

The quantile-based classifier with variable-wise parameters

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Abstract

Quantile-based classifiers can classify high-dimensional observations by minimising a discrepancy of an observation to a class based on suitable quantiles of the within-class distributions, corresponding to a unique percentage for all variables. The present work extends these classifiers by introducing a way to determine potentially different optimal percentages for different variables. Furthermore, a variable-wise scale parameter is introduced. A simple greedy algorithm to estimate the parameters is proposed. Their consistency in a nonparametric setting is proved. Experiments using artificially generated and real data confirm the potential of the quantile-based classifier with variable-wise parameters.

Keywords: High-dimensional data; Supervised classification; Asymmetric Laplace distributions.

1 Introduction

Quantiles can be useful for both supervised and unsupervised classification. They can be used to focus on tails of the class conditional distributions rather than only their central moments. For supervised classification, Hennig and Viroli (2016) proposed a quantile-based classifier, referred to as (original) quantile classifier (OQC), that considers a sum of variable-wise discrepancies of the components of a new observation (to be classified) to the within-class

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univariate quantiles. The method is a generalization of the centroid-based (Tibshirani et al., 2002, 2003) and the median-based (Hall et al., 2009) classifiers. It has been shown that the strategy works quite well for skewed data and heavy tails compared to other classifiers. A recent extension based on a regularised loss function (Lai and McLeod, 2020) performs comparatively well in different scenarios, especially for limited sample sizes or dimensionality.

Hennig and Viroli (2016) included a somewhat surprising empirical result, namely that using the same percentage for defining quantiles for all variables (only involving a check whether a variable is left- or right-skewed) worked better than any approach explored at the time to choose different percentages for different variables, even where these variables had different distributional shapes.

Hennig et al. (2019) developed a method for unsupervised classification based on quantiles, involving an idea for determining variable-wise percentages that can be adapted to unsupervised classification. In this paper we use a similar approach to propose a generalised classifier with variable-wise percentages and different weights. Here we show that the proposed quantile classifier with variable-wise percentage parameters can outperform the earlier version with a common quantile for all variables. Numerical studies provide very good results compared to a number of popular classification methods.

In Section 2, we introduce the original quantile classifier and describe its extension by introducing variable-wise percentage and scale parameters. We also address the computation of the new classifier. In Section 3 we prove in a general nonparametric situation that the parameter estimates employed by the new classifier are consistent for their corresponding population versions. Section 4 compares the proposed quantile classifier to several other popular classifiers based on simulated and real data sets. Section 5 concludes the paper.

2 The quantile classifier

In this section we describe the quantile classifier and its main properties.

Let Π_1 and Π_2 be two populations with probability densities F_1 and F_2 on \mathbb{R}^p . Quantile-based classifiers (Hennig and Viroli, 2016) assign a new observation $\mathbf{z} = (z_1, \dots, z_p) \in \mathbb{R}^p$ to the population from which it has lowest quantile discrepancy, as defined below, along all variables. Formally, \mathbf{z} is

allocated to Π_1 if

$$\sum_{j=1}^p \{\Phi_{2jn}(\mathbf{z}, \theta) - \Phi_{1jn}(\mathbf{z}, \theta)\} > 0, \quad (1)$$

and otherwise to Π_2 . Here, the quantile discrepancy Φ_{kjn} is defined for any variable $j = 1, \dots, p$, classes $k = 1, 2$, and a percentage θ , as

$$\Phi_{kjn}(\mathbf{z}, \theta) = \left(\theta + (1 - 2\theta) \mathbb{1}_{[z_j \leq q_{kjn}(\theta)]} \right) |z_j - q_{kjn}(\theta)|, \quad (2)$$

where $q_{kjn}(\theta)$ is the empirical marginal quantile function of variable j within class k evaluated at $\theta \in (0, 1)$, i.e., the sample θ -quantile. The index n indicates an underlying sample of $n_1 + n_2 = n$ training observations, n_1 of which stem from Π_1 , and n_2 stem from Π_2 .

