

Stark Many-Body Localisation Under Periodic Driving

Christian Duffin,¹ Aydin Deger,^{1,2} and Achilleas Lazarides¹

¹*Interdisciplinary Centre for Mathematical Modelling and Department of Mathematical Sciences,
Loughborough University, Loughborough, Leicestershire LE11 3TU, United Kingdom*

²*Department of Physics and Astronomy, University College London, London WC1E 6BT, United Kingdom*

We study stability of localisation under periodic driving in many-body Stark systems. We find that localisation is stable except near special resonant frequencies, where resonances cause delocalisation. We provide approximate analytical arguments and numerical evidence in support of these results. This shows that disorder-free broken ergodicity is stable to driving, opening up the way to studying nonequilibrium driven physics in a novel setting.

I. INTRODUCTION

Generic ergodic quantum systems are expected to thermalise according to the predictions of statistical mechanics. Such ergodic systems are understood to be in agreement with the eigenstate thermalisation hypothesis (ETH) [1–3], which states that the eigenstate expectation values of any local observable form a smooth function of energy. It is therefore interesting to study systems that violate the ETH, thus exhibiting non-ergodic behaviour. Ergodicity can be broken in various ways including glassiness, prethermalisation [3], or kinetic constraints [4–7]. More recently there has been considerable attention devoted to disorder-induced many-body localisation (MBL) [8–11]. In Anderson’s original work, it was shown that transport in a tight-binding model for a single particle is suppressed by inherent disorder [12]. MBL extends this phenomenon to the case of interacting systems. Similarly, it has been long known that transport can instead be suppressed by the presence of a tilted potential, leading to so-called Wannier-Stark localisation [13]. Here, the single-particle eigenstates become localised in space and equidistant in energy by a spacing that corresponds to the gradient of the tilt. Only much more recently was this shown to also persist in the presence of interactions, giving what is known as Stark MBL [14–19].

Periodic driving of a noninteracting Wannier-Stark system has been studied thoroughly. It was found that driving at a resonant frequency determined by the tilt angle results in delocalisation for arbitrarily weak driving [20, 21]. Here, we investigate the analogous situation in an interacting system using both approximate analytical and exact numerical methods, finding that delocalisation does still occur near certain resonant frequencies, but only above a finite driving strength. The situation is therefore analogous to that for disordered MBL systems [22, 23].

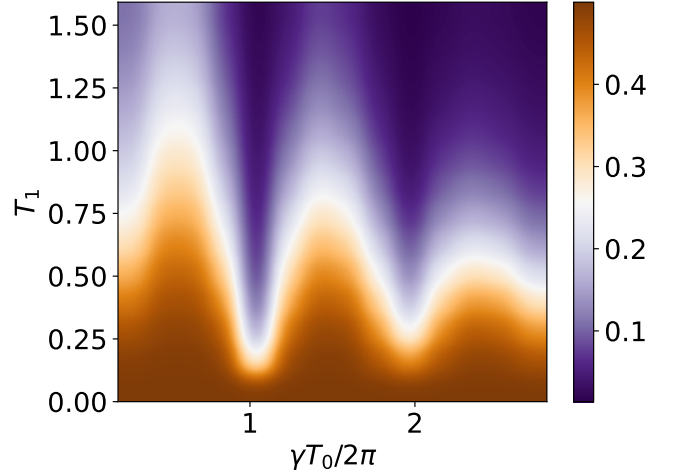


FIG. 1. Phase diagram of a driven Stark MBL system for the unkicked (diagonal) period T_0 vs kicked (off-diagonal) period T_1 . The metric used is the standard deviation σ of the probability distribution for the EEVs $\langle n_6 \rangle$, where n_6 is the number operator for a particle on site 6 of a 1D lattice. The ergodic regime is represented by the purple region $\sigma \approx 0$, corresponding to all EEVs equaling 0.5. The MBL regime is represented by the orange region $\sigma \approx 0.5$, corresponding to all EEVs equaling 0 or 1. The valleys in the MBL regions at integer multiples of $\gamma T_0 / 2\pi$ indicate resonant points, where driving occurs at a frequency that eliminates energy separations caused by the tilted potential with gradient γ . Parameters used are $L = 14$, $J = V = 0.5$, $\gamma = 5$, $w = 0.05$, and 14 data points are used for both axes.

