Chern-Simmons electrodynamics and torsion dark matter axions

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Abstract

In this paper, we delve into the influence of torsion axial pseudo vector on dark photons in an axion torsionic background, as investigated previously by Duncan et al[Nucl Phys B 387:215 (1992)]. Notably, axial torsion, owing to its significantly greater mass compared to axions, gives rise to magnetic helicity in torsionful Chern-Simons (CS) electrodynamics, leading to the damping of magnetic fields. In QCD scale the damping from dark massive photons leads us to obtain a magnetic field of 10^{-8} Gauss, which is approximated the order of magnitude of magnetic fields at present universe. This result is obtained by considering that torsion has the value of the 1 MeV at the early universe, and can be improved to the higher value of 10^{-3} Gauss when the axial torsion 0-component is given by 10^{8} MeV and the mass of dark photon is approximated equal to the axion. The axion plays a crucial role in achieving CS dynamo action arising from axions. This study is useful in deepening our understanding of fundamental physics, from nuclear interactions to the nature of dark matter.

Key-words: Chern-Simmons electrodynamics, axions, dark photon.

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1 Introduction

The axion, a fascinating particle, plays multifaceted roles spanning from solving fundamental problems in particle physics to influencing astrophysical phenomena and serving as a potential dark matter candidate, althrough its detection remains an active area of research, with ongoing efforts worldwide. Previously, Garretson, Field, and Carroll [1] used Pseudo-Goldstone bosons instead of the quantum chromodynamics (QCD) axions, attempting to obtain primordial magnetic fields strong enough to be amplified by dynamo mechanism. Unfortunatly, they did not achieve success. Subsequently, Duncan et al [2] investigated the Cartan torsion character of the axion hair of the black holes, by associating the hair to dilaton and axion of string inspiring theories [3–7], where the coupling of torsion to fermions remains minimal. The axion serves a dual role: its serves as a dynamical degree of freedom of the torsion. In their analysis,they proposed that a board class of torsionful theories could lead to the transmutation of torsion into axions [8–10]. Remarkably, even a simple QED model in a torsion-rich background can give rise to this transformation from torsion to axion [11]. These axion hair in black holes may be obtained from dynamical torsional anomalies [12].

More recently Agrawal et al [13], have investigated a mechanism to obtain the relic abundance of dark photon dark matter (DM) without torsion [14] and considered a very light spin-1 massive dark photon,where the photon mass with respect to the axion is given by $m_{\gamma} = m_{\mathrm{a}}\mathcal{O}(10^{-3} - 1)$, and $m_{\mathrm{a}} \approx 10^{-17}$ GeV. The decay coupling constant of the axion $f_{\mathrm{a}} \sim 10^{14}$ GeV. Inspired by the above references, we'll explore the intriguing physical properties arising from the minimal coupling of dark photons to torsion. The first reference demonstrates that torsion can be transmuted into an axion, which plays a crucial role in the production of dark photons and DM. Additionally, axions are significant contributors to dark energy and DM dynamics [15]. Our investigation involves torsionful Chern-Simons (CS) electrodynamics, a framework that naturally includes magnetic helicity density. Through this approach, we establish relationships between magnetic energy density, magnetic helicity density, and the Beltrami-Maxwell helicity magnetic field parameter. The CS electrodynamics with torsion is very important. Some authors have been able to place new stringent bounds in Lorentz symmetry breaking [16, 17].

