

Hypernuclear constraints on the existence and lifetime of a deeply bound H dibaryon

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Abstract

We study to what extent the unique observation of $\Lambda\Lambda$ hypernuclei by their weak decay into known Λ hypernuclei, with lifetimes of order 10^{-10} s, rules out the existence of a deeply bound doubly-strange ($\mathcal{S}=-2$) H dibaryon. Treating ${}_{\Lambda\Lambda}^6\text{He}$ (the Nagara emulsion event) in a realistic $\Lambda - \Lambda - {}^4\text{He}$ three-body model, we find that the ${}_{\Lambda\Lambda}^6\text{He} \rightarrow H + {}^4\text{He}$ strong-interaction lifetime increases beyond 10^{-10} s for $m_H < m_\Lambda + m_n$, about 176 MeV below the $\Lambda\Lambda$ threshold, so that such a deeply bound H is not in conflict with hypernuclear data. Constrained by Λ hypernuclear $\Delta\mathcal{S}=1$ nonmesonic weak-interaction decay rates, we follow EFT methods to evaluate the $\Delta\mathcal{S}=2$ $H \rightarrow nn$ weak-decay lifetime of H in the mass range $2m_n \lesssim m_H < m_\Lambda + m_n$. The resulting H lifetime is of order 10^5 s, many orders of magnitude shorter than required to qualify for a dark-matter candidate.

1. Introduction

The deuteron, with mass only 2.2 MeV below the sum of masses of its proton and neutron constituents, is the only particle-stable six-quark (hexaquark) dibaryon known so far. Here stability is regarded with respect to the lifetime of the proton, many orders of magnitude longer than the lifetime of the Universe (13.8 billion years [1]). Extending the very-light ud quark sector by the light strange quark s , Lattice-QCD (LQCD) calculations suggest two strong-interaction stable hexaquarks. Both are $J^\pi=0^+$ near-threshold s -wave dibaryons with zero spin and isospin: (i) a maximally strange $\mathcal{S}=-6$ $ssssss$ hexaquark classified as $\Omega\Omega$ dibaryon member of the SU(3) flavor $\mathbf{28}_f$

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multiplet, and (ii) a strangeness $\mathcal{S}=-2$ $uuddss$ hexaquark, a $\mathbf{1}_f$ H dibaryon, which is the subject of the present study. Whereas the LQCD calculation of $\Omega\Omega$ reached m_π values close to the physical pion mass [2], the most recent H LQCD calculation [3] was done at $m_\pi \approx 420$ MeV and within the $SU(3)_f$ symmetry limit, where

$$H = -\sqrt{\frac{1}{8}}\Lambda\Lambda + \sqrt{\frac{3}{8}}\Sigma\Sigma + \sqrt{\frac{4}{8}}N\Xi. \quad (1)$$

It is likely that in the $SU(3)_f$ -broken physical world a slightly bound $\mathbf{1}_f$ H becomes unbound with respect to the $\Lambda\Lambda$ threshold, lying possibly a few MeV below the $N\Xi$ threshold [4, 5].

The H dibaryon was predicted in 1977 by Jaffe [6] to lie about 80 MeV below the $2m_\Lambda=2231$ MeV $\Lambda\Lambda$ threshold. Dedicated experimental searches, beginning as soon as 1978 with a $pp \rightarrow K^+K^-X$ reaction [7] at BNL, have failed to observe a $\mathcal{S}=-2$ dibaryon signal over a wide range of dibaryon masses below $2m_\Lambda$ [8, 9, 10], notably BABAR's recent search at SLAC looking for a $\Upsilon(2S, 3S) \rightarrow \bar{\Lambda}\Lambda X$ decay [10]. Furthermore, a simple argument questioning the existence of a strong-interaction stable $\mathcal{S}=-2$ dibaryon was put forward by several authors, notably Dalitz et al. [11]. It relates to the few established $\Lambda\Lambda$ hypernuclei [12, 13], foremost the lightest known one ${}_{\Lambda\Lambda}^6\text{He}$ (the Nagara emulsion event) where a $\Lambda\Lambda$ pair is bound to ${}^4\text{He}$ by 6.91 ± 0.17 MeV [14] exceeding twice the separation energy of a single Λ in ${}^5_\Lambda\text{He}$ by merely $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^6\text{He})=0.67\pm 0.17$ MeV. If H existed deeper than about 7 MeV below the $\Lambda\Lambda$ threshold, ${}_{\Lambda\Lambda}^6\text{He}$ could decay *strongly*,

$${}_{\Lambda\Lambda}^6\text{He} \rightarrow {}^4\text{He} + H, \quad (2)$$

considerably faster than the $\Delta\mathcal{S}=1$ *weak-interaction* decay by which it has been observed and uniquely identified [14]:

$${}_{\Lambda\Lambda}^6\text{He} \rightarrow {}^5_\Lambda\text{He} + p + \pi^-. \quad (3)$$

Further arguments questioning an H bound by less than about 7 MeV were put forward by Gal [15].

Arguments of this kind, questioning the existence of a strong-interaction stable H dibaryon, were challenged 20 years ago by Farrar [16] who suggested that the H dibaryon may be a long-lived compact object with as small radius as 0.2 fm and as small mass as 1.5 ± 0.2 GeV, in which case it becomes

absolutely stable, without disrupting the observed stability of nuclei. Once sufficiently abundant, relic H dibaryons would qualify as a cold Dark Matter (DM) candidate. Farrar’s present estimate for the mass m_H of such a compact dibaryon, often termed Sexaquark in these works, is between 1850 and 2050 MeV [17]. Here, $m_H \lesssim 1850$ MeV is disfavored by the stability of oxygen [18], whereas the mass value 2050 MeV stems from the threshold value $(m_n + m_\Lambda) = 2055$ MeV, above which the $\Delta\mathcal{S}=1$ strangeness changing weak decay $H \rightarrow n\Lambda$ would make H definitely short lived with respect to a lifetime of cosmological origin expected for a DM candidate.

