Nakanishi covariant operator formalism for higher derivative systems: Vector spin-0 dual model as a prelude to generalized QED_4

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In this work we extend the Kugo-Ojima-Nakanishi covariant operator formalism to quantize two higher derivative systems, taking into account their extended phase space structures. More specifically, the one describing spin-0 particles by a vector field and the generalized electrodynamics. We investigate the commutator structure of these theories and present the definition of their physical Hilbert subspaces. Remarkably, the establishment of a second-class nature for the primary constraints of such models demands a higher derivative structure for the auxiliary field Lagrangian following previous claims. Regarding the first model, it presents a reducible gauge symmetry implying the necessity of two sets of auxiliary fields. We also discuss its massless limit. For the case of the generalized QED_4, we derive a set of suitable definitions for the positive-definite Hilbert subspace in order to eliminate contributions from spurious modes and also the problematic negative norm transverse excitation. We show that the Hamiltonian operator taken within the domain of this subspace presents no instabilities. Finally, a set of discussions on the interacting regime are developed to ensure that the scattering processes restricted to the physical Hilbert subspace remain unitary even at this context.

I. INTRODUCTION

Higher derivative field theories can hardly be overestimated if one considers the wide range of their applications in different areas of physics. Regarding field theories, the inclusion of higher derivative operators can also be understood as quantum corrections [1]. Accordingly, as in [2–9], the Lee-Wick model, a higher derivative theory, can be regarded as a system obtained by elevating the status of the Pauli-Villars regulators to dynamical degrees of freedom. Another pertinent model correlated to this context is the so-called Bopp-Podolsky Lagrangian [10–15]. It leads to a

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generalized electrodynamics whose associated static potential is well-behaved at the origin [16–18]. Moreover, as an ultraviolet improvement [19–26], it also leads to a finite self-energy for the electron.

This kind of features based on a higher derivative Lagrangian can occur in a set of alternative contexts such as in one loop renormalized linearized quantum gravity, for example. The generation of higher derivative quantum corrections can be directly verified from the functional renormalization group analysis [27–29]. One can also cite the alternative framework of the string theory in which a generalized gravitational theory including higher derivative terms, associated with the product of curvatures, emerges in the low energy limit [30, 31]. Finally, although these aforementioned models may be associated with the presence of extra ghost particles with negative norms [32, 33], it is possible to show that non-linear interactions provide a way to overcome these Ostrogadskian instabilities. The quantum corrections imply an evanescent nature for these modes, eliminating them from the asymptotic spectrum [34, 35]. We also mention the importance of the Lee-Wick models in gravity [36–45] due to the mechanisms that conciliate renormalizability with unitarity. Finally, it is worth mentioning the rich and longevous discussion on these ghosts [46–56], in which its general properties are investigated.

The other fundamental issue of the paper is closely related to the issue of duality in field theories. Namely, using the master action approach one can derive dual relations between theories describing spinless particles using a scalar field and the ones associated with a vector field description [57, 58]. The latter model presents no particle content at the massless limit. Additionally, one can also consider another alternative description based on an antisymmetric field, the so-called massless Kalb-Ramond (KR) model [59–61]. Interestingly, the massive version of the latter describes spin-1 particles meaning that a degree of freedom discontinuity occurs at the zero mass limit. Regarding spin-1 particles, there is also an alternative dual description in terms of a symmetric tensor field [57]. The latter also presents the previously mentioned kind of degree of freedom jump in the massless limit. This is a topic of current research, see the following papers addressing this discontinuity for the (KR) model [62, 63] and also for the case of non-Abelian theories [64].

Regarding the higher derivative spin-0 model studied in this paper, it can be related to the previously mentioned vector second-order derivative one through the master action technique [57]. Interestingly, this higher derivative (UV) completion also contributes to overcoming the massless discontinuity presented by the parent model. This target model also has a reducible gauge symmetry structure with direct implications in the definition of the path integral formulation. Here, we intend to be the first ones to investigate these issues using a complementary Heisenberg operator quantization in an indefinite metric Hilbert space.

Here, we extend the Kugo-Ojima-Nakanishi (KON) covariant operator formalism [65, 66] to quantize two Ostrogadskian systems, taking into account their generalized phase space structures [67, 68]. More specifically, the one describing spin-0 particles by a vector field [57] and the Bopp-Podolsky generalized electrodynamics [19]. We discuss a set of features related to the reducible structure of the local freedom [69] and a well-defined massless limit [62] for the first, and topics related to the correct definition of the positive norm subspace for the latter model, which is known to present transverse modes with a negative norm.

The (KON) formalism is suitable to be applied for Lagrangian systems with a singular Hessian matrix. A set of auxiliary fields is properly included in such a way to provide a covariant model whose primary constraints are of second-class in the Dirac classification [70]. In this manner, the Dirac brackets can be introduced to correctly project the model into the physical surface in which the constraints are valid in their strong form, eliminating quantization ambiguities. Therefore, considering the correspondence principle, the whole set of commutator's initial conditions can be obtained. From the operator equations of motion, the full commutator structure at unequal times can be provided. The quantization formalism is based on the Heisenberg description of an indefinite metric Hilbert space. This is a useful setting since the auxiliary fields as well as the negative norm spurious gauge field modes can be excluded from the physical subspace by an appropriate definition of the latter. This is achieved by a condition imposed by the auxiliary fields or even the BRST charges acting on the states, depending on the specific model considered.

This formalism has a relevant variety of applications ranging from Abelian and non-Abelian gauge theories to quantum gravity [65]. It is possible to cite a variety of research associated with this formalism, such as the investigation of quantum gravity in tetrad formalism[1] [71], the definition of a complementary tool for BRST symmetry extensions [72], the discussion of mass generation mechanisms [73], in QCD confinement research [74] and to unveil formal aspects of QED_4 in the so-called non-linear t'Hooft gauge [75, 76]. One can also mention the application in topological condensed matter describing subtle aspects of the statistical interaction [77]. Interestingly, it can directly provide the exact quantization of the two-dimensional non-Abelian BF model [78].

This formalism also has a perturbative counterpart. The general structure of the Wightman functions can be inferred by a method that consists of extracting the truncated n point functions from the n point commutators by requiring the positive energy condition. For a review including applications in the perturbative quantization of QED_4, a toy one-loop model, string theory, and

[1] For which the standard approaches generally fail.

the two-dimensional quantum gravity, see [79]. Here, we provide a set of straightforward extensions of such formalism. Since both models investigated in this paper are of a higher derivative nature, one must consider the extended phase space and new fundamental Poisson brackets that characterize the Ostrogadskian systems. It has a direct impact on the set of initial conditions obtained by the correspondence principle. Moreover, regarding the higher derivative description of the spinless vector model, issues related to the reducible spurious sector and the necessity of adding two auxiliary fields are novelties introduced here. Then, in connection with the previously mentioned recent correlated achievements, we show how the Hilbert space structure behaves in the massless limit [62]. Regarding the generalized QED_4, we explicitly show how the successful extension of the formalism indeed demands a higher derivative gauge fixing Lagrangian, being a full quantum completion to the previous efforts [80] implicated by semi-classical considerations. Moreover, we show how to consistently define the positive-definite Hilbert subspace for the intricate case of the Bopp-Podolsky model presenting transverse massive negative norm modes. The interacting regime is also considered and a suitable definition of the physical subspace in this context is derived. We demonstrate that the scattering processes of the interacting model can be established in a unitary manner if the correct definition of the asymptotic physical Hilbert space is considered. This can provide a suitable background to address the renormalized linearized gravity system in which negative norm modes that are not pure gauge arise.

