Hall mass and transverse Noether spin currents in noncollinear antiferromagnets

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Noncollinear antiferromagnets (AFMs) in the family of Mn_3X (X=Ir, Sn, Ge, Pt, etc.) have recently attracted attention in the emerging field of antiferromagnetic spintronics because of their various interesting transport, magnetic, and optical properties. Due to the noncollinear magnetic order, the localized electron spins on different magnetic sublattices are not conserved even when spin-orbit coupling is neglected, making it difficult to understand the transport of spin angular momentum. Here we study the conserved Noether current associated with spin-rotation symmetry of the local spins in noncollinear AFMs. We found that a Hall component of the d.c. spin current can be created by a longitudinal driving force associated with a propagating spin wave, and is proportional to a response coefficient that we denote as the Hall (inverse) mass. Such a Hall spin current can be generated by spin pumping in a ferromagnet (FM)-noncollinear AFM bilayer structure as we demonstrated numerically. Finally we showed that the Hall mass is an isotropic quantity, similar to the isotropic spin Hall conductivity, and should generally exist in noncollinear AFMs and their polycrystals. Our results shed light on the potential of noncollinear AFMs in manipulating the polarization and flow of spin currents in general spintronic devices.

Antiferromagnets (AFMs) with noncollinear magnetic order have recently become a topic of interest in spintronics. In spite of the usual challenge associated with vanishing net magnetization pertinent to all AFMs, which has been significantly mitigated in recent years due to the rapid development of antiferromagnetic spintronics [1–6], the complex magnetic ordering of noncollinear AFMs leads to exotic transport phenomena that open up opportunities otherwise unavailable in common collinear ones. A prominent example is the anomalous Hall effect (AHE) in the noncollinear AFM family Mn_3X (where X = Ir, Sn, Ge, etc.)[4, 7–13] as well as other transport and optical properties with the same symmetry requirements as the AHE [10–15], such as the anomalous Nernst effect [14] and the magneto-optical Kerr effect [16, 17]. The low symmetry of the magnetic structure also allows the existence of the magnetic spin-Hall (MSHE) and inverse spin-Hall (MISHE) effects [18–21], anisotropic magnetorestriction and piezomagnetic effects [22, 23], and nontrivial spin-transfer torques [24, 25], etc. Interesting features of the electronic structure in Mn_3X such as Weyl nodes near the Fermi surface and their associated transport signatures have also been extensively studied theoretically and experimentally [9, 26–28]. In addition to Mn_3X, transport phenomena in other noncollinear AFMs such as antiperovskite Mn_3AB with A = Ga, Ni, Cu and B = C, N [29–31], orthoferrites [32], vector spin Seebeck effect [33], and many effective Ising magnets [34–38] have also garnered significant interest recently.

In a magnet, spin current can be carried by itinerant electrons or magnons. Both mechanisms have been well characterized for ferromagnets (FMs) and collinear AFMs with isotropic (exchange) interactions [39]. The magnetic order parameter in the form of uniform or staggered magnetization lowers the SO(3) symmetry of global spin rotations to rotations about the magnetization direction. As a result, only the longitudinal component of spin current carried by quasiparticle excitations is usually considered, with spin superfluids in easy-plane magnets an exception, where it is carried by the magnetic ground state understood as magnon condensates. In the presence of finite spin-orbit coupling, an approximate diffusive picture of spin transport can usually be established, which has historically played a powerful role in discovering and understanding many remarkable phenomena in FM-based spintronics [40–43] and more recently in collinear AFM spintronics as well [6, 44, 45]. In contrast, in noncollinear AFMs, the magnetic order parameter breaks spin rotation symmetry completely. It is not clear whether a conserved spin current can even be defined for AFMs with noncollinear magnetic order. In spite of this conceptual difficulty, recent experimental and theoretical discoveries in noncollinear AFMs, including MSHE/MISHE [18, 19], and tunneling magnetoresistance [46, 47] have been interpreted using a spin current language. These heuristic arguments suggest that some form of a conserved spin current might nonetheless exist in noncollinear AFMs.