In this study, we demonstrate the presence of a damping effect arising from axial torsion dark mass photons within magnetic fields at the QCD scales. These scales correspond to intensities of approximately 10^{-8} Gauss. Notably, this differs from the purely axionic electrodynamics without torsion, which predicts a damping effect at a much lower field strength of 10^{-13} Gauss, as reported by Miniati et al [18]. The fascinating interplay between axion oscillation and dark photons lies at the heart of our investigation. Specifically, we explore how energy transfers occur through a tachyonic instability. Notably, the coupling between the dark photon mass and the axial 0-component of the torsion pseudo-vector plays a crucial role in the observed damping phenomenon. Consider the magnetic field seeds initially present at the QCD scale, characterized by a formidable strength of $B_{\text{seeds}} = 10^{17}$ Gauss. Over cosmic time, these intense fields are gradually damped down to more modest values around $B_{\text{QCD}} \sim 10^{-3}$ Gauss—a remarkable deviation from the earlier estimate of 10^{-13} Gauss proposed by Miniati et al [13]. However, an intriguing possibility emerges: If the dark mass can be further reduced to an astonishingly low scale of 10^{-29} GeV, the damping effect could become even more pronounced, potentially reaching as low as 10^{-12} Gauss. This unexpected result stems from an axial torsion 0-component that aligns with the string-inspired Kalb-Ramond field at an energy scale of 10^{8} GeV.

Additionally, our findings suggest the possibility of obtaining a dynamo mechanism and other axion-related solutions. The amplitude of axion oscillation may grow via torsion. Thus, unlike Garretson et al [19], our findings extend beyond mere theoretical curiosity, and hint at the feasibility of a dynamo mechanism driven by torsion, dark photon DM, and the massive axion. In particular, we emphasize the phenomenon of axion-torsion transmutation. Our focus lies on a spacetime characterized by torsion but devoid of curvature, where the covariant Riemannian derivative simplifies to $\nabla = \partial$, rather than the full Riemann-Cartan spacetime.

The reminder of this paper is organised as follows: In section 2, we address the action considered here of the CS electrodynamics with torsion coupling to axions generalised to include axionic kinectic terms and a dark photon potential. Variations of this action with respect to axions and magnetic vector potential is given in section 3, whereas conclusions and discussions are left to section 4.

2 Spin-1 dark massive photons torsion transmutation into axions and CS electrodynamics

Here we propose that in similar way as the determination of pp decay into torsion given by the cross section [20]

$$\sigma(pp \to TS). \tag{1}$$

Here, the cross section represents the decay of four-fermions into torsion (TS). By transitivity, we infer the presence of a corresponding decay rate between dark photons and TS. The decay rate of axions into dark photons, as proposed by Agrawal et al [21], can be expressed as:

$$\Gamma(a \to \gamma\gamma) \approx \frac{\beta^2}{64\pi}\frac{{m^3}_a}{{f^2}_a}. \tag{2}$$

Considering the work by Duncan et al [2], which discusses torsion transmutation into axions, we can conjecture a similar decay rate for axions into dark photons

$$\Gamma(TS \to a). \tag{3}$$

Based on the universal mathematical properties of transitivity, we propose the following decay rates

$$\Gamma(TS \to \gamma\gamma), \tag{4}$$

which is the main idea behind this section. Although we do not compute the last two decays in this work, we begin this section by investigating the kinematics of the decay process. Specifically, we consider the action of torsionful CS electrodynamics with minimal coupling,

$$\mathcal{S}_{\mathrm{DM}} = \int d^4x[\frac{1}{2}\partial_i\phi\partial^i\phi - V(\phi) - \frac{1}{4}F^2 + \frac{1}{2}{m^2}_\gamma A^2 - \frac{\beta\phi}{4f_a}F\tilde{F}]. \tag{5}$$

which leads to torsion transmutation into the axion, where the last term corresponds to the chiral term, $\tilde{F}$ represents the dual of the electromagnetic field 2-form $F = F_{ij}dx^i \wedge dx^j$ in the Cartan's language of exterior differential forms [14]. The indices $(i, j = 0, 1, 2, 3)$ are used to introduce the magnetic helicity term, resulting in a CS electrodynamics with dark massive photons in DM. We achieve this by taking the minimal coupling between the electromagnetic field tensor and torsion through the expression

$$\nabla_{[i}A_{j]} = F_{ij} + 2T_{ijk}A^k, \tag{6}$$

where $T^{ijk} = \epsilon^{ijkm}T_m$ is the totally skew-symmetric torsion tersor, and T_m is the axial pseudo-vector. From these expressions, one obtains

