bit

automata B_1 , B_2 ... B_n like in ID algorithm. In first iteration of all tables E_0 when $vo = \varepsilon$, $E(d_0) = \varphi$ and $E_0(\alpha) = \varepsilon$ when $\alpha \in L(A)$. Otherwise $E_0(\alpha) = \varphi$. After that IKL searches for a pair such that $E_{ic}{}^c(\alpha) = E_{ic}{}^c(\beta)$ but $E_{ic}{}^c(f(\alpha, \sigma))$ $/= E_{ic}{}^c(f(\beta, \sigma))$ whereas $\alpha, \beta \in P'$ and $\sigma \in \Sigma$. This expression shows that if IKL finds a pair from set P' which shows the same behavior i.e. either both $E_{ic}{}^c(\alpha)$ and $E_{ic}{}^c(\beta)$ accepted both lie in rejected block and when we concatenate $E_{ic}{}^c(\alpha)$ and $E_{ic}{}^c(\beta)$ with some alphabet σ from the input set Σ then their behavior changes i.e. one accepted block and other is rejected. Then the IKL algorithm chooses some string $\gamma \in E^c(f(\alpha, \sigma)) \oplus E^c(f(\beta, \sigma))$ and a new distinguishing

string is defined as $\sigma\gamma$. The IKL algorithm performs next iteration i + 1 to further partition the blocks, by reading distinguishing string $\sigma\gamma \in \Sigma$ from all elements of $E_i^c(\alpha)$. For this, it asks membership queries as $\alpha vi + 1 \in L(A)$?

If the adequate teacher answers as Yes then $E^{c}(\alpha)$ becomes $E^{c}(\alpha) \cup \{v_{i}\}$, i_{c} i_{c-1} cif the adequate teacher answers as No then $E^{c}(\alpha)$ is set to $E^{c}(\alpha)$. The i_{c} i_{c-1} IKL repeats the above process until all tables corresponding to $B_{1}, B_{2}..., B_{n}$ become consistent. After that it constructs the product automata H_{m} . If H_{m} is behaviorally equivalent to the target automata A and input string set S is empty then the IKL algorithm stops its execution.

Example

с

An example run of the IKL algorithm is given below: Kripke structure = 3 bits $\sum_{n=1}^{\infty} (x_n, b)$

 $\Sigma = \{a, b\}$ File S contain = a, ba Target automata A = Odd number of a's

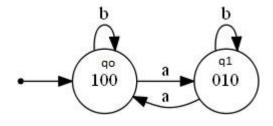


Figure 2.5.1: Target Automata

In the first step, the IKL algorithm finds the bit slicing of the target automata A in form of B_1 , B_2 , B_3 which are given below in Figure 2.5.2.

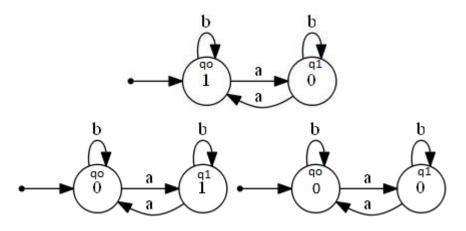


Figure 2.5.2: B₁, B₂, B₃

 $\begin{array}{l} c=1\rightarrow 3\\ i_{1}=0 \end{array}$

 $i_{2} = 0$ i3= 0 $V_1 = \varepsilon$ $V_2 = \varepsilon$ $V_3 = \varepsilon$ k = 0, t = 0 $P_0 = \{\varepsilon\}$ $P_0' = P_0 \cup \{d_0\} = \{\varepsilon, d_0\}$ $T_0 = P_0 \cup \Sigma = \{\varepsilon, a, b\}$ $E_0^1(d_0) = \varphi$ $E_0^2(d_0) = \varphi$ $E_0^3(d_0) = \varphi$ Now suppose the IKL algorithm reads the input string a then: k = 1, t = 1 $P_1 = P_0 \cup Pref(\alpha) = \{\varepsilon, \alpha\}$ $T_1 = T_0 \cup Pref(\alpha) \cup \{\alpha.b\} = \{\varepsilon, a, b\} \cup \{\varepsilon\} \cup \{a, aa, ab\} = \{\varepsilon, a, b, aa, ab\}$ $T_1 = \{ d_0, \varepsilon, a, b, aa, ab \}$