Following Farrar’s conjecture of a deeply-bound compact $\mathcal{S}=-2$ dibaryon, we present a realistic calculation of ${}_{\Lambda\Lambda}^6\text{He}$ lifetime owing to the two-body strong-interaction decay reaction Eq. (2). Treating ${}_{\Lambda\Lambda}^6\text{He}$ in a $\Lambda - \Lambda - {}^4\text{He}$ three-body model, it is found that the ${}_{\Lambda\Lambda}^6\text{He} \rightarrow H + {}^4\text{He}$ strong-interaction lifetime is correlated strongly with m_H , increasing upon decreasing m_H such that it exceeds the hypernuclear $\Delta\mathcal{S}=1$ weak-decay lifetime scale of order 10^{-10} s for $m_H < (m_\Lambda + m_n)$. Therefore, hypernuclear physics by itself does not rule out an H -like dibaryon in this mass range.

Constrained by Λ hypernuclear $\Delta\mathcal{S}=1$ nonmesonic weak-interaction decay rates within leading-order (LO) effective field theory (EFT) approach, we present a realistic calculation of the $\Delta\mathcal{S}=2$ weak decay $H \rightarrow nn$ for H mass satisfying $2m_n \lesssim m_H < (m_n + m_\Lambda)$. The resulting H lifetimes are of order 10^5 s, 10 orders of magnitude shorter than the order of 10^8 yr reached in the 2004 calculation by Farrar and Zaharijas (FZ) [19]. Hence, a deeply bound H dibaryon would be far from qualifying for a DM candidate.

2. H dibaryon wavefunction

2.1. Spatial part

Here we follow the simple ansatz for the H dibaryon six-quark ($6q$) fully symmetric spatial wavefunction Ψ_H given by FZ [19]:

$$\Psi_H = N_6 \exp\left(-\frac{\nu}{6} \sum_{i<j}^6 (\mathbf{r}_i - \mathbf{r}_j)^2\right), \quad (4)$$

where N_6 is a normalization constant and ν is related to the H ‘size’ as detailed below. To transform this $6q$ Ψ_H to a two-baryon form where each baryon B_a and B_b is described as a $3q$ cluster, we define relative coordinates

$\boldsymbol{\rho}$, $\boldsymbol{\lambda}$ and center-of-mass (c.m.) coordinates \mathbf{R} :

$$B_a: \quad \boldsymbol{\rho}_a = \mathbf{r}_2 - \mathbf{r}_1, \quad \boldsymbol{\lambda}_a = \mathbf{r}_3 - \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2), \quad \mathbf{R}_a = \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3), \quad (5)$$

$$B_b: \quad \boldsymbol{\rho}_b = \mathbf{r}_5 - \mathbf{r}_4, \quad \boldsymbol{\lambda}_b = \mathbf{r}_6 - \frac{1}{2}(\mathbf{r}_4 + \mathbf{r}_5), \quad \mathbf{R}_b = \frac{1}{3}(\mathbf{r}_4 + \mathbf{r}_5 + \mathbf{r}_6), \quad (6)$$

plus a total cm coordinate $\mathbf{R} = \frac{1}{2}(\mathbf{R}_a + \mathbf{R}_b) = \frac{1}{6} \sum_i^6 \mathbf{r}_i$. Using these $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ intrinsic quark coordinates, plus a relative coordinate $\mathbf{r} = (\mathbf{R}_b - \mathbf{R}_a)$ between the baryonic $3q$ clusters B_a and B_b , Eq. (4) assumes the form

$$\Psi_H = \psi_{B_a}(\boldsymbol{\rho}_a, \boldsymbol{\lambda}_a) \times \psi_{B_b}(\boldsymbol{\rho}_b, \boldsymbol{\lambda}_b) \times \psi_{B_a B_b}(\mathbf{r}), \quad (7)$$

where

$$\psi_{B_j} = \left(\frac{4\nu^2}{3\pi^2} \right)^{\frac{3}{4}} \exp\left(-\frac{\nu}{2}\rho_j^2 - \frac{2\nu}{3}\lambda_j^2\right) \quad (8)$$

provides normalized $3q$ baryonic spatial wf for baryon B_j , $j = a, b$, and

$$\psi_{B_a B_b} = \left(\frac{3\nu}{\pi} \right)^{\frac{3}{4}} \exp\left(-\frac{3\nu}{2}r^2\right) \quad (9)$$

provides a normalized spatial wavefunction in the relative coordinate \vec{r} of the dibaryon $B_a B_b$. Note that all three components of H in Eq. (7) share the *same* root-mean-square (r.m.s.) radius value:

$$\langle r_{B_a}^2 \rangle = \langle r_{B_b}^2 \rangle = \langle r_{B_a B_b}^2 \rangle = \frac{9}{8\nu}, \quad (10)$$

while for Ψ_H it is given by

$$\langle r_H^2 \rangle = \frac{5}{8\nu} \quad (11)$$

(correcting an error: $\langle r_H^2 \rangle = \nu^{-1}$ in Ref. [19]), so that the radial extent of H is about 75% of the radial extent of each of its three components $B_a, B_b, B_a B_b$. We also note that a $3q$ baryon wavefunction, similar to Ψ_H for $6q$,

$$\Psi_B = N_3 \exp\left(-\frac{\nu}{6} \sum_{i<j}^3 (\mathbf{r}_i - \mathbf{r}_j)^2\right), \quad (12)$$

implies a r.m.s. radius squared of

$$\langle r_B^2 \rangle = \frac{1}{\nu}, \quad (13)$$

Table 1: $\sqrt{\langle r_{\Lambda\Lambda}^2 \rangle}$ (fm units) vs. $B_{\Lambda\Lambda}$ (MeV units) for a short-range Gaussian $\Lambda\Lambda$ potential, Eq. (14) with $\lambda = 4 \text{ fm}^{-1}$. I'm indebted to Martin Schäfer for providing me with this table.