The quantile function for a one-dimensional distribution P with cdf F is defined as $q_P(\theta) = \inf\{x : F(x) \geq \theta\}$. This is the infimum of the not necessarily unique values of ξ that minimises the following variability measure,

$$\theta \int_{x>\xi} |x - \xi| dP(x) + (1 - \theta) \int_{x<\xi} |x - \xi| dP(x). \quad (3)$$

Since for $\theta = 0.5$ the quantile discrepancy is twice the L_1 -distance from the median, the decision rule (1) includes the median classifier (Hall et al., 2009) when $\theta = 0.5$. For all values θ , the quantile discrepancy not being based on squares shares with the L_1 -distance its better resistance against outliers compared to Euclidean L_2 -distance.

(Hennig and Viroli, 2016) propose to choose the θ -value as optimal that produces the optimal correct classification rate within the training data.

2.1 Variable-wise θ and a scale parameter

The original quantile classifier as defined above is based on finding a single optimal θ for all variables simultaneously. Considering that each variable might have different skewness, tail rate and shape, it appears to be promising to choose variable-wise values $\theta_1, \dots, \theta_p$. Using them, the new observation \mathbf{z} can be allocated to population Π_1 if

$$\sum_{j=1}^p \{\Phi_{2jn}(\mathbf{z}, \theta_j) - \Phi_{1jn}(\mathbf{z}, \theta_j)\} > 0. \quad (4)$$

Hennig and Viroli (2016) report on having tried out several strategies for choosing variable-wise θ , but all these resulted in a worse classification performance on independent data than just choosing a single θ . The problem with variable-wise θ_j is that plugging different θ_j for different variables into (2)) will induce relative weights $(\theta_j + (1 - 2\theta_j)\mathbb{1}_{[z_j \leq q_{kjn}(\theta_j)]})$ for the different variables, regardless of their information content for classification. In particular, variables with θ_j close to 0 or 1 will have more influence than other variables regardless of their effective discriminative power. Instead, Hennig and Viroli (2016) proposed to set the within-class skewness averaged over classes of all variables in the same direction by just multiplying variables that have the wrong skewness direction by -1, so that the information of the different variables for choosing the joint θ becomes more compatible. This preprocessing step resulted in good classification performances.

But this is a very rough adjustment, and in the present work we show that there is a sensible way to choose variable-wise θ s together with re-scaling the variables in such a way that the contributions of different variables are still properly balanced.

From now on we consider K classes where not necessarily $K = 2$. The decision rule in (1) can be easily generalised to $K > 2$ classes by just allocating an observation \mathbf{z} to the population which gives the lowest quantile discrepancy $\sum_{j=1}^p \Phi_{kjn}(\mathbf{z}, \theta)$, with $k = 1, \dots, K$.

Given an observed sample $\tilde{\mathbf{y}}_n = (\mathbf{y}_1, \dots, \mathbf{y}_n)$, where $\mathbf{y}_i = (\mathbf{x}_i, c_i)$, $i = 1, \dots, n$, $c_i \in \{1, \dots, K\}$ being the known number of the class to which $\mathbf{x}_i \in \mathbb{R}^p$ belongs, the aim is to find first the optimal θ_j , $j = 1, \dots, p$, which minimise the sum of variable-wise quantile discrepancies

$$\hat{\theta} = \arg \min_{\theta_1, \dots, \theta_p} \sum_{i=1}^n \sum_{j=1}^p \Phi_{c_{ijn}}(\mathbf{x}_i, \theta_j), \quad (5)$$

and then to allocate a new observation \mathbf{z} according to the minimum $\sum_{j=1}^p \Phi_{kjn}(\mathbf{z}, \hat{\theta}_j)$ for all $k = 1, \dots, K$.