iı	0	i2	0	i3	0
V 1	ε	<i>V</i> ₂	ε	V 3	ε
$E^1(d_0)$	φ	$E^{2}(d_{0})$	φ	$E^{3}(d_{0})$	φ
$E^1(\varepsilon)$	φ	$E^2(\varepsilon)$	φ	E ³ (ε)	φ
$E^1(a)$	φ	E ² (a)	{ ɛ }	E ³ (a)	φ
<i>E</i> ¹ (<i>b</i>)	{ ɛ }	E ² (b)	φ	E ³ (b)	φ
E ¹ (aa)	{ ɛ }	E²(aa)	φ	E ³ (aa)	φ
<i>E</i> ¹ (<i>ab</i>)	φ	E²(ab)	{ <i>ɛ</i> }	E ³ (ab)	φ

Table 2.5.1: For B1 Table 2.5.2: For B2 Table 2.5.3: For B3

Table 2.5.1 shows that $E_0^1(\varepsilon) = E_0^1(a)$ but $E_0^1(\varepsilon, a) /= E_0^1(a, a)$ so a is a distinguishing string for Table 2.5.1. Table 2.5.2 shows that $E_0^1(\varepsilon) = E_0^1(d_0)$ but $E_0^1(\varepsilon, a) /= E_0^1(d_0, a)$ so a is a distinguishing string for Table 2.5.2. Whereas in Table 2.5.3, the corresponding values for all strings belonging to Tare φ so it shows that it is consistent and having only one state that is φ denoted as q_0 for the Table 2.5.3.

Now IKL maintain the consistency of Table 2.5.1 and Table 2.5.2 by updating these table with distinguishing string a.

Updated Table 2.5.1 denoted as Table 2.5.1(a) and updated Table 2.5.2 denoted as Table 2.5.2(a) are given below:

i1	0	1	i2	0	1
V 1	ε	а	V 2	ε	а
$E^1(d_0)$	φ	φ	$E^{2}(d_{0})$	φ	φ
$E^1(\varepsilon)$	φ	φ	E²(ε)	φ	{ <i>a</i> }
$E^1(a)$	φ	{ <i>a</i> }	E ² (a)	{ ɛ }	$\{\varepsilon\}$
<i>E</i> ¹ (<i>b</i>)	$\{ \boldsymbol{\varepsilon} \}$	$\{\varepsilon\}$	E ² (b)	φ	{ <i>a</i> }
E ¹ (aa)	$\{ \boldsymbol{\varepsilon} \}$	$\{ \boldsymbol{\varepsilon} \}$	E²(aa)	φ	{ <i>a</i> }
E ¹ (ab)	φ	{ <i>a</i> }	E²(ab)	$\{ \boldsymbol{\varepsilon} \}$	$\{\varepsilon\}$

Table 2.5.1(a): For B1 Table 2.5.2(a): For B2

i1	0	1	2
V 1	ε	А	b
$E^{1}(d_{0})$	φ	Φ	φ
$E^1(\varepsilon)$	φ	Φ	{ <i>b</i> }
E ¹ (a)	φ	{ a }	{ <i>a</i> }
E ¹ (b)	$\{\boldsymbol{\varepsilon}\}$	$\{ \boldsymbol{\varepsilon} \}$	{ <i>ε</i> , <i>b</i> }
E ¹ (aa)	$\{ \boldsymbol{\varepsilon} \}$	$\{ \boldsymbol{\varepsilon} \}$	{ <i>ε</i> , <i>b</i> }
E¹(ab)	Φ	{ a }	{ <i>a</i> }

Table 2.5.1(a'): For *B*1

Table 2.5.1(a') is now consistent as it finds no such pair that is $E_i(\alpha) = E_i(\beta)$ but $E_i(f(\alpha, \sigma)) /= E_i(f(\beta, \sigma))$. So, it constructs the hypothesis DFA H_m by taking product of β_1 , B_2 , B_3 . For this, the IKL algorithm constructs the 1 bit automata B_1 , B_2 , B_3 for each corresponding table which are given below in Figure 2.5.3.