$B_{\Lambda\Lambda}$	5	20	50	100	200	300	400	1000
$\sqrt{\langle r_{\Lambda\Lambda}^2 \rangle}$	2.134	1.206	0.854	0.689	0.560	0.501	0.463	0.366

slightly smaller according to Eq. (10) than when embedded within the H dibaryon.

Although derived for a specific spatially *symmetric* wavefunction, Eq. (4), the relationships noted above between various $\langle r^2 \rangle$ values hold for *any* spatially symmetric form chosen for H . Establishing physically one such ‘size’ value determines necessarily all other ‘size’ values. However, the choice of a specific ‘size’ value is constrained by the choice of H binding energy value, as demonstrated in Table 1 for $B_a = B_b = \Lambda$. In this particular case, the Schroedinger equation in the relative coordinate $\mathbf{r}_{\Lambda\Lambda}$ was solved for assumed binding energy values $B_{\Lambda\Lambda}$, using attractive Gaussian $\Lambda\Lambda$ potential of the form $C_0^{(\lambda)}\delta_\lambda(\mathbf{r})$, where $C_0^{(\lambda)}$ is a strength parameter fitted to given values of $B_{\Lambda\Lambda}$ and

$$\delta_\lambda(\mathbf{r}) = \left(\frac{\lambda}{2\sqrt{\pi}}\right)^3 \exp\left(-\frac{\lambda^2}{4}\mathbf{r}^2\right) \quad (14)$$

is a zero-range Dirac $\delta^{(3)}(\mathbf{r})$ function in the limit $\lambda \rightarrow \infty$, smeared over distance of $\sqrt{\langle r^2 \rangle_\lambda} = \sqrt{6}/\lambda$ (0.612 fm for $\lambda = 4 \text{ fm}^{-1}$ chosen here). As expected, once $B_{\Lambda\Lambda}$ increases beyond a nuclear physics scale of roughly 20 MeV or so, $\sqrt{\langle r_{\Lambda\Lambda}^2 \rangle}$ decreases below 1 fm down to ≈ 0.5 fm. The corresponding H r.m.s. radius values are even smaller: $\sqrt{\langle r_H^2 \rangle}/\sqrt{\langle r_{\Lambda\Lambda}^2 \rangle} = (\sqrt{5}/3) \approx 0.745$ according to Eqs. (10,11). Taking a shorter-range Gaussian potential, say with $\lambda = 5 \text{ fm}^{-1}$, has a relatively small effect on $\sqrt{\langle r_{\Lambda\Lambda}^2 \rangle}$ which decreases between 3% to 12% as $B_{\Lambda\Lambda}$ increases from 5 to 1000 MeV. In passing we comment that the constraint imposed on $\sqrt{\langle r_H^2 \rangle}$ by assuming a definite value of $B_{\Lambda\Lambda}$, or vice versa, was overlooked in Ref. [19].

In the calculations reported below we use the fully symmetric spatial H wavefunction (4) or equivalently (7) by choosing $\nu = (9/8)\langle r_{\Lambda\Lambda}^2 \rangle^{-1}$, see Eq. (10), where $\langle r_{\Lambda\Lambda}^2 \rangle$ is obtained by solving the Schroedinger equation with attractive $\Lambda\Lambda$ potential shape, Eq. (14) for $\lambda = 4 \text{ fm}^{-1}$, and variable strength determined by assuming given values of $B_{\Lambda\Lambda}$, as listed in Table 1.

2.2. Spin-Flavor-Color part

To complete the discussion of the H dibaryon wavefunction we note that the fully symmetric spatial $6q$ wavefunction Ψ_H , Eq. (4), needs to be supplemented by a singlet $\mathbf{1}_S$ total spin $S=0$ component, represented by a $6q$ SU(2) Young tableau

$$\begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline \end{array} S, \quad (15)$$

and by singlet $\mathbf{1}_F$ total flavor (F) and $\mathbf{1}_C$ total color (C) components, each represented by its own $6q$ SU(3) Young tableau

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline & \\ \hline \end{array} F \text{ or } C. \quad (16)$$

Each of these S , F and C tableaux accommodates five components. In spin space, only one component corresponds to $S_a(uds) = S_b(uds) = \frac{1}{2}$ and $S_a(ud) = S_b(ud) = 0$ implied by a $\Lambda\Lambda$ dibaryon component, and in flavor space, again, only one component corresponds to $\mathbf{8}_a(uds)$ and $\mathbf{8}_b(uds)$ with isospin $I_a(ud) = I_b(ud) = 0$ for a $\Lambda\Lambda$ dibaryon component. In color space, too, only one component corresponds to colorless $\mathbf{1}_a(uds)$ and $\mathbf{1}_b(uds)$ baryons. Hence, up to a phase, we assign in each of these three spaces a coefficient of fractional parentage $\sqrt{1/5}$ to Ψ_H of Eq. (7). Finally, having chosen the $\Lambda\Lambda$ component over the $\Sigma\Sigma$ and $N\Xi$ components of H , see Eq. (1), involves a Clebsch-Gordan coefficient of magnitude $\sqrt{1/8}$ which together with the former coefficients amounts to supplementing the spatially symmetric $\Lambda\Lambda$ wavefunction $\psi_{\Lambda\Lambda}$, Eq. (7) for $B_a = B_b = \Lambda$, by a flavor-color-spin factor $\sqrt{1/1000}$.