The work is organized as follows. In Sec. II , the Lagrangian structure and the pertinent auxiliary fields for the higher derivative spin-0 model are provided. Later, at Sec. III, the complete set of equations of motion for the system is derived. The Sec. IV is devoted to characterizing the phase space of the system as well as obtaining the primary constraint structure which is second-class due to the presence of the previously mentioned auxiliary fields. In Sec. V, the correspondence principle and the complete set of commutators are derived. Moreover, the definition of the physical subspace and the massless limit are provided. The Sec. VI introduces the Lagrangian of the higher derivative electrodynamics, its auxiliary fields, equations of motion, primary constraints, and the Dirac Brackets. Then, in Sec. VII the full commutator structure is derived, as well as the definition of a physical Hilbert space capable of avoiding spurious gauge projections and also the massive transverse modes with negative norms. It is demonstrated that the Hamiltonian becomes positive-definite in this subspace. The Sec. VIII discusses the theory in the presence of a fermionic interaction. A prescription for obtaining the positive - definite subspace associated with unitary scattering processes is provided. Finally, we conclude in Sec. IX.

The metric signature $(+,-,-,-)$ is used throughout this work.

II. ON THE LAGRANGIAN STRUCTURE AND AUXILIARY FIELDS FOR A DUAL MODEL DESCRIBING SPIN-0 PARTICLES

The Lagrangian describing a spin-0 particle using a vector field embedded in a higher derivative structure reads [57]

$$\mathcal{L} = \frac{1}{2}\Big[\partial_\beta\big(\partial_\mu B^\mu\big)\partial^\beta\big(\partial_\mu B^\mu\big) - m^2\big(\partial_\mu B^\mu\big)^2\Big] + \epsilon^{\mu\nu\rho}\partial_\nu\phi_\rho\big(\Box + m^2\big)B_\mu + \partial^\mu\Omega\big(\Box + m^2\big)\phi_\mu \tag{1}$$

in which the auxiliary fields are explicitly highlighted. To establish a scalar Lagrangian, both auxiliary fields $\phi_\mu(x)$ and $\Omega(x)$ must be pseudo vector and pseudo scalar fields, respectively. We choose to work in $D = 2+1$ dimensions since, in this case, just two auxiliary fields are necessary to provide a second-class system in compliance with the correspondence principle. Therefore, the model has well-defined Dirac Brackets suitable for developing the quantization procedure. We will explicitly prove it in the Sec. IV .

The auxiliary fields are responsible for providing a gauge condition valid in all Hilbert space. This is a demand due to the singular nature of the theory composed just by the gauge vector field. In this case, the gauge field presents the following local freedom

$$B_\mu(x) \to B_\mu(x) + \epsilon_{\mu\nu\sigma}\partial^\nu\Lambda^\sigma(x) \tag{2}$$

with $\Lambda_\sigma(x)$ being a pseudo vector field representing the symmetry parameter. This transformation has a reducible structure presenting the zero modes

$$\Lambda_\alpha(x) = \partial_\alpha\rho(x) \tag{3}$$

in which $\rho(x)$ is an arbitrary pseudo scalar field.

However, the presence of the whole set of auxiliary fields breaks the symmetry through the condition [2] [67]

$$\big(\Box + m^2\big)\big(\epsilon^{\mu\nu\beta}F^B_{\mu\nu}(x) - 2\partial^\beta\Omega\big) = 0 \tag{4}$$

Due to the existence of the zero modes, this extra scalar auxiliary field is demanded to eliminate the freedom associated with any kind of first-class quantities [65]. Although performed in

[2] with the definition $F^B_{\mu\nu}(x) \equiv \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x)$

$D = 2+1$, this process can be carried out for any dimension. We focus on this particular one since, in this case, just one extra auxiliary field is necessary, defining a straightforward generalization of the well-known (KON) quantization. In the reference [81], an analogous quantization process is generalized for D-dimensions for the case of a dual description of a spin 1 model. Finally, beyond this point associated with the reducible nature, the system is also of a higher derivative structure, implying the necessity of introducing an extended phase space.

III. ON THE EQUATIONS OF MOTION

The model's equations of motion are given below

$$\begin{aligned}
-\partial^\mu\big(\Box+m^2\big)\big(\partial_\nu B^\nu(x)\big)+\big(\Box+m^2\big)\epsilon^{\mu\nu\rho}\partial_\nu\phi_\rho(x) &=0\ ,\\
\big(\Box+m^2\big)\partial_\mu\phi^\mu &=0\ ,\\
-\epsilon^{\mu\nu\rho}\partial_\nu\big(\Box+m^2\big)B_\mu(x)+\partial^\rho\big(\Box+m^2\big)\Omega(x) &=0
\end{aligned}\tag{5}$$

Taking the divergence of the first equation yields

$$\Box\big(\Box+m^2\big)\partial_\mu B^\mu(x)=0\tag{6}$$

. This equation has a deep analogy with the two pole structure of the Bopp-Podolsky generalized electrodynamics, despite the facts that there is no negative residue and the pole field is a scalar combination $\partial_\mu B^\mu(x)$.

From the divergence of the last equation of (5), it is possible to derive the pole structure for the auxiliary field $\Omega(x)$

$$\Box\big(\Box+m^2\big)\Omega(x)=0\tag{7}$$

Applying the differential operator $\epsilon_{\mu\nu\alpha}\partial^\nu$ on the first equation and using the auxiliary vector field transverse nature, implies the equation

$$\Box\big(\Box+m^2\big)\phi^\mu(x)=0\tag{8}$$

defining the pole structure of the vector auxiliary field.

IV. ON THE EXTENDED PHASE SPACE STRUCTURE

After this digression, we introduce the generalized canonical momenta structure which, for a fourth derivative model, is defined below

$$
\begin{aligned}
p_\Phi(x) \equiv& \Big[\frac{\partial\mathcal{L}}{\partial(\partial_0\Phi(x))} - 2\partial_i\Big(\frac{\partial\mathcal{L}}{\partial(\partial_i\partial_0\Phi(x))}\Big) - \partial_0\frac{\partial\mathcal{L}}{\partial(\partial_0\partial_0\Phi(x))}\Big]\\
\pi_\Phi(x) \equiv& \frac{\partial\mathcal{L}}{\partial(\partial_0\partial_0\Phi(x))}
\end{aligned}
\tag{9}
$$

with $\Phi(x)$ being a general notation for all fields appearing in the Lagrangian.

Then, for this specific system, the generalized momenta definition is expressed below

$$
\begin{aligned}
&\pi^B_\mu(x) = \big(\ddot{B}^0(x) + \partial_i\dot{B}^i(x)\big)\delta^0_\mu + \epsilon_{\mu\nu\rho}\partial_\nu\phi_\rho(x),\\
&p^B_\mu(x) = -m^2\big(\dot{B}^0(x) + \partial_i B^i(x)\big)\delta^0_\mu + 2\partial^k\partial_k\big(\dot{B}^0(x) + \partial_i B^i(x)\big)\delta^0_\mu\\
&- 2\partial^k\big(\ddot{B}^0(x) + \partial_i\dot{B}^i(x)\big)\delta^\mu_k, -\big(\dddot{B}^0(x) + \partial_i\ddot{B}^i(x)\big)\delta^0_\mu - \epsilon_{\mu\nu\rho}\partial^\nu\ddot{\phi}_\rho(x),\\
&p^\phi_\mu(x) = \epsilon^{k0\mu}\big(\Box + m^2\big)B_k(x) - \partial^\mu\ddot{\Omega}(x)\ ,\ \pi^\phi_\mu(x) = \partial_\mu\Omega(x)\ ,\\
&p^\Omega(x) = \big(\Box + m^2\big)\phi_0(x)\ ,\ \pi^\Omega(x) = 0
\end{aligned}
\tag{10}
$$

In order to define the nature of the primary constraints, we first introduce the fundamental Poisson brackets in this extended phase space $\Big(\Phi(x), \dot{\Phi}(x), p^\Phi(x), \pi^\Phi(x)\Big)$

$$
\Big\{\Phi(x), p^\Phi(y)\Big\} = \mathcal{I}\delta^2(\vec{x}-\vec{y})\ ,\ \Big\{\dot{\Phi}(x), \pi^\Phi(y)\Big\} = \mathcal{I}\delta^2(\vec{x}-\vec{y})
\tag{11}
$$

with $\mathcal{I}$ denoting the identity written in terms of the specific field tensor structure.

