# A NOTE ON THE ERDŐS MATCHING CONJECTURE 

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#### Abstract

The Erdős Matching Conjecture states that the maximum size $f(n, k, s)$ of a family $\mathcal{F} \subseteq\binom{[n]}{k}$ that does not contain $s$ pairwise disjoint sets is $\max \left\{\left|\mathcal{A}_{k, s}\right|,\left|\mathcal{B}_{n, k, s}\right|\right\}$, where $\mathcal{A}_{k, s}=$ $\binom{[s k-1]}{k}$ and $\mathcal{B}_{n, k, s}=\left\{B \in\binom{[n]}{k}: B \cap[s-1] \neq \emptyset\right\}$. The case $s=2$ is simply the Erdős-Ko-Rado theorem on intersecting families and is well understood. The case $n=s k$ was settled by Kleitman and the uniqueness of the extremal construction was obtained by Frankl. Most results in this area show that if $k, s$ are fixed and $n$ is large enough, then the conjecture holds true. Exceptions are due to Frankl who proved the conjecture and considered variants for $n \in\left[s k, s k+c_{s, k}\right]$ if $s$ is large enough compared to $k$. A recent manuscript by Guo and Lu considers non-trivial families with matching number at most $s$ in a similar range of parameters.

In this short note, we are concerned with the case $s \geq 3$ fixed, $k$ tending to infinity and $n \in\{s k, s k+1\}$. For $n=s k$, we show the stability of the unique extremal construction of size $\binom{s k-1}{k}=\frac{s-1}{s}\binom{s k}{k}$ with respect to minimal degree. As a consequence we derive $\lim _{k \rightarrow \infty} \frac{f(s k+1, k, s)}{\binom{s k+1}{k}}<$ $\frac{s-1}{s}-\varepsilon_{s}$ for some positive constant $\varepsilon_{s}$ which depends only on $s$.


## 1. Introduction

We use standard notation. Let $[n]$ denote the set of the first $n$ positive integers. For a set $X$, $\binom{X}{k}$ stands for the family of all $k$-element subsets of $X$ and a $k$-graph (or $k$-uniform hypergraph) on $X$ is a subset of $\binom{X}{k}$. The members of $X$ are called vertices and the $k$-sets are called $k$-edges or hyperedges or just edges. A $k$-graph is intersecting if every pair of edges has a nonempty intersection. A $k$-graph on $X$ for which there is an $x \in X$ that is in every edge is called a star.

One of the major unsolved problems in extremal set theory is due to Erdős: The matching number $\nu(\mathcal{F})$ of a family $\mathcal{F}$ of sets is the size of the largest set of pairwise disjoint sets that $\mathcal{F}$ contains. Let $f(n, k, s)$ denote the maximum size of a $k$-graph on $n$ vertices with matching number strictly smaller than $s$. Because $\nu(\mathcal{F})=1$ if and only if $\mathcal{F}$ is intersecting, the well-known Erdős-Ko-Rado theorem (5) gives

$$
f(n, k, 2)= \begin{cases}\binom{n-1}{k-1}, & \text { if } n \geq 2 k \\ \binom{n}{k}, & \text { if } n \leq 2 k-1\end{cases}
$$

The special case of $f(s k, k, s)$ was proved by Kleitman.
Theorem 1 (Kleitman [22]). If $k \geq 2$ and $s \geq 2$ are positive integers, then $f(s k, k, s)=\binom{s k}{k}-$ $\binom{s k-1}{k-1}=\frac{s-1}{s}\binom{s k}{k}$ with equality if and only if there is an element $x \in[n]$ such that every $k$-edge fails to contain $x$.

There are two natural families of $k$-sets with matching number $s-1$ :

$$
\mathcal{A}_{k, s}=\binom{[s k-1]}{k} \quad \mathcal{B}_{n, k, s}=\left\{B \in\binom{[n]}{k}: B \cap[s-1] \neq \emptyset\right\} .
$$

[^0]Erdős conjectured that for any values of $n, k, s$, one of these families achieves the maximum value.
Conjecture 2 (Erdős Matching Conjecture (EMC) [4]). For all $n, k, s$ with $n \geq s k$, we have $f(n, k, s)=\max \left\{\left|\mathcal{A}_{k, s}\right|,\left|\mathcal{B}_{n, k, s}\right|\right\}$.