3. ${}_{\Lambda\Lambda}^6\text{He}$ wavefunction

3.1. Three-body approximation

Given the tight binding of ${}^4\text{He}$, we treat the six-body ${}_{\Lambda\Lambda}^6\text{He}$ as a three-body $\Lambda\Lambda\alpha$ system with spatial coordinates \mathbf{r}_α , \mathbf{r}_{Λ_1} , \mathbf{r}_{Λ_2} . Starting from the two relative $\Lambda\alpha$ vector coordinates $\mathbf{r}_{\Lambda_1\alpha} = \mathbf{r}_{\Lambda_1} - \mathbf{r}_\alpha$ and $\mathbf{r}_{\Lambda_2\alpha} = \mathbf{r}_{\Lambda_2} - \mathbf{r}_\alpha$, we transform to their relative and c.m. coordinates

$$\mathbf{r}_{\Lambda\Lambda} = \mathbf{r}_{\Lambda_2\alpha} - \mathbf{r}_{\Lambda_1\alpha}, \quad \mathbf{R}_{\Lambda\Lambda} = \frac{1}{2}(\mathbf{r}_{\Lambda_1\alpha} + \mathbf{r}_{\Lambda_2\alpha}). \quad (17)$$

A reasonable simple approximation of the Pionless-EFT ($\not\chi$ EFT) ${}_{\Lambda\Lambda}{}^6\text{He}$ wavefunction calculated in Ref. [20] is then to use a factorized ansatz:

$$\Phi_{{}_{\Lambda\Lambda}{}^6\text{He}} = \phi_{\Lambda\Lambda}(r_{\Lambda\Lambda}) \Phi_{\Lambda\Lambda}(R_{\Lambda\Lambda}) \phi_{\alpha}, \quad (18)$$

where the wavefunctions $\phi_{\Lambda\Lambda}$ and $\Phi_{\Lambda\Lambda}$ are chosen as Gaussians constrained by requiring that $\phi_{\Lambda\Lambda}$ reproduces the r.m.s. radius of the coordinate $\mathbf{r}_{\Lambda\Lambda}$ in the 6-body ${}_{\Lambda\Lambda}{}^6\text{He}$ $\not\chi$ EFT calculation [20] as discussed below. Note that the r.m.s. radius value of the c.m. Gaussian $\Phi_{\Lambda\Lambda}$ is half that of the Gaussian $\phi_{\Lambda\Lambda}$. Finally, the ${}^4\text{He}$ core wavefunction ϕ_{α} within ${}_{\Lambda\Lambda}{}^6\text{He}$ is approximated by a free-space ${}^4\text{He}$ wavefunction identical with that for ${}^4\text{He}$ in the ${}_{\Lambda\Lambda}{}^6\text{He} \rightarrow H + {}^4\text{He}$ strong-interaction decay.

Studying the $\not\chi$ EFT ${}^5_{\Lambda}\text{He}$ five-body calculation [21] we note that $B_{\Lambda}^{\text{exp}}({}^5_{\Lambda}\text{He})$ is nearly reproduced by choosing Eq. (14) for ΛN contact terms, with cutoff values $\lambda = 1.25 \text{ fm}^{-1}$ or $\lambda = 1.50 \text{ fm}^{-1}$ for ΛN scattering length versions Alexander(B) and χ EFT(NLO19). Going over to the $\not\chi$ EFT ${}_{\Lambda\Lambda}{}^6\text{He}$ six-body calculation [20], the $\Lambda\Lambda$ r.m.s. distance computed for these cutoff values is $\sqrt{\langle r_{\Lambda\Lambda}^2 \rangle} = 3.65 \pm 0.10 \text{ fm}$, which we adopt for the r.m.s. radius of the Gaussian $\phi_{\Lambda\Lambda}$ in Eq. (18). Note that $\phi_{\Lambda\Lambda}$ appears as a bound-state wavefunction in spite of the $\Lambda\Lambda$ interaction being much too weak to form a bound state; it is the ${}^4\text{He}$ nuclear core that stabilizes the two Λ s in ${}_{\Lambda\Lambda}{}^6\text{He}$. We also note that since this value refers to a weakly ‘bound’ $\Lambda\Lambda$ pair in ${}_{\Lambda\Lambda}{}^6\text{He}$, it is considerably larger than $\sqrt{\langle r_{\Lambda\Lambda}^2 \rangle}$ values listed in Table 1 for a tightly bound H dibaryon.

3.2. Short-range behavior

Eq. (18) provides a simple wavefunction for two loosely bound Λ hyperons held together by ${}^4\text{He}$, disregarding the short-range repulsive component of the $\Lambda\Lambda$ interaction which is manifest in LQCD calculations [22] and is probed by a relatively compact H dibaryon. To check the effect exerted by short-range repulsion on the ${}_{\Lambda\Lambda}{}^6\text{He} \rightarrow H + {}^4\text{He}$ decay rate, we modify $\phi_{\Lambda\Lambda}$ in Eq. (18) by introducing a short-range correlation (SRC) factor $[1 - j_0(\kappa r)]$, where j_0 is a spherical Bessel function of order zero:

$$\tilde{\phi}_{\Lambda\Lambda}(r_{\Lambda\Lambda}) = (1 - j_0(\kappa r_{\Lambda\Lambda})) \phi_{\Lambda\Lambda}(r_{\Lambda\Lambda}). \quad (19)$$

Choosing $\kappa = 2.534 \text{ fm}^{-1}$, corresponding to 500 MeV/c in momentum space, nearly reproduces the $\Lambda\Lambda$ G -matrix calculation in Ref. [23] (Fig. 2 there and

related text). We therefore replace $\Phi_{\Lambda\Lambda}{}^6\text{He}$, Eq. (18), by

$$\Psi_i = \tilde{\phi}_{\Lambda\Lambda}(r_{\Lambda\Lambda}) \Phi_{\Lambda\Lambda}(R_{\Lambda\Lambda}) \phi_\alpha \quad (20)$$

for use as initial $\Lambda\Lambda$ ${}^6\text{He}$ wavefunction in the $\Lambda\Lambda$ ${}^6\text{He} \rightarrow H + {}^4\text{He}$ decay rate calculation reported below.