In this Letter, we study the spin current carried by the magnetization fields in a general noncollinear AFM. A noteworthy aspect of the spin current in a noncollinear AFM is the relationship between the spatial gradient of the magnetic order parameter and the direction of the flow of the spin current. In FMs and collinear AFMs, the spin current flows along the spatial gradient. We find that this is generally not the case in noncollinear AFMs. A spatial gradient of the magnetic order in the x spatial direction may induce a spin current flowing along the y direction.

We first derive a conserved current for global spin rotations by applying Noether's theorem to a continuum theory [48–51] of a Heisenberg antiferromagnet with 3 sublattices in 2 spatial dimensions with a hexagonal lattice symmetry exemplified by the Mn_3X family [52–60]. The conserved Noether charge is the total spin. Most interestingly, the Noether spin current can have a component transverse to the gradient of the order parameter, i.e., a Hall spin current. The theory is verified by simulating the spin dynamics in a lattice model consisting of an interface between an FM and a noncollinear kagome AFM, where the dynamical d.c. spin current on the AFM side driven by spin pumping on the FM side has a component flowing parallel to the interface. Such Hall spin currents originate from off-diagonal components of the inverse effective mass tensor in the continuum Lagrangian, which survives under averaging the Lagrangian over SO(3) rotations. Such off-diagonal components of the inverse effective mass tensor, named as the Hall (inverse) mass in this Letter, therefore exist in polycrystalline, or effectively isotropic, noncollinear AFM systems in general.

Noether spin current of noncollinear AFMs. We first use a prototypical two-dimensional kagome lattice model [51, 52] to study some generic properties of Noether spin currents in noncollinear AFMs. The model has classical spins of length S with antiferromagnetic nearest-neighbor exchange coupling J on a kagome lattice. We consider two representative noncollinear antiferromagnetic ground states in this model, direct [Fig. 1 (a) inset] and inverse triangular order [Fig. 1 (b) inset], relevant to that in Mn_3X.

The low-energy behavior of the model is captured by a continuum Lagrangian obtained through gradient expansion [51, 52, 55, 58–61]. The magnetization fields of the three sublattices are coplanar and point at angles of 120° to each other, thus forming a rigid body. The orientation of this object can be encoded in terms of a spin frame—three mutually orthogonal unit vectors $\mathbf{n}_\alpha$, where $\alpha = x, y, z$, rigidly attached to the magnetic order parameter [60, 61]. The low-energy Lagrangian is

$$\mathcal{L} = \frac{\rho}{4}\partial_t \mathbf{n}_\alpha \cdot \partial_t \mathbf{n}_\alpha - \frac{1}{4}\Gamma^{\alpha\beta}_{ab}\partial_a \mathbf{n}_\alpha \cdot \partial_b \mathbf{n}_\beta \tag{1}$$

Here the inertia density $\rho = \frac{1}{2JA_c}$ is related to the paramagnetic susceptibility. $A_c = 2\sqrt{3}a_0^2$ is the unit cell area and a_0 is the nearest-neighbor distance. Latin indices a and $b = x, y$ label spatial directions and Greek indices denote spin components. Summation over doubly repeated Greek or Latin indices is implied. The spin-frame vectors can be chosen as certain superpositions of sublattice magnetizations in such a way that $\mathbf{n}_x$ and $\mathbf{n}_y$ transform in terms of each other under point-group symmetries of the lattice in the same way as spatial gradients ∂_x and ∂_y (hence the labels) [60]. The third spin-frame vector $\mathbf{n}_z = \mathbf{n}_x \times \mathbf{n}_y$ then has the meaning of the vector spin chirality. The fourth-rank tensor Γ is generally symmetric under the simultaneous exchange of the Latin and Greek indices [61]. For the kagome models considered above we found,