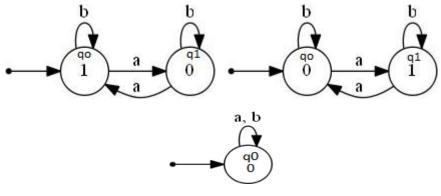


Figure 2.5.3: *B*₁, *B*₂, *B*₃

Product of all bit slice automata's B_1 , B_2 , B_3 is given below in Figure 2.5.4.

, , ,

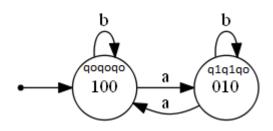


Figure 2.5.4: Product Automata Hm

We can see that product automaton H_m is behaviorally equivalent to the target automata A.

Now suppose the IKL algorithm reads the input string ba as it is consistent with H_m so $H_{m+1} = H_m \equiv A$. As the le *S* is now empty so the IKL algorithm stops its execution.

2.6 RPNI Algorithm

The RPNI algorithm is a passive learning algorithm proposed by Jose Oncina and Pedro Garcia in 1992 [3]. It uses a tree structure instead of table and does not maintain consistency. It takes the input as set of positive examples and set of negative examples S_+ and S_- respectively. It first writes the elements of S_+ and its prefixes in lexicographical order then from set of positive examples and their prefixes, it constructs the prefix tree PT(S_+). After that it recursively partitions the branches of the tree into blocks. The partition is represented as π and the target automata is represented as A. At first step each element of PT(S_+) belongs to its self-containing block. The RPNI algorithm recursively applies joint operation on these blocks so that they can be merged into two final blocks. One is accepting state block and second is non-accepting state block.

Let π be a partition over $PT(S_+)$ and blocks B_i , $B_j \in \pi$ then joint operation over any two blocks B_i , B_j is $J(\pi, B_i, B_j) = \{B \in \pi | B /= B_i, B /= B_j\} \cup \{B_i \cup B_j\}$. Initial automaton A_0 produced by $PT(S_+) = \pi_0 = \{u_0, u_1, \dots, u_r\}$ and $\pi_n = J(\pi_{n-1}, B, u_n)$ i $S_- L(A_0 / J(\pi_{n-1}, B, u_n)) = \varphi$ otherwise $\pi_n = \pi_{n-1}$. Detailed explanation of the RPNI algorithm is given below with the help of example.

Example

An example run of the RPNI algorithm is given below:

Target Automaton A: Odd number of a's

 $S_+ = \{ a, ab, bab, abaa \}$

 $S_{-} = \{$ b, baba, baa $\}$

The lexicographical order of pre xes of S_+ is: $\langle \lambda, a, b, ab, ba, bab, aba, abaa \rangle$ Initial automaton $A_0 = PT(S_+)$ is given in Figure 2.6.1.

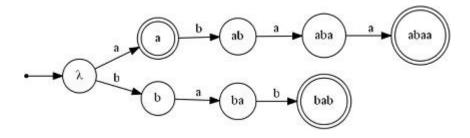


Figure 2.6.1: Initial Automata A_0 / π_0

To obtain π_1 where $u_1 = "a"$, the RPNI algorithm perform operation J (π_0, λ, a) which is given in Figure 2.6.2.

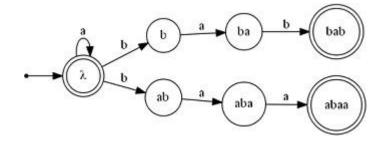


Figure 2.6.2: A_0 / J (π_0, λ, a)

In Figure 2.6.2, we can see that $S_{-T} L(A_0 / J(\pi_0, \lambda, a)) /= \varphi$ as above automaton accepts the string *baa* which belongs to the set S_{-} . As there are no more states to try to merge with $u_1 = "a"$ therefore $\pi_1 = \pi_0$

To obtain π_2 where $u_2 = b''$, the RPNI algorithm performs operation J (π_1, λ, b) which is given in Figure 2.6.3.