$$F'_{ij} = \nabla_{[i}A_{j]} = F_{ij} + 2\epsilon_{ijkm}A^kT^m. \tag{7}$$

Note that even if we have not placed by hand the dark matter massive term it would appear by the breaking of symmetry due to the following expression

$$F'^2 = F^2 + 4T^2A^2. \tag{8}$$

As previously demonstrated by De Sabbata, Sivaram, and Garcia de Andrade [13], this expression implies that the massive dark photon mass could be interpreted through torsion, as indicated by the decay rate. Before delving into the main topic, two crucial points must be addressed: Firstly, the minimal coupling to torsion introduces an intriguing aspect: the action described above can naturally give rise to the massive torsion mode and contribute to the dark mass photon. Secondly, in this paper we shall, however, keep axial torsion freeze and independent as a torsionful background to axions and massive dark photon. The generation of interaction between torsion and magnetic potential A of dark photon dark matter, of the dark photon is very important in the sense that the effective action of dark photon becomes

$$e^{i\Gamma_{\text{eff}}[A,T]} = \int [d\psi][d\bar{\psi}]e^{[i\int dx\mathcal{L}_{\text{QED}}(A,T,\psi,\bar{\psi})]}det\mathcal{O}. \tag{9}$$

Here the operator under the determinant det inside the integral sign is given by

$$\mathcal{O}_{xy} = (i\gamma D_x - M)\delta_{xy} \tag{10}$$

where γ represents the Dirac matrices, and the operator D is defined as

$$D_k\psi = \partial_k - ieA_k - igT_k. \tag{11}$$

The axial torsion is then introduced in the effective action. The expression for the effective action of the dark photon can be expanded as

$$e^{i\Gamma_{\text{eff}}[A,T]} = \int [d\psi][d\bar{\psi}]e^{[-i\int dxT^2A^2]}e^{[i\int dx\mathcal{L}_{\text{QED}}(A,\psi\bar{\psi})]}det\mathcal{O}. \tag{12}$$

From this expression, one notices that the axial torsion pseudo-vector in second order is now present in the action and may represent the dark photon itself. This is consistent with our assumption of the decay of torsion T_s into the dark photon pair $\gamma\gamma$. A more detailed account of this process in the Riemannian case can be found in the book [22] on effective Lagrangians for the Standard Model (SM) of particle physics and in the work by Shapiro [15] on torsionful anomalies. By expanding the dual-like invariant

$$F'\tilde{F}' = F'\tilde{F} + 2\tilde{F}TA, \tag{13}$$

which represents the coupling between axial torsion and the dark massive photon in DM, and substituting it into the dark photon action, one obtains

$$\begin{aligned}\mathcal{S}_{\mathrm{DM}} = \int d^4x[\frac{1}{2}\dot{\phi}^2 + \frac{1}{4}(S_0)^2\phi^0 + S^0\dot{\phi}\phi - V(\phi) - \frac{1}{4}F^2 + \frac{1}{2}m^2{}_\gamma A^2 \\ - \frac{\beta\phi}{4f_a}[F\tilde{F} + \tilde{F}^{0c}S_0A_c + S_0\mathbf{E}\cdot\mathbf{B} + S_0\mathbf{A}\cdot\mathbf{B}]].\end{aligned} \tag{14}$$

Let us now take a moment to analyse the physics behind this action: First of all, the action we're examining involves torsion, which is only homogeneous. The axial torsion has a non-vanishing component: the time component denoted as S_0. Interestingly, this squared axial torsion component could be proportional to the axion mass. Next, we utilize the concept of chirality decoupling, as proposed by Dobado et al [22]. Specifically, we focus on the vanishing of the chirality term involving electric and magnetic fields. Notably, the last term on the right-hand side of the action naturally includes DM magnetic helicity. Now, let's proceed by substituting the Lagrangian associated with the action (Equation 14) into the dark photon action. We'll use the Euler-Lagrange (EL) equation

$$\partial_t\frac{\partial\mathcal{L}}{\partial\dot{X}} - \frac{\partial\mathcal{L}}{\partial X} = 0 \tag{15}$$

since $X = (A, \phi)$. Applying the Euler-Lagrange equation to the four potential variable A of a magnetic field in a torsionful spacetime yields

$$\partial_i[F^{ik}(1 - \frac{\beta\phi}{4f_a})] = J^k + \frac{1}{2}m_\gamma A^k, \tag{16}$$

where J represents the Ohm current given by

$$\mathbf{J}_{\mathrm{Ohm}} = \sigma[\mathbf{E} + \mathbf{v}\times\mathbf{B}]. \tag{17}$$

