Explaining Fermions Mass and Mixing Hierarchies through $U(1)_X$ and Z_2 Symmetries

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Abstract

For understanding the hierarchies of fermion masses and mixing, we extend the standard model gauge group with $U(1)_X$ and Z_2 symmetry. The field content of the Standard model is augmented by three heavy right-handed neutrinos and two new scalar singlets. $U(1)_X$ charges of different fields are considered after satisfying anomaly cancellation conditions. In this scenario, the fermion masses are generated through higher dimensional effective operators. The small neutrino masses are obtained through type-1 seesaw mechanism using the heavy right handed neutrino fields. We discuss the flavor-changing neutral current processes which is originated due to the sequential nature of $U(1)_X$ symmetry. We have written effective higher dimensional operators in terms of renormalizable dimension four operators by introducing vector like fermions.

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I. INTRODUCTION

The Standard Model (SM) of particle physics is the most successful theory which explains how the fundamental particles of our universe, like quarks and leptons interact with each other through gauge bosons and also explain how these fundamental particles get mass through the Higgs mechanism.

In the SM all the fermions are directly coupled to a single scalar doublet known as Higgs which after spontaneous symmetry breaking gets vacuum expectation value (vev) and the fermions of the SM get masses. The mass of the fermion is equal to vev times corresponding Yukawa coupling of the particular fermion field with Higgs. But experimentally we know the masses of the different fermions in the SM vary over a wide range of magnitude, as such the Yukawa couplings corresponding to different fermions are also required to vary over wide range. As for example the Yukawa coupling corresponding to top quark and that of electron differ by about 10^6. Neutrino masses are expected to be even smaller, of the order of 0.1 eV [1]. If one wants to get neutrino mass in the SM scenario, similar to charged fermions then light right handed field may be introduced. But the corresponding Yukawa couplings for neutrino with Higgs are required to be much smaller, almost 10^{-12} times top quarks Yukawa coupling. Such a significant differences in order of Yukawa couplings for different fermion masses are not suitable for perturbative approach of calculation in quantum field theory. One may note that the mass of the top quark is of the same order as the vev of Higgs and its Yukawa coupling is of $\mathcal{O}(1)$. But all other fermion masses are way below of the top quarks mass and their Yukawa couplings are very small. In the SM, for quarks the mixing between the 1^{st} and 2^{nd} generation are high but mixing between the 2^{nd} and 3^{rd} and mixing between 1^{st} and 3^{rd} is very small. There is no explanation of that mixing pattern in the SM. Also in SM the neutrinos are mass less but now from neutrino oscillation data it is confirmed that the neutrino has very little mass and there are mixing in different flavour of neutrino. A complete flavour theory should explain all those problems collectively known as flavour puzzle of standard model. For basic introduction of flavour physics and mass matrix model see e.g [2–7] The smallness of Yukawa couplings of fermions except top quarks indicate that they may not be directly interacting with the Higgs and their masses are not directly connected with Higgs vev. These small Yukawa couplings of the fermions could be realized by introducing higher dimensional operator involving some new scalar fields. These operator will have some inverse power of mass dimension and then the corresponding dimensionless Yukawa Couplings will be of $\mathcal{O}(1)$[8, 9].

Flavour problems for charged fermions with different symmetries involving different fields have been studied by different authors [2, 7, 9–14] but they only discussed the mass of charged fermions and mixing of quarks but not discuss about neutrino mass and mixing. Later on in [15–18] authors explain the neutrino masses and mixing. But their charge assignment is random without considering the anomaly cancellation of that symmetry. This type of work for different kind of Abelian flavour symmetry have been studies in different context [19–28]. Apart from Abelian symmetry there are other ways to explain the hierarchies of fermion masses like context of left right symmetric model [29, 30], non abelian gauge symmetry[31],by hierarchical fermion wave function [32] and by using discrete symmetry[33–35].

In this work, higher dimensional operator are introduced with some flavour symmetry in such a way that only top quark has direct coupling the SM Higgs and direct coupling between all the light fermions and Higgs are forbidden by the new flavour symmetry and they will get mass from some higher dimensional effective operators generated by some new scalar fields such a way that different element of the mass matrices have have get values from different order of higher dimensional effective operators which are suppress by different power of cutoff scale of the theory and when the new scalars get *vev* this flavour symmetry is broken and the effective higher dimensional operators will give back the SM interaction multiplied by some power of a small number ϵ, which is equal to *vev* of the new scalar divided by cutoff scale of the theory. In this way we can generate the mass matrix which can explain fermion masses and mixing by taking the Yukawa coupling of $\mathcal{O}(1)$.

We extended the fermion sector of SM by adding 3 heavy right handed neutrinos which is necessary for generating small neutrino masses in seesaw mechanism. We have added a extra $U(1)_X$ symmetry with the SM symmetry group. Using the anomaly cancellation conditions[21, 36] we assign charges of fermions in such a way that the allowed interaction provided the suitable mass matrices which give the proper masses as shown in [37]. This anomaly free extra $U(1)_X$ naturally explained the inter generation mass hierarchy of quarks and charged leptons but if we consider only $U(1)_X$ symmetry the inter- generational hierarchies of fermions. But to get the intra-generational mass hierarchies of fermions doublet we need to introduce a Z_2 symmetry. The discrete Z_2 symmetry is used in extensively in different context of models building in beyond SM like in two-Higgs-doublet model (2HDM)[38] and the minimal super symmetric SM (MSSM) [39]. When the new scalars χ_1 and χ_2 acquires *vev* those extra symmetries will be broken. The cut-off scale Λ can not be directly related to the fermion masses hierarchies of the SM, they only determine by a small parameter(ϵ) equal to the ratio of the *vev* of the new scalars and the cutoff scale. In this model, tree level flavour changing neutral current effect is predicted but they are with in the experimental bounds. In section II, we have discussed the details of the model with $U(1)_X$ and Z_2 symmetry and various fields content with their charges based on the anomaly cancellation conditions. The allowed higher dimensional operators based on the symmetries are also discussed. In section III, we have calculated the masses and mixing of quarks and leptons and hence discuss the CKM matrix for quarks and PMNS matrix associated with leptons. In section IV we have discussed the phenomenological implication of the various decay channels of new gauge boson Z' and various rare decay through Z'. In section , we discuss a possible ultra-violet (UV) complete theory by introducing some vector like fermions with their charge assignment. In section VI we present concluding remarks. In appendix VII, we have presented the best fit values of Yukawa couplings for fitting fermions masses and mixing.