It can be computed that $\left|\mathcal{A}_{k, s}\right|<\left|\mathcal{B}_{n, k, s}\right|$ for all $n \geq s(k+1)$. Most results concerning Conjecture 2 determine that $f(n, k, s)=\left|\mathcal{B}_{n, k, s}\right|$ if $n$ is large enough compared to $k$ and $s$ [15, 20, with the current best bounds due to Frankl [8 for $n \geq(2 s+1) k-s$ and Frankl and Kupavskii [14] for $n \geq \frac{5}{3} s k-\frac{2}{3} s$ if $s$ is large enough. The case $k=3$ is also completely settled by Frankl 9$]$. Minimum degree versions of the problem are also studied [1, 18, 21 and general inequalities on $f(n, k, s)$ are known [14, 19] for $n=c k s$ with $c>1$ and $k$ and/or $s$ is large enough.

Not too many results are known in the region when $n$ is very close to $s k$ : Frankl [10 proved that $f(n, k, s)=\left|\mathcal{A}_{k, s}\right|$, if $s k \leq n \leq s(k+\varepsilon)$ where $\varepsilon$ depends only on $k$, but $s$ has to be large enough with respect to $k$ ( $s \geq k^{2}+k+1$ suffices). Frankl [12] and recently Guo and Lu [17] considered non-trivial families (i.e. those with no isolated vertices) in a similar range of parameters.

Our aim is to obtain bounds on $f(n, k, s)$ when $n=s k+1$, but with $s$ fixed and $k$ being large. Observe that the EMC states that there should be a huge difference between the cases $s=2$ and $s \geq 3$ : on the one hand $\frac{f(2 k, k, 2)}{\binom{2 k}{k}}=\frac{1}{2}$ and $\frac{f(2 k+1, k, 2)}{\binom{2 k+1}{k}} \rightarrow \frac{1}{2}$ as $k$ tends to infinity. On the other hand, $\frac{f(s k, k, s)}{\binom{k}{k}}=\frac{s-1}{s}$, while $\frac{\left|\mathcal{A}_{k, s}\right|}{\binom{k+1}{k}} \rightarrow\left(\frac{s-1}{s}\right)^{2}$ and $\frac{\left|\mathcal{B}_{s k+1, k, s}\right|}{\binom{k+1}{k}} \rightarrow 1-\left(1-\frac{1}{s}\right)^{s-1}$ as $k$ tends to infinity, thus the EMC would yield an immediate drop in the limiting constant. Our main result shows that there is indeed a gap between $\frac{f(s k, k, s)}{\binom{k}{k}}=\frac{s-1}{s}$ and $\frac{f(s k+1, k, s)}{\binom{s k+1}{k}}$.
Theorem 3. For any $s \geq 3$ there exists a positive real $\varepsilon_{s}$ such that $f(s k+1, k, s) \leq\left(\frac{s-1}{s}-\varepsilon_{s}\right)\binom{s k+1}{k}$ holds for all $k$.

To prove Theorem 3, as an intermediate step, we will show the following stability version of Kleitman's bound on $f(s k, k, s)$ with respect to the minimum degree.
Theorem 4. There exist absolute constants $C$ and $\delta_{0}$ such that, if $s \geq 3$ and $\delta \leq \delta_{0}$, then any family $\mathcal{F} \subseteq\binom{[s k]}{k}$ with $\nu(\mathcal{F}) \leq s-1$ and minimum degree at least $\delta\binom{s k-1}{k-1}$ satisfies $|\mathcal{F}| \leq$ $\left(\frac{s-1}{s}-\frac{4(s-2) \delta}{s^{2} C}\right)\binom{s k}{k}$.

## 2. Tools of the proofs: Removal lemmas and shifting

In this section, we introduce some terminology and results from the literature that we will use in our proofs of Theorem 3 and Theorem 4. Removal lemmas are widely used in combinatorics, they state that if some combinatorial structure $\mathcal{S}$ contains only a few copies of some pattern $P$, then $\mathcal{S}$ can be made $P$-free while almost keeping the structure $\mathcal{S}$. We will need a removal lemma for intersecting families. A general result is the following.
Theorem 5 (Friedgut, Regev [16]). Let $\gamma>0$ and $n, k$ positive integers with $\gamma n<k<(q / 2-\gamma) n$. Then for any $\varepsilon>0$ there exists $\delta>0$ such that any $\mathcal{F} \subseteq\binom{[n]}{k}$ with at most $\delta|\mathcal{F}|\binom{n-k}{k}$ disjoint pairs can be made intersecting by removing at most $\varepsilon\binom{n-1}{k-1}$ sets from $\mathcal{F}$.