II. MODEL AND ANALYTICAL ARGUMENTS

A. Hamiltonian

We will focus on the time-periodic Hamiltonian

$$H(t) = \begin{cases} H_0, & 0 < \text{mod}(t, T) \leq T_0 \\ H_1, & T_0 < \text{mod}(t, T) \leq T \end{cases} \quad (1)$$

with period $T = T_0 + T_1$ and

$$H_0 = -\gamma \sum_j^L j n_j + \sum_j^L h_j n_j + V \sum_j^{L-1} n_j n_{j+1}$$

diagonal in the basis of product (Fock) states, while

$$H_1 = -J \sum_j^{L-1} (b_j b_{j+1}^\dagger + \text{h.c.}).$$

Here the b_j are hard-core bosonic operators and our system lives on a 1D lattice of size L . The parameters J and V denote the amplitudes of nearest-neighbour hopping and interactions, respectively, while γ denotes the gradient of the tilted potential. The times T_0 and T_1 correspond to the diagonal H_0 and off-diagonal H_1 respectively, such that T_1 controls the strength of the driving and the system is trivially localised in the $T_1 = 0$ limit.

In the limit $h_j = 0$, the static system $H_0 + H_1$ is not localised [24], suggesting that the Floquet system described by Eq. 1 will also be delocalised. We therefore add a small disorder term where the h_j are drawn from a zero-mean normal distribution with variance w . To ensure that the disorder itself does not cause MBL, we set $w \ll V$. This is enough for the static system to be localised [24].¹

Throughout this work, the parameters used will be $J = V = T_1 = 0.5$, $\gamma = 5$, $w = 0.05$ (See Appendix A for motivation). Finally, we work at half-filling and with open boundary conditions.

The Floquet operator for this Hamiltonian, propagating the system over a period, is then

$$\mathcal{F} = e^{-iH_0 T_0} e^{-iH_1 T_1}. \quad (2)$$

B. Mapping to RZS

One can obtain insight into the behaviour of this system by recasting the problem in the so-called Repeated Zone Scheme (RZS) [25]. To facilitate this, we recast the Hamiltonian of Eq. 1 as

$$H(t) = \bar{H} + \delta H(t), \quad (3)$$

where the time-independent part $\bar{H}$ is the time average of $H(t)$ over a single period while $\delta H(t)$ is the rest. We have

$$\begin{aligned} \bar{H} &= H_0 \frac{T_0}{T} + H_1 \frac{T_1}{T} \\ &= \sum_j \left[\tilde{J} (b_j b_{j+1}^\dagger + \text{h.c.}) + \tilde{V} n_j n_{j+1} + (\tilde{\gamma} j + \tilde{h}_j) n_j \right] \end{aligned} \quad (4)$$

with $\tilde{J} = JT_1/T$, $\tilde{V} = VT_0/T$, $\tilde{\gamma} = \gamma T_0/T$, and $\tilde{h}_j = h_j T_0/T$ and

$$\delta H(t) = \begin{cases} \delta H_1 = \bar{H} \frac{T}{T_0} - H_1 \frac{T_1}{T_0}, & \text{mod } (t, T) \leq T_0 \\ \delta H_2 = \bar{H} \frac{T}{T_1} - H_0 \frac{T_0}{T_1}, & T_0 < \text{mod } (t, T) \leq T \end{cases}. \quad (5)$$

One can eliminate the time dependence at the price of working with infinitely many coupled undriven systems [25]. Expanding in Fourier components, one finds that our system corresponds to a time-independent system on an infinite ladder, the n^{th} rung of which hosts a copy of the static part of the Hamiltonian $\bar{H}$ shifted in energy by $n\omega$. The rungs are then coupled by the Fourier components of the driving Hamiltonian, $\delta H(t)$ which cause hopping between the rungs. Denoting by $|\alpha\rangle$ a basis state of the Hilbert space of $\bar{H}$, the time-independent system can be written in terms of the states $|\alpha, n\rangle = |\alpha\rangle \otimes |n\rangle$ localised in rung n and the physical basis state α in the form

$$\langle \alpha, n | H_F | \beta, m \rangle = H_{\alpha, \beta}^{(n-m)} - \omega n \delta_{n, m} \delta_{\alpha, \beta}. \quad (6)$$

Here, $H_{\alpha, \beta}^{(p)}$ are the matrix elements $\langle \alpha | H^{(p)} | \beta \rangle$ of the p^{th} Fourier component of the physical Hamiltonian, $H^{(p)}$. Writing $H^{(0)} = \bar{H}$ and $H^{(p)} = \delta H^{(p)}$ for $p \neq 0$ (since these are the Fourier components of δH of Eq. 3), H_F can be represented as follows:

$$H_F = \begin{pmatrix} \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\ \dots & \bar{H} - 2\omega & \delta H^{(1)} & \delta H^{(2)} & \delta H^{(3)} & \delta H^{(4)} & \dots \\ \dots & \delta H^{(1)} & \bar{H} - \omega & \delta H^{(1)} & \delta H^{(2)} & \delta H^{(3)} & \dots \\ \dots & \delta H^{(2)} & \delta H^{(1)} & \bar{H} & \delta H^{(1)} & \delta H^{(2)} & \dots \\ \dots & \delta H^{(3)} & \delta H^{(2)} & \delta H^{(1)} & \bar{H} + \omega & \delta H^{(1)} & \dots \\ \dots & \delta H^{(4)} & \delta H^{(3)} & \delta H^{(2)} & \delta H^{(1)} & \bar{H} + 2\omega & \dots \\ \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad (7)$$

Physically, the hopping between different rungs corresponds to the system gaining or losing energy in integer multiples of ω , and is mediated by the components

$$\delta H^{(n)} = \frac{\delta H_1}{2\pi i n} \left(e^{2\pi i n T_1/T} - 1 \right) + \frac{\delta H_2}{2\pi i n} \left(e^{2\pi i n} - e^{2\pi i n T_2/T} \right). \quad (8)$$

We note for later reference that the components decay algebraically with n .