II. THE MODEL AND FORMALISM

In our model, the top quark is directly coupled to Higgs doublet and its mass is proportional to the *vev* of the Higgs and all the other fermions masses are suppressed by a small parameter ϵ such that all the Yukawa couplings becomes $\mathcal{O}(1)$. If we normalized the top quark mass as 1 then mass of all other charged fermions and quarks mixing angles can be written in terms a small parameter ϵ. For example, if we consider $0.02 < \epsilon < 0.03$ then masses of quarks and charged lepton along with quarks mixing angles can be written in the power of ϵ as

$$\begin{array}{cccccc} m_t \approx 1 & m_b \approx \epsilon & m_s \approx \epsilon & m_s \approx \epsilon^2 & m_u \approx \epsilon^3 & m_d \approx \epsilon^3 \\ m_\tau \approx \epsilon & m_\mu \approx \epsilon^2 & m_e \approx \epsilon^3 & s^q_{12} \approx \epsilon & s^q_{23} \approx \epsilon & s^q_{13} \approx \epsilon^2 \end{array} \quad (1)$$

where, $s_{ij} = \sin\theta_{ij}$ and θ_{ij} is the mixing angle between i^{th} and j^{th} flavour of fermions. We can generate this kind of suppression in masses of quarks and charged leptons and mixing angles of quarks if the mass matrix of up-type, down-type quark and charged leptons takes the following form

$$m_u = \begin{pmatrix} \epsilon^3 & \epsilon & 1 \\ \epsilon^3 & \epsilon & 1 \\ \epsilon^3 & \epsilon & 1 \end{pmatrix} \qquad m_d = \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon \end{pmatrix} \qquad m_l = \begin{pmatrix} \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon \end{pmatrix} \tag{2}$$

The neutrino mass and mixing can naturally be explained by type-1 seesaw mechanism. We will discuss this in section III. For getting mass from these mass matrices we have to diagonalized them. As theses matrices are not hermitian, we have to diagonalize these matrices by bi-unitary transformations. By following the notation of [37] we get

$$m_D = S^\dagger m R \qquad m_D^2 = S^\dagger m m^\dagger S \qquad m_D^2 = R^\dagger m^\dagger m R \tag{3}$$

Where, m_D is the diagonalized matrix corresponding to m and S and R are the two unitary matrices. In other words we can say that the eigenvalues of $m^\dagger.m$ or $m.m^\dagger$ give mass square of the fermions and the physical mixing matrix for quarks and leptons are CKM and PMNS matrix given by

$$V_{CKM} = R^{u\dagger} R^d \qquad V_{PMNS} = R^{\nu\dagger} R^l \tag{4}$$

where, R^u, R^d, R^l, R^ν correspond to R matrix in (3) for m corresponding to up, down type quarks, charged leptons and neutrino respectively. This CKM and PMNS matrix can be parameterize by three angle and one phase in standard parameterization as [1]:

$$V_{CKM}/V_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \tag{5}$$

where, $s_{ij} = sin(\theta_{ij})$ and $c_{ij} = cos(\theta_{ij})$ and δ is CP-violating phase as here our main goal is to explain the hierarchy of fermion masses and mixing. For simplicity we will not consider CP violation here. So, we will take all the Yukawa coupling to be real. Our main motivation for this work to generate this kind of mass matrix as in Eq.(2) by some symmetry principle in a minimal setup. For this, fermion sector of SM is extended by three right handed neutrinos and gauge group of SM is extended by an extra $U(1)_X$ symmetry and calculate the anomaly cancellation condition for this. The three right handed neutrino are singlet under the SM gauge group. Let $U(1)_X$ charges of the fermions are given by

$$Q_{Li} \to n_1^i, \quad u_{Ri} \to n_2^i, \quad d_{Ri} \to n_3^i \quad L_{Li} \to n_4^i \quad e_{Ri} \to n_5^i \quad N_{Ri} \to n_6^i \tag{6}$$

where $Q_{Li} = (u,d)_{Li}$ are left handed quark doublets ($i = 1,2,3$ for 1^{st}, 2^{nd} and 3^{rd} generation respectively), u_{Ri} are right handed up type quarks, d_{Ri} are right handed down type quarks, $L_{Li} = (\nu, e)_{Li}$ are left handed lepton doublets, e_{Ri} are the right handed charged leptons and N_{Ri} are the right handed heavy neutrinos. The anomaly cancellation conditions are given as :

$$
\begin{aligned}
[U(1)_Y]^2 U(1)_X : 1/6 n_1^i - 4/3 n_2^i - 1/3 n_3^i + 1/2 n_4^i - n_5^i &= 0 \\
[U(1)_X]^2 U(1)_Y : {n_1^i}^2 - 2{n_2^i}^2 + {n_3^i}^2 - {n_4^i}^2 + {n_5^i}^2 &= 0 \\
[SU(3)_c]^2 U(1)_X : 2n_1^i - n_2^i - n_3^i &= 0 \\
[SU(2)_L]^2 U(1)_X : 9/2 n_1^i + 3/2 n_4^i &= 0 \\
U(1)_X : 6n_1^i - 3n_2^i - 3n_3^i + 2n_4^i - n_5^i - n_6^i &= 0 \\
[U(1)_X]^3 : 6{n_1^i}^3 - 3{n_2^i}^3 - 3{n_3^i}^3 + 2{n_4^i}^3 - {n_5^i}^3 - {n_6^i}^3 &= 0
\end{aligned} \tag{7}
$$