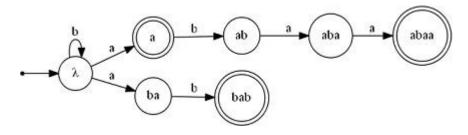


Figure 2.6.3: A_0 /J (π_1 , λ , b)

In Figure 2.6.3, we can see that $S_{-T} L(A_0 / J(\pi_1, \lambda, b)) = \varphi$ as above automata accepts all strings belonging to the set S_+ and rejects all negative data belonging to the set S_- . So $\pi_2 = J(\pi_1, \lambda, b)$.

To obtain π_3 where $u_3 = ab''$, the RPNI algorithm performs operation *J* (π_2 , *a*, *ab*) which is given in Figure 2.6.4.

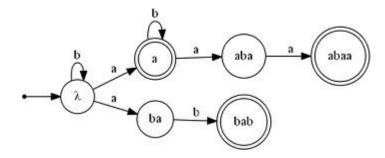


Figure 2.6.4: $A_0 / J (\pi_2, a, ab)$

In Figure 2.6.4, we can see that $S_{-T} L(A_0 / J(\pi_2, a, ab)) = \varphi$ as above automaton accepts all strings belonging to the set S_+ and rejects all negative data belonging to the set S_- . Therefore $\pi_3 = J(\pi_2, a, ab)$.

To obtain π_4 where $u_4 = "ba"$, the RPNI algorithm performs operation J (π_3 , *a*, *ba*) which is given in Figure 2.6.5.

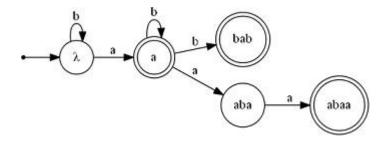


Figure 2.6.5: *A*₀ / *J* (*π*₃, *a*, *ba*)

In Figure 2.6.5, we can see that $S_{-T} L(A_0 / J (\pi_3, a, ba)) = \varphi$ as above automaton accepts all strings belonging to the set S_+ and rejects all negative data belonging to the set S_- . Therefore $\pi_4 = J (\pi_3, a, ba)$.

To obtain π_5 where $u_5 = "bab"$, the RPNI algorithm performs operation J (π_4 , *a*, *bab*) which is given in Figure 2.6.6.

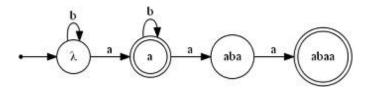


Figure 2.6.6: A_0 / J (π_4 , *a*, *bab*)

In Figure 2.6.6, we can see that $S_{-T} L(A_0 / J(\pi_4, a, bab)) = \varphi$ as above automaton accepts all strings belonging to the set S_+ and rejects all negative data belonging to the set S_- . Therefore $\pi_5 = J(\pi_4, a, bab)$.

To obtain π_6 where $u_6 = "aba"$, the RPNI algorithm performs operation *J* (π_5 , λ , *aba*) which is given in Figure 2.6.7.

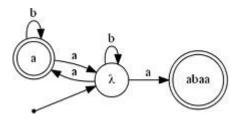


Figure 2.6.7 : A_0 / J (π_5, λ, aba)

In Figure 2.6.7, we can see that $S_{-T} L(A_0 / J(\pi_5, \lambda, aba)) = \varphi$ as above automaton accepts all strings belonging to the set S_+ and rejects all negative data belonging to the set S_- . Therefore $\pi_6 = J(\pi_5, \lambda, aba)$.

To obtain π_7 where $u_7 = "abaa"$, the RPNI algorithm performs operation *J* (π_6 , *a*, *abaa*) which is given in Figure 2.6.8.

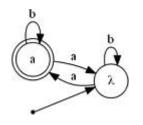


Figure 2.6.8 : A_0 / J ($\pi_6, a, abaa$)

In Figure 2.6.8, we can see that $S_{-T} L(A_0 / J(\pi_6, \lambda, abaa)) = \varphi$ as above automaton accepts all strings belonging to the set S_+ and rejects all negative data belonging to the set S_- . Therefore $\pi_7 = J(\pi_6, a, abaa)$.

As there are total eight elements in lexicographical order in the pre xes of S_+ set so after seven recursive partitions π_7 the RPNI algorithm stops its execution and we can see that this partition is behaviorally equivalent to the target automaton A.