$$\Gamma^{\alpha\beta}_{ab} = \pm\frac{\sqrt{3}}{4}JS^2(\delta_{a\alpha}\delta_{b\beta} + \delta_{a\beta}\delta_{b\alpha}), \tag{2}$$

where $+(-)$ corresponds to the direct (inverse) triangular order, α and β take on values x or y; it vanishes if either $\alpha = z$ or $\beta = z$. Alternatively, the spin-frame vectors $(\mathbf{n}_\alpha)_m$ can be understood as column vectors of a rotation matrix $R_{m\alpha}$ [61], which leads to an equivalent form of Eq. (1) [51, 52]:

$$\mathcal{L} = -\frac{\rho}{4}\,\mathrm{Tr}\,[(R^{-1}\partial_t R)^2] + \frac{1}{4}\,\mathrm{Tr}[\Gamma_{ab}(R^{-1}\partial_a R)(R^{-1}\partial_b R)] \tag{3}$$

where the trace is over spin indices. The latter will be omitted below for brevity when possible.

The equation of motion for the spin frame is obtained by minimizing the action with respect to the fields $\mathbf{n}_\alpha$ while maintaining the orthonormality constraints $\mathbf{n}_\alpha \cdot \mathbf{n}_\beta = \delta_{\alpha\beta}$ [60]. They read

$$\frac{\rho}{2}\mathbf{n}_\alpha \times \partial_t^2 \mathbf{n}_\alpha - \frac{1}{2}\Gamma^{\alpha\beta}_{ab}\,\mathbf{n}_\alpha \times \partial_a\partial_b \mathbf{n}_\beta = 0. \tag{4}$$

Lagrangian (1) is invariant under global spin rotations, $\delta\mathbf{n}_\alpha = \boldsymbol{\theta} \times \mathbf{n}_\alpha$ for an infinitesimal rotation angle $\boldsymbol{\theta}$. By Noether's theorem, the spin current flowing along spatial direction a is

$$\boldsymbol{\mathcal{J}}_a = \mathbf{n}_\alpha \times \frac{\partial \mathcal{L}}{\partial(\partial_a \mathbf{n}_\alpha)} = -\frac{1}{2}\Gamma^{\alpha\beta}_{ab}\,\mathbf{n}_\alpha \times \partial_b \mathbf{n}_\beta. \tag{5}$$

The spin density is

$$\begin{aligned}\boldsymbol{\mathcal{J}}_0 &= \mathbf{n}_\alpha \times \frac{\partial \mathcal{L}}{\partial(\partial_t \mathbf{n}_\alpha)} = \frac{\rho}{2}\mathbf{n}_\alpha \times \partial_t \mathbf{n}_\alpha = \rho\boldsymbol{\Omega} \\ &= \frac{1}{A_c}\sum_{i=1}^{3}\mathbf{S}_i,\end{aligned} \tag{6}$$

where $\boldsymbol{\Omega}$ is the rotation frequency of the spin frame. The last equality comes from the equation of motion for the canting field [61]. The continuity equation for the spin current,

$$\partial_t \boldsymbol{\mathcal{J}}_0 + \partial_a \boldsymbol{\mathcal{J}}_a = 0, \tag{7}$$

follows directly from the equation of motion (4).

The Noether spin current Eq. (5) generally becomes nonzero whenever the spin configuration is nonuniform, even if it is static, similar to the case of spin supercurrents in collinear FMs and AFMs [58, 62–68]. By way of example, consider a static spin configuration with the spin frame twisting about $\mathbf{n}_x$ as one moves along the spatial x direction, $\partial_x \mathbf{n}_\alpha = \partial_x\phi\,\mathbf{n}_x \times \mathbf{n}_\alpha$, where $\phi(x)$ is a twist angle. The only nonzero spin current components are:

$$\boldsymbol{\mathcal{J}}_y = \pm\frac{\sqrt{3}}{8}JS^2\,\partial_x\phi\,\mathbf{n}_y, \tag{8}$$

whose spatial direction is orthogonal to the gradient of ϕ, and are therefore reminiscent of a Hall current. This will be our focus below.