Using the above anomaly cancellation conditions one may write different $U(1)_X$ charges in term of n_1 and n_2 in the following way :

$$
n_3^i = 2n_1^i - n_2^i \qquad n_4^i = -3n_1^i \qquad n_5^i = -(2n_1^i + n_2^i) \qquad n_6^i = (n_2^i - 4n_1^i) \tag{8}
$$

To create the matrix structure in (2) we need to assign a sequential $U(1)_X$ charges. If we take $(n_1^1, n_1^2, n_1^3) = (0,0,0)$ and $(n_2^1, n_2^2, n_2^3) = (3,1,0)$ for 1^{st} , 2^{nd} and 3^{rd} generation respectively, then using Eq. (8) the charges of other fields in Eq.(6)

$$
(n_3^1, n_3^2, n_3^3) = (-3,-1,0) \qquad (n_4^1, n_4^2, n_4^3) = (0,0,0) \tag{9}
$$

$$
(n_5^1, n_5^2, n_5^3) = (-3,-1,0) \qquad (n_6^1, n_6^2, n_6^3) = (3,1,0) \tag{10}
$$

If the scalar sector of the model have one Higgs doublet and one scalar singlet χ_1 with $U(1)_X$ charges 0 and -1 respectively, then only 3rd generation of quarks and charged lepton get masses from renormalizable dimension 4 operators through scalar doublet ϕ, 2^{nd} and 3^{rd} generation of quarks and lepton will get mass from dimension 5 and dimension 7 operators respectively through scalar singlet χ_1. In this way, we can generate a hierarchy in masses of different generations of up type quarks, down type quarks and charged leptons. But in SM up and down sector of fermions have also possess a strong hierarchy among them self, these type of hierarchies could be obtained by introducing another scalar singlet χ_2 in our model which couples to only 2^{nd} and 3^{rd} generation of quarks and leptons. This can be achieved by introducing Z_2 symmetry with proper charges. The charges of all the particles in our model with respect to different symmetries has been shown in Table I. With respect to this

TABLE I. Charges of scalars and fermions particles

Particles	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	Z_2
$Q_{iL} = (u,d)_{iL}$	3	2	1/6	(0,0,0)	(+,+,+)
u_{iR}	3	1	2/3	(3,1,0)	(+,+,+)
d_{iR}	3	1	- 1/3	(-3,-1,0)	(+,-,+)
$L_{iL} = (\nu,l)_{iL}$	1	2	-1/2	(0,0,0)	(+,+,+)
e_{iR}	1	1	-1	(-3,-1,0)	(+,-,+)
N_{iR}	1	1	0	(3,1,0)	(+,+,+)
ϕ	1	2	1/2	0	+
χ_1	1	1	0	-1	+
χ_2	1	1	0	0	-

charge assignment the Yukawa Lagrangian of quarks is given by

$$\mathcal{L}^Y = \sum_{i=1}^{3} \left(\frac{\chi_1}{\Lambda}\right)^3 h^u_{i1}\bar{Q}_{iL}\tilde{\phi}u_R + \left(\frac{\chi_1}{\Lambda}\right) h^u_{i2}\bar{Q}_{iL}\tilde{\phi}c_R + h^u_{i3}\bar{Q}_{iL}\tilde{\phi}t_R + \left(\frac{\chi_1^*}{\Lambda}\right)^3 h^d_{i1}\bar{Q}_{iL}\phi d_R + \left(\frac{\chi_1^*\chi_2}{\Lambda^2}\right)$$
$$h^d_{i2}\bar{Q}_{iL}\phi s_R + \left(\frac{\chi_2}{\Lambda}\right) h^d_{i3}\bar{Q}_{iL}\phi b_R + \left(\frac{\chi_1^*}{\Lambda}\right)^3 h^l_{i1}\bar{L}_{1L}\phi e_{iR} + \left(\frac{\chi_1^*\chi_2}{\Lambda^2}\right) h^l_{i2}\bar{L}_{2L}\phi\mu_{iR} + \left(\frac{\chi_2}{\Lambda}\right)$$
$$h^l_{i3}\bar{L}_{3L}\phi\tau_{iR} + \left(\frac{\chi_1}{\Lambda}\right)^3 h^\nu_{i1}\bar{L}_{1L}\tilde{\phi}N_{iR} + \left(\frac{\chi_1}{\Lambda}\right) h^\nu_{i2}\bar{L}_{2L}\tilde{\phi}N_{iR} + h^\nu_{i3}\bar{L}_{3L}\tilde{\phi}N_{iR} + \frac{M_i}{2}\bar{N}^c_{iR}N_{iR} \quad (11)$$

where, h_{ij} are the Yukawa couplings and the superscripts u, d, l, and ν are for up type quarks, down type quarks, charged leptons and neutrinos respectively.