Solutions of the time-dependent Schroedinger equation $(i\partial_t - H(t))|\phi(t)\rangle = 0$ can be written as $|\phi_a(t)\rangle = e^{i\Omega_a t} \sum_n |\varphi_a^{(n)}\rangle e^{in\omega t}$, where $|\varphi_a\rangle = \sum_{n=-\infty}^{\infty} |\varphi_a^{(n)}\rangle \otimes |n\rangle$ is an eigenstate of the time-independent Hamiltonian (7) with eigenvalue Ω_a . Here $|n\rangle$ indexes the rung, so that $|\varphi_a^{(n)}\rangle$ is the part of the state living in a single rung. When considering stroboscopic dynamics, $|\phi_a(pT)\rangle = e^{i\Omega_a pT} \sum_n |\varphi_a^{(n)}\rangle$ (for integer p); this implies that all rungs (all n) are equivalent, and one sums up the eigenvectors across all rungs.

¹ Alternatively, in the limit of $T_1 = h_j = V = 0$, conservation of dipole moment means that all Fock states with the same dipole moment (centre of mass) will be degenerate. If then $T_1 > 0$, these states will strongly hybridise, delocalising the system. Adding weak disorder lifts the degeneracy.

What does this picture say about the system of Eqn. 3? In the limit we are working in ($h_j = V = 0$) and $\delta H(t)$, the $|\varphi_a^{(n)}\rangle$ are localised and independent of n ; they are the eigenstates $|\alpha\rangle$ of $\bar{H}$, which are many-body Stark localised states with eigenvalues $q_\alpha \tilde{\gamma}$ where the q_α are integers.²

To delocalise the system the driving must couple $|\varphi_a^{(n)}\rangle$ with different a in different rungs n , so that the stroboscopic dynamics of the resulting state will be delocalised. Perturbatively in δH , there are two leading-order ways this can happen.

The first is that two eigenstates $|\alpha\rangle$ and $|\beta\rangle$ in neighbouring rungs are coupled by the matrix element $\langle\alpha|\delta H^{(1)}|\beta\rangle$; the energy difference is $\bar{\epsilon}_\alpha - \bar{\epsilon}_\beta = m\tilde{\gamma}$ with integer m ; since they are on neighbouring rungs, the condition on ω for this is $\bar{\epsilon}_\alpha - \bar{\epsilon}_\beta = m\tilde{\gamma} = \omega$ from which we find $\frac{\gamma T_0}{2\pi} = \frac{1}{m}$.

Now the matrix element $\langle\alpha|\delta H^{(1)}|\beta\rangle$ is exponentially suppressed whenever $|\alpha\rangle$ and $|\beta\rangle$ differ by moving a particle by more than one site (because $\delta H^{(1)}$ has only nearest-neighbour hopping and the $|\alpha\rangle$ are localised). Thus $m = 1$ is dominant and we expect delocalisation to be clearly visible at $\frac{\gamma T_0}{2\pi} = 1$.

The second is that two eigenstates $|\alpha\rangle$ and $|\beta\rangle$ in rungs n and $n + p$, with $p > 1$, respectively are coupled by $\langle\alpha|\delta H^{(p)}|\beta\rangle$. The resonance condition is then $\bar{\epsilon}_\alpha - \bar{\epsilon}_\beta = m\tilde{\gamma} = p\omega$ leading to $\frac{\gamma T_0}{2\pi} = \frac{p}{m}$. Like before, $m > 1$ is exponentially suppressed, while the matrix element $\langle\alpha|\delta H^{(p)}|\beta\rangle$ decays linearly with p ; since $p > 1$, $p = 2$ dominates and we expect strong signs of delocalisation at $\frac{\gamma T_0}{2\pi} = 2$, with weaker delocalisation for increasing p .

Thus there are resonances at

$$\gamma T_0 / 2\pi = p/m \quad (9)$$

for integer p, m , with $p = m = 1$ the dominant one, followed by $p = 2, m = 1$, then $p = 3, m = 1$ and so on. The ones for $m > 1$ are strongly suppressed because of the locality of $\delta H(t)$ and will not be visible in our numerics. These conclusions are in broad agreement with the results of Holthaus and Hone on single-particle systems [20].