4. $\Lambda\Lambda$ ${}^6\text{He} \rightarrow H + {}^4\text{He}$ decay rate

We assume that the strong-interaction decay $\Lambda\Lambda$ ${}^6\text{He} \rightarrow H + {}^4\text{He}$ of a loosely ‘bound’ $\Lambda\Lambda$ pair in $\Lambda\Lambda$ ${}^6\text{He}$ into a $\Lambda\Lambda$ pair constituent of a tightly bound $\mathbf{1}_F$ H dibaryon, flying off ${}^4\text{He}$ with momentum \mathbf{k}_H in their c.m. system, is triggered by the $\Lambda\Lambda$ strong interaction $V_{\Lambda\Lambda}$ extracted near threshold. The spatial dependence of the decay matrix element is given by $\langle \Psi_f | V_{\Lambda\Lambda} | \Psi_i \rangle$, where Ψ_i stands for the initial $\Lambda\Lambda$ ${}^6\text{He}$ wavefunction, Eq. (20), and

$$\Psi_f = \tilde{\psi}_{\Lambda\Lambda}(r_{\Lambda\Lambda}) \exp(i\mathbf{k}_H \cdot \mathbf{R}_H) \phi_\alpha, \quad \tilde{\psi}_{\Lambda\Lambda} = \sqrt{1/1000} \times \psi_{\Lambda\Lambda}, \quad (21)$$

where $\psi_{\Lambda\Lambda}$, Eq. (9), is renormalized by the flavor-color-spin factor $\sqrt{1/1000}$ discussed at the end of subsection 2.2, thereby accounting for the elimination of $\Sigma\Sigma$ and $N\Xi$ components of H . Note that in agreement with the overall attraction of the BB interaction in the $\mathbf{1}_F$ channel [22] no SRC factor was introduced in Eq. (21) for Ψ_f . We note that the calculations reported below disregard the slight difference between the inner $3q$ structure of each Λ hyperon in the H dibaryon to that in $\Lambda\Lambda$ ${}^6\text{He}$. For a $3q$ baryon size of about 0.5 fm [24], this neglect is well justified in the range of $B_{\Lambda\Lambda}$ values considered here.

The $\Lambda\Lambda$ ${}^6\text{He} \rightarrow H + {}^4\text{He}$ decay rate, or equivalently the corresponding strong-interaction width of $\Lambda\Lambda$ ${}^6\text{He}$, is given by [25]:

$$\Gamma(\Lambda\Lambda \text{ } {}^6\text{He} \rightarrow H + {}^4\text{He}) = \frac{\mu_{H\alpha} k_H}{(2\pi\hbar c)^2} \int |\langle \Psi_f | V_{\Lambda\Lambda} | \Psi_i \rangle|^2 d\hat{\mathbf{k}}_H, \quad (22)$$

where $\mu_{H\alpha}$ is the $H - {}^4\text{He}$ reduced mass and $\langle \Psi_f | V_{\Lambda\Lambda} | \Psi_i \rangle$ is a product of two factors, as follows.

One factor is the $\Lambda\Lambda$ interaction matrix element $\langle \tilde{\psi}_{\Lambda\Lambda} | V_{\Lambda\Lambda} | \tilde{\phi}_{\Lambda\Lambda} \rangle$ in the $r_{\Lambda\Lambda}$ relative distance with $V_{\Lambda\Lambda}$ connecting $\tilde{\phi}_{\Lambda\Lambda}$, Eq. (19), for the initial $\Lambda\Lambda$ ${}^6\text{He}$ wavefunction component to the final H dibaryon renormalized wavefunction component $\tilde{\psi}_{\Lambda\Lambda}$, Eq. (21). A normalized Gaussian $\delta_{\lambda=4}(\mathbf{r})$, see Eq. (14),

was used for $V_{\Lambda\Lambda}$ with strength parameter $C_0^{(\lambda=4)} = -152 \text{ MeV}\cdot\text{fm}^3$ fitted in Ref. [20] to the HAL-QCD scattering length $a_{\Lambda\Lambda} = -0.8 \text{ fm}$ [26]. The calculated matrix element $\langle V_{\Lambda\Lambda} \rangle$ depends weakly on the chosen value of λ within $\delta\lambda = \pm 1 \text{ fm}^{-1}$. As for dependence on SRC, Eq. (19), it introduces a multiplicative factor

$$1 - \exp(-\kappa^2/4\chi) \quad (23)$$

to $\langle V_{\Lambda\Lambda} \rangle$, with $\chi = \frac{1}{2}(\nu_i + \nu_f) + \frac{1}{4}\lambda^2$ for wavefunctions of Gaussian shape $\exp(-\nu r^2/2)$. This factor varies from 0.25 to 0.19 as $B_{\Lambda\Lambda}$ is increased from 100 to 400 MeV. Altogether, the matrix element $\langle \tilde{\psi}_{\Lambda\Lambda}|V_{\Lambda\Lambda}|\tilde{\phi}_{\Lambda\Lambda} \rangle$ varies slightly from -59 to -53 keV in the same $B_{\Lambda\Lambda}$ range.

Table 2: ${}_{\Lambda\Lambda}^6\text{He} \rightarrow H + {}^4\text{He}$ decay rate Γ , Eq. (22), and decay time \hbar/Γ for some representative values of H binding energy $B_{\Lambda\Lambda}$ and their associated k_H and $I(k_H; a_\Phi)$ values for $a_\Phi = 1.49 \text{ fm}$, see text. $B_{\Lambda\Lambda} = 176 \text{ MeV}$ corresponds to $m_H = m_\Lambda + m_n$.