III. MASS AND MIXING OF FERMIONS

After the flavour symmetry breaking the χ_1 and χ_2 get *vev* and if we define $\frac{v_1}{\Lambda} = \epsilon$ and $\frac{v_2}{\Lambda} = \epsilon'$ where, v_1 and v_2 are the *vev* of χ_1 and χ_2 respectively, then the effective Lagrangian for Yukawa interactions for up and down type quarks are as follows :

$$\mathcal{L}^Y_Q = \epsilon^3 h^u_{i1}\bar{Q}_{iL}\tilde{\phi}u_R + \epsilon h^u_{i2}\bar{Q}_{iL}\tilde{\phi}c_R + h^u_{i3}\bar{Q}_{iL}\tilde{\phi}t_R + \epsilon^3 h^d_{i1}\bar{Q}_{iL}\phi d_R + \epsilon\epsilon' h^d_{i2}\bar{Q}_{iL}\phi s_R + \epsilon' h^d_{i3}\bar{Q}_{iL}\phi b_R \quad (12)$$

The mass matrices of the quarks after χ_1 and χ_2 get *vev*

$$m_u = \begin{pmatrix} h^u_{11}\epsilon^3 & h^u_{12}\epsilon & h^u_{13} \\ h^u_{21}\epsilon^3 & h^u_{22}\epsilon & h^u_{23} \\ h^u_{31}\epsilon^3 & h^u_{32}\epsilon & h^u_{33} \end{pmatrix} \frac{v}{\sqrt{2}}; \qquad m_d = \begin{pmatrix} h^d_{11}\epsilon^3 & h^d_{12}\epsilon\epsilon' & h^d_{13}\epsilon' \\ h^d_{21}\epsilon^3 & h^d_{22}\epsilon\epsilon' & h^d_{23}\epsilon' \\ h^d_{31}\epsilon^3 & h^d_{32}\epsilon\epsilon' & h^d_{33}\epsilon' \end{pmatrix} \frac{v}{\sqrt{2}}; \tag{13}$$

where, v is the *vev* of the Higgs field. We get masses of quarks by diagonalizing the above mass matrices. Following Eq.(3), we write

$$m_u^\dagger m_u = \begin{pmatrix} x^u_{11}\epsilon^6 & x^u_{12}\epsilon^4 & x^u_{13}\epsilon^3 \\ x^u_{12}\epsilon^4 & x^u_{22}\epsilon^2 & x^u_{23}\epsilon \\ h^u_{13}\epsilon^3 & h^u_{23}\epsilon & x^u_{33} \end{pmatrix} \qquad m_d^\dagger m_d = \begin{pmatrix} x^d_{11}\epsilon^6 & x^d_{12}\epsilon^4\epsilon' & x^d_{13}\epsilon^3\epsilon' \\ x^d_{12}\epsilon^4\epsilon' & x^d_{22}\epsilon^2\epsilon'^2 & x^d_{23}\epsilon\epsilon'^2 \\ x^d_{13}\epsilon^3\epsilon' & x^d_{23}\epsilon\epsilon'^2 & x^d_{33}\epsilon'^2 \end{pmatrix} \tag{14}$$

where,

$$\begin{aligned} x_{11} &= h_{11}^2 + h_{21}^2 + h_{31}^2 & x_{12} &= h_{12}h_{11} + h_{22}h_{21} + h_{32}h_{31} \\ x_{22} &= h_{12}^2 + h_{22}^2 + h_{32}^2 & x_{23} &= h_{13}h_{12} + h_{23}h_{22} + h_{33}h_{32} \\ x_{33} &= h_{13}^2 + h_{23}^2 + h_{33}^2 & x_{13} &= h_{11}h_{13} + h_{21}h_{23} + h_{31}h_{33} \end{aligned} \tag{15}$$

For up and down type quark replace x with $x^u$and x^d respectively and h are replaced with h^u and h^d respectively. Following [37] we get the masses of the quarks as :

$$(m_u, m_c, m_t) \approx \left(\sqrt{\frac{x^u_{11}{x^u_{23}}^2 + {x^u_{12}}^2 x^u_{33} - x^u_{11}x^u_{22}x^u_{33}}{{x^u_{23}}^2 - x^u_{22}x^u_{33}}}\epsilon^3, \sqrt{x^u_{22} - \frac{{x^u_{23}}^2}{x^u_{33}}}\epsilon, \sqrt{x^u_{33}} \right) \frac{v}{\sqrt{2}} \tag{16}$$

$$(m_d, m_s, m_b) \approx \left(\sqrt{\frac{x^d_{11}{x^d_{23}}^2 + {x^d_{12}}^2 x^d_{33} - x^d_{11}x^d_{22}x^d_{33}}{{x^d_{23}}^2 - x^d_{22}x^d_{33}}}\epsilon^3, \sqrt{x^d_{22} - \frac{{x^d_{23}}^2}{x^d_{33}}}\epsilon\epsilon', \sqrt{x^d_{33}}\epsilon' \right) \frac{v}{\sqrt{2}} \tag{17}$$

and the mixing in up type quarks and down type quarks are given as :

$$(s^u_{12}, s^u_{23}, s^u_{13}) \approx \left(\left| \frac{x^u_{13}x^u_{23} - x^u_{12}x^u_{33}}{{x^u_{23}}^2 - x^u_{22}x^u_{33}} \right| \epsilon^2, \frac{x^u_{23}}{x^u_{33}}\epsilon, \frac{x^u_{13}}{x^u_{33}}\epsilon^3 \right) \tag{18}$$

$$(s^d_{12}, s^d_{23}, s^d_{13}) \approx \left(\left| \frac{x^d_{13}x^d_{23} - x^d_{12}x^d_{33}}{{x^d_{23}}^2 - x^d_{22}x^d_{33}} \right| \frac{\epsilon^2}{\epsilon'}, \frac{x^d_{23}}{x^d_{33}}\epsilon, \frac{x^d_{13}}{x^d_{33}}\frac{\epsilon^3}{\epsilon'} \right) \tag{19}$$

We can write the mixing matrices for up type and down type quarks in the leading order as :

$$R^u = \begin{pmatrix} 1 & s^u_{12} & s^u_{13} \\ -s^u_{12} & 1 & s^u_{23} \\ (s^u_{12}s^u_{23} - s^u_{13}) & s^u_{23} & 1 \end{pmatrix} \qquad R^d = \begin{pmatrix} 1 & s^d_{12} & s^d_{13} \\ -s^d_{12} & 1 & s^d_{23} \\ (s^d_{12}s^d_{23} - s^d_{13}) & s^d_{23} & 1 \end{pmatrix} \tag{20}$$

