Recoil heating of a dielectric particle illuminated by a linearly polarized plane wave within the Rayleigh regime

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We investigate the recoil heating phenomenon experienced by a dielectric spherical particle when it interacts with a linearly polarized plane wave within the Rayleigh regime. We derive the fluctuating force acted upon the particle arising from the fluctuations of the electromagnetic fields. Our derivations reveal that the spectral density of the fluctuating force along the propagation direction is $7\hbar\omega_0 P_{\mathrm{scat}}/5c^2$. Meanwhile, along the direction of the electric and magnetic fields, it is $11\hbar\omega_0 P_{\mathrm{scat}}/5c^2$ and $2\hbar\omega_0 P_{\mathrm{scat}}/5c^2$, respectively. Here, P_{scat} denotes the power scattered by the particle, $\hbar\omega_0$ represents the energy of a photon, and c is the speed of light. Recoil heating imposes fundamental limitations in levitated optomechanics, constraining the minimum temperatures achievable in cooling processes, the coherence time of the system, and the sensitivity of force measurements.

The quantum fluctuations of electromagnetic fields induce a heating mechanism in particle motion, resulting from random momentum transfer during photon scattering[1–4]. This phenomenon, referred to as photon recoil heating, sets a fundamental limit on the achievable final temperature in cooling processes[5, 6]. It also imposes constraints on the coherence time and force sensitivity in levitated optomechanical systems[1, 7, 8]. In addition, when a particle interacts with an electromagnetic field, it experiences damping of its motion, referred to as radiation damping[9].The equilibrium energy of the particle is determined by the balance between recoil heating and radiation damping, as dictated by the fluctuation-dissipation theorem[1, 10].

Previous studies have estimated recoil heating by drawing an analogy with shot noise[1, 11]. In this letter, we present a rigorous derivation of recoil heating for a dielectric particle interacting with linearly polarized plane waves within the Rayleigh regime. We calculate the fluctuating force induced to the particle motion due to the quantum fluctuations of the electromagnetic fields. We utilize the quantization of electromagnetic fields within a dissipative medium, which is based on the principles of polarization and magnetization quantum noises[12–14].

Consider a dielectric spherical particle illuminated by a monochromatic plane wave. The plane wave is assumed to be x-polarized and propagating along the z-direction. The electromagnetic fields experienced by the particle can be written as:

$$\hat{\mathbf{E}}(\mathbf{r},t) = \frac{E_0}{2}\mathbf{e}_x e^{i(k_0 z-\omega_0 t)} + \int_0^\infty \hat{\mathbf{E}}_N(\mathbf{r},\omega)e^{-i\omega t}d\omega + \mathrm{H.c.}, \tag{1a}$$

$$\begin{aligned}\hat{\mathbf{B}}(\mathbf{r},t) =& \frac{E_0}{2c}\mathbf{e}_y e^{i(k_0 z-\omega_0 t)} \\ &+ \int_0^\infty \frac{1}{i\omega}\nabla\times\hat{\mathbf{E}}_N(\mathbf{r},\omega)e^{-i\omega t}d\omega + \mathrm{H.c.}.\end{aligned} \tag{1b}$$

The first terms in the above expressions represent the incident plane wave, while the second terms are dedicated to describing the quantum fluctuations of the electromagnetic fields. Here, ω_0 and k_0 represent the angular frequency and wave number of the incident wave, respectively, and c denotes the speed of light.

We assume that the incident wave is in the classical limit, which implies that the fluctuations are identical to those for vacuum. Consequently, the electromagnetic noise is assumed to have a vanishing average and obeys the following correlation relations[12–14]:

$$\langle\hat{\mathbf{E}}_N(\mathbf{r},\omega)\hat{\mathbf{E}}_N^\dagger(\mathbf{r}',\omega')\rangle = \frac{\hbar\mu_0\omega^2}{\pi}\mathrm{Im}\left[\mathbf{G}_0(\mathbf{r},\mathbf{r}',\omega)\right]\delta(\omega-\omega'), \tag{2a}$$

$$\begin{aligned}\langle\hat{\mathbf{E}}_N^\dagger(\mathbf{r},\omega)\hat{\mathbf{E}}_N(\mathbf{r}',\omega')\rangle &= \langle\hat{\mathbf{E}}_N(\mathbf{r},\omega)\hat{\mathbf{E}}_N(\mathbf{r}',\omega')\rangle \\ &= \langle\hat{\mathbf{E}}_N^\dagger(\mathbf{r},\omega)\hat{\mathbf{E}}_N^\dagger(\mathbf{r}',\omega')\rangle = 0.\end{aligned} \tag{2b}$$

Here, $\mathbf{G}_0$ represents the dyadic Green's function of the free space[15].