Away from the limit $h_j = V = 0$, this picture is no longer exact. We therefore use numerics to study to what extent the conclusions survive away from this limit. To check whether the system is localised we implement two numerical probes of localisation, and find that they do indeed confirm our conclusions.

III. EIGENSTATE PROPERTIES

A localised system violates the ETH, while delocalisation via periodic driving is expected to coincide with its reestablishment [22, 23]. We therefore probe directly ETH by checking whether the eigenstate expectation values (EEVs) form a smooth function of the eigenenergies or not. Selecting our local observable to be the number density of a particle on some fixed site j , we calculate the EEV for each eigenstate $|\epsilon_\alpha\rangle$ as $\langle n_i \rangle = \langle \epsilon_\alpha | n_i | \epsilon_\alpha \rangle$, and plot these versus the quasienergies $\epsilon_\alpha = -(\ln \Theta_\alpha) / iT$ where Θ_α are the eigenvalues of the propagator $\mathcal{F}$ corresponding to $|\epsilon_\alpha\rangle$.

In a maximally localised system half of the EEVs will equal 1 and the rest 0, while in an infinite-temperature system, as is expected for a driven, delocalised, ergodic system [26], they will all be equal to each other and to the infinite-temperature result $\text{tr}(n_j) = 1/2$ [22].

In Figure 2 we plot both the density of states (DOS) as a function of quasienergy (left column) and the probability densities of the EEVs (right column) for various system sizes, both on resonance and off resonance, with the resonant points as determined earlier.

In the off-resonant plots, the EEVs are evenly distributed between both $\langle n_6 \rangle = 0$ and $\langle n_6 \rangle = 1$ as anticipated for the localised case. On the other hand, for resonant driving the EEVs become sharply peaked at $\langle n_6 \rangle = 0.5$, consistent with Floquet ETH. Crucially, the standard deviation decreases with system size, indicating that the system is indeed delocalised in the thermodynamic limit.

To further assess whether our results are reliable, we turn to the DOS plots. In the thermodynamic limit, we expect the DOS to be uniform due to the folding of the quasienergies and the mixing of the states caused by the driving. Indeed, in the off-resonant cases the DOS fluctuates around a constant value, with the amplitude of the fluctuations decreasing with system size. This shows that our driving protocol is able to mix the bands that are present in the undriven $T_1 = 0$ (discussed in the Appendix B).

In the resonant cases, the DOS shows instead strong quasienergy dependence. Nevertheless, this is not an issue because this would usually suggest that the system is less localised than appears—but here, the EEVs already show that the system is delocalised. In any case, the DOS becomes more evenly distributed with system size so this is clearly a finite-size effects.

IV. MULTIFRACTALITY

Another signature of broken ergodicity is accessed by studying the structure of the eigenstates of $\mathcal{F}$. In an ergodic Floquet system, an eigenstate is simply a uniform superposition of all Fock states [26]. Conversely, in MBL systems the eigenstates are multifractal in Fock space [27]. This multifractal property can be studied by

² This is because the single-particle states have energies $m\tilde{\gamma}$ with m integer, and a many-body state is simply a Slater determinant of such states thus having an energy that is a sum of these.