From Eq. (4) the CKM matrix is given by

$$V_{CKM} = R^{u\dagger}.R^d = \begin{pmatrix} 1 & s^d_{12} & s^d_{13} \\ -s^d_{12} & 1 & (s^d_{23}+s^u_{23}) \\ (s^d_{12}(s^d_{23}+s^u_{23})-s^d_{13}) & (s^d_{23}+s^u_{23}) & 1 \end{pmatrix} \tag{21}$$

By following the same approach we can write the Yukawa interactions for charged leptons after the new scalar field χ_1 and χ_2 get vev as :

$$\mathcal{L}^Y_l = \epsilon^3 h^l_{i1}\bar{L}_{iL}\phi e_{1R} + \epsilon\epsilon' h^l_{i2}\bar{L}_{iL}\phi\mu_{2R} + \epsilon' h^l_{i3}\bar{L}_{iL}\phi\tau_{3R} \tag{22}$$

Then we can write the mass matrix of charged leptons as :

$$m_l = \begin{pmatrix} h^l_{11}\epsilon^3 & h^l_{12}\epsilon\epsilon' & h^l_{13}\epsilon' \\ h^l_{21}\epsilon^3 & h^l_{22}\epsilon\epsilon' & h^l_{23}\epsilon' \\ h^l_{31}\epsilon^3 & h^l_{32}\epsilon\epsilon' & h^l_{33}\epsilon' \end{pmatrix}\frac{v}{\sqrt{2}}; \qquad m^\dagger_l m_l = \begin{pmatrix} x^l_{11}\epsilon^6 & x^l_{12}\epsilon^4\epsilon' & x^l_{13}\epsilon^3\epsilon' \\ x^l_{12}\epsilon^4\epsilon' & x^l_{22}\epsilon^2\epsilon'^2 & x^l_{23}\epsilon\epsilon'^2 \\ x^l_{13}\epsilon^3\epsilon' & x^l_{23}\epsilon\epsilon'^2 & x^l_{33}\epsilon'^2 \end{pmatrix} \tag{23}$$

where, x^l_{ij} is same as defined in Eq.(15) only replace x_{ij} by x^l_{ij} and h_{ij} by h^l_{ij}. After diagonalizing we get the masses and mixing angles of charged leptons as

$$(m_e, m_\mu, m_\tau) \approx \left(\sqrt{\frac{x^l_{11}{x^l_{23}}^2 + {x^l_{12}}^2 x^l_{33} - x^l_{11}x^l_{22}x^l_{33}}{{x^l_{23}}^2 - x^l_{22}x^l_{33}}}\epsilon^3, \sqrt{x^l_{22} - \frac{{x^l_{23}}^2}{x^l_{33}}}\epsilon\epsilon', \sqrt{x^l_{33}}\epsilon'\right)\frac{v}{\sqrt{2}} \tag{24}$$

$$(s^l_{12}, s^l_{23}, s^l_{13}) \approx \left(\left|\frac{x^l_{13}x^l_{23} - x^l_{12}x^l_{33}}{{x^l_{23}}^2 - x^l_{22}x^l_{33}}\right|\frac{\epsilon^2}{\epsilon'}, \frac{x^l_{23}}{x^l_{33}}\epsilon, \frac{x^l_{13}}{x^l_{33}}\frac{\epsilon^3}{\epsilon'}\right) \tag{25}$$

$$R^l = \begin{pmatrix} 1 & s^l_{12} & s^l_{13} \\ s^l_{12} & 1 & s^l_{23} \\ (s^l_{12}s^l_{23} - s^l_{13}) & s^l_{23} & 1 \end{pmatrix} \tag{26}$$

Unlike mass generation of up quarks we need to consider some other mechanism to explain very small masses of neutrinos. Here we consider type-1 seesaw mechanism for explaining small neutrino masses. The effective Yukawa interactions for neutrinos after the new scalar getting vev is given by

$$\mathcal{L}^Y_\nu = \epsilon^3 h^\nu_{i1}\bar{L}_{1L}\tilde{\phi}N_{iR} + \epsilon h^\nu_{i2}\bar{L}_{2L}\tilde{\phi}N_{iR} + h^\nu_{i3}\bar{L}_{3L}\tilde{\phi}N_{iR} + \frac{M_i}{2}\bar{N}^c_{iR}N_{iR} \tag{27}$$

where M_i are the Majorana mass of the neutrinos. Then corresponding Dirac and Majorana mass matrix for neutrino are given by :

$$M_D = \begin{pmatrix} h^\nu_{11}\epsilon^3 & h^\nu_{12}\epsilon & h^\nu_{13} \\ h^\nu_{21}\epsilon^3 & h^\nu_{22}\epsilon & h^\nu_{23} \\ h^\nu_{31}\epsilon^3 & h^\nu_{32}\epsilon & h^\nu_{33} \end{pmatrix} \frac{v}{\sqrt{2}} \qquad M_R = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \tag{28}$$

where the m_1, m_2 and m_3 are three heavy right handed neutrino masses. Now by using the seesaw formula [40–43] we get the light neutrino mass matrix as :