If the particle is much smaller than the wavelength of the incident wave, it can be modeled as an electric dipole[16–18]. This equivalent dipole moment $\hat{\mathbf{p}}$ can be expressed by:

$$\hat{\mathbf{p}} = \frac{E_0}{2}\alpha(\omega_0)\mathbf{e}_x e^{-i\omega_0 t} + \int_0^\infty \alpha(\omega)\hat{\mathbf{E}}_N(\mathbf{r}_p,\omega)e^{-i\omega t}d\omega + \mathrm{H.c.}, \tag{3}$$

where $\alpha(\omega)$ represents the polarizability of the particle, defined as:

$$\alpha(\omega) = \frac{\alpha_0}{1-i\omega^3\alpha_0/6\pi\epsilon_0 c^3}. \tag{4}$$

Here, $\alpha_0 = 4\pi\epsilon_0 R_p^3(\epsilon_p-1)/(\epsilon_p+2)$ is the quasi-static polarizability of the particle, with R_p being the particle's radius and ϵ_p being its dielectric constant[18].

The force exerted upon the particle can be calculated from[15, 16]:

$$\hat{\mathbf{F}} = (\hat{\mathbf{p}}\cdot\nabla)\,\hat{\mathbf{E}} + \frac{\partial\hat{\mathbf{p}}}{\partial t}\times\hat{\mathbf{B}}. \tag{5}$$

This force can be decomposed to two parts: $\hat{\mathbf{F}} = \bar{\mathbf{F}} + \delta\hat{\mathbf{F}}$. The first term, $\bar{\mathbf{F}}$, represents the deterministic part of the force exerted by the incident wave on the particle. Since the incident wave is a plane wave, $\bar{\mathbf{F}}$ is equivalent to the radiation pressure, which is given by:

$$\bar{\mathbf{F}} = \frac{k_0 E_0^2}{2}\mathrm{Im}\left[\alpha(\omega_0)\right]\mathbf{e}_z. \tag{6}$$

The second term, $\delta\hat{\mathbf{F}}$ stands for the fluctuating force acted upon the particle, which will be discussed in detail in the following.

Fluctuating force along x-direction.— As indicated by Eq. 5, the fluctuating force along the x-direction, which aligns with the polarization direction of the incident wave, is given by:

$$\begin{aligned}\delta\hat{F}_x = {} & \bar{p}_x\int_0^\infty \partial_x \hat{E}_{N_x}(\mathrm{r}_p,\omega)e^{-i\omega t}d\omega \\ & + \partial_z\bar{E}_x\int_0^\infty \alpha(\omega)\hat{E}_{N_z}(\mathrm{r}_p,\omega)e^{-i\omega t}d\omega \\ & + \bar{B}_y\int_0^\infty i\omega\alpha(\omega)\hat{E}_{N_z}(\mathrm{r}_p,\omega)e^{-i\omega t}d\omega \\ & + \mathrm{H.c.},\end{aligned} \tag{7}$$

where

$$\bar{p}_x = \mathrm{Re}\left[\alpha(\omega_0)E_0 e^{-i\omega_0 t}\right], \tag{8a}$$

$$\partial_z\bar{E}_x = \mathrm{Re}\left[\frac{i\omega_0}{c}E_0 e^{-i\omega_0 t}\right], \tag{8b}$$

$$\bar{B}_y = \mathrm{Re}\left[\frac{E_0}{c}e^{-i\omega_0 t}\right]. \tag{8c}$$

By utilizing the following relations regarding $\mathbf{G}_0$:

$$\mathrm{Im}\left[\mathbf{G}_0(\mathbf{r}_p,\mathbf{r}_p,\omega)\right] = \frac{\omega}{6\pi c}\mathbf{I}, \tag{9a}$$

$$\mathrm{Im}\left[\partial_i\mathbf{G}_0(\mathbf{r}_p,\mathbf{r}_p,\omega)\right] = \mathrm{Im}\left[\partial_i'\mathbf{G}_0(\mathbf{r}_p,\mathbf{r}_p,\omega)\right] = 0, \tag{9b}$$