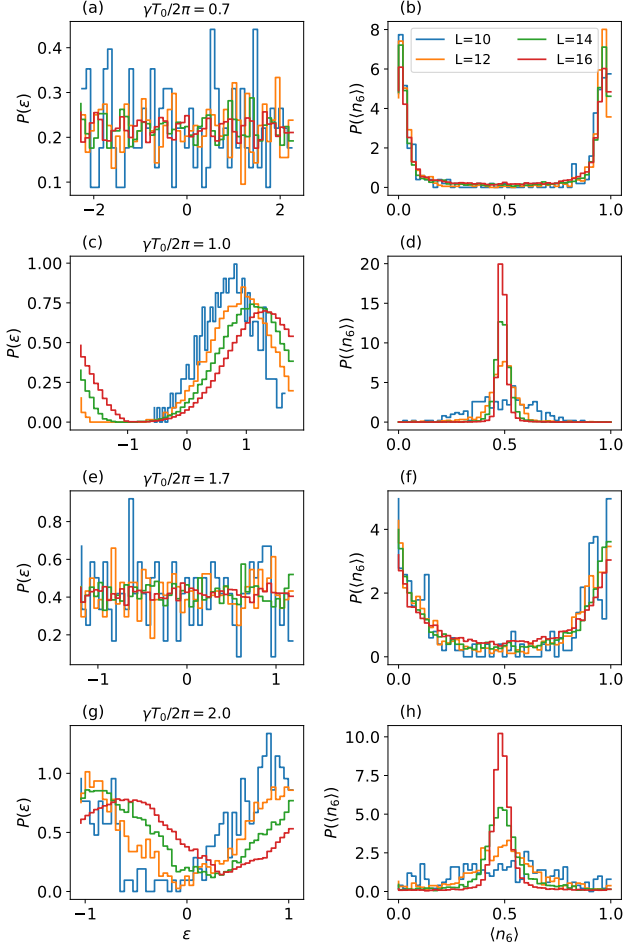


FIG. 2. Left column: Density of states as a function of quasienergy ϵ . Right column: Probability distributions of eigenstate expectation values of the particle number density on site $j = 6$, $\langle n_6 \rangle$ for a periodically driven Stark MBL system, at various system sizes L and unkicked periods T_0 . The second and fourth rows correspond to resonant driving at the resonant points $n = 1.0$ and $n = 2.0$ respectively, where the strongly peaked behaviour around $\langle n_6 \rangle = 0.5$ in figures (d) and (h) indicates a strong lack of energy dependence between the EEVs, which corresponds to the infinite temperature ensemble that is associated with systems exhibiting Floquet ETH. The remaining figures show off-resonant driving, where the EEVs are strongly peaked around $\langle n_6 \rangle = 0$ and $\langle n_6 \rangle = 1.0$. This indicates violation of the ETH and thus the presence of MBL. Parameters used are $J = V = T_1 = 0.5$, $\gamma = 5$, $w = 0.05$.

invoking a metric called generalised inverse participation ratios (IPRs) [28]. This is defined by

$$I_q = \sum_j |\langle \epsilon | j \rangle|^{2q}. \quad (10)$$

At $q = 2$ this is the usual IPR. This depends on the Hilbert space dimension $\mathcal{D}$ via $I_q \sim \mathcal{D}^{-\tau_q}$ with $\tau_q = -\frac{\ln I_q}{\ln \mathcal{D}}$ the scaling exponent. The τ_q are related to the

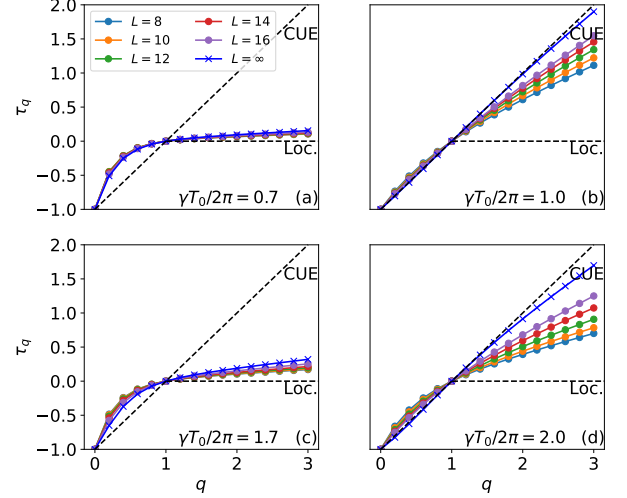


FIG. 3. Averaged scaling exponent τ_q at finite sizes and infinite size for the driven Stark MBL system under periodic driving. Averaging takes place over all eigenvectors and m disorder realisations, where $m = \text{ceil}(10000/\mathcal{D})$. Figures (a) and (c) correspond to off-resonant driving, which shows the expected multifractal behaviour for MBL eigenstates, specifically that the τ_q sit above the thermal prediction at $0 < q < 1$ and below it at $q > 1$. Figures (b) and (d) shows resonant driving at two different resonant points, which shows agreement with the thermal prediction up to values above $q = 1$, indicating that the eigenstates lie in the ETH regime. Parameters used are $J = V = T_1 = 0.5$, $\gamma = 5$, $w = 0.05$.

multifractal dimension D_q , via $D_q = \frac{\tau_q}{q-1}$.

In a maximally localised system (in Fock space), $D_q = 0$ for $q > 0$, which translates to $\tau_q = 0$. In a completely delocalised system, $D_q = 1$, which leads to the random matrix theory (RMT) prediction $\tau_q = q - 1$. The general behaviour for disordered interacting systems is intermediate [28–30]: close to the transition point between MBL and ergodic phase, the eigenstates are multifractal in both regimes.

In Fig. 3 we show the τ_q for a given q at several size, as well as our thermodynamic limit prediction. This was obtained by plotting two finite size values against $1/\ln \mathcal{D}$ and extrapolating to $1/\ln \mathcal{D} = 0$ [28]. In these plots we average over all eigenvectors, as well as a number m of disorder realisations; we select $m = \text{ceil}(10000/\mathcal{D})$.

On resonance (panels (b) and (d)), our extrapolated result for the thermodynamic limit agrees with the thermal prediction up to moments $q^* > 1$. This indicates that the system is in the ETH phase, with a residual multifractality confined to high order moments. Off resonance, the τ_q sit above the thermal prediction in the interval $0 < q < 1$, and below it for all $q > 1$, thus exhibiting behaviour similar to that observed in MBL systems.