$B_{\Lambda\Lambda}$ (MeV)	k_H (fm^{-1})	$I(k_H; a_\Phi)$ (fm^3)	Γ (eV)	$\tau = \hbar/\Gamma$ (s)
100	2.547	$1.002 \cdot 10^{-3}$	$0.782 \cdot 10^{-2}$	$0.841 \cdot 10^{-13}$
200	3.612	$4.742 \cdot 10^{-10}$	$0.501 \cdot 10^{-8}$	$1.315 \cdot 10^{-7}$
300	4.377	$5.996 \cdot 10^{-16}$	$0.679 \cdot 10^{-14}$	$0.970 \cdot 10^{-1}$
400	4.980	$2.157 \cdot 10^{-21}$	$2.436 \cdot 10^{-20}$	$2.703 \cdot 10^4$
176	3.393	$1.521 \cdot 10^{-8}$	$1.550 \cdot 10^{-7}$	$4.245 \cdot 10^{-9}$

The other factor in $\langle \Psi_f|V_{\Lambda\Lambda}|\Psi_i \rangle$ is an overlap matrix element between the initial $\Lambda\Lambda - \alpha$ Gaussian wavefunction $\Phi_{\Lambda\Lambda}(R_{\Lambda\Lambda})$ in ${}_{\Lambda\Lambda}^6\text{He}$ and the final outgoing $H - \alpha$ plane-wave $\exp(i\mathbf{k}_H \cdot \mathbf{R}_H)$. Note that \mathbf{R}_H has nothing to do with the relatively small size of H . In the following we identify \mathbf{R}_H with the corresponding argument $\mathbf{R}_{\Lambda\Lambda}$ in Eq. (18), both defined relative to ${}^4\text{He}$ and denoted below simply by \mathbf{R} . The square of this overlap matrix element times 4π from $d\hat{\mathbf{k}}_H$ in Eq. (22) is given by

$$I(k_H; a_\Phi) \equiv 4\pi \left| \int \exp(i\mathbf{k}_H \cdot \mathbf{R}) \Phi_{\Lambda\Lambda}(R) d^3\mathbf{R} \right|^2 = 32\pi^{5/2} a_\Phi^3 \exp(-a_\Phi^2 k_H^2), \quad (24)$$

where $a_\Phi = \sqrt{2 \langle R_{\Lambda\Lambda}^2 \rangle / 3} = 1.49 \pm 0.04 \text{ fm}$. As shown in Table 2, $I(k_H; a_\Phi)$ varies strongly with k_H over 18 decades as $B_{\Lambda\Lambda}$ is varied from 100 MeV (76 MeV above $m_\Lambda + m_n$) to 400 MeV (47 MeV below $2m_n$). This is caused by the increased oscillations of $\exp(i\mathbf{k}_H \cdot \mathbf{R})$ vs. the smoothly varying $\Phi_{\Lambda\Lambda}(R)$

in Eq. (24). And on top of that, the ± 0.04 fm uncertainty of a_Φ makes $I(k_H; a_\Phi)$ uncertain by a factor of 4 larger/smaller values than for the mean value $a_\Phi = 1.49$ fm at $B_{\Lambda\Lambda} = 176$ MeV corresponding to the $m_\Lambda + m_n$ threshold, increasing to a factor about 20 at $B_{\Lambda\Lambda} = 400$ MeV.

The final values of ${}_{\Lambda\Lambda}^6\text{He} \rightarrow H + {}^4\text{He}$ decay rate Γ listed in Table 2 account for the two factors in $\langle \Psi_f | V_{\Lambda\Lambda} | \Psi_i \rangle$ considered above. Notably, Γ decreases over 17 decades, reflecting the strong variation of $I(k_H; a_\Phi)$ with $B_{\Lambda\Lambda}$, and the ${}_{\Lambda\Lambda}^6\text{He}$ strong-interaction lifetime $\tau = \hbar/\Gamma$ increases as strongly over this range of $B_{\Lambda\Lambda}$ values. In particular, for H binding energy $B_{\Lambda\Lambda}$ somewhat lower than 176 MeV ($m_\Lambda + m_n$ threshold), τ starts to exceed the weak-interaction lifetime scale set by the free- Λ lifetime $\approx 2.6 \times 10^{-10}$ s, so that the robust observation of $\Lambda\Lambda$ hypernuclei by their weak-interaction decay modes does not rule out the existence of a deeply bound H dibaryon with mass below $m_\Lambda + m_n$.

5. $\Lambda\Lambda$ nonmesonic weak decays

Having realized that a deeply bound H dibaryon lying below $m_\Lambda + m_n$ is not in conflict with the weak-decay lifetime scale $\sim 10^{-10}$ s of all observed $\Lambda\Lambda$ hypernuclei, we now estimate the leading $\Delta\mathcal{S} = 2$ weak-interaction decay rate of H , that of the $H \rightarrow nn$ two-body decay. H is represented here, as above, by its deeply bound $\Lambda\Lambda$ component. Although $\Delta\mathcal{S} = 2$ $\Lambda\Lambda \rightarrow nn$ transitions are not constrained directly by experiment, they are related to $\Delta\mathcal{S} = 1$ $\Lambda n \rightarrow nn$ transitions which are constrained by ample lifetime data in Λ hypernuclei [12]. These $\Delta\mathcal{S} = 1$ transitions, including the pion-exchange transition depicted in Fig. 1(b), proceed in Λ hypernuclei with a total rate comparable to the $\Lambda \rightarrow n\pi^0$ free-space decay rate associated with Fig. 1(a). The weak-interaction coupling constant g_w extracted from the free-space Λ lifetime, and proved to be relevant in the $\Lambda n \rightarrow nn$ nonmesonic decay of Λ hypernuclei, could then be used as shown in Fig. 1(c) to estimate the strength of the $\Delta\mathcal{S} = 2$ $\Lambda\Lambda \rightarrow nn$ weak-decay transition.