The primary constraints are the ones generating non-dynamical relations between the phase space degrees of freedom. They are the following $\pi^B_j(x) = \epsilon^{j0k}\dot{\phi}_k(x) + \epsilon^{jl0}\partial_l\phi_0(x)$, $\pi^\phi_\mu(x) = \partial^\mu\Omega(x)$ and $\pi^\Omega(x) = 0$. Then, it proves that the correct introduction of the complete set of auxiliary fields, indeed leads to a second-class system from the beginning even in this higher derivative scenario. The constraints can be suitably grouped as

$$
\begin{aligned}
&\mathcal{C}^{(1)}_\mu(x) \equiv \big(-\pi^j_B(x) - \epsilon^{ij}\dot{\phi}_j(x) + \epsilon^{ij}\partial_j\phi_0(x)\big)\delta^\mu_i + \pi^\Omega(x)\delta^\mu_0 \approx 0\\
&\mathcal{C}^{(2)}_\mu(x) \equiv \pi^\phi_\mu(x) - \partial^\mu\Omega(x) \approx 0
\end{aligned}
\tag{12}
$$

being associated with the following inverse matrix of constraints

$$\mathcal{G}^{\mu\nu}_{IJ}(x,y) = \{\mathcal{C}^I_\mu(x), \mathcal{C}^J_\nu(y)\}^{(-1)} = \begin{pmatrix} 0 & -\epsilon_{ij}\delta^i_\mu\delta^j_\nu + \delta^0_\mu\delta^0_\nu \\ \epsilon_{ij}\delta^i_\mu\delta^j_\nu - \delta^0_\mu\delta^0_\nu & 0 \end{pmatrix} \delta^2(\vec{x}-\vec{y}) \tag{13}$$

enabling one to establish the Dirac brackets, the key object to provide the system's quantization

$$\Big\{F(x), G(y)\Big\}_D = \Big\{F(x), G(y)\Big\} - \int d^3z\; d^3w \Big\{F(x), \mathcal{C}^I_\mu(z)\Big\}\mathcal{G}^{\mu\nu}_{IJ}(z,w)\Big\{\mathcal{C}^J_\nu(w), G(y)\Big\} \tag{14}$$

for which the constraints are valid in their strong form.

V. FROM THE COMMUTATOR INITIAL CONDITIONS TO ITS FINAL FORM AT UNEQUAL TIMES

From the Dirac brackets, the correspondence principle can be performed yielding the following set of non-vanishing commutators

$$\begin{aligned}
&\Big[B_\mu(x), p^\nu_B(y)\Big]_0 = i\delta^\nu_\mu\delta^2(\vec{x}-\vec{y})\ , \quad \Big[\dot{B}_\mu(x), \pi^\nu_B(y)\Big]_0 = i\delta^\nu_\mu\delta^2(\vec{x}-\vec{y}) \\
&\Big[\Omega(x), p^\Omega(y)\Big]_0 = i\delta^2(\vec{x}-\vec{y})\ , \qquad \Big[\phi_\mu(x), p^\nu_\phi(y)\Big]_0 = i\delta^\nu_\mu\delta^2(\vec{x}-\vec{y}) \\
&\Big[\partial_0\phi_\mu(x), \pi^\nu_\phi(y)\Big]_0 = i\delta^\nu_\mu\delta^2(\vec{x}-\vec{y})
\end{aligned}$$

leading to a set of commutator's initial conditions due to the momentum definition and the imposition of the primary constraints

$$\begin{aligned}
&\Big[\dot{B}_0(x), \ddot{B}_0(y)\Big]_0 = i\delta^2(\vec{x}-\vec{y})\ , \quad \Big[\dot{B}_j(x), \epsilon^{ik}\dot{\phi}_k(y)\Big]_0 = -i\delta^i_j\delta^2(\vec{x}-\vec{y}), \\
&\Big[B_0(x), \ddot{B}_0(y)\Big]_0 = -i\delta^2(\vec{x}-\vec{y})\ , \qquad \Big[\phi_0(x), \ddot{\Omega}(y)\Big]_0 = -i\delta^2(\vec{x}-\vec{y}), \\
&\Big[\dot{\phi}_0(x), \dot{\Omega}(y)\Big]_0 = i\delta^2(\vec{x}-\vec{y})\ , \quad \Big[\phi_i(x), \epsilon^{jk}\ddot{B}_k(x)\Big]_0 = i\delta^j_i\delta^2(\vec{x}-\vec{y})
\end{aligned} \tag{15}$$

The first commutator to be addressed here is associated with the auxiliary scalar field. Since it obey

$$\Box\big(\Box + m^2\big)^x\Big[\Omega(x), \Omega(y)\Big] = 0 \tag{16}$$

then, using the properties of the distributions below [65]

$$\begin{aligned}
&\Box\Delta(x-y;s) = -s\Delta(x-y;s), \quad \Delta(x-y;s)\Big|_0 = 0, \quad \dot{\Delta}(x-y;s)\Big|_0 = \delta^2(\vec{x}-\vec{y}), \\
&\big(\Box + s\big)E(x-y;s) = \Delta(x-y;s), \quad E(x-y;s)\Big|_0 = 0, \quad \ddot{E}(x-y;s)\Big|_0 = \delta^2(\vec{x}-\vec{y}).
\end{aligned} \tag{17}$$

representing their initial conditions, one proves that the general solution must be of the form

$$\Big[\Omega(x), \Omega(y)\Big] = a\Delta(x-y, m) + b\Delta(x-y, 0) \tag{18}$$

Owing to the canonical momenta definition (10), one can obtain $\dot{\Omega}(x)$ in terms of $\pi_0^\phi(x)$. Then, considering the general Ostrogadskian phase space structure (11), it is possible to derive the initial condition

$$\Big[\dot{\Omega}(x), \Omega(y)\Big]_0 = 0 \tag{19}$$

implying $a + b = 0$. Since $p_0^\phi(x)$ and $\pi_0^\phi(x)$ are not phase space conjugate variables, we also have

$$\Big[\dot{\Omega}(x), \ddot{\Omega}(y)\Big]_0 = 0 \tag{20}$$

leading to $a = 0$ and, therefore, a vanishing commutator at unequal times

$$\Big[\Omega(x), \Omega(y)\Big] = 0 \tag{21}$$

Following similar reasoning, one obtains

$$\Big[\Omega(x), B_\nu(y)\Big] = 0 \tag{22}$$

Now, we derive the commutator between the gauge and vector auxiliary field. As already mentioned, this and the previous commutator are the key ones to understand how a consistent definition of the positive Hilbert space can be established. The general form is given below

$$\Big[B_\mu(x), \phi_\rho(y)\Big] = i\epsilon_{\mu\rho\nu}\partial^\nu\big(a\Delta(x-y, m^2) + b\Delta(x-y, 0)\big) \tag{23}$$

taking into account the pseudo-vector nature of $\phi_\mu(x)$ as well as its transverse condition implicated by the operator equations of motion.

This commutator complies with the fact that $\phi_\rho(x)$ lies in the kernel of the $\Box\big(\Box + m^2\big)$ operator and obeys the gauge condition $(\Box + m^2)\partial_\mu\phi^\mu(x) = 0$. From the initial conditions

$$\Big[B_j(x), \ddot{\phi}_m(y)\Big]_0 = i\epsilon_{mj}\delta^2(\vec{x} - \vec{y}) \tag{24}$$

the relation $a = -b$ with $a = \frac{i}{m^2}$ is achieved.