The second current in the above expression corresponds to a London-like current for the dark massive photon in DM. Taking the equations of Maxwell-Cartan-Proca electrodynamics, we obtain the Ampere-like equation

$$\partial_0[F^{0j} + 2S^0 A^j](1 - \frac{\beta\phi}{4f_a}) = J^j + \frac{1}{2}{m_\gamma}^2 A^j. \tag{18}$$

This equation is an Ampere-like equation, and a Coulomb-like equation comes from the other equation

$$\partial_i[(1 - \frac{\beta\phi}{4f_a})(E^i + 2S^0 A^0)] = \rho_\gamma + \frac{1}{2}{m_\gamma}^2 A^0. \tag{19}$$

This equation explicitly shows the Coulomb-like behavior, with the first term on the right-hand side representing the mass density ofthe dark photon

$$(1 - \frac{\beta\phi}{4f_a})\nabla \cdot \mathbf{E} = \rho_\gamma + \frac{1}{2}{m_\gamma}^2 A^0. \tag{20}$$

The Ampere-like law is given by

$$\partial_t[(1 - \frac{\beta\phi}{4f_a})\mathbf{E} - \frac{\beta\dot{\phi}}{4f_a}\mathbf{E} + ({m_\gamma}^2 + \frac{\beta\phi}{4f_a})\mathbf{A} = \mathbf{J}_{\text{Ohm}} \tag{21}$$

Taking the curl on both sides of the last expression and making use of the Faraday equation

$$\partial_t \mathbf{B} = -\nabla\times\mathbf{E} \tag{22}$$

and the magnetic Beltrami equation obeyed by the magnetic helical fields

$$\nabla\times\mathbf{B} = \lambda\mathbf{B}, \tag{23}$$

one obtains finally the magnetic wave equation for the evolution of the magnetic field as

$$\partial^2{}_t[(1 - \frac{\beta\phi}{4f_a})\mathbf{B}] - \frac{\beta\dot{\phi}}{4f_a}\partial_t\mathbf{B} - ({m_\gamma}^2 + \frac{\beta\dot{\phi}}{4f_a}S^0)\lambda\mathbf{B} = 0 \tag{24}$$

where $\nabla \times \mathbf{A} = \mathbf{B}$ is the magnetic field definition, and we have used in the last equation the convective dynamo term [23]

$$\nabla\times\mathbf{J}_{\text{Ohm}} = \sigma[\mathbf{B} + \nabla\times(\mathbf{v}\times\mathbf{B})]. \tag{25}$$

The truly convective second term on the right-hand side may be dropped, we show in what follows that even in the absence of the convective term, we may obtain a dynamo action

$$[(1-\frac{\beta\phi}{4f_a})]\partial^2{}_t\mathbf{B}-(\frac{\beta\dot{\phi}}{4f_a}+\sigma)\partial_t\mathbf{B}+[(m_\gamma{}^2+\frac{\beta\dot{\phi}}{4f_a}S^0-2(1-\frac{\beta\dot{\phi}}{4f_a}S^0)]\lambda\mathbf{B}=0. \quad (26)$$

Note that in the absence of torsion, an oscillating magnetic field emerges. Observing the equation of the CS electrodynamics, we find that a homogeneous axion $\phi(t)$ is necessary for its solution, with t denoting cosmic time. To derive the dynamical equation for the axion, we turn to the EL equation

$$\ddot{\phi}+\frac{1}{2}S^0\dot{\phi}+\partial_\phi V(\phi)+S_0\mathcal{H}=0. \quad (27)$$