Hall mass and transverse Noether spin currents.—In this section we illustrate the richness of the Hall spin currents mentioned above by considering dynamical noncollinear spins which are more relevant to typical spintronics experiments. We consider d.c. spin currents due to spin waves, for which $\mathbf{n}_\alpha(\mathbf{r},t) = \mathbf{n}^0_\alpha + \boldsymbol{\theta}(\mathbf{r},t)\times\mathbf{n}^0_\alpha$, $\mathbf{n}^0_\alpha$ being the ground state spin frame vectors and $\boldsymbol{\theta}(\mathbf{r},t) = \mathrm{Re}[\boldsymbol{\theta}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}]$. Taking a time average on both sides of Eq. (5) and subtracting any static contributions in equilibrium, we obtain

$$\begin{aligned}\langle\mathcal{J}^\alpha_a\rangle &= \Gamma^{\gamma\beta}_{ab}\left[-\frac{1}{2}\left\langle\mathbf{n}_\gamma\times\partial_b\mathbf{n}_\beta\right\rangle_\alpha\right] \\ &\equiv \mathrm{Tr}\left(\Gamma_{ab}\langle\mathcal{P}^\alpha_b\rangle\right)\end{aligned} \tag{9}$$

which resembles a linear response. Indeed, we show in [61] that $\mathcal{P}^\alpha_b$ can be understood as an SO(3) gauge potential [69] due to spatial translation along b projected onto the α-th spin angular momentum component, and can therefore be viewed as a spin current driving force [57]. Moreover, $\langle\mathcal{P}\rangle$ can be compactly expressed in terms of time-averaged densities of energy, linear momentum, and spin for spin waves, which are separately conserved and can hence be controlled externally, as presented below.

From Eq. (6), the time averaged spin density is

$$\langle\mathcal{J}^\gamma_0\rangle = -\frac{\rho\omega}{4}\mathrm{Im}(\boldsymbol{\theta}\times\boldsymbol{\theta}^*)_\gamma \tag{10}$$

Note that due to the last equality of Eq. (6), the time-averaged canting of the noncollinear spins is equal to the $\langle\mathcal{J}^\gamma_0\rangle$ above and represents the other part of the angular momentum that is carried by the spin waves, which will be discussed in a future work. Eq. (10) suggests that linearly polarized AFM spin waves, for which $\boldsymbol{\theta}$ are all real and $\boldsymbol{\theta}\times\boldsymbol{\theta}^* = 0$, cannot carry spin. However, the kagome AFM models considered here all have degenerate spin wave modes that can be circularly polarized [60]. More general cases will be discussed in the next section. The time-averaged linear momentum ($\mathcal{T}_{b0}$) and energy ($\mathcal{T}_{00}$) densities are [61]

$$\langle\mathcal{T}_{b0}\rangle = \frac{\rho\omega}{2}|\boldsymbol{\theta}|^2 k_b,\ \langle\mathcal{T}_{00}\rangle = \frac{\rho\omega^2}{2}|\boldsymbol{\theta}|^2 \tag{11}$$

where $|\boldsymbol{\theta}|^2 = \boldsymbol{\theta}\cdot\boldsymbol{\theta}^*$. As a result,

$$\langle\mathcal{P}^\alpha_b\rangle_{\beta\gamma} = \frac{1}{\rho}\frac{\langle\mathcal{J}^\gamma_0\rangle\langle\mathcal{T}_{b0}\rangle}{\langle\mathcal{T}_{00}\rangle}\delta_{\alpha\beta} \equiv P^\gamma_b\delta_{\alpha\beta}, \tag{12}$$

We therefore have $\langle\mathcal{J}^\alpha_a\rangle = \Gamma^{\alpha\gamma}_{ab}P^\gamma_b$ which even better illustrates the meaning of Γ as a response tensor than Eq. (9). It is then remarkable that, due to the off-diagonal components of Γ, neither the spin nor the spatial directions of the spin current have to be aligned with that of the driving force, or equivalently with that of the spin wave's spin and linear momentum, suggesting the existence of Hall spin currents.