V. CONCLUSIONS

We have showed that Stark-MBL systems may remain localised under periodic driving. Such systems are clean and translationally-invariant, in contrast to the usual disordered-MBL systems, which also have this property. We expect that phenomena usually studied in disordered cases should have analogues for Stark systems [31]. There also remain several open questions: How quickly does delocalisation set in at resonances, and how does it depend

on the detuning? Similarly, how slow is heating in the absence of a non-linear component to the potential, when the static system is known to be delocalised, but correlations spread very slowly [24]?

ACKNOWLEDGMENTS

A.D. and A.L. acknowledge support from EPSRC Grant No. EP/V012177/1. During completion of this work our attention was drawn to Ref. [32], which reports results consistent with ours.

-
- [1] J. M. Deutsch, Phys. Rev. A **43**, 2046 (1991).
 - [2] M. Srednicki, Phys. Rev. E **50**, 888 (1994).
 - [3] L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, Advances in Physics **00**, 122 (2015).
 - [4] Z. Lan, M. van Horssen, S. Powell, and J. P. Garrahan, Physical Review Letters **121** (2018).
 - [5] S. Roy and D. Logan, arxiv (2019).
 - [6] A. Deger, S. Roy, and A. Lazarides, Phys. Rev. Lett. **129**, 160601 (2022).
 - [7] A. Deger, A. Lazarides, and S. Roy, Physical Review Letters **129**, 10.1103/physrevlett.129.190601 (2022).
 - [8] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, Ann. Phys. (N. Y.) **321**, 1126 (2006).
 - [9] V. Oganesyan and D. A. Huse, Phys. Rev. B **75**, 155111 (2007).
 - [10] M. Znidaric, T. Prosen, and P. Prelovšek, Phys. Rev. B **77**, 64426 (2008).
 - [11] A. Pal and D. A. Huse, Phys. Rev. B **82**, 174411 (2010).
 - [12] P. W. Anderson, Phys. Rev. **109**, 1492 (1958).
 - [13] G. H. Wannier, Reviews of Modern Physics **34**, 645 (1962).
 - [14] M. Schulz, C. Hooley, R. Moessner, and F. Pollmann, Physical review letters **122**, 040606 (2019).
 - [15] W. Morong, F. Liu, P. Becker, K. Collins, L. Feng, A. Kyprianidis, G. Pagano, T. You, A. Gorshkov, and C. Monroe, Nature **599**, 393 (2021).
 - [16] S. R. Taylor, M. Schulz, F. Pollmann, and R. Moessner, Physical Review B **102**, 054206 (2020).
 - [17] Q. Guo, C. Cheng, H. Li, S. Xu, P. Zhang, Z. Wang, C. Song, W. Liu, W. Ren, H. Dong, *et al.*, Physical review letters **127**, 240502 (2021).
 - [18] L. Zhang, Y. Ke, W. Liu, and C. Lee, Physical Review A **103** (2021).
 - [19] E. V. Doggen, I. V. Gornyi, and D. G. Polyakov, Physical Review B **103**, L100202 (2021).
 - [20] M. Holthaus and D. W. Hone, *Localization Effects in ac-driven Tight-Binding Lattices*, Vol. 2812(March) (Cornell University Library, 1996) pp. 105–137.
 - [21] N. H. Shon and H. Nazareno, Journal of Physics: Condensed Matter **4**, L611 (1992).
 - [22] A. Lazarides, A. Das, and R. Moessner, Phys. Rev. Lett. **115**, 30402 (2015).
 - [23] P. Ponte, Z. Papić, F. Huveneers, and D. A. Abanin, Phys. Rev. Lett. **114**, 140401 (2015).
 - [24] B. Kloss, J. C. Halimeh, A. Lazarides, and Y. Bar Lev, Nature Communications **14**, 3778 (2023).
 - [25] J. H. Shirley, Phys. Rev. **138**, B979 (1965).
 - [26] A. Lazarides, A. Das, and R. Moessner, Phys. Rev. E **90**, 12110 (2014).
 - [27] A. De Luca and A. Scardicchio, Europhysics Letters **101**, 37003 (2013).
 - [28] D. J. Luitz, I. Khaymovich, and Y. Bar Lev, SciPost Physics Core **2** (2020).
 - [29] D. J. Luitz, N. Laflorencie, and F. Alet, Phys. Rev. B **91**, 81103 (2015).
 - [30] M. Serbyn, Z. Papić, and D. A. Abanin, Physical Review B **96**, 104201 (2017).
 - [31] Y. Bar Lev and A. Lazarides, *Discrete time-crystals in linear potentials* (2024), arXiv:2403.01912 [cond-mat].
 - [32] D. S. Bhakuni, R. Nehra, and A. Sharma, Physical Review B **102** (2020).

