Strong decays of the $P_{cs}(4338)$ and its high isospin cousin via the QCD sum rules

Xiu-Wu Wang, Zhi-Gang Wang[1]
Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In the present work, the strong decays of the newly observed $P_{cs}(4338)$ as well as its high isospin cousin $P_{cs}(4460)$ are studied via the QCD sum rules. According to conservation of isospin, spin and parity, the hadronic coupling constants in four decay channels are obtained, then the partial decay widths are obtained. The total width of the $P_{cs}(4338)$ coincides with the experimental data nicely, while the predictions for the $P_{cs}(4460)$ can be testified in the future experiment, and shed light on the nature of the $P_{cs}(4338)$.

PACS number: 12.39.Mk, 14.20.Lq, 12.38.Lg
Key words: Strong decays, Pentaquark molecular states, QCD sum rules

1 Introduction

Recent years, several P_c and P_{cs} exotic states, such as the $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$, $P_c(4337)$, $P_{cs}(4459)$ and $P_{cs}(4338)$, were observed by the LHCb collaboration [1, 2, 3, 4, 5], they are the hidden-charm pentaquark (molecule) candidates with or without strangeness. Except for the $P_c(4337)$, the P_c and P_{cs} exotic states lie near the thresholds of the $\Sigma_c^{(*)}\bar{D}^{(*)}$ and $\Xi_c\bar{D}^{(*)}$ pairs, respectively, it is natural to consider them as the meson-baryon molecules. In the present study, we will focus on the exotic $P_{cs}(4338)$, its measured Breit-Wigner mass and width are $4338.2 \pm 0.7 \pm 0.4\,\text{MeV}$ and $7.0 \pm 1.2 \pm 1.3\,\text{MeV}$, respectively, and the preferred spin-parity is $J^P = \frac{1}{2}^-$ [5].

In Ref.[6], the $P_{cs}(4338)$ is considered as the $\Xi_c\bar{D}$ molecule via the effective field theory. In Ref.[7], the mass and width of the $P_{cs}(4338)$ are studied by assigning it as the meson-baryon molecule with the $IJ^P = 0\frac{1}{2}^-$ based on the constituent quark model, and detailed partial decay widths of its strong decays are obtained. In Ref.[8], the $P_{cs}(4338)$ is interpreted as the $\Xi_c\bar{D}$ molecular state with the $J^P = \frac{1}{2}^-$ via the quasipotential Bethe-Salpeter equation, and new structures are predicted. In Ref.[9], the mass and decays of the $P_{cs}(4338)$ are studied with the QCD sum rules by considering it as the $\Xi_c\bar{D}$ molecule with the spin-parity $J^P = \frac{1}{2}^-$. For some other interesting works focusing on the $P_{cs}(4338)$, one can consult Refs.[10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. Along with the popular acceptance of the meson-baryon molecule assignment for the $P_{cs}(4338)$, debates do exist about its nature, for example, in Refs.[25, 26], the $P_{cs}(4338)$ is assigned as the compact pentaquark state.

For our research work on the $P_{cs}(4338)$, we considered it have the definite isospin, spin and parity $IJ^P = 0\frac{1}{2}^-$ and identified it as the $\Xi_c\bar{D}$ hadronic molecule via the QCD sum rules [11], moreover, our results showed that there maybe exist a high isospin cousin $P_{cs}(4460)$, which is assigned as the $\Xi_c\bar{D}$ resonant state. In Refs.[27, 28], we distinguish the isospin, spin and parity, study the mass spectrum of the hidden-charm pentaquark molecular states without strangeness and with strangeness in a systematic way, and make possible assignments of the existing pentaquark candidates and predict many new exotic states. If those predicted states could be observed in the future experiment, it would testify our interpretation about the nature of the $P_{cs}(4338)$, etc. In Ref.[29], we study the strong decays of the pentaquark molecule candidate $P_c(4312)$ and its higher isospin cousin $P_c(4330)$ with the QCD sum rules to examine their nature. Now we study the strong decays of the $P_{cs}(4338)$ and $P_{cs}(4460)$ in detail to provide some useful information for the future high energy experiment.

[1]E-mail: zgwang@aliyun.com.

This paper is arranged as follows, the QCD sum rules for the strong decays are derived in Section 2, the QCD sum rules for the Λ and Σ baryons are studied in Section 3, numerical calculations and discussions are presented in Section 4, and Section 5 is reserved for the conclusions.

2 QCD sum rules for the strong decays of the $P_{cs}(4338)$ and $P_{cs}(4460)$

In Ref.[11], the quantum numbers IJ^P of the exotic pentaquark states with strangeness $P_{cs}(4338)$ and $P_{cs}(4460)$ are assigned as $0\frac{1}{2}^-$ and $1\frac{1}{2}^-$, respectively, the currents $J_P(x)$ and $J_{P'}(x)$ are applied to interpolate $P_{cs}(4338)$ and $P_{cs}(4460)$, respectively,

$$
\begin{aligned}
J_P(x) &= \frac{1}{\sqrt{2}}\varepsilon^{ijk}\left[u^{iT}(x)C\gamma_5 s^j(x)c^k(x)\bar{c}(x)\mathrm{i}\gamma_5 d(x) - d^{iT}(x)C\gamma_5 s^j(x)c^k(x)\bar{c}(x)\mathrm{i}\gamma_5 u(x)\right] , \\
J_{P'}(x) &= \frac{1}{\sqrt{2}}\varepsilon^{ijk}\left[u^{iT}(x)C\gamma_5 s^j(x)c^k(x)\bar{c}(x)\mathrm{i}\gamma_5 d(x) + d^{iT}(x)C\gamma_5 s^j(x)c^k(x)\bar{c}(x)\mathrm{i}\gamma_5 u(x)\right] , \quad (1)
\end{aligned}
$$

where $\mathrm{i}^2=-1$, i, j, k are the color indices, the C represents the charge conjugation matrix. We consider conservation of the IJ^P in the strong decays, and study the typical decay channels,

$$
\begin{aligned}
P_{cs}(4338) &\to \eta_c + \Lambda \, , \\
P_{cs}(4338) &\to J/\psi + \Lambda \, , \\
P_{cs}(4460) &\to \eta_c + \Sigma \, , \\
P_{cs}(4460) &\to J/\psi + \Sigma \, , \quad (2)
\end{aligned}
$$

where the IJ^P of the η_c, J/ψ, Λ and Σ are 00^-, 01^-, $0\frac{1}{2}^+$ and $1\frac{1}{2}^+$, respectively. The interpolating currents of these mesons and baryons are written as,

$$
\begin{aligned}
J_{\eta_c}(x) &= \bar{c}(x)\mathrm{i}\gamma_5 c(x) \, , \\
J_{J/\psi,\mu}(x) &= \bar{c}(x)\gamma_\mu c(x) \, , \\
J_\Lambda(x) &= \sqrt{\frac{2}{3}}\varepsilon^{ijk}\left[u^{iT}(x)C\gamma_\alpha s^j(x)\gamma_5\gamma^\alpha d^k(x) - d^{iT}(x)C\gamma_\alpha s^j(x)\gamma_5\gamma^\alpha u^k(x)\right] , \\
J_\Sigma(x) &= \varepsilon^{ijk}u^{iT}(x)C\gamma_\alpha d^j(x)\gamma^\alpha\gamma_5 s^k(x) \, . \quad (3)
\end{aligned}
$$