To demonstrate the Hall spin currents explicitly, we consider a bilayer system consisting of an FM layer interfaced with a noncollinear AFM layer in 2D, both on a kagome lattice (Fig. 1 insets). The ground state configurations are calculated using an LLG solver to let the system relax [61]. We then solve the linearized LLG equation for harmonic excitations created by an external a.c. magnetic field applied on a few leftmost layers on the FM side. In the FM layer, such excitations correspond to FM spin waves propagating towards the interface, carrying a spin angular momentum whose direction is determined by that of the static magnetization, set to $\hat{\mathbf{x}}$. This is clearly shown by the d.c. spin currents on the FM side in Fig. 1. However, the spin currents on the AFM side have several components not naively expected from that on the FM side, and the transversely flowing $\mathcal{J}^y_y$ component has opposite signs for the direct and inverse triangular noncollinear states.

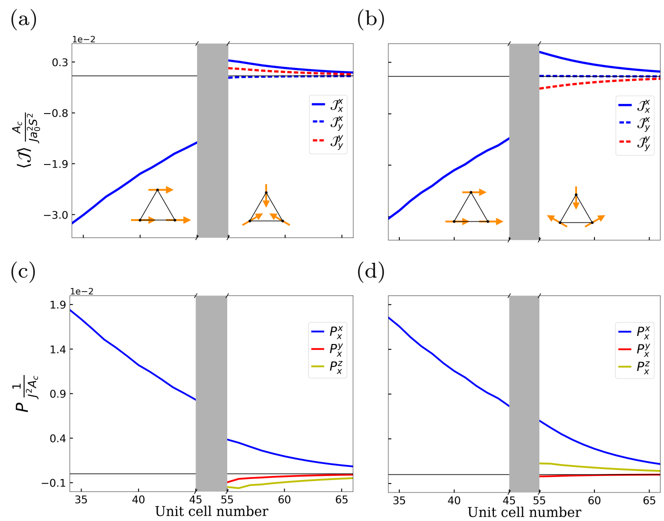

FIG. 1. (a) and (b), Dynamical d.c. Noether spin currents in (a) FM-Mn_3Ir (direct triangular order) and (b) FM-Mn_3Sn (inverse triangular order) interfaces outside of the interface region (gray rectangles). (c) and (d), Spin current driving force P for (c) FM-Mn_3Ir and (d) FM-Mn_3Sn interfaces outside of the interface region. All numerical LLG calculations were performed with the following parameters: gyromagnetic ratio $\gamma = 1$, damping $\alpha = 0.05$, $\omega = 1$, and linear polarization along $\hat{\mathbf{y}}$ for the external a.c. field. The interface region has a width of approximately 10 unit cells.

To understand this nontrivial behavior, we plot in Figs. 1 (c) and (d) the corresponding driving force P in both systems. One can see that different from $\langle\mathcal{J}\rangle$, both sides of the interface have the same dominant component of P, P^x_x. (P^y_x and P^z_x arise due to uncontrolled scattering at the interface region.) Moreover, the nonzero components of $\langle\mathcal{J}\rangle$ for the two noncollinear AFMs considered here are straightforwardly obtained from P_x^x and P_x^y together with the nontrivial components of Γ in Eq. (2). (P_x^z does not contribute since Γ has no z-spin indices.) For example, the $\langle \mathcal{J}_y^y \rangle$ component is due to $\Gamma_{yx}^{yx} = \pm\frac{\sqrt{3}JS^2}{4}$, whose different signs for the two types of order leads to the opposite $\langle \mathcal{J}_y^y \rangle$ in Figs. 1 (a) and (b). Conversely, the $\langle \mathcal{J}_y^x \rangle$ component has the same sign in both systems due to P_x^y and $\Gamma_{xy}^{yx} = \Gamma_{yx}^{yx}$ both changing sign.