Appendix A: Scanning Parameters

One question is how the behaviours seen depend on other parameters, such as J/V . In particular, we are interested in understanding whether there is an optimum value of J/V for which the distinction between MBL and ETH becomes most apparent for a given driving amplitude. One way to explore this is to calculate the standard deviation of the EEVs both on resonance and off resonance, and calculate their difference as a function of J/V . This is demonstrated in Fig. 7(a), with the quantity

$$\Delta\sigma = |\sigma(n = 2.0) - \sigma(n = 0.7)|, \quad (\text{A1})$$

where σ is the standard deviation of the EEV probability distribution $P(\langle n_6 \rangle)$. Here J is the quantity being changed, while $V = 0.5$. The resulting distribution demonstrates the interplay between the limits of trivial localisation $J = 0$ and delocalisation $J \gg V, \gamma$, where the peak indicates the point of clearest distinction between resonant and off-resonant driving.

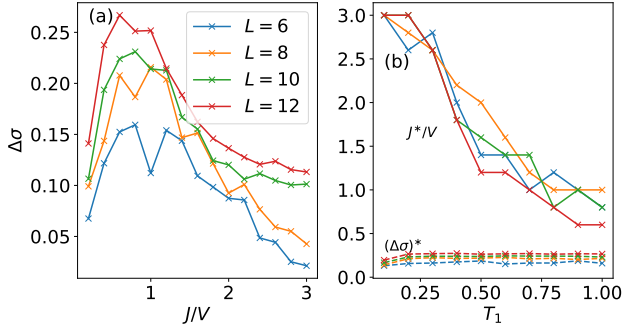


FIG. 4. (a) Difference in the standard deviation of $P(\langle n_6 \rangle)$ for resonant ($n = 2.0$) and off-resonant ($n = 0.7$) driving as defined in equation A1, plotted as a function of J/V . Remaining parameters are $V = 0.5$, $\gamma = 5$, $T_1 = 1.0$. (b) Maximum $\Delta\sigma$ and corresponding J/V , denoted as $(\Delta\sigma)^*$ and J^*/V , respectively, plotted as a function of T_1 .

Next, we obtain the quantities $(\Delta\sigma)^*$ and J^*/V , which are defined as the maximum value of $\Delta\sigma$ and the value of J/V at which this exists, respectively. These quantities are then scanned against T_1 , as shown in Fig. 4(b). J^*/V can be roughly seen to decrease with T_1 . This is unsurprising, as both T_1 and J delocalise the system, meaning that at larger T_1 , lower J is needed in order for MBL to persist off resonance. Importantly, we can see that $(\Delta\sigma)^*$ is independent of T_1 , suggesting that for any amplitude of driving, we will see a similar distinction provided $J/V = J^*/V$. Finally, we note that the values $J = V = 0.5$ correspond approximately to J^*/V at $T_1 = 0.5$, which are the parameters used in Fig. 2, suggesting that the distinction demonstrated is optimum.

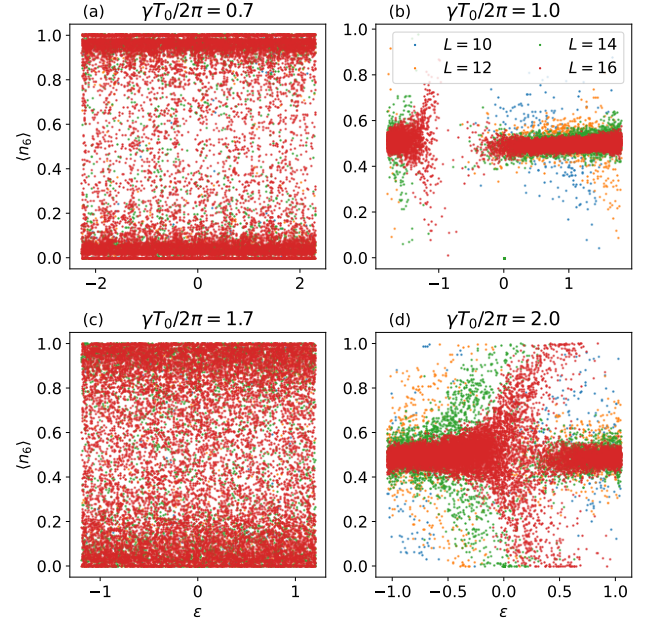


FIG. 5. Scatter plot of the EEVs $\langle n_6 \rangle$ corresponding to the probability distributions shown in the right column of figure 2. The left column shows cases of off-resonant driving, while the right columns correspond to resonant driving. Violation of the ETH is clearly demonstrated off resonance, with the strong fluctuation of EEVs. Meanwhile, the resonant plots showcase agreement with the ETH, except at specific regions within the quasienergy spectrum, where the EEVs are significantly more scattered.

Appendix B: Quasienergy Structure

In the left column in Fig. 2, we can see that the density of states exhibits a weak dependence on the quasienergy while off resonance, and a strong dependence while on resonance. One question of interest is whether this band-structure will have a significant impact on the structure of the EEVs.