The commutator at unequal times reads

$$\Big[B_\mu(x), \phi_\rho(y)\Big] = \frac{i}{m^2}\epsilon_{\mu\rho\nu}\partial^\nu\big(\Delta(x-y, m^2) - \Delta(x-y, 0)\big) \tag{25}$$

It means that just the non-physical transverse part of $B_\mu(x)$ has a non-zero commutator with $\phi_\mu(x)$. It furnishes a background to further derive the subsidiary condition defining the positive-definite Hilbert subspace.

In order to achieve this objective, the commutator between the different auxiliary fields must be obtained. Owing to Lorentz covariance, the general form for the mentioned commutator is given below

$$\Big[\phi_\rho(x), \Omega(y)\Big] = \partial_\rho\big(a\Delta(x-y, m^2) + b\Delta(x-y, 0)\big) \tag{26}$$

since both field operators are in the kernel of $\Box\big(\Box + m^2\big)$ and the scalar one is coupled to the longitudinal part of $\phi_\mu(x)$. Then, the initial conditions

$$\Big[\phi_0(x), \ddot{\Omega}(y)\Big]_0 = -i\delta^2(\vec{x} - \vec{y}) \tag{27}$$

and the distribution properties imply $a = -b$ and $a = -\frac{i}{m^2}$, leading to

$$\Big[\phi_\rho(x), \Omega(y)\Big] = -\frac{i}{m^2}\partial_\rho\big(\Delta(x-y, m^2) - \Delta(x-y, 0)\big) \tag{28}$$

It is worth mentioning that the last two commutators present well-defined massless limits since $\Delta(x-y, m^2) = \big(\Delta(x-y, 0) - m^2 E(x-y, 0) + ...\big)$ for $m \to 0$. This is an important distinguishing feature since there are other dual models with reducible local symmetries presenting a kind of DVZ discontinuity in their massless limit. Concretely, one can cite the Kalb-Ramond model describing scalar particles or spin-1 ones in terms of anti-symmetric tensor fields in the massless/massive phases, respectively.

The derivation of the commutator between the auxiliary vector fields is the final goal to achieve a complete description of the auxiliary sector. Considering the gauge fixing equation and the initial conditions, one obtains

$$\Big[\phi_\mu(x), \phi_\nu(y)\Big] = 0 \tag{29}$$

defining the zero norm nature of such field operators.

The last step before defining the physical Hilbert space is the establishment of the vector gauge field commutator. Considering its equation of motion, a compatible general form for the commutator reads

$$\begin{aligned}\Big[B_\mu(x), B_\nu(y)\Big] =& a\partial_\mu\partial_\nu\Delta(x-y, m^2) + d\eta_{\mu\nu}\Delta(x-y, m^2) + c\partial_\mu\partial_\nu E(x-y, 0) + \\ & e\partial_\mu\partial_\nu\Delta(x-y, 0) + g\eta_{\mu\nu}\Delta(x-y, 0)\end{aligned} \tag{30}$$

in which a, c, d, e, g are undetermined constants. This is the most general commutator compatible with the equation

$$\Box\big(\Box + m^2\big)\partial_\mu B^\mu(x) = 0 \tag{31}$$

Since it must also be in accordance with the subsidiary condition imposed on the $B_\mu(x)$ field, one should fix $g = 0$.

Considering the initial condition

$$\Big[\dot{B}_0(x), \ddot{B}_0(y)\Big]_0 = i\delta^2(\vec{x} - \vec{y}) \tag{32}$$

and also another one associated with the extended phase space due to the Ostrogadskian structure

$$\Big[B_i(x), \dot{B}^j(y)\Big]_0 = 0 \tag{33}$$

reduces all the remaining freedom implying the following commutator structure

$$\Big[B_\mu(x), B_\nu(y)\Big] = \frac{i}{m^4}\partial_\mu\partial_\nu\big(\Delta(x-y, m^2) - \Delta(x-y, 0)\big) + \frac{i}{m^2}\partial_\mu\partial_\nu E(x-y, 0) \tag{34}$$

It furnishes with the exact scalar commutator for the pole operators

$$\Big[\partial^\mu B_\mu(x), \partial^\nu B_\nu(y)\Big] = i\Delta(x-y, m^2) \tag{35}$$

It is worth mentioning that this structure defines a spin-0 particle even at the massless limit. An alternative check to this conclusion is the fact that the inter-particle potential derived for this model in [82] is well-defined at this limit. This theory is a higher derivative version of the second-order vector spin-0 one [57]. Since the latter loses its particle content at $m \to 0$, it indicates that the use of a higher derivative structure may avoid these discontinuities. A full discussion on this theme is provided in the appendix concerning the full Hamiltonian analysis of the model's Ostrogadskian phase space.

Having established the commutator structure for all the fields, one can derive a suitable subsidiary condition to avoid the emergence of the auxiliary fields in the positive Hilbert subspace. Therefore, a good subsidiary condition to define the positive semi-definite Hilbert subspace is

$$\phi^+_\mu(x)|\text{phys}\rangle = 0, \quad \forall|\text{phys}\rangle \in \mathcal{V}_{\text{phys}}. \tag{36}$$

in which $\phi^+_\mu(x) = \phi^{+(m)}_\mu(x) + \phi^{+(0)}_\mu(x)$ denotes the sum of the positive frequency parts of the massive and massless solutions of the $\phi(x)$ field equations of motion. According to the whole set of the previously derived commutators, this definition eliminates the spurious non-positive norm gauge field projections as well as both the auxiliary fields from the positive-definite metric Hilbert subspace

$$\mathcal{H}_{phys} = \frac{\mathcal{V}_{phys}}{\mathcal{V}_0} \tag{37}$$

defined as the completion of the quotient space above. The zero norm states associated with the action of the auxiliary fields on the vacuum, spanning the space $\mathcal{V}_0$, are suitably avoided by this definition.

It is straightforward to show that the scalar operator $\partial_\mu B^\mu(x)$ commutes with the vector auxiliary field, fulfilling the subsidiary condition. It means that the negative frequency part of this field creates a scalar particle with a positive norm when acting on the vacuum state, defining a physical state. Therefore, the present developments provide an extension of the (KON) formalism for systems with reducible gauge symmetry structure and of fourth order in derivatives.

VI. THE BOPP-PODOLSKY HIGHER DERIVATIVE ELECTRODYNAMICS

The Lagrangian for the higher order electrodynamics reads [19–21]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2m^2}\partial_\lambda F^{\alpha\lambda}\partial^\rho F_{\alpha\rho} + \partial^\mu B\big(\frac{\Box}{m^2}+1\big)A_\mu \tag{38}$$

We consider a higher derivative structure for the gauge fixing sector due to two reasons. First, as we are going to see, it is the most general condition compatible with a pole equation for the vector field in the physical subspace [80]. The other is; considering the previously introduced Ostrogadskian phase space structure, this higher-order term contributes to generating a set of non-vanishing generalized momenta responsible for turning the system into a second-class one, enabling the establishment of the correspondence principle by means of Dirac Brackets. This is one of the fundamental underlying principles of the covariant operator formalism [65, 79].