As we will point out in the last section, Theorem 5 could also be used to derive our results, but the following statement is better suited for our purposes.
Theorem 6 (Das, Tran [3). There exists an absolute constant $C$ such that if $n, k, \ell$ satisfy $n>2 k \ell^{2}$, then for any $\mathcal{F} \subseteq\binom{[n]}{k}$ with at most $\left(\binom{\ell}{2}+\beta\right)\binom{n-1}{k-1}\binom{n-k-1}{k-1}$ disjoint pairs where $\max \{2 \ell|\alpha|,|\beta|\} \leq \frac{n-2 k}{(20 C)^{2} n}$ there exists a family $\mathcal{S}$ that is a union of $\ell$ stars such that $|\mathcal{F} \triangle \mathcal{S}| \leq$ $C((2 \ell-1) \alpha+2 \beta) \frac{n}{n-2 k}\binom{n-1}{k-1}$.

We state the case where $\ell=1, \alpha=0, n=s k$ as a corollary, since this is what we will use in our proofs.

Corollary 1. There exists an absolute constant $C$ such that for any $\mathcal{F} \subseteq\binom{[s k]}{k}$ with at most $s \beta\binom{s k-1}{k-1}\binom{(s-1) k}{k}$ disjoint pairs where $\beta \leq \frac{s-2}{s(20 C)^{2}}$ there exists a star $\mathcal{S}$ such that $|\mathcal{F} \triangle \mathcal{S}| \leq$ $\frac{2 s}{s-2} C \beta\binom{s k-1}{k-1}$.

Or equivalently, if the maximum degree $\Delta(\mathcal{F})$ of a family $\mathcal{F} \subseteq\binom{[s k]}{k}$ is at most $(1-\beta)\binom{s k-1}{k-1}$ where $\beta \leq \frac{s-2}{s(20 C)^{2}}$, then $\mathcal{F}$ contains at least $\frac{2 \beta(s-2)}{C}\binom{s k-1}{k-1}\binom{(s-1) k}{k}$ disjoint pairs.

Let us now turn to our other tool in proving our results. The shifting operation was introduced by Erdős, Ko, and Rado [5] and is a very frequently used powerful tool in extremal set theory. For a set $F$ and family $\mathcal{F}$ of sets we define

$$
S_{i, j}(F)= \begin{cases}F \backslash\{j\} \cup\{i\}, & \text { if } i \notin F, j \in F, F \backslash\{j\} \cup\{i\} \notin F \\ F, & \text { otherwise }\end{cases}
$$

$S_{i, j}(\mathcal{F})=\left\{S_{i, j}(F): F \in \mathcal{F}\right\}$. A family $\mathcal{F} \subseteq\binom{[n]}{k}$ is left-compressed if $S_{i, j}(\mathcal{F})=\mathcal{F}$ for all $1 \leq i<j \leq n$. We will use the following lemma [6, Lemma 4.2(iv)].

Lemma 7 (Frankl [6]). If $\mathcal{F}^{\prime} \subseteq\binom{[n]}{k}$ has matching number $\nu(\mathcal{F}) \leq s$, then there is a leftcompressed $\mathcal{F} \subseteq\binom{[n]}{k}$ with $\nu(\mathcal{F}) \leq s$ and $|\mathcal{F}|=\left|\mathcal{F}^{\prime}\right|$.

We will need some more properties of left-compressed families. To state them, we need some definitions: for a family $\mathcal{F} \subseteq\binom{[n]}{k}$ and $x, y \in[n]$, we write $\mathcal{F}_{x}=\{F \in \mathcal{F}: x \in F\}, \mathcal{F}_{\bar{x}}=\{F \in \mathcal{F}$ : $x \notin F\}, \mathcal{F}_{x, y}=\{F \in \mathcal{F}: x, y \in F\}$ and $\mathcal{F}_{x, \bar{y}}=\{F \in \mathcal{F}: x \in F, y \notin F\}$.

Lemma 8. For any left-compressed family $\mathcal{F} \subseteq\binom{[n]}{k}$, we have
(a) $\frac{n-k}{k}\left|\mathcal{F}_{n}\right| \leq\left|\mathcal{F}_{\bar{n}}\right|$,
(b) $\frac{\left|\mathcal{F}_{n}\right|}{\binom{n-1}{k-1}} \leq \frac{\mid \mathcal{F}_{n-1, \bar{n} \mid}}{\binom{n-2}{k-1}}$.