In unsupervised quantile-based clustering, Hennig et al. (2019) established the connection between the quantile discrepancy and the asymmetric Laplace distribution. More specifically, for $p = 1$ and $K = 1$, minimising the quantile discrepancy on a sample of data by choice of θ is equivalent to maximising the likelihood of the asymmetric Laplace distribution with $\lambda = 1$,

defined by

$$f(x; \lambda, \theta) = \lambda \theta (1 - \theta) e^{-\lambda \{ \theta + (1 - 2\theta) \mathbb{1}_{[x < q(\theta)]} \} |x - q(\theta)|}. \quad (6)$$

λ is a scale parameter, and its impact on the quantile classifier is that its introduction assigns a weight to the contribution of a variable. The general quantile-based classifier can be defined as the problem of assigning a new observation to the class minimising the discrepancy

$$\sum_{j=1}^p \lambda_j \Phi_{kjn}(\mathbf{z}, \theta_j),$$

where λ_j and θ_j are estimated on the training set $\tilde{\mathbf{y}}_n$ as

$$T_n(\tilde{\mathbf{y}}_n) = \arg \min_{\boldsymbol{\lambda}, \boldsymbol{\theta}} \left(\sum_{i=1}^n \sum_{j=1}^p \lambda_j \Phi_{cijn}(\mathbf{x}_i, \theta_j) - n \sum_{j=1}^p \log \lambda_j \theta_j (1 - \theta_j) \right), \quad (7)$$

where $-n \sum_{j=1}^p \log \lambda_j \theta_j (1 - \theta_j)$ derives from the normalization constant of the density (6). It can be interpreted as a penalty term that penalises θ_j -values that are too close to 0 or 1, and small scale parameters λ_j . Let

$$\Psi(\boldsymbol{\lambda}, \boldsymbol{\theta}, \tilde{\mathbf{y}}_n) = \sum_{i=1}^n \sum_{j=1}^p \lambda_j \Phi_{cijn}(\mathbf{x}_i, \theta_j) - n \sum_{j=1}^p \log \lambda_j \theta_j (1 - \theta_j).$$

2.2 Estimation

Parameter estimation can be obtained through a greedy algorithm that starts with an initialization step and then alternates between updating θ_j and λ_j until convergence of the objective function $\Psi(\boldsymbol{\lambda}, \boldsymbol{\theta}, \tilde{\mathbf{y}}_n)$ in (7). To this aim we need the scores of the objective function with respect to the parameters of interest.

By equating the score of $\Psi(\boldsymbol{\lambda}, \boldsymbol{\theta}, \tilde{\mathbf{y}}_n)$ to zero with respect to each scale parameter λ_j , we get an estimate in closed form given by

$$\lambda_j = \frac{n}{\sum_{i=1}^n \Phi_{cijn}(\mathbf{x}_i, \theta_j)}. \quad (8)$$

The estimates of θ_j can be obtained similarly and are given in closed form by taking a root of the quadratic equation:

$$\theta_j^2 \lambda_j \sum_{i=1}^n (x_{ij} - q_{cijn}(\theta_j)) - \theta_j \left(2n + \lambda_j \sum_{i=1}^n (x_{ij} - q_{cijn}(\theta_j)) \right) + n = 0. \quad (9)$$

Thus, the algorithm is very fast. It is represented in the following scheme:

1. *Initialization*: For each variable, choose randomly a value θ_j and compute the class-conditional quantiles $q_{kjn}(\theta_j)$; set $\lambda_j = 1$.
 2. Repeat the following until $\Psi(\boldsymbol{\lambda}, \boldsymbol{\theta})$ stops changing:
 - (a) For $j = 1, \dots, p$ compute θ_j by equation (8).
 - (b) For $j = 1, \dots, p$ compute λ_j by equation (9).
-

We call the resulting classifier variable-wise quantile classifier (VWQC).

3 Consistency

We show here that in a nonparametric situation with data from classes $1, \dots, K$ with probabilities π_1, \dots, π_K generated by distributions Q_1, \dots, Q_K , respectively, for $n \rightarrow \infty$ the estimated parameters $(\boldsymbol{\lambda}, \boldsymbol{\theta})$ converge almost surely (a.s.) to their population counterparts. This is somewhat different from the consistency result in Hennig and Viroli (2016), where in the probabilistic limit a parameter was chosen that minimised the misclassification probability in the population. This was possible because the parameter θ was one-dimensional in Hennig and Viroli (2016), and it was feasible to choose it minimising the in-sample misclassification probability. This is numerically too tedious here. Consequently, we can show that the class-wise distributions are “asymptotically correctly” represented in the classification rule, but not that this is asymptotically optimal for classification. Note that due to the nonparametric nature of the result, it is particularly not assumed that the variables are independent within classes, even though the classifier does not make use of potential dependence.