The equations of motion are the following

$$(\Box + m^2)\partial^\mu A_\mu = 0\ , \quad (\Box + m^2)\big(\partial^\nu F_{\nu\mu} + \partial_\mu B\big) = 0\ , \quad \Box\big(\Box + m^2\big)B = 0 \tag{39}$$

From the previous relations, the vector field equation can be properly decoupled

$$\Box^2\big(\Box + m^2\big)A_\mu(x) = 0 \tag{40}$$

According to (10), the phase space variables in this Ostrogadskian [67, 68] system reads

$$
\begin{aligned}
p_\alpha(x) &= -F_{0\alpha}(x) - \frac{1}{m^2}\big(\partial_k\partial_\lambda F^{0\lambda}(x)\delta^k_\alpha - \partial_0\partial_\lambda F_\alpha{}^\lambda - \partial_\alpha\partial_0 B\big), \\
\pi_\alpha(x) &= \frac{1}{m^2}\big(\partial_\lambda F^{0\lambda}(x)\delta^0_\alpha - \partial_\lambda F_\alpha{}^\lambda(x)\big) + \frac{1}{m^2}\partial_\alpha B(x), \\
p_B(x) &= (\frac{\Box}{m^2}+1)A_0\ , \quad \pi_B(x) = 0
\end{aligned}
\tag{41}
$$

The momenta definition furnishes two constraints which are of second-class [70], see [80]

$$
\mathcal{C}^{(1)} \equiv \pi_0(x) - \frac{1}{m^2}\dot{B}(x) \approx 0 \quad , \quad \mathcal{C}^{(2)} \equiv \pi_B(x) \approx 0 \tag{42}
$$

implying that the gauge fixing sector has a suitable form to provide the quantization process. The inverse of the constraint matrix reads

$$
\mathcal{G}^{IJ}(x,y) = \{\mathcal{C}^I(x), \mathcal{C}^J(y)\}^{(-1)} = \begin{pmatrix} 0 & +m^2 \\ -m^2 & 0 \end{pmatrix} \delta^3(\vec{x}-\vec{y}) \tag{43}
$$

which can be used to build the Dirac brackets, the fundamental object to define the quantization process by the correspondence principle

$$
\{F(x), G(y)\}_D = \{F(x), G(y)\} - \int d^3z d^3w \{F(x), \mathcal{C}_I(z)\}\mathcal{G}^{IJ}(z,w)\{\mathcal{C}^J(w), G(y)\} \tag{44}
$$

Then, it is possible to obtain a well-defined bracket for which the constraints are valid in the strong form.

VII. ON THE COMMUTATOR STRUCTURE

In order to define all the field commutators at unequal times, one must consider the equations of motion and the initial conditions provided by the correspondence principle. The extended phase space for a system of fourth order in derivatives is $\epsilon_\mu \equiv \big(A_\mu(x), \dot{A}_\mu(x), B(x), \dot{B}(x), p_\alpha(x), \pi_\alpha(x), p_B(x), \pi_B(x)\big)$ and the fundamental Poisson brackets structure is the one defined in (11). With this knowledge, the Dirac brackets (44) can be established. Then, considering the correspondence principle, the following set of initial conditions can be derived

$$
\begin{aligned}
&\Big[\dot{A}_i(x), \ddot{A}^j(y)\Big]_0 = im^2\delta^j_i\delta^3(\vec{x}-\vec{y})\ , \quad &&\Big[A_0(x), \ddot{B}(y)\Big]_0 = im^2\delta^3(\vec{x}-\vec{y}), \\
&\Big[A_\mu(x), \dot{A}^\nu(y)\Big]_0 = 0\ , &&\Big[A_i(x), \dddot{A}^j(y)\Big]_0 = -im^2\delta^j_i\delta^3(\vec{x}-\vec{y})
\end{aligned}
\tag{45}
$$

In order to infer the general structure of the commutator involving the gauge and the auxiliary field, we use the operator equations of motion to obtain an equation valid at unequal times

$$\Box^y\big(\Box + m^2\big)^y\Big[A_\mu(x), B(y)\Big] = 0 \tag{46}$$

Then, owing to Lorentz covariance, we establish the most general commutator in the kernel of the above equation

$$\Big[A_\mu(x), B(y)\Big] = a\partial_\mu\Delta(x-y;0) + b\partial_\mu\Delta(x-y;m^2) \tag{47}$$

in which a and b are undetermined constants.

Then, considering the initial condition below

$$\Big[A_0(x), \ddot{B}(y)\Big]_0 = im^2\delta^3(\vec{x}-\vec{y}) \tag{48}$$

one concludes that that $a = -b$ and $a = i$, leading to

$$\Big[A_\mu(x), B(y)\Big] = i\big(\partial_\mu\Delta(x-y;0) - \partial_\mu\Delta(x-y;m^2)\big) \tag{49}$$

Then, we conclude that the auxiliary field does not commute with the spurious longitudinal projections of the gauge field, as it should be.

Following similar reasoning, one can also derive

$$\Big[B(x), B(y)\Big] = 0 \tag{50}$$

expressing the zero-norm character of the auxiliary field.

Considering the equation for the vector field $A_\mu(x)$,

$$\Box^2\big(\Box + m^2\big)A_\mu(x) = 0 \tag{51}$$

it is possible to infer the most general commutator for the vector field

$$\begin{aligned}\Big[A_\mu(x), A_\nu(y)\Big] &= a\big(\eta_{\mu\nu}\Delta(x-y;0) - \partial_\mu\partial_\nu E(x-y;0)\big) + b\big(\eta_{\mu\nu}\Delta(x-y,m^2)\\ &+ \frac{1}{m^2}\partial_\mu\partial_\nu\Delta(x-y,m^2)\big) + d\partial_\mu\partial_\nu E(x-y,0) + e\partial_\mu\partial_\nu E(x-y,m^2)\\ &+ r\eta_{\mu\nu}E(x-y,m^2) + n\eta_{\mu\nu}E(x-y,m^2) + f\partial_\mu\partial_\nu\Delta(x-y,m^2) + c\partial_\mu\partial_\nu\Delta(x-y,0)\end{aligned} \tag{52}$$

written in a specific convenient manner.

The gauge condition imposes a restriction on the commutator

$$\big(\Box + m^2\big)\partial^\mu\Big[A_\mu(x), A_\nu(y)\Big] = 0 \tag{53}$$

implying the following constraints $d = e = n = r = 0$.

From the initial condition

$$\left[A_i(x), \dot{A}^j(y)\right]_0 = 0 \tag{54}$$

the following relations are imposed $a + b = 0$ and $\frac{b}{m^2} + c + f = 0$.

The compatibility with the condition

$$\left[\dot{A}_i(x), \ddot{A}^j(y)\right]_0 = im^2 \delta_i^j \delta^3(\vec{x} - \vec{y}) \tag{55}$$

eliminates the remaining freedom, revealing the structure of the vector field commutator

$$\begin{aligned}\left[A_\mu(x), A_\nu(y)\right] &= -i\big(\eta_{\mu\nu}\Delta(x-y,0) - \partial_\mu\partial_\nu E(x-y,0)\big) \\ &+ i\big(\eta_{\mu\nu}\Delta(x-y,m^2) + \frac{1}{m^2}\partial_\mu\partial_\nu\Delta(x-y,m^2)\big) \\ &- i\frac{1}{m^2}\partial_\mu\partial_\nu\Delta(x-y,0)\end{aligned} \tag{56}$$

Using the fact that the simple and double pole massive Pauli-Jordan distributions tend to zero at $m \to \infty$, the Maxwellian form for the commutator is recovered at this limit [65]

$$\left[A_\mu(x), A_\nu(y)\right] = -i\big(\eta_{\mu\nu}\Delta(x-y;0) - \partial_\mu\partial_\nu E(x-y;0)\big) \tag{57}$$

Now, considering the full commutator structure of (56), the subsidiary condition can be defined. Let's try to apply the same one used to characterize the positive semi-definite Hilbert subspace of QED_4

$$B^+(x)|\text{phys}\rangle = 0, \quad \forall|\text{phys}\rangle \in \mathcal{V}_{\text{phys}}. \tag{58}$$

with $B^+(x) = B^{+(0)}(x) + B^{+(m)}(x)$ being the sum of massive and massless positive frequency parts of the auxiliary field operator.

Although this condition prevents spurious gauge field projections from appearing in the physical subspace, it does not eliminate transverse massive negative norm states present in (56).