$$M_\nu = M_D M_R^{-1} {M_D}^T \approx \begin{pmatrix} \frac{{h^\nu_{13}}^2}{m_3} + \frac{{h^\nu_{12}}^2\epsilon^2}{m_2} & \frac{h^\nu_{13}h^\nu_{23}}{m_3} + \frac{h^\nu_{12}h^\nu_{22}\epsilon^2}{m_2} & \frac{h^\nu_{13}h^\nu_{33}}{m_3} + \frac{h^\nu_{12}h^\nu_{32}\epsilon^2}{m_2} \\ \frac{h^\nu_{13}h^\nu_{23}}{m_3} + \frac{h^\nu_{12}h^\nu_{22}\epsilon^2}{m_2} & \frac{{h^\nu_{23}}^2}{m_3} + \frac{{h^\nu_{22}}^2\epsilon^2}{m_2} & \frac{h^\nu_{23}h^\nu_{33}}{m_3} + \frac{h^\nu_{22}h^\nu_{32}\epsilon^2}{m_2} \\ \frac{h^\nu_{13}h^\nu_{33}}{m_3} + \frac{h^\nu_{12}h^\nu_{32}\epsilon^2}{m_2} & \frac{h^\nu_{23}h^\nu_{33}}{m_3} + \frac{h^\nu_{22}h^\nu_{32}\epsilon^2}{m_2} & \frac{{h^\nu_{33}}^2}{m_3} + \frac{{h^\nu_{32}}^2\epsilon^2}{m_2} \end{pmatrix} \tag{29}$$

After diagonalizing this effective light neutrino mass matrix we get the three light neutrino masses as :

$$\begin{aligned}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}) \approx \Big(0, & \frac{\epsilon^2 (h_{12}^2\,(h_{23}^2 + h_{33}^2) - 2h_{12}h_{13}(h_{22}h_{23} + h_{32}h_{33})}{m_2(h_{13}^2 + h_{23}^2 + h_{33}^2)} + \\ & \frac{h_{13}^2\,(h_{22}^2 + h_{32}^2) + (h_{23}h_{32} - h_{22}h_{33})^2}{m_2\,(h_{13}^2 + h_{23}^2 + h_{33}^2))}, \frac{h_{13}^2 + h_{23}^2 + h_{33}^2}{m_3}\Big)\end{aligned} \tag{30}$$

$$(s_{12}, s_{23}, s_{13}) \approx \Big(\frac{m_2 h^\nu_{13}h^\nu_{23} + m_3 h^\nu_{12}h^\nu_{22}\epsilon^2}{m_2{h^\nu_{23}}^2 + m_3{h^\nu_{22}}^2\epsilon^2}, \frac{m_2 h^\nu_{23}h^\nu_{33} + m_3 h^\nu_{22}h^\nu_{32}\epsilon^2}{m_2{h^\nu_{33}}^2 + m_3{h^\nu_{23}}^2\epsilon^2}, \frac{m_2 h^\nu_{13}h^\nu_{33} + m_3 h^\nu_{12}h^\nu_{32}\epsilon^2}{m_2{h^\nu_{33}}^2 + m_3{h^\nu_{32}}^2\epsilon^2}\Big) \tag{31}$$

As all off diagonal elements of R^l are at least suppressed by one power of ϵ's so, we can say the rotation matrix correspond to charged lepton are essentially identity matrix. So, the PMNS matrix becomes

$$V_{PMNS} = R^{\nu\dagger} R^l \approx R^{\nu\dagger} \tag{32}$$

where, the $R^{\nu\dagger}$ is parameterized by Eqn.(31). If we take the heavy right handed heavy neutrino mass scale at around 10^{11} to 10^{14} GeV then these masses and mixing angles will satisfy the the observed values of neutrino mass square differences and mixing angles with $\mathcal{O}(1)$ value of dimensional less Yukawa couplings. We will discuss a bench mark values of Yukawa coupling for all fermions in appendix.

IV. PHENOMENOLOGY OF THE MODEL

As all the fermions in our model couple to only one Higgs doublet, so the Yukawa matrices and fermion mass matrices can be simultaneously diagonalized. Because of this, there are no tree level flavour changing neutral current (FCNC) in the Higgs portal. [44, 45] which is consistent with the phenomenological bound [1]. But as the different generations of up, down type quarks and charged leptons have different charge so the there a is possibility of FCNC through new gauge boson ($Z^{'}$) interactions. There will be no mass mixing of $Z-Z^{'}$ as there is no common *vev* which spontaneously break both $U(1)_X$ and $SU(2)_L \times U(1)_Y$ [46, 47] and we consider that the kinetic mixing between $Z-Z^{'}$ is very small. Since Z' couples to quarks and leptons according to $U(1)_X$ charges given in Table I, the branching ratio of $Z' \to e^- e^+$ and $Z' \to \mu^- \mu^+$ is for proportional to $(n_5^{1,2})^2/\sum_{i=1}^{3} 3((n_2^i)^2 + (n_3^i)^2) + ((n_5^i)^2 + (n_6^i)^2$.

$$\frac{\Gamma(Z' \to e^- e^+)}{\Gamma(Z' \to \mu^- \mu^+)} = \frac{(n_5^1)^2}{(n_5^2)^2} = 9 \tag{33}$$

This ratio can be used to distinguish this model with other $U(1)_X$ models. One interesting thing is that the τ has zero $U(1)_X$ charge, so $Z' \to \tau^+ \tau^-$ decay is not possible in this model. Observation of $Z' \to \tau^+ \tau^-$ decay will rule out our model. Since the $U(1)_X$ for different family of fermion are not diagonal the gauge interactions of the new gauge boson with the charged fermions are given as :

$$\mathcal{L}_{int} = g_F^{'} Z' \left[3\bar{u}_R^{'} \gamma^\mu u_R^{'} + \bar{c}_R^{'} \gamma^\mu c_R^{'} - 3\bar{d}_R^{'} \gamma^\mu d_R^{'} - \bar{s}_R^{'} \gamma^\mu s_R^{'} - 3\bar{e}_R^{'} \gamma^\mu e_R^{'} - \bar{\mu}_R^{'} \gamma^\mu \mu_R^{'} \right] + h.c \tag{34}$$

where, $u_R^{'}, d_R^{'}, c_R^{'}, s_R^{'}, e_R^{'}, \mu_R^{'}$ are in interaction eigenstates. To get their mass eigenstates we need to rotate them by (20). Then the flavour changing interaction in leading orders is given by