Pion exchange is not the only contributor to the nonmesonic weak decay of Λ hypernuclei. Owing to the large momentum transfer in Fig. 1(b), pion exchange generates mostly a tensor ${}^3S_1 \rightarrow {}^3D_1$ transition which is Pauli forbidden for nn final states, so shorter-range meson exchanges contribute instead. It is useful then to follow an EFT approach initiated by Parreño et al. [28] and updated recently in Ref. [29]. In this approach, Fig. 1(b) and Fig. 1(c) are replaced by Fig. 2(a) and Fig. 2(b), respectively, where the

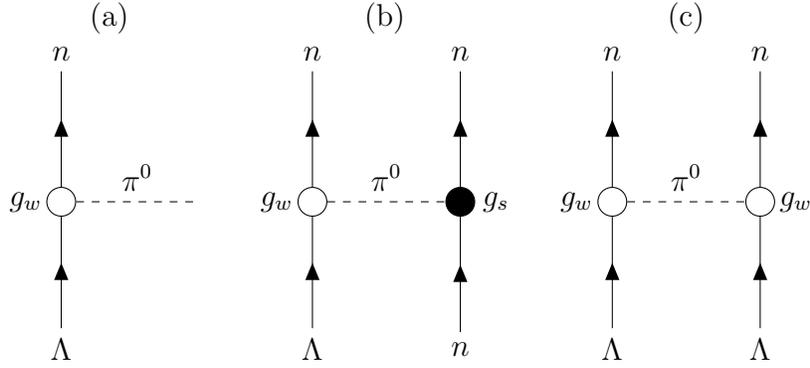


Figure 1: $\Delta\mathcal{S} = 1$ $\Lambda \rightarrow n$ weak-interaction diagrams in free space (a) or in Λ hypernuclei (b), see Dalitz [27], and $\Delta\mathcal{S} = 2$ $\Lambda\Lambda \rightarrow nn$ weak-interaction diagram (c), all involving emission (a) or exchange (b,c) of a π^0 meson. Weak-interaction and strong-interaction coupling constants g_w and g_s , respectively, are denoted by circles.

square vertices stand at LO for ${}^1S_0 \rightarrow {}^1S_0$ low-energy constants (LECs) that replace the coupling-constant schematic combinations $g_w g_s$ and g_w^2 in these figures, respectively.

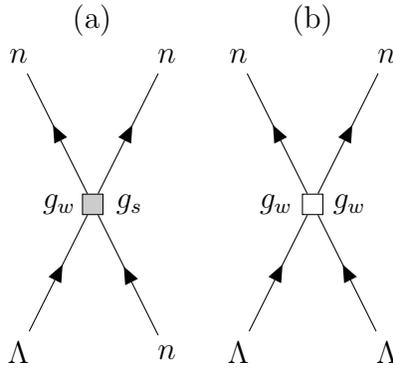


Figure 2: ${}^1S_0 \rightarrow {}^1S_0$ LO EFT $\Delta\mathcal{S} \neq 0$ weak-interaction diagrams: (a) $\Delta\mathcal{S} = 1$ $\Lambda n \rightarrow nn$, (b) $\Delta\mathcal{S} = 2$ $\Lambda\Lambda \rightarrow nn$.

Since the momentum $p_f \approx 420$ MeV/c of each of the final neutrons in Fig. 2(a) is much larger than the Fermi momentum of the initial neutron, the Λ hypernuclear decay rate induced by this diagram is well approximated by a quasi-free expression tested in studies of Σ hypernuclear widths [30, 31],

$$\Gamma_n = v_{\Lambda n} \sigma_{\Lambda n \rightarrow nn} \rho_n, \quad (25)$$

where ρ_n is the neutron density in nuclear matter. To calculate the $\Delta\mathcal{S} = 1$ two-body reaction cross section $\sigma_{\Lambda n \rightarrow nn}$ at LO, we use a $\Delta\mathcal{S} = 1$ ${}^1S_0 \rightarrow {}^1S_0$ contact interaction $C_1^{(\lambda)} \delta_\lambda(\mathbf{r})$, see Eq. (14), with a LEC $C_1^{(\lambda)}$ determined by fitting the r.h.s. of Eq. (25) to $\Gamma_n \approx \frac{1}{2} \Gamma_{hyp}$, where the total hypernuclear decay rate Γ_{hyp} is the inverse of the hypernuclear lifetime τ_{hyp} determined experimentally in medium-heavy hypernuclei [32, 33]: $\tau_{hyp} \approx 210$ ps. We note that Γ_{hyp} is dominated by non-mesonic hypernuclear decays [12]. Following this approach for a representative value $\lambda = 4$ fm $^{-1}$, and using a plane-wave (PW) approximation to evaluate $\sigma_{\Lambda n \rightarrow nn}$ at rest, yields a value (up to a sign) of $C_1^{(\lambda=4)} = -63.4$ eV \cdot fm 3 ; Λn initial-state interaction could increase it by up to a factor of two. Considering $\Delta\mathcal{S} = 0$ 1S_0 LECs from Ref. [20], $C_0^{(\lambda=4)}(\Lambda N) = -239$ or $C_0^{(\lambda=4)}(NN) = -301$ MeV \cdot fm 3 , we find that the ratio $C_1^{(\lambda=4)}/C_0^{(\lambda=4)}$ assumes values of 2.65×10^{-7} or 2.11×10^{-7} . These values are approximately the same as g_w/g_s when g_w is identified with the Fermi constant $G_F m_\pi^2 = 2.21 \times 10^{-7}$ and g_s is of order one, well below the pion-exchange diagram value $g_\pi \approx 13.6$. In passing we note the weak dependence of $C_1^{(\lambda)}$ on λ : choosing a shorter range parameter, say $\lambda \sim 8$ fm $^{-1}$ often practised in weak-interaction studies [29], results in $C_1^{(\lambda=8)} = -51.5$ eV \cdot fm 3 , a value smaller by only 19% than $C_1^{(\lambda=4)}$.