Proof. Inequality (a) follows from counting the pairs $(F, j)$ where $F \in \mathcal{F}_{n}$ and $j \notin F$. The number of such pairs is $(n-k)\left|\mathcal{F}_{n}\right|$. Moreover, the set $S_{j, n}(F)$ is in $\mathcal{F}$ and any set $F \in \mathcal{F}_{\bar{n}}$ can be the $S_{j, n}$-image of at most $k$ sets in $\mathcal{F}_{n}$. Hence the number of such pairs is at most $k\left|\mathcal{F}_{\bar{n}}\right|$

To see (b), a similar double counting argument establishes $\frac{n-k}{k-1}\left|\mathcal{F}_{n, n-1}\right| \leq\left|\mathcal{F}_{n, \overline{n-1}}\right|$ and thus $\left|\mathcal{F}_{n}\right|=\left|\mathcal{F}_{n, n-1}\right|+\left|\mathcal{F}_{n, \overline{n-1}}\right| \leq \frac{n-1}{n-k}\left|\mathcal{F}_{n, \overline{n-1}}\right|$. Also, applying $S_{n-1, n}$, we obtain $\left|\mathcal{F}_{n, \overline{n-1}}\right| \leq\left|\mathcal{F}_{n-1, \bar{n}}\right|$. Therefore

$$
\frac{\left|\mathcal{F}_{n}\right|}{\binom{n-1}{k-1}} \leq \frac{\frac{n-1}{n-k}\left|\mathcal{F}_{n, n-1}\right|}{\frac{n-1}{n-k}\binom{n-2}{k-1}} \leq \frac{\left|\mathcal{F}_{n-1, \bar{n}}\right|}{\binom{n-2}{k-1}}
$$

as claimed.
For any $r$-graph $H$, the sequence $\frac{\operatorname{ex}_{r}(n, H)}{\binom{n}{r}}$ is monotone decreasing.

## 3. Proofs

Proof of Theorem 4. Let $\mathcal{F} \subseteq\binom{[s k]}{k}$ with $\nu(\mathcal{F}) \leq s-1$ and $\delta(\mathcal{F}) \geq \delta\binom{s k-1}{k-1}$. Let us write $\mathcal{G}=$ $\binom{[s k]}{k} \backslash \mathcal{F}$ and consider an arbitrary subfamily $\mathcal{G}^{\prime} \subseteq \mathcal{G}$ with $\left|\mathcal{G}^{\prime}\right|=\frac{1}{s}\binom{s k}{k}=\binom{s k-1}{k-1}$. (By Theorem 11, we know $|\mathcal{G}| \geq \frac{1}{s}\binom{s k}{k}$, and so such a $\mathcal{G}^{\prime}$ exists.)

The minimum degree condition on $\mathcal{F}$ implies $\Delta\left(\mathcal{G}^{\prime}\right) \leq(1-\delta)\binom{s k-1}{k-1}$. Corollary 1 yields that if $\delta$ is sufficiently small, then $\mathcal{G}^{\prime}$ (and thus $\mathcal{G}$ ) contains at least $\frac{2 \delta(s-2)}{C}\binom{s k-1}{k-1}\binom{(s-1) k}{k}$ disjoint pairs.

Every set $G$ is contained in $M=\frac{1}{(s-1)!} \prod_{j=1}^{s-1}\binom{j k}{k} s$-partitions of [sk]. Also, every disjoint pair $G, G^{\prime}$ is contained in $M^{\prime}=\frac{1}{(s-2)!} \prod_{j=1}^{s-2}\binom{j k}{k} s$-partitions, and clearly one fixed $s$-partition contains $\binom{s}{2}$ disjoint pairs. Let us count the pairs $(G, \pi)$ with $\pi$ being an $s$-partition of $[s k]$ into $k$-sets, and $G \in \mathcal{G}$ being one of those $k$-sets. On the one hand the number of such pairs is $|\mathcal{G}| \cdot M$. On the other hand, by definition of $\mathcal{F}$ and $\mathcal{G}$, every $\pi$ contains at least one set of $\mathcal{G}$. Also, by the above observations, the number of $\pi$ 's containing at least two sets from $\mathcal{G}$ is at least $\frac{2 \delta(s-2)}{C}\binom{s k-1}{k-1}\binom{(s-1) k}{k} \cdot \frac{M^{\prime}}{\binom{s}{2}}$. Putting these together,

$$
|\mathcal{G}| \cdot M \geq \frac{1}{s!} \prod_{j=1}^{s}\binom{j k}{k}+\frac{2 \delta(s-2)}{C}\binom{s k-1}{k-1}\binom{(s-1) k}{k} \cdot \frac{M^{\prime}}{\binom{s}{2}}
$$