Now the three-point correlation functions for those decay channels can be written as,

$$\Pi_P(p,q) = \mathrm{i}^2\int d^4x d^4y e^{\mathrm{i}p\cdot x}e^{\mathrm{i}q\cdot y}\langle 0|\mathtt{T}\left\{J_{\eta_c}(x)J_\Lambda(y)\bar{J}_P(0)\right\}|0\rangle \, , \quad (4)$$

$$\Pi_{P,\mu}(p,q) = \mathrm{i}^2\int d^4x d^4y e^{\mathrm{i}p\cdot x}e^{\mathrm{i}q\cdot y}\langle 0|\mathtt{T}\left\{J_{J/\psi,\mu}(x)J_\Lambda(y)\bar{J}_P(0)\right\}|0\rangle \, , \quad (5)$$

$$\Pi_{P'}(p,q) = \mathrm{i}^2\int d^4x d^4y e^{\mathrm{i}p\cdot x}e^{\mathrm{i}q\cdot y}\langle 0|\mathtt{T}\left\{J_{\eta_c}(x)J_\Sigma(y)\bar{J}_{P'}(0)\right\}|0\rangle \, , \quad (6)$$

$$\Pi_{P',\mu}(p,q) = \mathrm{i}^2\int d^4x d^4y e^{\mathrm{i}p\cdot x}e^{\mathrm{i}q\cdot y}\langle 0|\mathtt{T}\left\{J_{J/\psi,\mu}(x)J_\Sigma(y)\bar{J}_{P'}(0)\right\}|0\rangle \, , \quad (7)$$

where the $\mathtt{T}$ is the time-order operator. At the hadronic sides, a complete set of intermediate hadron states which have the same quantum numbers IJ^P as the corresponding currents are inserted [30, 31, 32], those correlation functions are shown as follows after the contributions of the ground states being isolated,

$$\Pi_P(p,q) = \frac{f_{\eta_c}m_{\eta_c}^2}{2m_c}\lambda_\Lambda\lambda_P g_{\eta\Lambda}\frac{(\not{q}+m_\Lambda)(\not{p}'+m_P)}{(m_P^2-p'^2)(m_{\eta_c}^2-p^2)(m_\Lambda^2-q^2)} + \cdots \, , \quad (8)$$

$$
\begin{aligned}
\Pi_{P,\mu}(p,q) &= f_{J/\psi}m_{J/\psi}\lambda_\Lambda\lambda_P \frac{-\mathrm{i}}{(m_P^2-p'^2)\left(m_{J/\psi}^2-p^2\right)(m_\Lambda^2-q^2)}\left(-g_{\mu\alpha}+\frac{p_\alpha p_\mu}{p^2}\right) \\
&\quad (\not{q}+m_\Lambda)\left(g_{J/\psi\Lambda,V}\gamma^\alpha-\frac{\mathrm{i}g_{J/\psi\Lambda,T}}{m_P+m_\Lambda}\sigma^{\alpha\beta}p_\beta\right)\gamma_5\left(\not{p}'+m_P\right)+\cdots\,, \qquad (9)
\end{aligned}
$$

$$
\Pi_{P'}(p,q) = \frac{f_{\eta_c}m_{\eta_c}^2}{2m_c}\lambda_\Sigma\lambda_{P'}g_{\eta\Sigma}\frac{(\not{q}+m_\Sigma)\left(\not{p}'+m_{P'}\right)}{(m_{P'}^2-p'^2)\left(m_{\eta_c}^2-p^2\right)(m_\Sigma^2-q^2)}+\cdots\,, \qquad (10)
$$

$$
\begin{aligned}
\Pi_{P',\mu}(p,q) &= f_{J/\psi}m_{J/\psi}\lambda_\Sigma\lambda_{P'} \frac{-\mathrm{i}}{(m_{P'}^2-p'^2)\left(m_{J/\psi}^2-p^2\right)(m_\Sigma^2-q^2)}\left(-g_{\mu\alpha}+\frac{p_\alpha p_\mu}{p^2}\right) \\
&\quad (\not{q}+m_\Sigma)\left(g_{J/\psi\Sigma,V}\gamma^\alpha-\frac{\mathrm{i}g_{J/\psi\Sigma,T}}{m_{P'}+m_\Sigma}\sigma^{\alpha\beta}p_\beta\right)\gamma_5\left(\not{p}'+m_{P'}\right)+\cdots\,, \qquad (11)
\end{aligned}
$$

where the $g_{\eta\Lambda}$, $g_{\eta\Sigma}$, $g_{J/\psi\Lambda,V}$, $g_{J/\psi\Lambda,T}$, $g_{J/\psi\Sigma,V}$ and $g_{J/\psi\Sigma,T}$ are the hadronic coupling constants, the λ_P, $\lambda_{P'}$, λ_Λ and λ_Σ are the pole residues of the pentaquark molecular states $P_{cs}(4338)$, $P_{cs}(4460)$, Λ and Σ, respectively, the f_{η_c} and $f_{J/\psi}$ are the decay constants of the mesons η_c and J/ψ, respectively, moreover, those constants satisfy the following definitions,

$$
\begin{aligned}
\langle 0|J_P(0)|\mathcal{P}_P(p')\rangle &= \lambda_P U_P(p')\,, \\
\langle 0|J_{P'}(0)|\mathcal{P}_{P'}(p')\rangle &= \lambda_{P'} U_{P'}(p')\,, \\
\langle 0|J_\Lambda(0)|\Lambda(q)\rangle &= \lambda_\Lambda U_\Lambda(q)\,, \\
\langle 0|J_\Sigma(0)|\Sigma(q)\rangle &= \lambda_\Sigma U_\Sigma(q)\,, \\
\langle 0|J_{J/\psi,\mu}(0)|J/\psi(p)\rangle &= f_{J/\psi}m_{J/\psi}\varepsilon_\mu\,, \\
\langle 0|J_{\eta_c}(0)|\eta_c(p)\rangle &= \frac{f_{\eta_c}m_{\eta_c}^2}{2m_c}\,, \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
\langle \eta_c(p)\Lambda(q)|\mathcal{P}_P(p')\rangle &= \mathrm{i}g_{\eta\Lambda}\overline{U}_\Lambda(q)U_P(p')\,, \\
\langle J/\psi(p)\Lambda(q)|\mathcal{P}_P(p')\rangle &= \overline{U}_\Lambda(q)\varepsilon^*_\alpha\left(g_{J/\psi\Lambda,V}\gamma^\alpha-\mathrm{i}\frac{g_{J/\psi\Lambda,T}}{m_P+m_\Lambda}\sigma^{\alpha\beta}p_\beta\right)\gamma_5 U_P(p')\,, \\
\langle \eta_c(p)\Sigma(q)|\mathcal{P}_{P'}(p')\rangle &= \mathrm{i}g_{\eta\Sigma}\overline{U}_\Sigma(q)U_{P'}(p')\,, \\
\langle J/\psi(p)\Sigma(q)|\mathcal{P}_{P'}(p')\rangle &= \overline{U}_\Sigma(q)\varepsilon^*_\alpha\left(g_{J/\psi\Sigma,V}\gamma^\alpha-\mathrm{i}\frac{g_{J/\psi\Sigma,T}}{m_{P'}+m_\Sigma}\sigma^{\alpha\beta}p_\beta\right)\gamma_5 U_{P'}(p')\,, \qquad (13)
\end{aligned}
$$

where the $|\mathcal{P}_P\rangle$, $|\mathcal{P}_{P'}\rangle$, $|\Lambda\rangle$, $|\Sigma\rangle$, $|\eta_c\rangle$ and $|J/\psi\rangle$ represent the ground states $P_{cs}(4338)$, $P_{cs}(4460)$, Λ, Σ, η_c and J/ψ, respectively, the U_P, $U_{P'}$, U_Λ and U_Σ are the Dirac spinors, the ε_μ stands for the polarization vector of the J/ψ meson, it satisfies the formula $\sum\varepsilon_\mu\varepsilon^*_\alpha=-g_{\mu\alpha}+\frac{p_\mu p_\alpha}{p^2}$.