Having determined $C_1^{(\lambda)}$, we multiply it by $(g_w/g_s) \sim G_F m_\pi^2$ to estimate the $\Delta\mathcal{S} = 2$ LEC: $C_2^{(\lambda)} = (2.21 \times 10^{-7}) C_1^{(\lambda)}$. Neglecting here too nn final-state interaction (FSI), the $H \rightarrow nn$ decay rate is given by

$$\Gamma(H \rightarrow nn) = \frac{\mu_{nn} k_n}{(2\pi\hbar c)^2} \int |\langle \exp(i\mathbf{k}_n \cdot \mathbf{r}) | C_2^{(\lambda)} \delta_\lambda(\mathbf{r}) | \tilde{\psi}_{\Lambda\Lambda}(r) \rangle|^2 d\hat{\mathbf{k}}_n. \quad (26)$$

Calculated $H \rightarrow nn$ decay rate values Γ_H and their associated lifetime values τ_H , using a cutoff value $\lambda = 4$ fm $^{-1}$ in Eq. (26), are listed in Table 3. A weak dependence of τ_H on the H mass ($m_H = 2m_\Lambda - B_{\Lambda\Lambda}$) is noted, in contrast to the strong dependence observed in Table 2 for the strong-interaction lifetime of ${}_\Lambda^6\text{He}$ caused by the rapid exponential decrease $\exp(-a_\Phi^2 k_H^2)$ upon

Table 3: $H \rightarrow nn$ decay rate Γ_H , Eq. (26) for $\lambda = 4 \text{ fm}^{-1}$, and H lifetime $\tau_H = \hbar/\Gamma_H$ for several representative values of H binding energy $B_{\Lambda\Lambda}$ and their associated neutron momentum release k_n values. Here, $C_2^{(\lambda)} = (2.21 \times 10^{-7})C_1^{(\lambda)}$ with $C_1^{(\lambda=4)} = -63.4 \text{ eV}\cdot\text{fm}^3$, see text.

$B_{\Lambda\Lambda}$ (MeV)	k_n (fm^{-1})	Γ_H (eV)	τ_H (s)
176	2.109	$2.366 \cdot 10^{-21}$	$2.782 \cdot 10^5$
200	1.955	$2.211 \cdot 10^{-21}$	$2.977 \cdot 10^5$
300	1.130	$1.365 \cdot 10^{-21}$	$4.820 \cdot 10^5$

increasing k_H in Eq. (24). In the present case, the relatively large value $a_\Phi \approx 1.5 \text{ fm}$ extracted from ${}_{\Lambda\Lambda}^6\text{He}$ is replaced by a considerably smaller value of less than 0.3 fm owing to the considerably smaller size parameters of the deeply bound H wavefunction $\tilde{\psi}$ and of the contact term $\delta_\lambda(\mathbf{r})$, resulting in a weak k_n dependence. The calculated $H \rightarrow nn$ lifetimes listed in Table 3 are then uniformly of order 10^5 s , less than 1 yr, many orders of magnitude shorter than cosmological time scales commensurate with the Universe age. Regarding the model dependence of the above result, $\tau_H \sim 10^5 \text{ s}$, we note that it depends weakly on the range parameter λ of the LECs $C_1^{(\lambda)}$ and $C_2^{(\lambda)}$. Furthermore, introducing FSI in *both* Γ_n and $\Gamma(H \rightarrow nn)$ calculations leads to almost the same decay rate $\Gamma(H \rightarrow nn)$ as calculated using PW nn states, the reason being that the increased value of $C_1^{(\lambda)}$ extracted then from Γ_n largely cancels the decrease of $\Gamma(H \rightarrow nn)$ when FSI replaces PW.

6. Concluding remarks

In this work we considered hypernuclear constraints on the existence and lifetime of an hypothetical deeply-bound doubly-strange H dibaryon. Regarding the existence of H , it was found by considering the unambiguously identified ${}_{\Lambda\Lambda}^6\text{He}$ double- Λ hypernucleus [13] that its strong-interaction lifetime for decay to ${}^4\text{He} + H$ would increase substantially upon decreasing m_H , exceeding for $m_H < m_\Lambda + m_n$ by far the 10^{-10} s weak-interaction hypernuclear lifetime scale. Thus, the unique observation of double- Λ hypernuclei through weak decay to single- Λ hypernuclei does not rule out on its own the existence of a deeply-bound H dibaryon in the mass range $2m_n$ to $m_\Lambda + m_n$, defying doubts raised by Dalitz et al. 35 years ago [11], but in agreement with the 20 years old claim by FZ [19]. Special attention was given in our

evaluation to constrain hadronic cluster sizes by their binding energies and, indeed, our own conclusion follows from respecting the large difference between the r.m.s. radius of the loosely bound $\Lambda\Lambda$ pair in ${}_{\Lambda\Lambda}^6\text{He}$ and that of the compact $\Lambda\Lambda$ pair within the H dibaryon. And furthermore, the anticipated $\Lambda\Lambda$ short-range repulsion was fully considered by incorporating the SRC factor $(1 - j_0(\kappa r))$, Eq. (19), into the $\Lambda\Lambda$ wavefunction.

Provided an H -like dibaryon exists in the mass range $2m_n$ to $m_\Lambda + m_n$, it was found that its $\Delta\mathcal{S} = 2$ decay lifetime $\tau(H \rightarrow nn)$ would be quite long, of the order of 10^5 s, but many orders of magnitude shorter than cosmological lifetimes comparable to the age of the Universe that H would need to qualify for a DM candidate. To reach this conclusion we used a weak-interaction EFT approach [28] constrained by experimentally known, largely nonmesonic, hypernuclear lifetimes [12]. Our conclusion is in stark disagreement with that reached by FZ [19] using outdated by now hard-core strong-interaction nuclear models and, furthermore, disregarding the constraint imposed on the H radius r_H by its binding energy.

We have not considered in the present work the case for an H -like dibaryon with mass below the nn threshold, a scenario likely to be ruled out by the established stability of several key nuclei, notably ${}^{16}\text{O}$. A straightforward calculation of the hypothetical two-body decay rate $\Gamma({}^{16}\text{O} \rightarrow H + {}^{14}\text{O})$ gives indeed a rate many orders of magnitude larger than the upper bound established for oxygen by Super-Kamiokande [18]. A report of this calculation is in preparation.

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