We consider the following setup. Let $(\mathbf{X}_i, C_i)_{i \in \mathbb{N}}$ be a sequence of i.i.d. random variables with $\mathcal{L}(\mathbf{X}_i, C_i) = Q$, where $Q\{C_i = k\} = \pi_k$ for $k \in 1, \dots, K$, and $\mathcal{L}(\mathbf{X}_i | C_i = k) = Q_k$. Let $\tilde{\mathbf{Y}}_n = ((\mathbf{X}_1, C_1), \dots, (\mathbf{X}_n, C_n))$. In the proof we will rely on uniform convergence and continuity. Sample quantiles are not continuous in θ . Therefore for the theory we will consider the “location” of a within-class distribution ($q(\theta)$ in (6)) as additional parameter,

even though this ultimately evaluates to the sample or population quantile by minimising (3). Therefore define for $j = 1, \dots, p$:

$$\begin{aligned}\Phi_j(\mathbf{z}, \theta, \xi) &= (\theta + (1 - 2\theta)\mathbb{1}_{[z_j \leq \xi]}) |z_j - \xi|, \\ V_n(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, \tilde{\mathbf{Y}}_n) &= \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^p \lambda_j \Phi_{C_ijn}(\mathbf{X}_i, \theta_j, \xi_{C_ij}) - \sum_{j=1}^p \log \lambda_j \theta_j (1 - \theta_j),\end{aligned}$$

where $\boldsymbol{\xi}$ is the vector collecting all ξ_{kj} . V_n is just Ψ divided by n for making laws of large numbers available. Therefore,

$$T_n(\tilde{\mathbf{Y}}_n) = \arg \min_{\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}} V_n(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, \tilde{\mathbf{Y}}_n).$$

Further let

$$\begin{aligned}V(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, Q) &= \sum_{k=1}^K \pi_k \int \sum_{j=1}^p \lambda_j \Phi_j(\mathbf{x}, \theta_j, \xi_{cj}) dQ(\mathbf{x}, c) - \sum_{j=1}^p \log \lambda_j \theta_j (1 - \theta_j), \\ T(Q) &= \arg \min_{\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}} V(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, Q).\end{aligned}$$

The consistency proof will rely on showing that parameter estimators for large n do not leave a compact set, but (considering a single variable) $\lambda \rightarrow \infty$ and $\theta \rightarrow 0$ or $\theta \rightarrow 1$ may happen together without constraints on the parameter space, causing trouble with compactness. This can be avoided by either constraining $\theta_j \in [\tau, 1 - \tau]$, $\tau > 0$, or $\lambda_j \leq \lambda^+ < \infty$ for $j \in \{1, \dots, p\}$. We will impose the latter constraint here. The parameter space used is

$$S = \{(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}) : \theta_j \in (0, 1), \lambda_j \in (0, \lambda^+], \xi_{kj} \in \mathbb{R}, j \in \{1, \dots, p\}\},$$

and the argmin in the definition of T and T_n is taken over S .

We require the following assumptions ($\|\bullet\|_1$ denotes the L1-norm):

A1 $B_k = \int \|\mathbf{x}\|_1 dQ_k(\mathbf{x}) < \infty$ for $k = 1, \dots, K$.

A2 $T(Q)$ is uniquely defined.