$$\mathrm{Im}\left[\partial_i\partial_j'\mathbf{G}_0(\mathbf{r}_p,\mathbf{r}_p,\omega)\right] = \frac{\omega^3}{15\pi c^3}\delta_{ij}\mathbf{I} - \frac{\omega^3}{60\pi c^3}(\mathbf{e}_i\mathbf{e}_j + \mathbf{e}_j\mathbf{e}_i), \tag{9c}$$

it can be easily shown that the auto-correlation function of $\delta\hat{F}_x$ can be expressed as:

$$\begin{aligned}\langle\delta\hat{F}_x(t)\delta\hat{F}_x(t+\tau)\rangle = {} & \\ & \frac{\hbar\mu_0}{2\pi}\mathrm{Re}\left[|\alpha(\omega_0)|^2E_0^2e^{i\omega_0\tau}\right]\int_0^\infty \frac{\omega^5}{30\pi c^3}e^{i\omega\tau}d\omega + \\ & \frac{\hbar\mu_0}{2\pi}\mathrm{Re}\left[\frac{\omega_0^2}{c^2}E_0^2e^{i\omega_0\tau}\right]\int_0^\infty \frac{\omega^3}{6\pi c}|\alpha(\omega)|^2e^{i\omega\tau}d\omega + \\ & \frac{\hbar\mu_0}{2\pi}\mathrm{Re}\left[\frac{E_0^2}{c^2}e^{i\omega_0\tau}\right]\int_0^\infty \frac{\omega^5}{6\pi c}|\alpha(\omega)|^2e^{i\omega\tau}d\omega.\end{aligned} \tag{10}$$

We can now determine the spectral density of $\delta\hat{F}_x$ by taking the Fourier transform of its auto-correlation function, defined as follows:

$$\begin{aligned}S_{F_xF_x}(\Omega) &= \int_{-\infty}^{+\infty}\langle\delta\hat{F}_x(t)\delta\hat{F}_x(t+\tau)\rangle e^{i\Omega\tau}d\tau \\ &= \int_0^{+\infty}\int_{-\infty}^{+\infty}\frac{\hbar\mu_0}{4\pi}\alpha_0^2E_0^2\left(\frac{\omega^5}{5\pi c^3} + \frac{\omega^3\omega_0^2}{6\pi c^3}\right) \\ &\quad\times\left[e^{i(\omega+\omega_0+\Omega)\tau} + e^{i(\omega-\omega_0+\Omega)\tau}\right]d\tau d\omega.\end{aligned} \tag{11}$$

It is important to note that we employ the approximation $|\alpha(\omega)|^2 \simeq \alpha_0^2$ in deriving the above expression. Evaluating the integral over τ yields:

$$\begin{aligned}S_{F_xF_x}(\Omega) = \int_0^{+\infty} & \frac{\hbar\mu_0}{2}\alpha_0^2E_0^2\left(\frac{\omega^5}{5\pi c^3} + \frac{\omega^3\omega_0^2}{6\pi c^3}\right)\times \\ & \left[\delta(\omega+\omega_0+\Omega) + \delta(\omega-\omega_0+\Omega)\right]d\omega.\end{aligned} \tag{12}$$

Since the mechanical frequencies are much smaller than the optical frequencies, i.e. $\Omega \ll \omega_0$, the spectral density of $\delta\hat{F}_x$ can be simplified to:

$$S_{F_xF_x}(\Omega) \simeq \frac{11\hbar\mu_0\alpha_0^2E_0^2\omega_0^5}{60\pi c^3} = \frac{11}{5}\frac{\hbar\omega_0}{c^2}P_{\mathrm{scat}}, \tag{13}$$

where $P_{\mathrm{scat}} = \omega_0^4\alpha_0^2E_0^2/12\pi\epsilon_0c^3$ denotes the power scattered by the particle.

Fluctuating force along y-direction.— We aim to determine the fluctuating force along the y-direction, which corresponds to the direction of the incident magnetic field. According to Eq. 5, the fluctuating force along the y-direction is given by:

$$\begin{aligned}\delta\hat{F}_y = {} & \bar{p}_x\int_0^\infty \partial_x\hat{E}_{N_y}(\mathrm{r}_p,\omega)e^{-i\omega t}d\omega \\ & - \frac{\partial\bar{p}_x}{\partial t}\int_0^\infty \frac{1}{i\omega}\left[\partial_x\hat{E}_{N_y}(\mathrm{r}_p,\omega) - \partial_y\hat{E}_{N_x}(\mathrm{r}_p,\omega)\right]e^{-i\omega t}d\omega \\ & + \mathrm{H.c.}.\end{aligned} \tag{14}$$