It is reasonable to suppose that the hadronic sides $\Pi_H(p,q)$ and QCD sides $\Pi_{QCD}(p,q)$ of the correlation functions should match with each other [29, 33, 34],

$$
Tr[\Pi_H(p,q)\cdot\Gamma] = Tr[\Pi_{QCD}(p,q)\cdot\Gamma]\,, \qquad (14)
$$

where the Γ is any matrix in the Dirac spinor space. In the present study, the Γ are chosen as $\sigma_{\mu\nu}$ and γ_μ for both the $\Pi_P(p,q)$ and $\Pi_{P'}(p,q)$, when setting $\Gamma=\sigma_{\mu\nu}$, the tensor structure is chosen as $p_\mu q_\nu-q_\mu p_\nu$, as for $\Gamma=\gamma_\mu$, the structure q_μ is picked out. For the correlation functions $\Pi_{P,\mu}(p,q)$ and $\Pi_{P',\mu}(p,q)$, the Γ are selected as $\gamma_5\not{z}$ and γ_5, the chosen tensor structures are $q_\mu p\cdot z$ and q_μ, respectively. For clarity, the chosen tensor structures are expressed as,

$$
\begin{aligned}
\frac{1}{4}Tr[\Pi_P(p,q)\sigma_{\mu\nu}] &= \Pi_a(p'^2,p^2,q^2)\mathrm{i}(p_\mu q_\nu - q_\mu p_\nu)+\cdots\,,\\
\frac{1}{4}Tr[\Pi_P(p,q)\mathrm{i}\gamma_\mu] &= \Pi_b(p'^2,p^2,q^2)\mathrm{i}q_\mu+\cdots\,,\\
\frac{1}{4}Tr[\Pi_{P,\mu}(p,q)\gamma_5\not{z}] &= \Pi_c(p'^2,p^2,q^2)\mathrm{i}q_\mu p\cdot z+\cdots\,,\\
\frac{1}{4}Tr[\Pi_{P,\mu}(p,q)\gamma_5] &= \Pi_d(p'^2,p^2,q^2)\mathrm{i}q_\mu+\cdots\,,\\
\frac{1}{4}Tr[\Pi_{P'}(p,q)\sigma_{\mu\nu}] &= \Pi_e(p'^2,p^2,q^2)\mathrm{i}(p_\mu q_\nu - q_\mu p_\nu)+\cdots\,,\\
\frac{1}{4}Tr[\Pi_{P'}(p,q)\mathrm{i}\gamma_\mu] &= \Pi_f(p'^2,p^2,q^2)\mathrm{i}q_\mu+\cdots\,,\\
\frac{1}{4}Tr[\Pi_{P',\mu}(p,q)\gamma_5\not{z}] &= \Pi_g(p'^2,p^2,q^2)\mathrm{i}q_\mu p\cdot z+\cdots\,,\\
\frac{1}{4}Tr[\Pi_{P',\mu}(p,q)\gamma_5] &= \Pi_h(p'^2,p^2,q^2)\mathrm{i}q_\mu+\cdots\,.
\end{aligned}
\tag{15}
$$

At the QCD sides, all the quark fields are contracted via the Wick theorem, and then the operator product expansions are performed, for the analytical calculation of the quark fields, the integrals of the light and heavy quarks are solved in the coordinate space and momentum space, respectively [33, 34]. Since the relation $p' = p + q$ holds for all the two-body decay channels considered in the present study, the p'^2 is set as ξp^2, where the ξ is a constant relied on the particular decay channel [29], taking the decay channel $P_{cs}(4338) \to \eta_c + \Lambda$ for example, $\xi = \frac{m_\Lambda^2}{m_{\eta_c^2}} + 1$. Followed by the rigorous quark-hadron duality below the continuum thresholds [35, 36, 37, 38, 39], double Borel transformations are applied, then the QCD sum rules for the hadronic coupling constants are derived as,

$$
\begin{aligned}
&\frac{f_{\eta_c}m_{\eta_c}^2\lambda_\Lambda\lambda_P g_{\eta\Lambda,a}}{2m_c\xi}\frac{1}{\frac{m_P^2}{\xi}-m_{\eta_c}^2}\left\{\exp\left(-\frac{m_{\eta_c}^2}{T_1^2}\right)-\exp\left(-\frac{m_P^2}{\xi T_1^2}\right)\right\}\exp\left(-\frac{m_\Lambda^2}{T_2^2}\right)\\
&+C_a\exp\left(-\frac{m_{\eta_c}^2}{T_1^2}-\frac{m_\Lambda^2}{T_2^2}\right)=\int_{4m_c^2}^{s_{\eta_c}^0}ds\int_0^{s_\Lambda^0}du\,\rho_a(s,u)\exp\left(-\frac{s}{T_1^2}-\frac{u}{T_2^2}\right)\,,
\end{aligned}
\tag{16}
$$

$$
\begin{aligned}
&\frac{f_{\eta_c}m_{\eta_c}^2\lambda_\Lambda\lambda_P g_{\eta\Lambda,b}}{2m_c\xi}\frac{m_P+m_\Lambda}{\frac{m_P^2}{\xi}-m_{\eta_c}^2}\left\{\exp\left(-\frac{m_{\eta_c}^2}{T_1^2}\right)-\exp\left(-\frac{m_P^2}{\xi T_1^2}\right)\right\}\exp\left(-\frac{m_\Lambda^2}{T_2^2}\right)\\
&+C_b\exp\left(-\frac{m_{\eta_c}^2}{T_1^2}-\frac{m_\Lambda^2}{T_2^2}\right)=\int_{4m_c^2}^{s_{\eta_c}^0}ds\int_0^{s_\Lambda^0}du\,\rho_b(s,u)\exp\left(-\frac{s}{T_1^2}-\frac{u}{T_2^2}\right)\,,
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
&\frac{f_{J/\psi}m_{J/\psi}\lambda_\Lambda\lambda_P}{\xi}\frac{g_{J/\psi\Lambda,T/V}}{\frac{m_P^2}{\xi}-m_{J/\psi}^2}\left\{\exp\left(-\frac{m_{J/\psi}^2}{T_1^2}\right)-\exp\left(-\frac{m_P^2}{\xi T_1^2}\right)\right\}\exp\left(-\frac{m_\Lambda^2}{T_2^2}\right)\\
&+C_{J/\psi\Lambda,T/V}\exp\left(-\frac{m_{J/\psi}^2}{T_1^2}-\frac{m_\Lambda^2}{T_2^2}\right)\\
&=\int_{4m_c^2}^{s_{J/\psi}^0}ds\int_0^{s_\Lambda^0}du\,\rho_{J/\psi\Lambda,T/V}(s,u)\exp\left(-\frac{s}{T_1^2}-\frac{u}{T_2^2}\right)\,,
\end{aligned}
\tag{18}
$$

$$
\frac{f_{\eta_c}m_{\eta_c}^2\lambda_\Sigma\lambda_{P^\prime}g_{\eta\Sigma,e}}{2m_c\xi}\frac{1}{\frac{m_{P^\prime}^2}{\xi}-m_{\eta_c}^2}\left\{\exp\left(-\frac{m_{\eta_c}^2}{T_1^2}\right)-\exp\left(-\frac{m_{P^\prime}^2}{\xi T_1^2}\right)\right\}\exp\left(-\frac{m_{\Sigma}^2}{T_2^2}\right)
$$

$$
+C_e\exp\left(-\frac{m_{\eta_c}^2}{T_1^2}-\frac{m_{\Sigma}^2}{T_2^2}\right)=\int_{4m_c^2}^{s_{\eta_c}^0}ds\int_0^{s_\Sigma^0}du\,\rho_e(s,u)\exp\left(-\frac{s}{T_1^2}-\frac{u}{T_2^2}\right)\,, \tag{19}
$$

$$
\frac{f_{\eta_c}m_{\eta_c}^2\lambda_\Sigma\lambda_{P^\prime}g_{\eta\Sigma,f}}{2m_c\xi}\frac{m_{P^\prime}+m_\Sigma}{\frac{m_{P^\prime}^2}{\xi}-m_{\eta_c}^2}\left\{\exp\left(-\frac{m_{\eta_c}^2}{T_1^2}\right)-\exp\left(-\frac{m_{P^\prime}^2}{\xi T_1^2}\right)\right\}\exp\left(-\frac{m_{\Sigma}^2}{T_2^2}\right)
$$