This feature is not a failure of the B field approach, this model indeed presents this kind of ghost particle as well as, for example, some higher derivative gravity models [83]. However, we can generalize this approach to correctly separate the positive semi-definite Hilbert subspace. Then, we consider two conditions [3]

$$B^{+(0)}(x)|\text{phys}\rangle = 0, \quad \Box^2 A_\mu^+(x)|\text{phys}\rangle = 0 \quad \forall|\text{phys}\rangle \in \mathcal{V}_{\text{phys}}. \tag{59}$$

[3] Here, $B^{+(0)}(x) \equiv (\Box + m^2)B^+(x)$, with the $+$ label denoting the positive frequency part of the field.

First of all, we notice that the two conditions are compatible. Moreover, the auxiliary field $B^{+(m)}(x)$ and the whole massive pole contribution to $A_\mu(x)$ are necessarily outside of the physical subspace due to commutations relations (49) and (56). The longitudinal spurious contribution from the massless sector is also excluded due to the condition generated by the B-field. Therefore, we can establish a well-defined positive-definite Hilbert subspace even for the Bopp-Podolsky model as the Cauchy completion of

$$\mathcal{H}_{\text{phys}} = \frac{\mathcal{V}_{\text{phys}}}{\mathcal{V}_0} \tag{60}$$

with $\mathcal{V}_0$ representing the zero norm subspace. This definition is Poincaré invariant.

A. On the positive-definite Hilbert subspace

The quantum vector field can be expressed in terms of creation and annihilation operators as [4]

$$A_\mu(x) = \int \frac{d^3p}{(2\pi)^{3/2}}\Big[\sum_\lambda \frac{1}{\sqrt{2|\vec{p}|}}\big(e^{-ipx}a_\lambda(\vec{p})\epsilon^\lambda_\mu(\vec{p}) + e^{ipx}a^\dagger_\lambda(\vec{p})\epsilon^{*\lambda}_\mu(\vec{p})\big) + \sum_\sigma \frac{1}{\sqrt{2\sqrt{|\vec{p}|^2+m^2}}}\big(e^{-i\bar{p}x}\bar{a}_\sigma(\vec{p})\chi^\sigma_\mu(\vec{p}) + e^{i\bar{p}x}\bar{a}^\dagger_\sigma(\vec{p})\chi^{*\sigma}_\mu(\vec{p})\big)\Big] \tag{61}$$

with $p_\mu = (\sqrt{\vec{p}^2}, \vec{p})$, $\bar{p}_\mu = (\sqrt{\vec{p}^2+m^2}, \vec{p})$ and $[a^\lambda(\vec{p}), (a^{\lambda'})^\dagger(\vec{q})] = [\bar{a}^\lambda(\vec{p}), (\bar{a}^{\lambda'})^\dagger(\vec{q})] = \eta^{\lambda\lambda'}\delta^3(\vec{p}-\vec{q})$.

This field operator structure is in compliance with equation (56) if the following polarization sums,

$$\eta_{\lambda\lambda'}\epsilon^\lambda_\mu(\vec{p})\epsilon^{*\lambda'}_\nu(\vec{p}) = -\theta_{\mu\nu}(p) - \frac{p_\mu p_\nu}{m^2} \tag{62}$$

$$\eta_{\lambda\lambda'}\chi^\lambda_\mu(\vec{\bar{p}})\chi^{*\lambda'}_\nu(\vec{\bar{p}}) = g_{\mu\nu} - \frac{\bar{p}_\mu\bar{p}_\nu}{m^2} \tag{63}$$

with $\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$, are considered.

Owing to the equation (56), for a given specific frame $p_\mu = (p,0,0,p)$, the physical modes with positive norm have the following commutator

$$\Big[\tilde{a}_i(p), \tilde{a}^\dagger_j(q)\Big] = \delta_{ij}(2\pi)^3\theta(p_0)\delta(p^2)\delta^4(p-q) \tag{64}$$

[4] In natural units.

with $i = 1, 2$ denoting spatial coordinates. They are associated with the alternative setting of the positive and negative frequency parts of the vector field [65]

$$A_\mu^{(+)}(x) = \int d^4p \big(\tilde{a}_\mu(p) e^{-ip.x} + a'_\mu(p) e^{-ip.x}\big) \ ; \ A_\mu(x) = A_\mu^{(+)}(x) + A_\mu^{(-)}(x) \tag{65}$$

The physical modes previously highlighted are associated with the annihilation operators as $\tilde{a}_\mu(p) \equiv \sum_\lambda a^\lambda(\vec{p})\epsilon_\mu^\lambda(\vec{p})\theta(p_0)\delta(p^2)\sqrt{2p_0}$. The operator $a'_\mu(p)$ is related to the massive sector.

It is worth mentioning that the generalized QED_4 has a Hermitian Hamiltonian operator $H = H^\dagger$, ensuring pseudo-unitarity [65]. However, it is not positive-definite as a consequence of its Ostrogadskian structure. Despite this fact, considering the definition of the physical Hilbert subspace (59), one can show that the matrix elements

$$\langle \mathrm{phys}A| : H : |B\mathrm{phys}\rangle = \langle \mathrm{phys}A| \sum_{i=1}^{2} \int \frac{d^3k}{(2\pi)^3} E(k) a_i a_i^\dagger |B\mathrm{phys}\rangle \tag{66}$$

are positive-definite with $E(k) = |\vec{k}|$. The labels A and B are used to identify a given pair of two different physical states. Therefore, Ostragadskian instabilities are absent in the physical subspace. We have defined $a_i(\vec{p}) \equiv \sum_\lambda a^\lambda(\vec{p})\epsilon_i^\lambda(\vec{p})$, with $i = 1, 2$ denoting the transverse spatial coordinates.

VIII. POSITIVE HILBERT SUBSPACE FOR AN INTERACTING THEORY

A natural question that arises is how to separate the physical Hilbert space in an interacting theory. In order to address this point, let's add to the free Bopp-Podolsky Lagrangian the following source term $A_\mu(x)J^\mu(x)$, written in terms of a fermionic current [5] $J_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$, and also the kinetic term for the matter field. Regarding the subsidiary condition, the commutation relations between the gauge field and the B-field are kept since $\partial_\mu J^\mu(x) = 0$.

It is possible to show that the asymptotic fields are still a combination of massive and massless excitations since the vacuum-corrected Feynman propagator has the form [6] [25]

$$P_{\mu\nu}(p^2) = \frac{im^2\ \theta_{\mu\nu}}{p^2(p^2 - m^2 - m^2\pi^R(p^2))} \tag{67}$$

with $\pi^R(p^2)$ denoting the scalar part of the complete polarization tensor structure. It can be renormalized to fulfill the condition $\pi^R(0) = 0$ ensuring a positive unitary norm for the massless pole. On the other hand, the massive pole is renormalized by the self-energy correction.

Therefore, considering asymptotic completeness, the physical states can be written in terms of

[5] $\psi(x)$ is the spinor field representing the electrons and positrons.
[6] We are highlighting just its transverse physical part.

a series of states generated by a linear combination of a set of powers of the transverse sector of the free asymptotic massless gauge field negative frequency part acting on the vacuum [65]. Then, we propose the following subsidiary condition based on the field taken at the zero coupling limit

$$B^{+(0)}(x)|\mathrm{phys}\rangle = 0, \quad \Box^2\big(\lim_{e\to 0} A^{+}_{\mu}(x)\big)|\mathrm{phys}\rangle = 0 \quad \forall|\mathrm{phys}\rangle \in \mathcal{V}_{\mathrm{phys}}. \tag{68}$$

in terms of the asymptotic field present in the Yang-Feldman equation

$$A_\mu(x) = A^{as}_\mu(x) + e\int d^4y P^{(0)R/A}_{\mu\nu}(x-y)\big(\bar{\psi}(y)\gamma^\nu\psi(y)\big) \tag{69}$$

with the R/A labels denoting the retarded and advanced versions of the free-propagator obtained by (67) in the limit $\pi^R(p)\to 0$. Then, the positive-definite Hilbert space is again defined as the completion of the quotient space eliminating the zero norm sector.