2.7 RPNII Algorithm

The RPNII algorithm is an incremental extension of the RPNI algorithm [3]. The RPNI algorithm takes the positive and negative examples as a whole and can't accommodate new labeled example unless it may start its whole execution from the scratch. The RPNII algorithm reduces this discrepancy as it has the ability to accommodate a new labeled example easily [4].

The RPNII algorithm initially takes the set of positive and negative examples, S_+ , S_- respectively. It also takes the prefix tree acceptor PTA(S_+), deterministic quotient automaton (DQA) and a new labeled example x.

If the new labeled example consistent with deterministic quotient automaton (DQA) then initial deterministic quotient automaton will be the final solution. Otherwise, the RPNII algorithm accommodates new labeled example by recursive splitting process in form of depth first search (in reverse lexicographical order). This process continuous until the quotient automaton becomes deterministic. After that when the quotient automaton becomes deterministic as well as consistant with S_+ and S_- then the RPNII algorithm applies the RPNI algorithm on it. Which we have brie y explained in the previous section.

Example

An example run of the RPNII algorithm is described below:

Let $S_{+} = \{ \lambda, ab, bab, babb \}$ $S_{-} = \{ a, baa \}$ The lexico-graphical order = $\langle \lambda, a, b, ab, ba, bab, babb \rangle$ PTA(S_{+}):

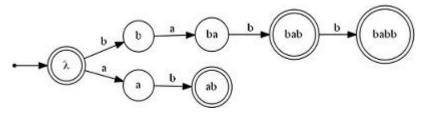


Figure 2.7.1: PTA(*S*+)

DQA:

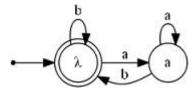


Figure 2.7.2: Initial Deterministic Quotient Automaton

let x = (b, -)

As initial DQA shows that b is accepting string while new labeled example x shows that string b belongs to S_{-} therefore the RPNII algorithm modifies the initial DQA to make it consistent with the sets S_{+} and S_{-} . For this purpose, the RPNII algorithm starts from the string *babb* and splits the initial deterministic quotient automaton which is given below in Figure 2.7.3.

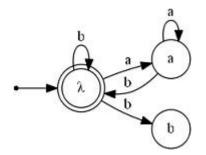


Figure 2.7.3: Splitting for the string babb

Figure 2.7.3 shows that due to splitting of initial DQA for the string *babb*, the initial automaton became non-deterministic as the initial state has two transitions for input symbol b. Therefore, to make it deterministic, the RPNII algorithm again splits this automaton on the basis of the string *bab* which is given below in Figure 2.7.4.

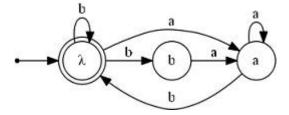


Figure 2.7.4: Splitting for the string bab

Figure 2.7.4 shows that splitting at the string *bab* also creates non-determination at the initial state, as this state has two transitions for input symbol b. Therefore, the RPNII algorithm again splits this automaton on the basis of the string *ba*. New quotient automaton is described below in Figure 2.7.5.

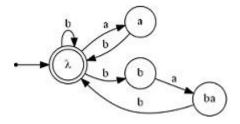


Figure 2.7.5: Splitting for the string ba

Figure 2.7.5 shows that splitting at the string *ba* also create non-determination at the initial state, as this state has two transitions for input symbol b. Therefore the RPNII algorithm again splits this automaton on the basis of the string *b*. New quotient automaton is described below in Figure 2.7.6.

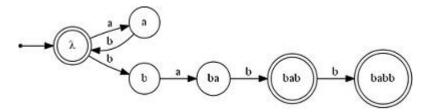


Figure 2.7.6: Splitting for the string b

As we can see that Figure 2.7.6 shows the quotient automaton A_0 which is now deterministic and consistent with the sets, S_+ and S_- . Therefore, the RPNII algorithm stops its recursive splitting process.

Here the RPNII algorithm applies the RPNI algorithm on deterministic quotient automaton which is given below.