Here, we neglect the chirality term while retaining the helicity density $\mathcal{H}=\mathbf{A}\cdot\mathbf{B}$. Assuming the axion potential takes the form

$$V=m^2{}_a f^2{}_a(1-\cos(\frac{\phi}{f_a})), \quad (28)$$

then we give a partial derivative of this potential

$$\partial_\phi V=m^2{}_a f^2{}_a\sin(\frac{\phi}{f_a}), \quad (29)$$

which approximatly becomes

$$\partial_\phi V\approx m^2{}_a f_a\phi. \quad (30)$$

The substitution of the approximate potential into the axion dynamical equation (27) yields the following expression

$$\ddot{\phi}+\frac{1}{2}S^0\dot{\phi}+m_a{}^2\phi+S_0\mathcal{H}=0. \quad (31)$$

To solve Equation (31) and determine the evolution of the magnetic field in DM, the last term on the left-hand side of this equation can be neglected since both the axial torsion and helicity are both very weak. This simplifies the equation to

$$\delta^2+\frac{1}{2}S^0\delta+m_a{}^2f_a=0, \quad (32)$$

where we have taken the ansatz $\phi = \phi_0 e^{\delta t}$ and substitution into expression (Equation 32). By solving the characteristic algebraic equation, we find

$$\delta_- = \frac{2m^2{}_a f_a}{S_0}. \tag{33}$$

Thus, the axion scalar spin-0 boson is expressed in terms of torsion, which, as described by Duncan et al, indicates that there is a torsion transmutation to the axion

$$\phi(t) = \phi_0 exp[(\frac{2m^2{}_a f_a}{S_0})t]. \tag{34}$$

The behavior of the axion, driven by torsion, is influenced by the chirality of the torsion or the sign of S_0. When the left-hand torsion chirality is negative, the axion cosmic scale decays over time, on the other hand when it is positive the axion cosmic scale is amplified. Let's now substitute this axion expression into the magnetic wave equation to obtain magnetogenesis due to the dark massive photon. But before that, we need to compute the time derivative of the axion using the following expression

$$\dot{\phi} = \frac{2m^2{}_a f_a}{S_0}\phi. \tag{35}$$

Differentiating Equation (35) with respect to cosmic time, we obtain

$$\ddot{\phi} - \omega^2 \phi = 0, \tag{36}$$

where $\omega = \frac{m^2 f_a}{S_0}$. Solving this differential equation yields

$$\phi = \phi_0 sinh[(\frac{m^2{}_a f_a}{S_0})t]. \tag{37}$$

Notably, this solution reveals that the axial torsion contributes to the damping of the axion. In the early universe, where time intervals are extremely short (e.g., at inflation $t \sim 10^{-35}$ s or at QCD scale $t \sim 10^{-5}$ s, we can consider $t \ll 1$. Under this approximation, Equation (37) simplifies to

$$\phi = \phi_0[(\frac{m^2{}_a f_a}{S_0})t], \tag{38}$$

which definetly shows that the axial torsion pseudo-vector 0-component causes a damping in the oscillating axion. This of course is not present in the reference [21]. To show how strong is this damping, we assume that: the axion decaying constant f_a is 10^{14} GeV, the axion mass m_a is approximately 10^{-17} GeV, and the torsion parameter S_0 is approximately 10^8 MeV (equivalent to 10^5 GeV or 10^{-7} TeV, as computed by Mavromatos [24]). Remarkably, this torsion parameter is significantly lower than the energy scales achievable at the Large Hadron Collider (LHC), which typically operates in the range of $7 - 14$ TeV. The damping effect is quantified by the ratio of f_a) and m_a^2, yielding an estimate of 10^{-22} GeV^2, indicating strong damping in the axion oscillation. However, if we consider the axial torsion as 1 MeV (equivalent to 10^{-3} GeV), the damping factor ω then becomes 10^{-3} GeV^2, corresponding to a much lighter damping effect. These axion torsion damping effects, particularly in the context of dark photons and dark matter, could inspire experimental proposals for axion-torsion detection. In summary, understanding the interplay between axion properties and torsion [25, 26] provides valuable insights into the behavior of these elusive particles.