The above observation suggests that it is meaningful to associate certain components in Γ with an analogue of the Hall effect for the Noether spin current. Since Γ has the units of inverse mass per area or volume (times $\hbar^2$), we name the part of Γ responsible for the Hall spin currents the Hall (inverse) mass. However, it is not as simple as deeming the off-diagonal components such as Γ_{xy}^{xy} the Hall mass since the individual components change their values under rotations of the coordinate system, while both the charge Hall effect and the spin Hall effect have certain invariance under coordinate transformations. More specifically, the charge Hall effect is a pseudovector $\sigma_{\mathrm{H}}^a = \frac{1}{2}\epsilon_{abc}\sigma_{bc}$ [70], with σ_{bc} the conductivity tensor, whose length is invariant under O(3) transformations; the spin Hall effect has an isotropic part $\frac{1}{6}\epsilon_{abc}\sigma_{abc}^{\mathrm{s}}$, with $\sigma_{abc}^{\mathrm{s}}$ the spin conductivity tensor, which is a scalar and invariant under O(3) transformations as well. In the next section we will propose an appropriate definition of the Hall mass based on a general field theory of noncollinear AFMs.

General definition of Hall mass in noncollinear AFMs.—We start by considering an arbitrary single-crystalline noncollinear AFM in three dimensions with spin S and Heisenberg exchange coupling between any two spins that depends on their spatial separation. The Γ tensor of such an AFM can be obtained through gradient expansion as [61]

$$\Gamma_{ab}^{\alpha\beta} = -\frac{S^2}{V_c}\sum_{j,pq} J_{0p,jq}(\mathbf{r}_{0p,jq})_a(\mathbf{r}_{0p,jq})_b m_p^\alpha m_q^\beta \qquad (13)$$

where V_c is the volume of the unit cell, i, j label unit cells, p, q label sublattices, $\hat{\mathbf{m}}_p$ is a unit vector along the spin direction on sublattice p, and $\mathbf{r}_{ip,jq}$ is the position vector of site jq relative to site ip. Such a Γ is symmetric under separate permutations of its spatial and spin indices and is also independent of unit cell choices [61].

The Γ as defined in Eq. (13) generally has off-diagonal components whose values will also change under rotations of the coordinate system. Moreover, the low-energy spin wave modes that depend on Γ are in general nondegenerate and are linearly polarized, different from that in ferromagnets, making them unable to carry spin according to Eq. (10). (Spin can also be carried by large-angle precession, or supercurrents, of the noncollinear spins, which we do not discuss in this work.) Both of these hurdles can be overcome if we consider a polycrystal of the given noncollinear AFM. When the grain sizes of the polycrystal are smaller than the typical wavelengths of the spin wave modes, its low-energy Lagrangian should have the same form as Eq. (3), but with ρ replaced by an effective isotropic paramagnetic susceptibility $\bar{\rho}$ and Γ by an angular-averaged $\bar{\Gamma}$ that depends only on two parameters [61, 71]:

$$\bar{\Gamma}_{ab}^{\alpha\beta} = g_{\mathrm{H}}(\delta_{a\alpha}\delta_{b\beta} + \delta_{a\beta}\delta_{b\alpha}) + g_0\delta_{ab}\delta_{\alpha\beta} \qquad (14)$$

We define the g_0 and g_{H} in Eq. (14) as the longitudinal and the Hall mass, respectively, since they are now invariant under rotations. (Note that $\delta_{ab}\delta_{\alpha\beta}$, $\delta_{a\alpha}\delta_{b\beta}$, and $\delta_{a\beta}\delta_{b\alpha}$ are the only three linearly independent isotropic rank-4 tensors in three dimensions.) In terms of the Γ of the corresponding single crystal, one can get $g_{\mathrm{H}} = \frac{1}{10}\left(\Gamma_{ab}^{ab} - \frac{1}{3}\Gamma_{bb}^{aa}\right)$, $g_0 = \frac{2}{15}\left(\Gamma_{bb}^{aa} - \frac{1}{2}\Gamma_{ab}^{ab}\right)$.