$$
+C_f\exp\left(-\frac{m_{\eta_c}^2}{T_1^2}-\frac{m_{\Sigma}^2}{T_2^2}\right)=\int_{4m_c^2}^{s_{\eta_c}^0}ds\int_0^{s_\Sigma^0}du\,\rho_f(s,u)\exp\left(-\frac{s}{T_1^2}-\frac{u}{T_2^2}\right)\,, \tag{20}
$$

$$
\frac{f_{J/\psi}m_{J/\psi}\lambda_\Sigma\lambda_{P^\prime}}{\xi}\frac{g_{J/\psi\Sigma,T/V}}{\frac{m_{P^\prime}^2}{\xi}-m_{J/\psi}^2}\left\{\exp\left(-\frac{m_{J/\psi}^2}{T_1^2}\right)-\exp\left(-\frac{m_{P^\prime}^2}{\xi T_1^2}\right)\right\}\exp\left(-\frac{m_{\Sigma}^2}{T_2^2}\right)
$$

$$
+C_{J/\psi\Sigma,T/V}\exp\left(-\frac{m_{J/\psi}^2}{T_1^2}-\frac{m_{\Sigma}^2}{T_2^2}\right)
$$

$$
=\int_{4m_c^2}^{s_{J/\psi}^0}ds\int_0^{s_\Sigma^0}du\,\rho_{J/\psi\Sigma,T/V}(s,u)\exp\left(-\frac{s}{T_1^2}-\frac{u}{T_2^2}\right)\,, \tag{21}
$$

where

$$
\begin{aligned}
C_{J/\psi\Lambda,T} &= \left[(m_P-m_\Lambda)C_c+C_d\right]\frac{m_P+m_\Lambda}{m_P^2-m_\Lambda^2-m_{J/\psi}^2}\,,\\
\rho_{J/\psi\Lambda,T}(s,u) &= \left[(m_P-m_\Lambda)\rho_c(s,u)+\rho_d(s,u)\right]\frac{m_P+m_\Lambda}{m_P^2-m_\Lambda^2-m_{J/\psi}^2}\,,\\
C_{J/\psi\Lambda,V} &= \left(\frac{m_{J/\psi}^2}{m_P+m_\Lambda}C_c+C_d\right)\frac{m_P+m_\Lambda}{m_P^2-m_\Lambda^2-m_{J/\psi}^2}\,,\\
\rho_{J/\psi\Lambda,V}(s,u) &= \left(\frac{m_{J/\psi}^2}{m_P+m_\Lambda}\rho_c(s,u)+\rho_d(s,u)\right)\frac{m_P+m_\Lambda}{m_P^2-m_\Lambda^2-m_{J/\psi}^2}\,,
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
C_{J/\psi\Sigma,T} &= \left[(m_{P^\prime}-m_\Sigma)C_g+C_h\right]\frac{m_{P^\prime}+m_\Sigma}{m_{P^\prime}^2-m_\Sigma^2-m_{J/\psi}^2}\,,\\
\rho_{J/\psi\Sigma,T}(s,u) &= \left[(m_{P^\prime}-m_\Sigma)\rho_g(s,u)+\rho_h(s,u)\right]\frac{m_{P^\prime}+m_\Sigma}{m_{P^\prime}^2-m_\Sigma^2-m_{J/\psi}^2}\,,\\
C_{J/\psi\Sigma,V} &= \left(\frac{m_{J/\psi}^2}{m_{P^\prime}+m_\Sigma}C_g+C_h\right)\frac{m_{P^\prime}+m_\Sigma}{m_{P^\prime}^2-m_\Sigma^2-m_{J/\psi}^2}\,,\\
\rho_{J/\psi\Sigma,V}(s,u) &= \left(\frac{m_{J/\psi}^2}{m_{P^\prime}+m_\Sigma}\rho_g(s,u)+\rho_h(s,u)\right)\frac{m_{P^\prime}+m_\Sigma}{m_{P^\prime}^2-m_\Sigma^2-m_{J/\psi}^2}\,,
\end{aligned} \tag{23}
$$

the T_1 and T_2 are the Borel parameters, the $\rho_Z(s,u)$ are spectral densities at the QCD sides derived from the corresponding correlation functions $\Pi_Z(p^{\prime 2},p^2,q^2)$, the subscript Z stands for $a,b,\cdots,h$, the C_Z are the unknown parameters determined by choosing flat Borel platforms for the related hadronic coupling constants in the numerical calculations.

3 QCD sum rules for the Λ and Σ baryons

We write down the two-point correlation functions for the baryons Λ and Σ,

$$\Pi_{\Lambda/\Sigma}(q) = \mathrm{i}\int d^4y e^{\mathrm{i}q\cdot y}\langle 0|\mathrm{T}\left\{J_{\Lambda/\Sigma}(y)\bar{J}_{\Lambda/\Sigma}(0)\right\}|0\rangle \, . \tag{24}$$

A complete set of baryon states with the same quantum numbers as the currents $J_{\Lambda/\Sigma}(y)$ are inserted into the correlation functions, and only considering the contributions from the ground states, the correlation functions at the hadronic sides are acquired as,

$$\begin{aligned}
\Pi_{\Lambda/\Sigma}(q) &= \lambda^2_{\Lambda/\Sigma}\frac{\not{q}+m_{\Lambda/\Sigma}}{m^2_{\Lambda/\Sigma}-q^2}+\cdots \, , \\
&= \Pi^1_{\Lambda/\Sigma}(q^2)\not{q}+\Pi^0_{\Lambda/\Sigma}(q^2)+\cdots \, .
\end{aligned} \tag{25}$$

The structures $\not{q}$ and 1 are chosen, therefore the four QCD sum rules are determined as,

$$\begin{aligned}
\lambda^2_{\Lambda/\Sigma}\mathrm{exp}\left(-\frac{m^2_{\Lambda/\Sigma}}{T^2}\right) &= \int_0^{s^0_{\Lambda/\Sigma}}\rho^1_{\Lambda/\Sigma,QCD}(u)\exp\left(-\frac{u}{T^2}\right)du \, , \\
m_{\Lambda/\Sigma}\lambda^2_{\Lambda/\Sigma}\mathrm{exp}\left(-\frac{m^2_{\Lambda/\Sigma}}{T^2}\right) &= \int_0^{s^0_{\Lambda/\Sigma}}\rho^0_{\Lambda/\Sigma,QCD}(u)\exp\left(-\frac{u}{T^2}\right)du \, ,
\end{aligned} \tag{26}$$

where the $s^0_{\Lambda/\Sigma}$ represent the threshold parameters of the Λ and Σ baryons, respectively, the $\rho^1_{\Lambda/\Sigma,QCD}(u)$ and $\rho^0_{\Lambda/\Sigma,QCD}(u)$ are the spectral densities derived from the components $\Pi^1_{\Lambda/\Sigma}(q^2)$ and $\Pi^0_{\Lambda/\Sigma}(q^2)$, respectively. Their detailed analytic expressions are,

$$\begin{aligned}
\rho^1_{\Lambda,QCD}(u) &= \frac{u^2}{64\pi^4}-\left(\frac{1}{3}m_s\langle\bar{q}q\rangle-\frac{1}{4}m_s\langle\bar{s}s\rangle\right)\frac{1}{\pi^2}+\langle g_s^2GG\rangle\frac{1}{128\pi^4} \\
&+\left(\frac{1}{12}m_s\langle\bar{q}g_s\sigma Gq\rangle-\frac{1}{12}m_s\langle\bar{s}g_s\sigma Gs\rangle\right)\frac{\delta(u)}{\pi^2} \\
&-\left(\frac{2}{9}\langle\bar{q}q\rangle^2-\frac{8}{9}\langle\bar{q}q\rangle\langle\bar{s}s\rangle\right)\delta(u)+\frac{1}{27}\langle\bar{q}q\rangle^2g_s^2\frac{\delta(u)}{\pi^2} \\
&+\frac{37}{15696}m_s\langle g_s^2GG\rangle\langle\bar{s}s\rangle\frac{\delta(u)}{\pi^2T^2} \\
&+\left(\frac{1}{6}\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle-\frac{2}{9}\langle\bar{q}q\rangle\langle\bar{s}g_s\sigma Gs\rangle-\frac{1}{9}\langle\bar{q}g_s\sigma Gq\rangle\langle\bar{s}s\rangle\right)\frac{\delta(u)}{T^2} \, ,
\end{aligned} \tag{27}$$