Upon utilizing the relations in Eq. 9, we can calculate the auto-correlation function of $\delta\hat{F}_y$, resulting in

$$\begin{aligned}\langle\delta\hat{F}_y(t)\delta\hat{F}_y(t+\tau)\rangle = {} & \\ & \frac{\hbar\mu_0}{2\pi}\mathrm{Re}\left[|\alpha(\omega_0)|^2E_0^2e^{i\omega_0\tau}\right]\int_0^\infty \frac{\omega^5}{15\pi c^3}e^{i\omega\tau}d\omega - \\ & \frac{\hbar\mu_0}{\pi}\mathrm{Re}\left[i\omega_0|\alpha(\omega_0)|^2E_0^2e^{i\omega_0\tau}\right]\int_0^\infty \frac{i\omega^4}{12\pi c^3}e^{i\omega\tau}d\omega + \\ & \frac{\hbar\mu_0}{2\pi}\mathrm{Re}\left[\omega_0^2|\alpha(\omega_0)|^2E_0^2e^{i\omega_0\tau}\right]\int_0^\infty \frac{\omega^3}{6\pi c^3}e^{i\omega\tau}d\omega.\end{aligned} \tag{15}$$

Then, we can derive the spectral density of $\delta\hat{F}_y$, as follows:

$$S_{F_yF_y}(\Omega)=\int_0^{+\infty}\int_{-\infty}^{+\infty}\frac{\hbar\mu_0}{4\pi}\alpha_0^2E_0^2\times \Big[\Big(\frac{\omega^5}{15\pi c^3}+\frac{\omega^4\omega_0}{6\pi c^3}+\frac{\omega^3\omega_0^2}{6\pi c^3}\Big)e^{i(\omega+\omega_0+\Omega)\tau}+ \Big(\frac{\omega^5}{15\pi c^3}-\frac{\omega^4\omega_0}{6\pi c^3}+\frac{\omega^3\omega_0^2}{6\pi c^3}\Big)e^{i(\omega-\omega_0+\Omega)\tau}\Big]d\tau d\omega. \tag{16}$$

If we perform the integration over τ, one obtains:

$$S_{F_yF_y}(\Omega)=\int_0^{+\infty}\frac{\hbar\mu_0}{2}\alpha_0^2E_0^2\times \Big[\Big(\frac{\omega^5}{15\pi c^3}+\frac{\omega^4\omega_0}{6\pi c^3}+\frac{\omega^3\omega_0^2}{6\pi c^3}\Big)\delta(\omega+\omega_0+\Omega)+ \Big(\frac{\omega^5}{15\pi c^3}-\frac{\omega^4\omega_0}{6\pi c^3}+\frac{\omega^3\omega_0^2}{6\pi c^3}\Big)\delta(\omega-\omega_0+\Omega)\Big]d\omega. \tag{17}$$

Eventually, we can simplify the spectral density of $\delta\hat{F}_y$ to

$$S_{F_yF_y}(\Omega)\simeq\frac{\hbar\mu_0\alpha_0^2E_0^2\omega_0^5}{30\pi c^3}=\frac{2}{5}\frac{\hbar\omega_0}{c^2}P_{\text{scat}}, \tag{18}$$

given that the mechanical frequencies are much smaller than the optical frequency ω_0.

Fluctuating force along z-direction.— We now aim to derive the fluctuating force along the propagation direction. As indicated by Eq. 5, the fluctuating force along the z-direction is given by:

$$\begin{aligned}\delta\hat{F}_z&=\bar{p}_x\int_0^{\infty}\partial_x\hat{E}_{N_z}(\mathrm{r}_p,\omega)e^{-i\omega t}d\omega\\&+\frac{\partial\bar{p}_x}{\partial t}\int_0^{\infty}\frac{1}{i\omega}\left[\partial_z\hat{E}_{N_x}(\mathrm{r}_p,\omega)-\partial_x\hat{E}_{N_z}(\mathrm{r}_p,\omega)\right]e^{-i\omega t}d\omega\\&-\bar{B}_y\int_0^{\infty}i\omega\alpha(\omega)\hat{E}_{N_x}(\mathrm{r}_p,\omega)e^{-i\omega t}d\omega\\&+\text{H.c..}\end{aligned} \tag{19}$$