The lexicographical order of automaton A_0 is $\langle \lambda, a, b, ba, bab, babb \rangle$ $u_1 = a$

 $J (\pi_0, \lambda, a)$

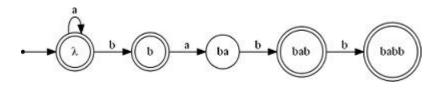


Figure 2.7.7: A_0/J (π_0, λ, a)

Figure 2.7.7 shows that $L(A_0/J(\pi_0, \lambda, a)) \cap S_- \neq \varphi$ as strings *a* and *b* are accepting here, according to Figure 2.7.7 but these belong to *S*_. Therefore $\pi_1 = \pi_0$

Now $u_2 = b$ J (π_1 , λ , b)

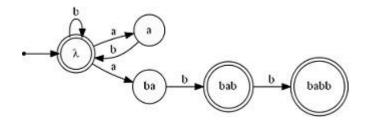


Figure 2.7.8: A_0/J (π_1, λ, b)

Figure 2.7.8 shows that $L(A_0/J(\pi_1, \lambda, b)) \cap S_- \neq \varphi$ as string *b* is accepting in Figure 2.7.8 but this is belonging to *S*_. Therefore $\pi_2 = \pi_1$ Now $u_3 = ba$

 $J(\pi_2, b, ba)$

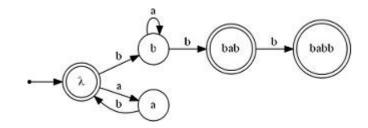


Figure 2.7.9: *A*₀ / *J* (*π*₂, *b*, *ba*)

Figure 2.7.9 shows that $L(A_0/J(\pi_2, b, ba)) \cap S_{-}=\varphi$ therefore we can say that it rejects all negative strings. So $\pi_3 = J(\pi_2, b, ba)$

Now $u_4 = bab$

 ${m J}$ ($\pi_2,$ ba, bab)

This operation is not suitable as if we will merge the strings ba and bab then in the next step, string b will be accepted but as it belongs to S_{-} so it should not be accepted.

Now $u_4 = babb$ J (π_2 , bab, babb)

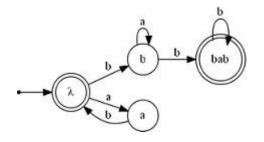


Figure 2.7.10: A_0 / J (π_2 , bab, babb)

Figure 2.7.10 shows that this automaton $H = A_0/J(\pi_2, bab, babb)$ is behaviorally equivalent to the target automaton A as well as it is also consistent with the S₊ and S₋ sets. Therefore, RPNII algorithm stops its execution and returns the automaton H as an output.

2.8 Analysis

Algorithm	Learning Type	Complexity	Learned Automata
L*	Complete	$O(\Sigma .N^2M)$	Moore
ID	Complete	Ο (Σ . Ρ .Ν)	Moore
IID	Incremental	$O(\Sigma . P_l .N)$	Moore
IDS	Incremental	$O(\Sigma . P_k .N)$	Moore
IKL	Incremental	Ο (Σ . Ρ . NΙ)	Moore
RPNI	Complete	$O((I_p + I_n) \cdot I_p ^2)$	Moore
RPNII	Incremental	$O((I_p + I_n) \cdot I_p ^2)$	Moore

2.8.1 Time Complexities of Learning Algorithms

Table 2.8.1: Complexities of Learning Algorithms

Above Table 2.8.1 shows that size of the input alphabet $|\mathcal{E}|$, number of nodes N in the target DFA A and the number of queries; M for L*, |P| for the ID and IKL algorithms, $|P_l|$ for IID algorithm, $|P_k|$ for IDS algorithm, $|I_p|$ (pos- itive sample) and $|I_n|$ (negative sample) for the RPNI and RPNII algorithms, contribute in the complexities of learning algorithms. If we analyze, we can see that number of queries have major contribution in the complexities of above mentioned algorithms as the size of input alphabet $|\mathcal{E}|$ and number of nodes N in the target DFA are nearly static factors.

Algorithm	Membership	Book-keeping	Lexicographical
	Queries	Queries	Order
L*	Yes	No	No
ID	Yes	No	No
IID	Yes	Yes	No
IDS	Yes	Yes	No
IKL	Yes	Yes	No
RPNI	No	No	Yes
RPNII	No	No	Yes

2.8.2 Query-Wise Analysis of Learning Algorithms

Table 2.8.2: Query wise Analysis of Learning Algorithms

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