$$\begin{aligned} \mathcal{L}_{int}^{'} = g_F^{'} Z' \Big[& \left(s_{23}^u \bar{c}_R \gamma_\mu t_R - s_{23}^d \bar{s} \gamma_\mu b_R - s_{23}^l \bar{\mu}_R \gamma_\mu \tau_R \right) - 4 \left(s_{12}^d \bar{d}_R \gamma_\mu s_R + s_{12}^l \bar{e}_R \gamma_\mu \mu_R \right) \\ & + 4 s_{12}^u \epsilon^2 \bar{c}_R \gamma_\mu u_R + \left((3 s_{13}^d - 4 s_{12}^d s_{23}^d) \bar{d}_R \gamma_\mu b_R + (3 s_{13}^l - 4 s_{12}^l s_{23}^l) \bar{e}_R \gamma_\mu \tau_R) \right) \Big] + h.c \end{aligned} \tag{35}$$

For an example using Eq.(35) we can study the rare $\mu^- \to e^- e^- e^+$ decay in the context of this model. The amplitude of this process based on interaction lagrangian (35) is given by

$$\mathcal{M}(\mu^- \to e^- e^- e^+) = \frac{3 s_{12}^l g_F^2}{M_{Z'}^2} \left[\bar{u}_e \gamma^\mu \left(1 + \gamma_5\right) u_\mu \right] \left[\bar{v}_e \gamma^\mu \left(1 + \gamma_5\right) u_e \right] \tag{36}$$

Experimentally we know that branching ratio of the decay is less than 1.0×10^{-12} [1]. This gives the constraint

$$s_{12}^l \left(\frac{g_F M_W}{M_{Z'}} \right)^4 < 2.5 \times 10^{-6} \tag{37}$$

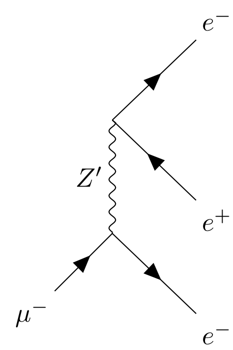

FIG. 1. Feynman diagram for $\mu \to eee$ via new gauge boson

where, M_W is the W boson mass. Then for $g_F = 0.1$ and $s^l_{12} \approx 10^{-3}$. The lower bound of $M_{Z'}$ is 10 TeV. In the quark sector the flavor changing neutral current process are introduced by the terms

$$(d_R\gamma_\mu b_R)^2 + h.c \qquad (s_R\gamma_\mu b_R)^2 + h.c \qquad (d_R\gamma_\mu s_R)^2 + h.c \tag{38}$$

These will contribute to the mixing of $\bar{B}^0 - B^0$, $\bar{B}^0_s - B^0_s$ and $\bar{K}^0 - K^0$ mixing data. As in [28] we can find the contribution of those operator in the mass splitting of the various mesons as :

$$\Delta M_B = 4.5 \times 10^{-2}(3s^d_{13} - 4s^d_{12}s^d_{23})^2(g^2_F/m^2_{Z'}) \tag{39}$$

$$\Delta M_{B_s} = 6.4 \times 10^{-2}s^d_{23}(4s^d_{12})^2(g^2_F/m^2_{Z'}) \tag{40}$$

$$\Delta M_K = 1.9 \times 10^{-3}(s^d_{23})^2(g^2_F/m^2_{Z'}) \tag{41}$$

If we take $M'_Z = 10$ TeV and $g_F = 0.1$then this contribution and SM contribution can explain the experimental values of this mass splittings [1].

V. RENOMALIZABLE DIMENSION 4 OPERATOR REALIZATION

We can describe the effective higher dimension operators of Eq.(11) in terms of dimension four operators by adding some vector-like fermions as shown in Table II. Due to the vector-like nature of the extra fermions, they will create no anomaly [48, 49]. With the above charge assignments of vector like fermions and along with charge assignment given in Table I for other fields, the renormalizable Yukawa interactions associated with up type quarks and down type quarks are respectively :

$$\mathcal{L}^u_Y = \bar{Q}_{iL}\tilde{\phi}f^u_0 + \bar{f}_0\chi_1 f^u_1 + \bar{f}^u_1\chi_1 f^u_2 + \bar{f}^u_2\chi_1 u_{1R} + \bar{Q}_{iL}\tilde{\phi}f^u_0 + \bar{f}_0\chi_1 f^u_1 + \bar{f}^u_1\chi_1 u_{2R} + \bar{Q}_{iL}\tilde{\phi}u_{3R} + h.c \tag{42}$$

TABLE II. Charges of vector-like fermions

Particles	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	Z_2
f_0^u	3	1	2/3	0	+
f_1^u	3	1	2/3	1	+
f_2^u	3	1	2/3	2	+
f_0^d	3	1	-1/3	0	-
f_1^d	3	1	-1/3	-1	+
f_2^d	3	1	-1/3	-2	-
f_0^l	1	1	-1	0	-
f_1^l	1	1	-1	-1	+
f_2^l	1	1	-1	-2	-

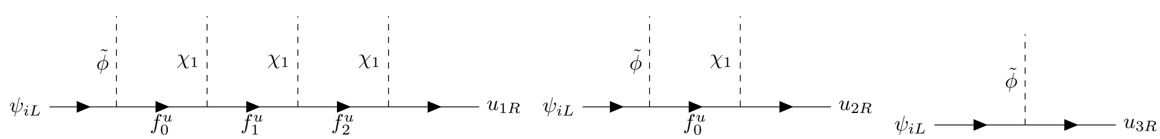

FIG. 2. Up type quarks mass generation through dimension four operators

$$\mathcal{L}_Y^d = \bar{Q}_{iL}\phi f_0^d + \bar{f}_0^d\chi_1^* f_1^d + \bar{f}_1^d\chi_1^* f_2^d + \bar{f}_2^d\chi_2 d_{1R} + \bar{Q}_{iL}\phi f_1^* + \bar{f}_0^d\chi_1^* f_2^d + \bar{f}_2^d\chi_2 d_{2R} + \bar{Q}_{iL}^d\phi f_0^d + \bar{f}_0^d\chi_2 d_{3R} + h.c \tag{43}$$