$$\begin{aligned}
\rho^0_{\Lambda,QCD}(u) &= -m_s\frac{u^2}{96\pi^4}-\left(\frac{1}{3}\langle\bar{q}q\rangle-\frac{1}{12}\langle\bar{s}s\rangle\right)\frac{u}{\pi^2}+m_s\langle g_s^2GG\rangle\frac{1}{384\pi^4} \\
&+\left(\langle\bar{q}g_s\sigma Gq\rangle-\langle\bar{s}g_s\sigma Gs\rangle\right)\frac{1}{24\pi^2} \\
&+\left(\frac{4}{3}m_s\langle\bar{q}q\rangle^2-\frac{4}{9}m_s\langle\bar{q}q\rangle\langle\bar{s}s\rangle\right)\delta(u)-m_s\langle\bar{q}q\rangle^2g_s^2\frac{\delta(u)}{243\pi^2} \\
&+\left(\frac{1}{96}\langle g_s^2GG\rangle\langle\bar{q}q\rangle-\frac{1}{288}\langle g_s^2GG\rangle\langle\bar{s}s\rangle\right)\frac{\delta(u)}{\pi^2} \, ,
\end{aligned} \tag{28}$$

$$
\begin{aligned}
\rho^1_{\Sigma,QCD}(u) &= \frac{u^2}{128\pi^4}+m_s\langle\bar{s}s\rangle\frac{1}{8\pi^2}+\langle g_s^2GG\rangle\frac{1}{256\pi^4} \\
&-m_s\langle\bar{s}g_s\sigma Gs\rangle\frac{\delta(u)}{24\pi^2} \\
&+\langle\bar{q}q\rangle^2\frac{\delta(u)}{3}+\left(\frac{1}{81}\langle\bar{q}q\rangle^2g_s^2+\frac{1}{162}\langle\bar{s}s\rangle^2g_s^2\right)\frac{\delta(u)}{\pi^2} \\
&-\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle\frac{\delta(u)}{12T^2}\,, \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
\rho^0_{\Sigma,QCD}(u) &= m_s\frac{u^2}{64\pi^4}-\langle\bar{s}s\rangle\frac{u}{8\pi^2}-m_s\langle g_s^2GG\rangle\frac{1}{256\pi^4} \\
&+m_s\langle\bar{q}q\rangle^2\frac{2\delta(u)}{3}+m_sg_s^2\langle\bar{q}q\rangle^2\frac{\delta(u)}{162\pi^2} \\
&+\langle\bar{s}s\rangle\langle g_s^2GG\rangle\frac{\delta(u)}{288\pi^2}\,. \qquad (30)
\end{aligned}
$$

Based on the analytical expressions of the spectral densities and Eq.(26), it is straightforward to obtain the analytical expressions of the masses of the Λ and Σ baryons,

$$
m^2_{\Lambda/\Sigma} = \frac{-\frac{\partial}{\partial\tau}\int_0^{s^0_{\Lambda/\Sigma}}\rho^{1/0}_{\Lambda/\Sigma,QCD}(u)\exp\left(-\frac{u}{T^2}\right)du}{\int_0^{s^0_{\Lambda/\Sigma}}\rho^{1/0}_{\Lambda/\Sigma,QCD}(u)\exp\left(-\frac{u}{T^2}\right)du}\,, \qquad (31)
$$

where $\tau=\frac{1}{T^2}$.

4 Numerical results and discussions

For the traditional QCD sum rules, the vacuum condensates are input parameters in the numerical calculations, their standard values are determined to be $\langle\overline{q}q\rangle=-(0.24\pm0.01\,\text{GeV})^3$, $\langle\overline{s}s\rangle=(0.8\pm 0.1)\langle\overline{q}q\rangle$, $\langle\overline{q}g_s\sigma Gq\rangle=m_0^2\langle\overline{q}q\rangle$, $\langle\overline{s}g_s\sigma Gs\rangle=m_0^2\langle\overline{s}s\rangle$, $m_0^2=(0.8\pm0.1)\,\text{GeV}^2$, $\langle\frac{\alpha_s}{\pi}GG\rangle=(0.33\,\text{GeV})^4$ at the energy scale $\mu=1\,\text{GeV}$ [30, 31, 32, 40], the $\overline{MS}$ masses $m_c(m_c)=(1.275\pm0.025)\,\text{GeV}$ and $m_s(\mu=2\,\text{GeV})=0.095\pm0.005\,\text{GeV}$ are taken from the Particle Data Group [41], the energy-scale dependence of the input parameters are shown as,

$$
\begin{aligned}
\langle\overline{q}q\rangle(\mu) &= \langle\overline{q}q\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}}\,, \\
\langle\overline{s}s\rangle(\mu) &= \langle\overline{s}s\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}}\,, \\
\langle\overline{q}g_s\sigma Gq\rangle(\mu) &= \langle\overline{q}g_s\sigma Gq\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}}\,, \\
\langle\overline{s}g_s\sigma Gs\rangle(\mu) &= \langle\overline{s}g_s\sigma Gs\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}}\,, \\
m_c(\mu) &= m_c(m_c)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{33-2n_f}}\,, \\
m_s(\mu) &= m_s(2\text{GeV})\left[\frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})}\right]^{\frac{12}{33-2n_f}}\,, \\
\alpha_s(\mu) &= \frac{1}{b_0t}\left[1-\frac{b_1}{b_0^2}\frac{\log t}{t}+\frac{b_1^2(\log^2t-\log t-1)+b_0b_2}{b_0^4t^2}\right]\,, \qquad (32)
\end{aligned}
$$

where $t=\log\frac{\mu^2}{\Lambda^2_{QCD}}$, $b_0=\frac{33-2n_f}{12\pi}$, $b_1=\frac{153-19n_f}{24\pi^2}$, $b_2=\frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3}$ and $\Lambda_{QCD}=213$ MeV, 296 MeV, 339 MeV for the flavors $n_f=5,4,3$, respectively [41, 42], for the strong decays studied in the present work, $n_f=4$, for the QCD sum rules of the Λ and Σ baryons, $n_f=3$. The energy scales μ are set as $\frac{m_{\eta_c}}{2}$ for the decay channels $P_{cs}(4338)\to\eta_c+\Lambda$ and $P_{cs}(4460)\to\eta_c+\Sigma$, $\mu=\frac{m_{J/\psi}}{2}$ for the decay channels $P_{cs}(4338)\to J/\psi+\Lambda$ and $P_{cs}(4460)\to J/\psi+\Sigma$, and the energy scale is set as $\mu=1\,\rm{GeV}$ for the QCD sum rules of the Λ and Σ baryons.