Upon using the relations given in Eq. 9, the auto-correlation function of $\delta\hat{F}_z$ can be written as:

$$\begin{aligned}&\langle\delta\hat{F}_z(t)\delta\hat{F}_z(t+\tau)\rangle=\\&\quad\frac{\hbar\mu_0}{2\pi}\text{Re}\left[|\alpha(\omega_0)|^2E_0^2e^{i\omega_0\tau}\right]\int_0^{\infty}\frac{\omega^5}{15\pi c^3}e^{i\omega\tau}d\omega-\\&\quad\frac{\hbar\mu_0}{\pi}\text{Re}\left[i\omega_0|\alpha(\omega_0)|^2E_0^2e^{i\omega_0\tau}\right]\int_0^{\infty}\frac{i\omega^4}{12\pi c^3}e^{i\omega\tau}d\omega+\\&\quad\frac{\hbar\mu_0}{2\pi}\text{Re}\left[\omega_0^2|\alpha(\omega_0)|^2E_0^2e^{i\omega_0\tau}\right]\int_0^{\infty}\frac{\omega^3}{6\pi c^3}e^{i\omega\tau}d\omega+\\&\quad\frac{\hbar\mu_0}{2\pi}\text{Re}\left[\frac{E_0^2}{c^2}e^{i\omega_0\tau}\right]\int_0^{\infty}|\alpha(\omega_0)|^2\frac{\omega^5}{6\pi c}e^{i\omega\tau}d\omega.\end{aligned} \tag{20}$$

Subsequently, we can derive the spectral density of $\delta\hat{F}_z$ by taking the Fourier transform from its auto-correlation function, resulting in

$$S_{F_zF_z}(\Omega)=\int_0^{+\infty}\int_{-\infty}^{+\infty}\frac{\hbar\mu_0}{4\pi}\alpha_0^2E_0^2\times \Big[\Big(\frac{7\omega^5}{30\pi c^3}+\frac{\omega^4\omega_0}{6\pi c^3}+\frac{\omega^3\omega_0^2}{6\pi c^3}\Big)e^{i(\omega+\omega_0+\Omega)\tau}+ \Big(\frac{7\omega^5}{30\pi c^3}-\frac{\omega^4\omega_0}{6\pi c^3}+\frac{\omega^3\omega_0^2}{6\pi c^3}\Big)e^{i(\omega-\omega_0+\Omega)\tau}\Big]d\tau d\omega. \tag{21}$$

Upon evaluating the integral over τ, one obtains

$$S_{F_zF_z}(\Omega)=\int_0^{+\infty}\frac{\hbar\mu_0}{2}\alpha_0^2E_0^2\times \Big[\Big(\frac{7\omega^5}{30\pi c^3}+\frac{\omega^4\omega_0}{6\pi c^3}+\frac{\omega^3\omega_0^2}{6\pi c^3}\Big)\delta(\omega+\omega_0+\Omega)+ \Big(\frac{7\omega^5}{30\pi c^3}-\frac{\omega^4\omega_0}{6\pi c^3}+\frac{\omega^3\omega_0^2}{6\pi c^3}\Big)\delta(\omega-\omega_0+\Omega)\Big]d\omega, \tag{22}$$

which can be further simplified to

$$S_{F_zF_z}(\Omega)\simeq\frac{7\hbar\mu_0\alpha_0^2E_0^2\omega_0^5}{60\pi c^3}=\frac{7}{5}\frac{\hbar\omega_0}{c^2}P_{\text{scat}}, \tag{23}$$

since the mechanical frequencies are much smaller than the optical frequency ω_0.

Particle dynamics.— The interaction of the particle with the incident wave gives rise to a drag force known as radiation damping, along with a fluctuating force corresponding to recoil heating. Additionally, the presence of a thermal bath induces mechanical damping and thermal noise. Therefore, we can describe the particle motion along the x-direction using the equation:

$$m\frac{d\hat{v}_x}{dt}=-m(\Gamma_m+\Gamma_x)\hat{v}_x+\hat{F}_x+\hat{\xi}, \tag{24}$$

where $\hat{F}_x$ represents the fluctuating force, satisfying Eq. 13, and $\Gamma_x=P_{\text{scat}}/mc^2$ is the radiation damping along x-direction[9]. Furthermore, Γ_m represents the mechanical damping rate, and ξ denotes the thermal noise, satisfying $\langle\xi(t)\xi(t')\rangle=2m\Gamma_mk_BT\delta(t-t')$. Here, m denotes the particle mass, k_B is the Boltzmann constant, and T represents the ambient temperature[12, 19]. We can now derive the variance of v_x, resulting in

$$\langle v_x^2\rangle=\frac{2m\Gamma_mk_BT+11\hbar\omega_0P_{\text{scat}}/5c^2}{2m^2(\Gamma_m+P_{\text{scat}}/mc^2)}. \tag{25}$$