Since the Hamiltonian is Hermitian, the system enjoys pseudo-unitarity, meaning that the projections in the whole Hilbert space are kept by the time evolution operator

$$S =: Te^{i\int H dt}: \tag{70}$$

with H denoting the complete Hamiltonian in compliance with the Heisenberg description.

Regarding the optical theorem, the polarization sums (62) and (63) are responsible for its fulfillment. It guarantees that projections are maintained in time evolution. The specific case associated with the case of the one-loop electron self-energy in the diagrammatic approach is suitable to verify this feature in a concrete example. However, the system is pseudo-unitary since, for example, the emission of a massive photon by an electron has an associated negative probability due to the previously mentioned polarization sums.

Then, if one considers transitions between physical states, the time evolution operator is unitary in the associated Hilbert subspace. This is in accordance with the theorem A.2-2 of [65] ensuring a unitary time evolution in the physical Hilbert subspace if the conditions previously highlighted above hold.

This discussion becomes clearer in the interaction description in which all the field operators obey the free field equations. The scattering processes are associated with the following kind of amplitudes

$$\langle physA| : T\int d^4x\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x)...\int d^4y\bar{\psi}(y)\gamma_\nu\psi(y)A^\nu(y) : |Bphys\rangle \tag{71}$$

with A, B labeling a general pair of physical states. Here ... denotes the product of a given set of vertex operators in the interaction description. Such amplitudes are of the type that appear at a given truncation of the S-matrix elements.

Due to the Wick theorem, as the physical asymptotic external states exclude massive photons, their only contribution comes from contractions between vector field operators associated with the Bopp-Podolsky propagator. Then, it contributes to improving renormalization aspects while keeping the unitary nature of the scattering between physical states. Therefore, even if the gauge field mass lies below the particle production threshold, avoiding the Merlin mode scenario [34, 35], the physical processes can be still consistently separated enjoying a unitary nature.

Finally, since a Bopp-Podolsky like structure arises in the linearized renormalized one-loop gravity [84] formulation, an interesting future perspective is defined by the possibility of applying the same investigation outlined here to separate its specific ghost contribution.

IX. CONCLUSION

Throughout this article the covariant operator quantization of two higher derivative systems was performed. The first is associated with a vector dual description of a spinless model. Issues on duality, reducible structure of the local freedom, and its massless limit were investigated. The analysis was restricted to $D = 2+1$ dimensions in order to incorporate just two auxiliary fields. The (KON) quantization framework was successfully applied considering the generalized Ostrogadskian phase space structure. The choice of the subsidiary condition was suitable to eliminate all auxiliary fields from the physical Hilbert subspace. It was explicitly verified that the scalar degree of freedom indeed fulfills the required conditions to define a physical mode. Moreover, the higher derivative structure ensures a smooth massless limit.

Regarding the generalized higher derivative electrodynamics, the same processes were carried out. In this case, just one auxiliary field was necessary to quantize the system. The full set of commutators was obtained considering the correspondence principle and the operator equations of motion. A careful discussion on a suitable new kind of subsidiary condition to define the physical subspace was provided. This was necessary since, beyond the spurious gauge modes, the model also presents negative norm excitations in the transverse sector. Therefore, a set of two compatible conditions were considered to define the positive Hilbert subspace.

The next achievement was related to the explicit verification of Hamiltonian positivity. Namely, although not positive-definite in the whole Hilbert space, it is indeed positive within the physical subspace. Moreover, discussions on the interacting phase were also performed. A suitable definition of the physical subspace was provided considering asymptotic completeness. In analyzing the renormalized system, we adopted the interaction description for a while to conclude that scattering processes between the previously defined physical states are unitary. This is a consequence of the Hermitian nature of the Hamiltonian as well as the fulfillment of the optical theorem ensuring pseudo-unitarity in the whole Hilbert space. Therefore, as a future perspective, we proposed in suitable points throughout the article the application of this whole program to address the problem of negative norm ghosts appearing in higher derivative systems such as the one loop renormalized linearized quantum gravity [85]. It allows a unitary definition of scattering processes even in this non-trivial context. Interestingly, based on the previous work on infrared completed Bopp-Podolsky model in the context of Debye screening [86], the introduction of a Fierz-Pauli mass term in the renormalized gravity lagrangian seems to possibly lead to a promising structure to provide a discussion on the possible emergence of a set of phases, depending on the magnitude of the model's parameters, displayed by the associated gravitational interaction.

X. APPENDIX

Here, we analyze the Hamiltonian formulation of the dual vector spin-0 model in order to reinforce some conclusions achieved throughout the paper. We consider the Dirac-Bergman algorithm to derive the Hamiltonian positivity, the correct degrees of freedom, and a continuous massless limit. Let's consider the theory without the auxiliary fields

$$\mathcal{L} = \frac{1}{2}\Big[\partial_\beta\big(\partial_\mu B^\mu\big)\partial^\beta\big(\partial_\mu B^\mu\big) - m^2\big(\partial_\mu B^\mu\big)^2\Big] \tag{72}$$

Considering the momentum definition (10), the Hamiltonian density obtained through the Legendre transform reads

$$\mathcal{H} = \frac{\big(\pi_0^B\big)^2}{2} - \pi_0^B\partial_i\dot{B}_i + p_i^B\dot{B}_i + p_0^B\dot{B}_0 + \partial_j\big(\dot{B}_0 + \partial_i B_i\big)\partial_j\big(\dot{B}_0 + \partial_i B_i\big) + \frac{m^2}{2}\big(\dot{B}_0 + \partial_i B_i\big)^2 \tag{73}$$

with the primary constraint $\pi_i^B \approx 0$. The Latin indices denote spatial coordinates. All sums present in the Hamiltonian density are Euclidian ones.

The Dirac-Bergman consistency algorithm leads to the following extra set of constraints

$$-p_i^B + \partial_i\pi_0^B \approx 0\ ;\ p_0^B \approx 0 \tag{74}$$

Therefore, due to their first-class nature, they remove 14 degrees of freedom from the entire sixteen-dimensional Ostrogadskian phase space, leading to one degree of freedom in the configuration space characterizing a spin-0 particle. The quantity and the classification of the constraints remain the same at the massless limit. It implies the aforementioned continuous behavior at this limit.

The Hamiltonian evaluated at the constraint surface is positive-definite

$$\mathcal{H}=\frac{\left(\pi_0^B\right)^2}{2}+\partial_j\left(\dot{B}_0+\partial_i B_i\right)\partial_j\left(\dot{B}_0+\partial_i B_i\right)+\frac{m^2}{2}\left(\dot{B}_0+\partial_i B_i\right)^2 \tag{75}$$

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[1] L.H.C. Borges, F.A. Barone, C.A.M. de Melo, F.E. Barone, Nucl. Phys. B 944, (2019), 114634.
[2] W. Pauli and F. Villars, Rev. Mod. Phys. 21, 434 (1949).
[3] J. Rayski, Acta. Phys. Pol. 9, 129 (1948).
[4] J. Rayski Phys. Rev. 75, 1961 (1949).
[5] C. R. Ji, A. T. Suzuki, J. H. Sales and R. Thibes, Eur. Phys. J. 79, (2019).
[6] T. D. Lee and G. C. Wick, Nucl. Phys. B 9, 209 (1969).
[7] T. D. Lee and G. C. Wick, Phys. Rev. D 2, 103 (1970).
[8] A. Accioly, P. Gaete, J. H. Neto, E. Scatena and R. Turcati, Mod. Phys. Lett. A 26, 26 (2011).
[9] L.Casarin, ArXiv: 1710.08021v2 (2017).
[10] F. Bopp, Ann. Phys. (Leipzig) 430, 345 (1940).
[11] B. Podolsky, Phys. Rev. 62, 68 (1942).
[12] B. Podolsky, C. Kikuchi, Phys. Rev. 65, 228 (1944).
[13] B. Podolsky, C. Kikuchi, Phys. Rev. 67, 184 (1945).
[14] B. Podolsky and P. Schwed, Rev. Mod. Phys. 20, 4 (1948).
[15] J. S. Chang, Proc. Camb. Philos. Soc. 44, 76 (1948).
[16] J. Frenkel, Phys. Rev. E 54, 5859 (1996).
[17] A. E. Zayats, Ann. Phys. 342, 11 (2014).
[18] F.A.Barone, G.F.Hidalgo, A.A. Nogueira, Phys. Rev. D 91 (2015), 027701.
[19] R. Bufalo, B. M. Pimentel, G. E. R. Zambrano Phys. Rev. D 83, 045007 (2011).
[20] R. Bufalo, B. M. Pimentel, and G. E. R. Zambrano, Phys. Rev. D 86, 125023 (2012).
[21] R. Bufalo and B. M. Pimentel, Phys. Rev. D 88, 125023 (2013).
[22] R. Bufalo, B.M. Pimentel and D.E. Soto, Phys. Rev. D, 90 (2014).
[23] R. Bufalo, B.M. Pimentel and D.E. Soto, Ann. of Phys. (2014).
[24] R. Bufalo, B.M. Pimentel and D.E. Soto,Int. Journ. Mod. Phys. A, 32 (2017).