For the mass of the $P_{cs}(4338)$, we use the experimental result $m_P=4.338\,\rm{GeV}$ [5], and for the mass of the $P_{cs}(4460)$, we follow the conclusion of Ref.[11], and set it as $m_{P'}=4.460\,\rm{GeV}$. From the Particle Data Group [41], the masses of the mesons and baryons are chosen as $m_{\eta_c}=2.984\,\rm{GeV}$, $m_{J/\psi}=3.097\,\rm{GeV}$, $m_\Lambda=1.116\,\rm{GeV}$ and $m_\Sigma=1.189\,\rm{GeV}$, respectively. For the pole residues of the $P_{cs}(4338)$ and $P_{cs}(4460)$, we use the numerical results in Ref.[11], that is, $\lambda_P=1.43\times10^{-3}\,\rm{GeV}^6$ and $\lambda_{P'}=1.37\times10^{-3}\,\rm{GeV}^6$. As for the threshold parameters $\sqrt{s^0_{\eta_c}}$ and $\sqrt{s^0_{J/\psi}}$, they are chosen as $\sqrt{s^0_{\eta_c}}=3.50\,\rm{GeV}$ and $\sqrt{s^0_{J/\psi}}=3.60\,\rm{GeV}$ [33]. Moreover, the decay constants $f_{J/\psi}=0.418\,\rm{GeV}$ and $f_{\eta_c}=0.387\,\rm{GeV}$ from the QCD sum rules combined with lattice QCD [43].

For the QCD sum rules of the Λ and Σ baryons, the masses and pole residues of the baryon states are phenomenologically solved via $m_{\Lambda/\Sigma}=\frac{1}{2}(m^1_{\Lambda/\Sigma}+m^0_{\Lambda/\Sigma})$ and $\lambda_{\Lambda/\Sigma}=\frac{1}{2}(\lambda^1_{\Lambda/\Sigma}+\lambda^0_{\Lambda/\Sigma})$, where the $m^{1/0}_{\Lambda/\Sigma}$ and $\lambda^{1/0}_{\Lambda/\Sigma}$ are the masses and pole residues derived from the spectral densities $\rho^{1/0}_{\Lambda/\Sigma,QCD}(u)$, respectively. The diagrams $m_\Lambda-T^2$ and $\lambda_\Lambda-T^2$ are shown in the Fig.1, based on the numerical calculations by setting the threshold parameter $\sqrt{s^0_\Lambda}=1.59\,\rm{GeV}$, the Borel window of the Λ baryon ranges from $T^2_{min}=1.10\,\rm{GeV}^2$ to $T^2_{max}=1.50\,\rm{GeV}^2$ with the pole contribution being $(43-62)\%$, moreover, the extracted mass from the Borel window is $1.118\,\rm{GeV}$, which coincides very well with the data $m_\Lambda=1.116\,\rm{GeV}$ from the Particle Data Group [41], thus the value of the extracted pole residue is determined as $\lambda_\Lambda=2.87\times10^{-2}\,\rm{GeV}^3$. The diagrams $m_\Sigma-T^2$ and $\lambda_\Sigma-T^2$ are shown in the Fig.2, for the Σ baryon, $\sqrt{s^0_\Sigma}=1.68\,\rm{GeV}$, the Borel window ranges from $T^2_{min}=1.15\,\rm{GeV}^2$ to $T^2_{max}=1.55\,\rm{GeV}^2$, the pole contribution in the Borel window is $(41-61)\%$, and the extracted mass from the center of the Borel window is $1.188\,\rm{GeV}$, which also coincides well with the data $m_\Sigma=1.189\,\rm{GeV}$ from the Particle Data Group [41], thus the pole residue of the Σ baryon is determined as $\lambda_\Sigma=2.17\times10^{-2}\,\rm{GeV}^3$.

For the spectral densities ρ_Z, $Z=a,b,\cdots,h$, they all contain two Borel parameters T_1^2 and T_2^2, namely, $\rho_Z=\rho_Z(T_1^2,T_2^2)$, obviously, the hadronic coupling constants also rely on T_1^2 and T_2^2, that is, $g=g(T_1^2,T_2^2)$. For the QCD sum rules, the error bounds due to the Borel parameters should be tiny, thus, flat Borel platform should be obtained to extract the physical quantities, then, it is straightforward to simply set $T_1^2=T_2^2=T^2$. Via trivial and error, the Borel platforms of each hadronic coupling constants are obtained by setting the free parameters C_Z, they are determined as, $C_a=5.95\times10^{-6}\,\rm{GeV}^9$, $C_b=3.86\times10^{-5}\,\rm{GeV}^{10}$, $C_{J/\psi\Lambda,T}=1.80\times10^{-5}\,\rm{GeV}^{11}$, $C_{J/\psi\Lambda,V}=2.50\times10^{-5}\,\rm{GeV}^{11}$, $C_e=-1.23\times10^{-5}\,\rm{GeV}^9+4.92\times10^{-7}T^2\,\rm{GeV}^7$, $C_f=-7.52\times10^{-5}\,\rm{GeV}^{10}+3.01\times10^{-7}T^2\,\rm{GeV}^8$, $C_{J/\psi\Sigma,T}=-2.87\times10^{-5}\,\rm{GeV}^{11}+1.18\times10^{-7}T^2\,\rm{GeV}^9$, $C_{J/\psi\Sigma,V}=-3.67\times10^{-5}\,\rm{GeV}^{11}-2.93\times10^{-8}T^2\,\rm{GeV}^9$.

In order to estimate the error bounds, the approximations $\frac{\delta\lambda_P}{\lambda_P}=\frac{\delta\lambda_{P'}}{\lambda_{P'}}=\frac{\delta\lambda_\Lambda}{\lambda_\Lambda}=\frac{\delta\lambda_\Sigma}{\lambda_\Sigma}=\frac{\delta f_{J/\psi}}{f_{J/\psi}}=\frac{\delta f_{\eta_c}}{f_{\eta_c}}$ are applied [34, 44, 45], moreover, the error bounds due to the parameters C_Z are not considered, under such considerations, the $g-T^2$ graphs are shown in the Figs.3-6, and their numerical results are extracted as,

$$\begin{aligned}
g_{\eta\Lambda,a} &= 0.184^{+0.048}_{-0.048}\,, \qquad T^2 = 5.5-6.5\,\rm{GeV}^2\,, \\
g_{\eta\Lambda,b} &= 0.181^{+0.054}_{-0.054}\,, \qquad T^2 = 5.5-6.5\,\rm{GeV}^2\,, \\
g_{J/\psi\Lambda,T} &= 0.135^{+0.042}_{-0.042}\,, \qquad T^2 = 5.5-6.5\,\rm{GeV}^2\,, \\
g_{J/\psi\Lambda,V} &= 0.371^{+0.107}_{-0.107}\,, \qquad T^2 = 6.0-7.0\,\rm{GeV}^2\,, \\
g_{\eta\Sigma,e} &= 0.577^{+0.096}_{-0.096}\,, \qquad T^2 = 5.5-6.5\,\rm{GeV}^2\,, \\
g_{\eta\Sigma,f} &= 0.577^{+0.106}_{-0.106}\,, \qquad T^2 = 6.0-7.0\,\rm{GeV}^2\,, \\
g_{J/\psi\Sigma,T} &= 1.012^{+0.215}_{-0.215}\,, \qquad T^2 = 6.0-7.0\,\rm{GeV}^2\,, \\
g_{J/\psi\Sigma,V} &= 0.129^{+0.314}_{-0.324}\,, \qquad T^2 = 6.0-7.0\,\rm{GeV}^2\,.
\end{aligned} \tag{33}$$