In Fig. 5, it can be seen that during resonant driving, the EEVs obey the eigenstate thermalisation hypothesis except at specific regions within the quasienergy spectrum. We propose that this effect is due to massive degeneracies induced by resonant driving in the $T_1 = w = 0$ limit, creating a clear bandstructure in the quasienergies of the Floquet operator $\mathcal{F} = e^{-iH_0T_0}$. Let us look at the case of the second resonant point, $n = 2.0$. Initially, if we also set $V = 0$, the eigenvalues of H_0 will have the form $E = k\gamma$, where k is an integer. The corresponding Floquet eigenvalues are then $\epsilon = e^{-4\pi i k} = 1$. Introducing nearest-neighbour interactions, we have $E = k\gamma + aV$, with $a \in \{0, L/2 - 1\}$ denoting the total number of interactions for a given Fock state. The Floquet eigenvalues are now $\epsilon = e^{-4\pi i(k+aV/\gamma)} = e^{-4\pi i a V/\gamma}$. Here the eigenvalues split into 5 different degeneracies (given $L \geq 12$). The size of each of these degeneracies is de-

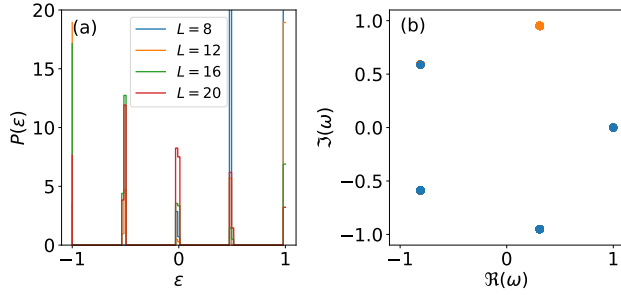


FIG. 6. (a) Distribution of eigenvalues of the Floquet operator $\mathcal{F}$ around the unit circle in the $T_1 = w = 0$ limit, at the second resonant point $n = 2.0$. (b) Probability distribution $P(\epsilon)$ of the corresponding quasienergies. Other parameters used are $V = 0.5$, $\gamma = 5$.

terminated by how many Fock states possess a given total nearest-neighbour interaction. For a total interaction number a , this is determined by the product of binomial coefficients

$$m(a) = \binom{L/2 + 1}{a + 1} \binom{L/2 - 1}{a}. \quad (\text{B1})$$

The distribution of the Floquet eigenvalues around the complex unit circle is shown in Fig. 6(b), along with the probability distribution of the corresponding quasienergies in (a). Next, if we re-introduce small disorder w , these degeneracies will split into bands. In Fig. 7 we plot $P(\epsilon)$ by calculating the eigenvalues analytically, which reveals how the dips in the distributions correspond to the scattering in the EEVs shown in Fig. 5(d) (in particular, the dips are shown to shift right with increasing L , which can be seen to match the shift of the EEVs.)

This behaviour can now be understood as follows: resonant driving eliminates energy differences introduced by the tilted potential term in the eigenvalues of $\mathcal{F}$, resulting in a bandstructure that is governed by the distribution of nearest-neighbour interactions in H_0 . This creates specific regions within the quasienergy spectrum that are sparsely populated, ensuring that when driving is introduced the eigenstates in this region are subject to less mixing with the rest of the eigenstates. As a result, the eigenstates outside of these regions become similar to each other in accordance with Floquet ETH, while the eigenstates inside the region remain relatively unchanged.

The question of physical interest now is whether this behaviour persists in the thermodynamic limit, or if it is a finite-size effect. We can see from Fig. 7 that $P(\epsilon)$ becomes more evenly distributed with increasing size, suggesting that the weakly mixed regions vanish at infinite size. This is also implied by the reduction in scattering with increasing size seen in the EEVs in Fig. 5.

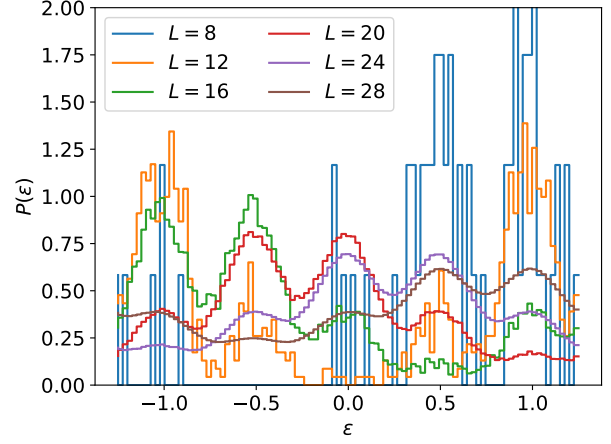


FIG. 7. (a) Probability distribution of quasienergies of F in the $T_1 = 0$ limit, at the second resonant point $n = 2.0$. Parameters used are $V = 0.5$, $\gamma = 5$, $w = 0.05$.