3 Dynamo action in dark bosons DM driven by torsion

Recently, C. H. Nam [27] investigated dark gauge bosons through the Einstein-Cartan portal, which resides in the hidden sector—an invisible world that couples to the SM, and explored the production of dark gauge bosons, similar to our approach, but via bremsstrahlung off the dark sector. Notably, Ref. [24] demonstrated that these gauge dark bosons remain sensitive to very small kinetic mixing, provided that the decay channel of the gauge bosons to dark gauge bosons remains inaccessible. In this section, to the best of our knowledge, we present the first evidence that dynamo action onset occurs for axial torsion on the order of 10^5 GeV or 10^{-4} GeV—energy scales well within the capabilities of the LHC at CERN. To support this claim, we solve the magnetic wave equation

$$[(1-\frac{\beta\phi}{4f_a})]\partial^2{}_t\mathbf{B}-(\frac{\beta\dot{\phi}}{4f_a}+\sigma)\partial_t\mathbf{B}+[(m_\gamma{}^2+\frac{\beta\dot{\phi}}{4f_a}S^0-2(1-\frac{\beta\dot{\phi}}{4f_a}S^0)]\lambda\mathbf{B}=0. \quad (39)$$

By taking the ansatz for the magnetic field as $B = B_{\text{seed}}exp[\gamma t]$ into Equation (26) yields

$$\gamma^2 - (\frac{\beta\phi_0 m^2{}_a}{4} + \sigma)\gamma + [(m_\gamma{}^2 + \frac{\beta\dot{\phi}}{4f_a}S^0 - 2(1 - \frac{\beta\dot{\phi}}{4f_a}S^0)]\lambda = 0. \quad (40)$$

Therefore, solving the characteristic algebraic equation above, we obtain

$$\gamma_{-} = -\frac{1}{8}\frac{\sqrt{\lambda}\beta\phi_0\sigma {m^2}_a}{S_0}. \tag{41}$$

In this case, we have suppressed the term in front of the second time derivative of the magnetic field by taking the early universe cosmic time of $t \ll 1$ approximation. Note that if the axial torsion is negative or left-chiral, we have a dynamo amplification in one of the branches of solutions. If the axial torsion is positive, the magnetic field decays as

$$\mathbf{B} \approx \mathbf{B}_{seed}(1 - \frac{1}{8}\frac{\sqrt{\lambda}\beta\phi_0\sigma {m^2}_a}{S_0}) \tag{42}$$

Therefore, the decay of the dynamo mechanism of the magnetic field in dark boson dark matter sect depends upon the torsion chirality. If one considers $\beta = 40$ and the magnetic helicity of the order of 10^{-26} cm^{-1}, along with an electrical conductivity of 10^{28} s^{-1}, the resulting magnetic field strength at the QCD scale is $B_{\text{QCD}} = 10^{-13}$ Gauss in the present universe. Additionally, the seed field at QCD scales, as discussed by Miniati et al [18], corresponds to $B_{\text{seed}} = 10^{17}$ G. By substituting these data into the expression $B_{\text{QCD}} = B \times 10^{-2}\phi_0$, we can estimate the initial cosmic axion boson mass to be approximately 0.1 GeV. For a more detailed phenomenological analysis, further investigation can be pursued elsewhere.

4 Summary

In this paper, we investigate the impact of the torsion axial pseudo-vector on dark photons within an axion-torsionic background. At the QCD scale, the damping effect from massive dark photons yields a magnetic field strength of approximately 10^{-8} Gauss, roughly consistent with magnetic fields at present universe. This intriguing result emerges when axial torsion's 0-component reaches 10^8 MeV, and the dark photon mass approximates that of the axion. Our research will illuminate the intricate connections between axion physics, torsion, and dark photon interactions, providing fresh insights into the fundamental forces shaping our universe.

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