Based on the hadronic coupling constants, we obtain the corresponding partial decay widths directly,

$$\begin{aligned}
\Gamma^a(P_{cs}(4338)\to\eta_c\Lambda) &= 0.95^{+0.50}_{-0.50}\,\rm{MeV}\,, \\
\Gamma^b(P_{cs}(4338)\to\eta_c\Lambda) &= 0.92^{+0.55}_{-0.55}\,\rm{MeV}\,, \\
\Gamma(P_{cs}(4338)\to J/\psi\Lambda) &= 5.21^{+8.00}_{-5.21}\,\rm{MeV}\,, \\
\Gamma^e(P_{cs}(4460)\to\eta_c\Sigma) &= 11.04^{+3.67}_{-3.67}\,\rm{MeV}\,, \\
\Gamma^f(P_{cs}(4460)\to\eta_c\Sigma) &= 11.04^{+4.06}_{-4.06}\,\rm{MeV}\,, \\
\Gamma(P_{cs}(4460)\to J/\psi\Sigma) &= 14.76^{+22.71}_{-14.76}\,\rm{MeV}\,,
\end{aligned} \tag{34}$$

where the $\Gamma^{a/b}$ are due to the hadronic coupling constants $g_{\eta\Lambda,a/b}$, respectively, the $\Gamma^{e/f}$ are due to the hadronic coupling constants $g_{\eta\Sigma,e/f}$, respectively. Taking the average value $\frac{1}{2}(\Gamma^a+\Gamma^b)$ for the decay channel $P_{cs}(4338)\to\eta_c\Lambda$, the width of the $P_{cs}(4338)$ is then 6.15 MeV, it is in good agreement with experimental data [5], moreover, the ratio of the partial decay widths,

$$\frac{\Gamma(P_{cs}(4338)\to\eta_c\Lambda)}{\Gamma(P_{cs}(4338)\to J/\psi\Lambda)} = 0.18\,. \tag{35}$$

As for the $P_{cs}(4460)$, which is the high isospin cousin of the $P_{cs}(4338)$, we also take the average value $\frac{1}{2}(\Gamma^e+\Gamma^f)$ for the decay channel $P_{cs}(4460)\to\eta_c\Sigma$, the width of this resonance state is then 25.80 MeV, the ratio of its partial decay widths,

$$\frac{\Gamma(P_{cs}(4460)\to\eta_c\Sigma)}{\Gamma(P_{cs}(4460)\to J/\psi\Sigma)} = 0.48\,. \tag{36}$$

5 Conclusions

In the present work, the hadronic coupling constants in the two-body strong decays of the $P_{cs}(4338)$ and $P_{cs}(4460)$ are studied via the QCD sum rules. The numerical results show that the theoretical calculations are in good agreement with the experimental data for the width of the $P_{cs}(4338)$, the ratio of the partial decay widths is waiting for more experimental data to testify, it will in return judge our interpretation of the physical nature of the exotic pentaquark candidate $P_{cs}(4338)$. The hadronic decays for the predicted state $P_{cs}(4460)$ are also obtained, which would present additional information for the possible observation of this state in the future experiment.

Acknowledgements

This work is supported by National Natural Science Foundation, Grant Number 12175068 and the Fundamental Research Funds for the Central Universities of China.

References

[1] R. Aaij et al, Phys. Rev. Lett. **115** (2015) 072001.

[2] R. Aaij et al, Phys. Rev. Lett. **122** (2019) 222001.

[3] R. Aaij et al, Sci. Bull. **66** (2021) 1278.

[4] R. Aaij et al, Phys. Rev. Lett. **128** (2022) 062001.

[5] R. Aaij et al, Phys. Rev. Lett. **131** (2023) 031901.

[6] M. J. Yan, F. Z. Peng, M. S. Sanchez and M. P. Valderrama, Phys. Rev. **D107** (2023) 074025.

[7] P. G. Ortega, D. R. Entem and F. Fernsndez, Phys. Lett. **B838** (2023) 137747.

[8] J. T. Zhu, S. Y. Kong and J. He, Phys. Rev. **D107** (2023) 034029.

[9] K. Azizi, Y. Sarac and H. Sundu, Phys. Rev. **D108** (2023) 074010.

[10] L. Meng, B. Wang and S. L. Zhu, Phys. Rev. **D107** (2023) 014005.

[11] X. W. Wang and Z. G. Wang, Chin. Phys. **C47** (2023) 013109.

[12] U. Ozdem, Phys. Lett. **B836** (2023) 137635.

[13] S. X. Nakamura and J. J. Wu, Phys. Rev. **D108** (2023) L011501.

[14] F. L. Wang and X. Liu, Phys. Rev. **D109** (2024) 014043.

[15] B. Wang, K. Chen, L. Meng and S. L. Zhu, arXiv: 2312.13591.

[16] T. J. Burns and E. S. Swanson, Phys. Lett. **B838** (2023) 137715.

[17] A. Giachino, A. Hosaka, E. Santopinto, S. Takeuchi, M. Takizawa and Y. Yamaguchi, Phys. Rev. **D108** (2023) 074012.

[18] K. Chen, Z. Y. Lin and S. L. Zhu, Phys. Rev. **D106** (2022) 116017.

[19] Z. Y. Yang, F. Z. Peng, M. J. Yan, M. S. Sanchez and M. P. Valderrama, arXiv: 2211.08211.

[20] A. Feijoo, W. F. Wang, C. W. Xiao, J. J. Wu, E. Oset, J. Nieves and B. S. Zou, Phys. Lett. **B839** (2023) 137760.

[21] E. Y. Paryev, Nucl. Phys. **A1037** (2023) 122687.

[22] G. Yang, J. L. Ping and J. Segovia, arXiv: 2311.01044.

[23] S. X. Nakamura and J. J. Wu, arXiv: 2312.01989.

[24] Q. Wu and D. Y. Chen, arXiv: 2402.14467.

[25] S. Y. Li, Y. R. Liu, Z. L. Man, Z. G. Si and J. Wu, Phys. Rev. **D108** (2023) 056015.

[26] W. X. Zhang, C. L. Liu and D. J. Jia, arXiv: 2312.12770.

[27] X. W. Wang, Z. G. Wang, G. L. Yu and Q. Xin, Sci. China Phys. Mech. Astron. **65** (2022) 291011.

[28] X. W. Wang and Z. G. Wang, Int. J. Mod. Phys. **A37** (2022) 2250189.

[29] X. W. Wang and Z. G. Wang, Chin. Phys. **C48** (2024) 053102.

[30] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979) 385.

[31] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. **B147** (1979) 448.

[32] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rept. **127** (1985) 1.

[33] Z. G. Wang and X. Wang, Chin. Phys. **C44** (2020) 103102.

[34] Z. G. Wang, H. J. Wang and Q. Xin, Chin. Phys. **C45** (2021) 063104.

[35] Z. G. Wang and J. X. Zhang, Eur. Phys. J. **C78** (2018) 14.

[36] Z. G. Wang, Eur. Phys. J. **C79** (2019) 184.

[37] Z. G. Wang and Z. Y. Di, Eur. Phys. J. **C79** (2019) 72.

[38] Z. G. Wang, Acta Phys. Polon. **B51** (2020) 435.

[39] Z. G. Wang, Int. J. Mod. Phys. **A34** (2019) 1950110.

[40] P. Colangelo and A. Khodjamirian, hep-ph/0010175.

[41] P. A. Zyla et al, Prog. Theor. Exp. Phys. **2020** (2020) 083C01.

[42] S. Narison and R. Tarrach, Phys. Lett. **B125** (1983) 217.

[43] D. Becirevic, G. Duplancic, B. Klajn, B. Melic and F. Sanfilippo, Nucl. Phys. **B883** (2014) 306.

[44] Z. G. Wang, Chin. Phys. **C46** (2022) 123106.

[45] Z. G. Wang, Chin. Phys. **C46** (2022) 103106.

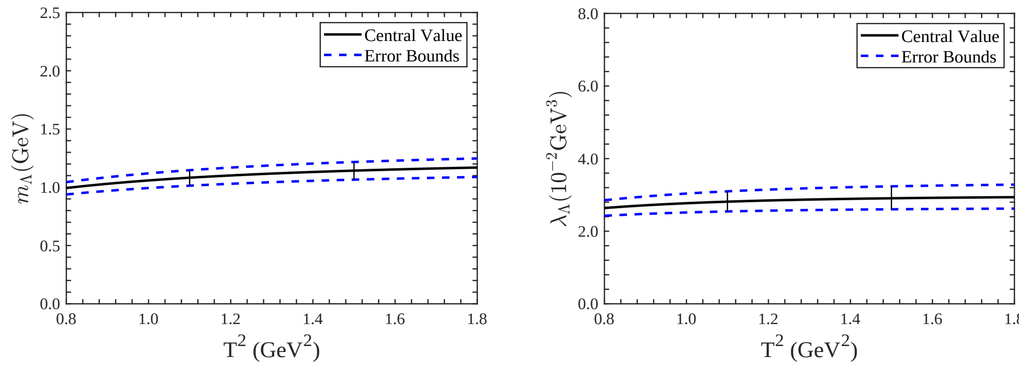

Figure 1: The numerical results of the mass (Left) and pole residue (Right) of the Λ baryon.