[25] R. Bufalo, T.R. Cardoso, A.A. Nogueira, B.M. Pimentel Phys. Rev. D 97, 105029 (2018).

[26] G. B. de Gracia, A. A. Nogueira, R. da Rocha Nucl. Phys. B 992, 116227 (2023).

[27] A. Satz, A. Codello, F.D. Mazzitelli, Phys. Rev. D 82, (2010).

[28] A. Codello, R.Percacci, C. Rahmede, Ann. Phys. 324, (2009).

[29] A. Codello, R.Percacci, L. Rachwal, A. Tonero, Eur. Phys. J. C, 76, (2016).

[30] M.Kaku, *Strings, Conformal Fields and Topology: An Introduction.*, Springer-Verlag, First Edition, New York, (1991).

[31] D. G. Boulware and S. Deser Phys. Rev. Lett. 55, 2656 (1985).

[32] L. Modesto, I. L. Shapiro, Phys. Lett. B 755, (2016), 279-284

[33] A.Salvio, Front. in Phys. 6, 77 (2018).

[34] J.F. Donoghue and G.Menezes, Phys. Rev. Lett. 123, 171601 (2019).

[35] J.F. Donoghue and G.Menezes, Phys. Rev. D 104, 045010 (2021).

[36] E. Tomboulis, Phys. Lett. B 70, 361 (1977).

[37] E. Tomboulis, Phys. Lett. B 97, 77 (1980).

[38] R. R. Cuzinatto, C. A. M. de Melo, P. J. Pompeia, Ann. Phys. (New York) 322, 1211 (2007).

[39] P. J. Pompeia, R. R. Cuzinatto, C. A. M. de Melo and L. G. Medeiros, Eur. Phys. J. C 53, 99 (2007).

[40] E. Bergshoeff, O. Hohm and P.K. Townsend, Phys. Rev. Lett. 102, 201301 (2009).

[41] L. Modesto and I.L. Shapiro, Phys. Lett. B 755, 279 (2016).

[42] L. Modesto, Nucl. Phys. B 909, 584 (2016).

[43] D. Anselmi and M. Piva, Phys. Rev. D 96, 045009 (2017).

[44] D. Anselmi, JHEP 06, 086 (2017).

[45] D. Anselmi, JHEP 02, 141 (2018).

[46] A. Pais and G.E. Uhlenbeck Phys. Rev. 79, 145 (1950).

[47] W. Heisenberg, Nuclear Phys. 4, 532 (1957).

[48] K. S. Stelle Phys. Rev. D 16, 953 (1977).

[49] S. W. Hawking and T. Hertog Phys. Rev. D 65, 103515 (2002).

[50] A. V. Smilga, Nucl. Phys. B 706, 598 (2005); J. Phys. Conf. Ser. 2038, 012023 (2021).

[51] M. Asorey, L. Rachwal and I. Shapiro, Galaxies 6 (1), 23 (2018)

[52] D. Anselmi and M. Piva, JHEP 06, 066 (2017).

[53] M. Raidal and H. Veermäe Nucl. Phys. B, 916 607 (2017).

[54] A. A. Nogueira, C. Palechor, A. F. Ferrari, Nucl. Phys. B 939, 372 (2019).

[55] D. S. Kaparulin, S. L. Lyakhovich, A. A. Sharapov, Eur. Phys. J. C 74 (10), 3072 (2014).

[56] Jialiang Dai, Eur. Phys. J. Plus (2020).

[57] D. Dalmazi and R.C. Santos, Phys. Rev. D 87, 085021 (2013).

[58] D. Dalmazi and R.C. Santos, Phys. Rev. D 84, 045027 (2011).

[59] A. Smailagic and E. Spallucci, J. Phys. A, 34, (2001).

[60] M.K. Dwivedi, Int. Journ. Mod. Phys. A, 56, (2017).

[61] M. Kalb and D. Ramond, Phys. Rev. D,9, (1974).

[62] A. Hell JCAP01 (2022) 056.

[63] G.B. de Gracia, Int. J. of Mod. Phys. A, 32, No. 06n07, 1750041 (2017).

[64] A. Hell, J. High Energ. Phys. 2022, 167 (2022).

[65] N. Nakanish and I. Ojima*Covariant operator formalism of gauge theories and quantum gravity*, Singapore, World Scientific, (1990).

[66] M. Abe and N. Nakanish, Prog. of Theor. Phys. 88, (1992).

[67] M. Ostrogadski, Mem. Ac. St. Petersbourg VI, 4, (1850) 385.

[68] J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, Phys. Rev. Lett., 114, (2015) 211101.

[69] I.A. Batalin and E.S. Fradkin, Phys. Lett. 122B 157 (1983).

[70] P. A.M. Dirac, *Lectures on quantum mechanics* New York:Yeshiva University (1964).

[71] Y. Kurihara, Nakanishi-Kugo-Ojima quantization of general relativity in Heisenberg picture, Eur. Phys. J. Plus, **136**, 462 (2021).

[72] T.tripathi, B. Chauhan, A.K. Rao and R.P. Malik, AHEP, **2021**, 2056629 (2021).

[73] C.A. Bonin and B.M. Pimentel, Phys. Rev. D **106**, 016003, (2022).

[74] D. Dudal, S. P. Sorella, N. Vandersickel, and H. Verschelde, Phys. Rev. D **79**, 121701 (2009).

[75] A. A. Nogueira and B. M. Pimentel, Phys. Rev. D 95, 065034 (2017).

[76] G.B. de Gracia, B.M. Pimentel, and L. Rabanal, Nucl. Phys. B **948** (2019) 114750.

[77] G. Gracia, B.M. Pimentel and R. da Rocha, Ann. of Phys. 459 (2023) 169545.

[78] M. Abe and N. Nakanish, Prog. of Theor. Phys. 89, (1993).

[79] N. Nakanishi, Prog. Theor. Phys. 111, (2004).

[80] C.A.P. Galvao and B.M. Pimentel, Can. Journ. Phys. 66, (1988) 460-466.

[81] G.B. de Gracia, Nucl. Phys. B, 1001, (2024), 116498.

[82] G.B. de Gracia and G.P. de Brito, IJMPA, 31, No. 12, 1650070 (2016).

[83] F.O.Salles and Ilya L. Shapiro, Univ. 4 (2018), 91.

[84] J.F.Donoghue, B.K.EL Menoufi, Phys. Rev. D 89, 104062 (2014).

[85] J. F. Donoghue and B. K. El-Menoufi, Phys. Rev. D 89, 104062 (2014).

[86] C.A. Bonin, G.B. de Gracia, A.A. Nogueira and B.M. Pimentel, IJMPA, 35, No. 28, 2050179 (2020).