The renormalizable Yukawa interactions associated with charged leptons are similar as down

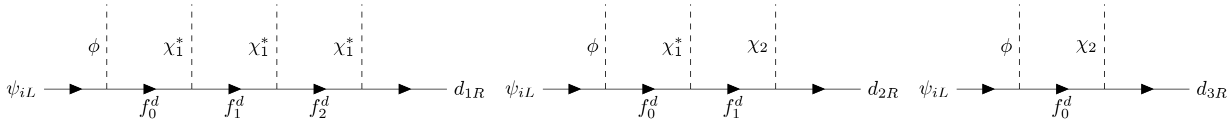

FIG. 3. Down type quarks mass generation through dimension four operators

type quarks only replace f_i^d with f_i^d, $\bar{Q}_{iL}$ with $\bar{L}_{iL}$ and d_{iR} with l_{iR}. Integrating out these fermions produces higher dimensional operators as discussed earlier.

VI. CONCLUSION

After extending the SM gauge symmetry by extra $U(1)_X$ and Z_2 symmetries we have successfully explained the fermions masses and mixing hierarchies by augmenting the SM field by three right handed neutrinos and two scalar singlets. The best fit values of all fermion masses and mixing can be achieved for all Yukawa couplings in the range of 0.1 to 4. As the new $U(1)_X$ considered to be of sequential type, it produces tree level FCNC process through the new gauge boson, but the decay width and cross sections of processes are with in the experimental upper bounds of those quantities . We have also discuss possible UV completion of the theory by considering some extra vector like fermions where all the interaction are the renormalizable dimension 4 operators.

VII. APPENDIX

For calculating the best fit value of the Yukawa couplings we define a χ^2 as :

$$\chi^2 = \sum_{i,j=1}^{3} \frac{(m_{ui} - m_{ui}^{model})^2}{\sigma_{m_{ui}}^2} + \frac{(m_{di} - m_{di}^{model})^2}{\sigma_{m_{di}}^2} + \frac{(m_{li} - m_{li}^{model})^2}{\sigma_{m_{li}}^2}$$
$$+ \frac{(\sin\theta_{ij} - \sin\theta_{ij}^{model})^2}{\sigma_{\sin\theta_{ij}}^2} + \frac{\Delta m_{ij}^2 - \Delta m_{ij}^{2\ model}}{\sigma_{\Delta m_{ij}^2}} + \frac{(\sin\theta_{ij}^{\nu} - \sin\theta_{ij}^{\nu\ model})^2}{\sigma_{\sin\theta_{ij}}^2} \quad (44)$$

The quantities with out superscripts indicates their observed values and the quantity with superscripts model indicate the value of the quantity according to our model and we have summed over all the 6 different quarks masses, 3 charged lepton masses, 3 quarks mixing angles, 2 mass square differences of neutrino and 3 lepton mixing angles. The masses of the charged fermion at about 1 TeV energy scale as given in [50] are :

$$(m_t, m_c, m_u) \approx (150.7 \pm 3.4, 0.532^{+0.074}_{-0.073}, (1.10^{+0.43}_{-0.37}) \times 10^{-3}) GeV$$
$$(m_b, m_s, m_d) \approx (2.43 \pm 0.08, 4.7^{+1.4}_{-1.3} \times 10^{-2}, 2.50^{+1.08}_{-1.03} \times 10^{-3}) GeV$$
$$(m_\tau, m_\mu, m_e) \approx (1.78 \pm 0.2, 0.105^{+9.4\times10^{-9}}_{-9.3\times10^{-9}}, 4.96 \pm 0.00000043 \times 10^{-4}) GeV \quad (45)$$

The mass square difference and $\sin^2\theta_{ij}$ corresponding to neutrino mixing in the case of normal hierarchy are given by [1]

$$\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} eV^2 \qquad \Delta m_{32}^2 = (2.45 \pm 0.0.033) \times 10^{-3} eV^2 \quad (46)$$

$$\sin^2(\theta_{12}) = 0.307 \pm 0.013 \quad \sin^2(\theta_{23}) = 0.546 \pm 0.021 \quad \sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2} \tag{47}$$

To calculate the best fit value of the Yukawa couplings for our desirable interval, we minimizing the χ^2. Considering $\epsilon_1 \approx \epsilon_2 \approx 0.0236$, $m_2 = 10^{11}$ GeV, $m_3 = 10^{14}$ GeV and considering the variation of Yukawa couplings in the range of 0.1 to 4 (which is suitable for validity of perturbation), we find the best fit value of the Yukawa couplings for quarks, charged leptons and neutrinos as

$$h^u = \begin{pmatrix} 0.10 & 0.30 & 0.46 \\ 3.35 & 0.10 & 0.35 \\ 0.11 & 0.44 & 0.64 \end{pmatrix} \qquad h^d = \begin{pmatrix} 3.99 & 0.16 & 0.19 \\ 1.10 & 1.39 & 0.50 \\ 4 & 0.13 & 0.18 \end{pmatrix}$$
$$h^l = \begin{pmatrix} 0.19 & 0.45 & 0.22 \\ 0.16 & 0.58 & 0.20 \\ 0.10 & 1.93 & 0.23 \end{pmatrix} \qquad y^\nu = \begin{pmatrix} 1.64 & 2.23 & 0.11 \\ 2.47 & 2.70 & 3.23 \\ 1.99 & 0.1 & 4 \end{pmatrix} \tag{48}$$

VIII. ACKOWLEDGEMENT

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