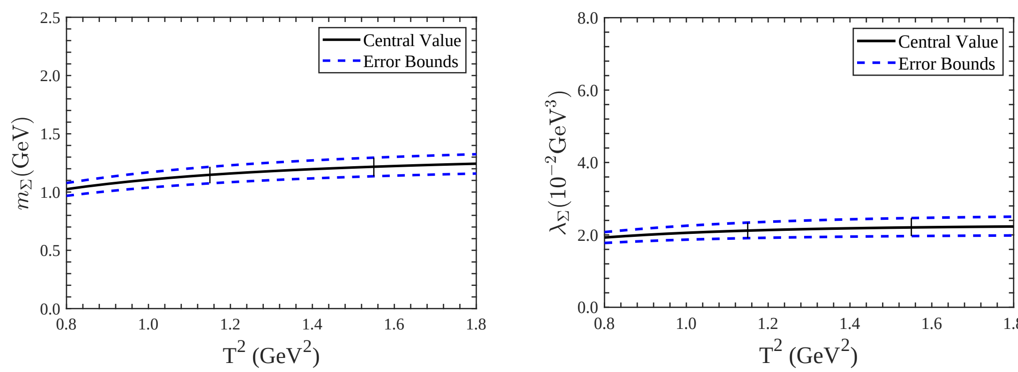

Figure 2: The numerical results of the mass (Left) and pole residue (Right) of the Σ baryon.

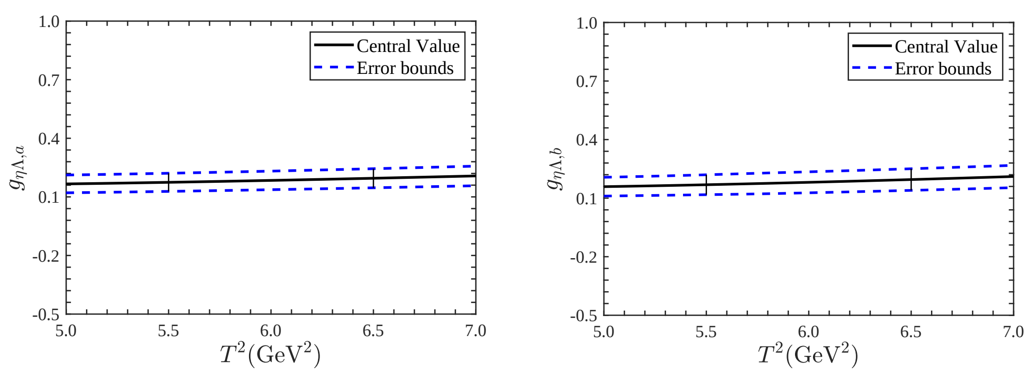

Figure 3: The $g_{\eta\Lambda,a} - T^2$ (Left) and $g_{\eta\Lambda,b} - T^2$ (Right) curves, where the region among the two short vertical lines of each graph represents the Borel platforms.

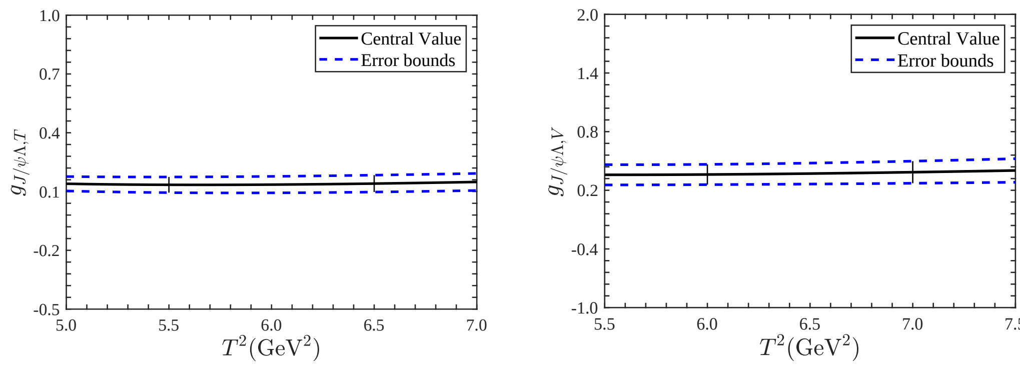

Figure 4: The $g_{J/\psi\Lambda,T} - T^2$ (Left) and $g_{J/\psi\Lambda,V} - T^2$ (Right) curves, where the region among the two short vertical lines of each graph represents the Borel platforms.

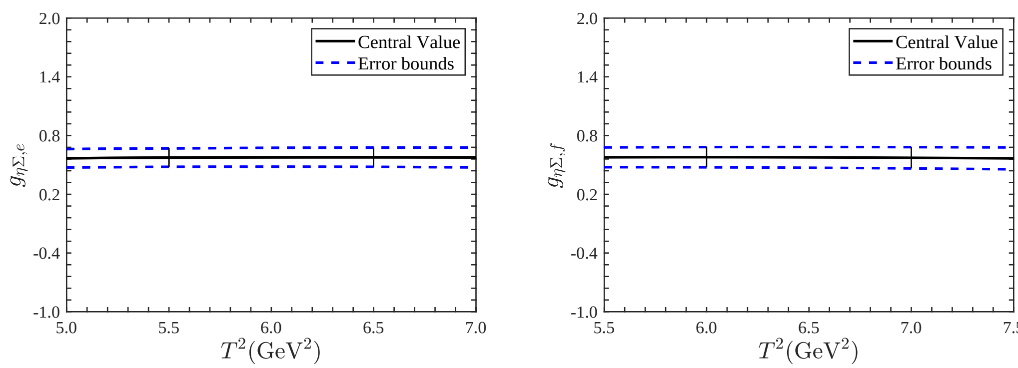

Figure 5: The $g_{\eta\Sigma,e} - T^2$ (Left) and $g_{\eta\Sigma,f} - T^2$ (Right) curves, where the region among the two short vertical lines of each graph represents the Borel platforms.

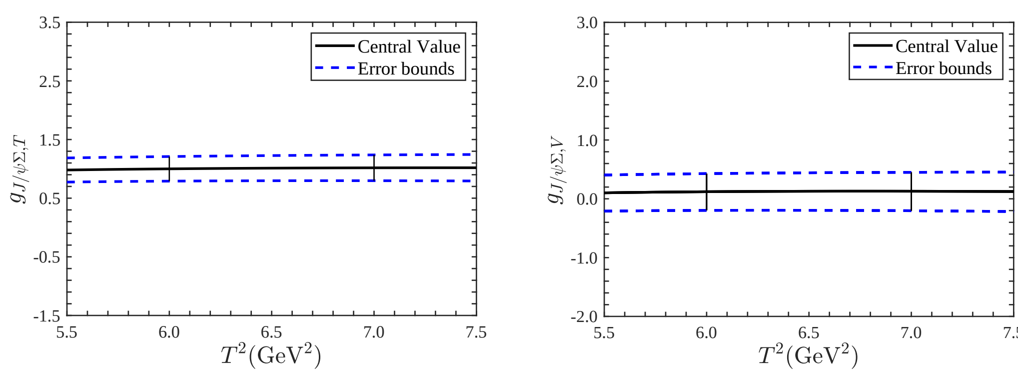

Figure 6: The $g_{J/\psi\Sigma,T} - T^2$ (Left) and $g_{J/\psi\Sigma,V} - T^2$ (Right) curves, where the region among the two short vertical lines of each graph represents the Borel platforms.