Using the isotropic $\bar{\Gamma}$ and the polycrystal Lagrangian, one can easily get the spin wave dispersions in terms of g_0, g_{H} and $\bar{\rho}$:

$$\begin{aligned} \omega_i &= c_i k, \ i = \mathrm{I}, \mathrm{II}, \mathrm{III} \\ c_{\mathrm{I}} &= \sqrt{g_0/\bar{\rho}} \\ c_{\mathrm{II,III}} &= \sqrt{(g_0 + g_{\mathrm{H}})/\bar{\rho}} \end{aligned} \qquad (15)$$

where the mode I has its polarization parallel to $\mathbf{k}$ while II, III have transverse polarizations, similar to phonons in isotropic elastic media [61]. The two transverse modes can thus be circularly polarized and carry the spin that is parallel to the propagation direction. For such spin waves, the Hall mass g_{H} gives rise to Hall spin currents with their spin polarization parallel to the directions of spatial flow, i.e., "axial" spin currents. Moreover, Eq. (15) suggests another way of determining the Hall mass from spectroscopic measurements on noncollinear AFM polycrystals.

Discussion.—A key prediction from our spin pumping calculation Fig. 1 is the transverse spin current $\langle \mathcal{J}_y^y \rangle$. Such an axial spin current, however, cannot be directly detected by ordinary inverse spin Hall effect, for which the directions of the spin current flow and of the spin polarization must be orthogonal to each other. A possible workaround is to use a low-symmetry crystal that has a spin Hall conductivity component $\sigma_{yya}^{\mathrm{s}}$, so that a charge current along a can be induced by $\langle \mathcal{J}_y^y \rangle$ [18, 24, 72–74]. Alternatively, if the noncollinear AFM under consideration has spin wave modes that can carry y-spin, one can pump y-spin from the FM side to create $\langle \mathcal{J}_y^x \rangle$ which is then detectable by ISHE, since $\Gamma_{yx}^{yx} = \Gamma_{yx}^{xy}$. Such a phenomenon is, however, more analogous to the spin swapping effect [75, 76] despite the different microscopic origins.

Separately, in real 3D noncollinear AFMs the Γ tensor can be quite different from those of our 2D kagome models. On the one hand, one can use Eq. (13) to calculate Γ for a given system once the spin Hamiltonian is determined from, e.g., inelastic neutron diffraction and linear spin-wave theory. On the other hand, nonzero components of Γ can be identified by symmetry analysis once the symmetry of the magnetic ground state is known. For example, we have used Eq. (13) to calculate Γ for cubic and hexagonal Mn_3X by keeping the nearest-neighbor exchange coupling only [61]. We found that even when the individual kagome planes in these materials have the same orientations as that in our 2D models, the 3D materials have different Γ components.

That the Hall mass g_H generally exists in isotropic noncollinear AFMs is remarkable. It suggests that all noncollinear AFMs can be classified by the signs of their Hall mass, which is also reflected by whether the two degenerate transverse-polarization spin wave modes are below or above the longitudinal-polarization mode as indicated by Eq. (15). We therefore propose inelastic magnetic neutron or x-ray measurements of powder samples of noncollinear AFMs which can reveal the existence and signs of their Hall mass.

Our work has ignored effects of conduction electrons for metallic noncollinear AFMs. When spin-orbit coupling is negligible, spin rotation symmetry is preserved for the whole system including conduction electrons and local moments. Therefore the Noether spin current is carried by the conduction electrons and local magnetic moments together, and our discussion on anisotropic spin current response induced by spin pumping, including the Hall mass, should qualitatively hold.

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