Dividing by $M$, we get $|\mathcal{G}| \geq\binom{ s k-1}{k-1}+\frac{4(s-2) \delta}{s C}\binom{s k-1}{k-1}$ and thus

$$
|\mathcal{F}|=\binom{s k}{k}-|\mathcal{G}| \leq\binom{ s k-1}{k}-\frac{4(s-2) \delta}{s C}\binom{s k-1}{k-1}=\left(\frac{s-1}{s}-\frac{4(s-2) \delta}{s^{2} C}\right)\binom{s k}{k}
$$

as claimed.
Proof of Theorem [3. Let $\mathcal{F} \subseteq\binom{[s k+1]}{k}$ be a family of sets with $\nu(\mathcal{F})<s,|\mathcal{F}|=f(s k+1, k, s)$ and let us write $n=s k+1$.

By Lemma 7, we can assume that $\mathcal{F}$ is left-compressed. By Theorem 1, we have $\left|\mathcal{F}_{\bar{n}}\right| \leq \frac{s-1}{s}\binom{s k}{k}$. Let $\delta_{0}, C$ be as in Theorem 4, and let $\varepsilon_{s}^{*}=\min \left\{\frac{4(s-2) \delta_{0}}{s^{2} C}, \frac{s-1}{s}-\delta_{0}\right\}$.

Suppose first $\left|\mathcal{F}_{n}\right| \leq\left(\frac{s-1}{s}-\varepsilon_{s}^{*}\right)\binom{s k}{k-1}$, then

$$
|\mathcal{F}|=\left|\mathcal{F}_{\bar{n}}\right|+\left|\mathcal{F}_{n}\right| \leq \frac{s-1}{s}\binom{s k}{k}+\left(\frac{s-1}{s}-\varepsilon_{s}^{*}\right)\binom{s k}{k-1} \leq\left(\frac{s-1}{s}-\frac{\varepsilon_{s}^{*}}{s+1}\right)\binom{s k+1}{k} .
$$

On the other hand, if $\left|\mathcal{F}_{n}\right| \geq\left(\frac{s-1}{s}-\varepsilon_{s}^{*}\right)\binom{s k}{k-1}$, then we use the fact that left-compression implies $\delta\left(\mathcal{F}_{\bar{n}}\right)=\left|\mathcal{F}_{n-1, \bar{n}}\right|$ and Lemma $[(\mathrm{b})$ to conclude that

$$
\frac{\delta\left(\mathcal{F}_{\bar{n}}\right)}{\binom{s k-1}{k-1}}=\frac{\mid \mathcal{F}_{n-1, \bar{n} \mid}}{\binom{s k-1}{k-1}} \geq \frac{\left|\mathcal{F}_{n}\right|}{\binom{s k}{k-1}} \geq \frac{s-1}{s}-\varepsilon_{s}^{*} .
$$

So one can apply Theorem 4 with $\delta=\delta_{0}$ and since $\frac{s-1}{s}-\epsilon_{s}^{*} \geq \delta_{0}$, it gives $\left|\mathcal{F}_{\bar{n}}\right| \leq\left(\frac{s-1}{s}-\varepsilon_{s}^{*}\right)\binom{s k}{k}$. According to Lemma $\mathbb{Z}\left((a)\right.$, we have $\left|\mathcal{F}_{n}\right| \leq \frac{k}{(s-1) k+1}\left|\mathcal{F}_{\bar{n}}\right|$ and thus

$$
|\mathcal{F}|=\left|\mathcal{F}_{n}\right|+\left|\mathcal{F}_{\bar{n}}\right| \leq\left(\frac{s-1}{s}-\varepsilon_{s}^{*}\right)\binom{s k+1}{k}
$$

So $\varepsilon_{s}=\varepsilon_{s}^{*} /(s+1)$ works in both cases.

## 4. Concluding Remarks

As we mentioned in Section 2, Theorem 3 and Theorem 4 can be proved using the " $\varepsilon-\delta$ removal lemma" of Theorem 5 with the help of Frankl's result [7] on the maximum size of intersecting families with bounded maximum degree. Also, there is a large literature on the size of intersecting families with constraints on some other parameters: minimum degree [21] and diversity [11, 13, [23, 24]. Supersaturation results on the minimum number of disjoint pairs in a family are also known [2]. Although, exact results are usually known for values of $n$ large enough compared to $k$, weaker bounds could be turned into bounds on $f(n, k, s)$ with the methods used in our note.

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