When Γ_m is much smaller than P_{scat}/mc^2, which necessitates ultra-high vacuum, the variance of v_x simplifies to $1.1\hbar\omega_0/m$. Under that condition, thermal decoherency is surpassed by recoil heating and radiation damping. Similarly, one can obtain the variance of v_y and v_z, that resulting in $0.2\hbar\omega_0/m$ and $7\hbar\omega_0/60m$, respectively. It should be noted that the radiation damping rate along y and z-directions are given by $\Gamma_y = P_{\mathrm{scat}}/mc^2$, and $\Gamma_z = 6P_{\mathrm{scat}}/mc^2$, respectively[9].

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[1] V. Jain, J. Gieseler, C. Moritz, C. Dellago, R. Quidant, and L. Novotny, Direct measurement of photon recoil from a levitated nanoparticle, Physical review letters **116**, 243601 (2016).

[2] C. Gonzalez-Ballestero, M. Aspelmeyer, L. Novotny, R. Quidant, and O. Romero-Isart, Levitodynamics: Levitation and control of microscopic objects in vacuum, Science **374**, eabg3027 (2021).

[3] D. E. Chang, C. Regal, S. Papp, D. Wilson, J. Ye, O. Painter, H. J. Kimble, and P. Zoller, Cavity optomechanics using an optically levitated nanosphere, Proceedings of the National Academy of Sciences **107**, 1005 (2010).

[4] J. Millen, T. S. Monteiro, R. Pettit, and A. N. Vamivakas, Optomechanics with levitated particles, Reports on Progress in Physics **83**, 026401 (2020).

[5] J. Gieseler, B. Deutsch, R. Quidant, and L. Novotny, Subkelvin parametric feedback cooling of a laser-trapped nanoparticle, Physical review letters **109**, 103603 (2012).

[6] M. Kamba, H. Kiuchi, T. Yotsuya, and K. Aikawa, Recoil-limited feedback cooling of single nanoparticles near the ground state in an optical lattice, Physical Review A **103**, L051701 (2021).

[7] F. Tebbenjohanns, M. L. Mattana, M. Rossi, M. Frimmer, and L. Novotny, Quantum control of a nanoparticle optically levitated in cryogenic free space, Nature **595**, 378 (2021).

[8] C. Gonzalez-Ballestero, J. A. Zielińska, M. Rossi, A. Militaru, M. Frimmer, L. Novotny, P. Maurer, and O. Romero-Isart, Suppressing recoil heating in levitated optomechanics using squeezed light, PRX Quantum **4**, 030331 (2023).

[9] M. A. Abbassi and L. Novotny, Radiation damping of a rayleigh scatterer illuminated by a plane wave, arXiv preprint arXiv:2403.16618 (2024).

[10] L. Novotny, Radiation damping of a polarizable particle, Physical Review A **96**, 032108 (2017).

[11] T. Seberson and F. Robicheaux, Distribution of laser shot-noise energy delivered to a levitated nanoparticle, Physical Review A **102**, 033505 (2020).

[12] P. W. Milonni, *An introduction to quantum optics and quantum fluctuations* (Oxford University Press, 2019).

[13] S. Y. Buhmann, *Dispersion Forces I: Macroscopic quantum electrodynamics and ground-state Casimir, Casimir–Polder and van der Waals forces*, Vol. 247 (Springer, 2013).

[14] S. Y. Buhmann and D.-G. Welsch, Dispersion forces in macroscopic quantum electrodynamics, Progress in quantum electronics **31**, 51 (2007).

[15] L. Novotny and B. Hecht, *Principles of nano-optics* (Cambridge university press, 2012).

[16] P. C. Chaumet and M. Nieto-Vesperinas, Time-averaged total force on a dipolar sphere in an electromagnetic field, Optics letters **25**, 1065 (2000).

[17] M. A. Abbassi and K. Mehrany, Inclusion of the backaction term in the total optical force exerted upon rayleigh particles in nonresonant structures, Physical Review A **98**, 013806 (2018).

[18] S. Albaladejo, M. I. Marqués, M. Laroche, and J. J. Sáenz, Scattering forces from the curl of the spin angular momentum of a light field, Physical review letters **102**, 113602 (2009).

[19] M. A. Abbassi and K. Mehrany, Green's-function formulation for studying the backaction cooling of a levitated nanosphere in an arbitrary structure, Physical Review A **100**, 023823 (2019).