Theorem 1 *If $(\mathbf{X}_1, C_1), (\mathbf{X}_2, C_2), \dots \sim Q$ i.i.d., and assumptions A1 and A2 hold, then, for $n \rightarrow \infty$: $T_n(\tilde{\mathbf{Y}}_n) \rightarrow T(Q)$ a.s.*

Proof: The principle of the proof follows the proof of Theorem 1 in Hennig et al. (2019) and is to show that $T_n(\tilde{\mathbf{Y}}_n)$ for large enough n has to lie in a compact set \mathcal{C} . In this compact set, by the uniform law of large numbers, $V_n(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, \tilde{\mathbf{Y}}_n)$ will converge uniformly to $V(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, Q)$, which in turn, together with continuity, will also enforce the minimiser to converge. $(\boldsymbol{\lambda}, \boldsymbol{\theta})$ optimizing V_n are enforced to eventually lie in a compact set by the penalty term $-\log \lambda \theta (1 - \theta)$.

To show that $T_n(\tilde{\mathbf{Y}}_n)$ for large enough n is a.s. in a compact set \mathcal{C} , first show that the minimum of V_n is asymptotically bounded from above. This is then used to constrain $\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}$ into a compact set. Define $(\boldsymbol{\lambda}_0, \boldsymbol{\theta}_0, \boldsymbol{\xi}_0)$ as follows: For $j = 1, \dots, p$, $k = 1, \dots, K$: $\theta_{0j} = \frac{1}{2}$, $\lambda_{0j} = 1$, $\xi_{0kj} = 0$. Then,

$$V_n(\boldsymbol{\lambda}_0, \boldsymbol{\theta}_0, \boldsymbol{\xi}_0, \tilde{\mathbf{Y}}_n) = \frac{1}{n} \sum_{i=1}^n \sum_{j=1}^p \frac{1}{2} |X_{ij}| - p \log \frac{1}{4}.$$

The first part converges a.s. to $B_1 = \frac{1}{2} \sum_{k=1}^K \pi_k B_k < \infty$ by A1.

If $\lambda_j \rightarrow 0$, $\theta_j \rightarrow 1$, or $\theta_j \rightarrow 0$, then $-\sum_{j=1}^p \log \lambda_j \theta_j (1 - \theta_j) \rightarrow \infty$, whereas $\frac{1}{n} \sum_{i=1}^n \sum_{j=1}^p \lambda_j \Phi_{C_{ij}}(\mathbf{X}_i, \theta_j, \xi_{C_{ij}})$ is bounded from below by $-\frac{1}{n} \sum_{i=1}^n \sum_{j=1}^p |X_{ij}|$, which converges a.s. to $-B_1 > -\infty$. This means that there is $\tau > 0$ so that $T_n(\tilde{\mathbf{Y}}_n) = (\lambda_{1n}, \dots, \lambda_{pn}, \theta_{1n}, \dots, \theta_{pn})$ must have $\lambda_{jn} \in [\tau, \lambda^+]$, $\theta_{jn} \in [\tau, 1 - \tau]$ for $j = 1, \dots, p$.

For given $\boldsymbol{\lambda}, \boldsymbol{\theta}$, the class-wise sample quantiles belonging to $\boldsymbol{\theta}$ are the values for $\boldsymbol{\xi}$ minimising $V_n(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, \tilde{\mathbf{Y}}_n)$; analogously the population quantiles minimise $V(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, Q)$. Mason (1982) showed that for $\tau > 0$ and $\theta \in [\tau, 1 - \tau]$, the sample quantiles converge uniformly a.s. to the population quantiles. Therefore, for large enough n , $\boldsymbol{\xi}_n$ of $T_n(\tilde{\mathbf{Y}}_n)$ must be in a bounded neighbourhood of the set of quantile vectors of Q_1, \dots, Q_K for $\theta \in [\tau, 1 - \tau]$, and therefore $T_n(\tilde{\mathbf{Y}}_n)$ as a whole will be in a compact set a.s.

According to van der Vaart (1998), Example 19.8 (sometimes referred to as “uniform law of large numbers”), if $\mathcal{F} = \{f_\eta : \eta \in \mathcal{C}\}$ is a set of measurable functions with $\eta \mapsto f_\eta(x)$ continuous for all x , \mathcal{C} compact, and $\exists F \geq |f_\eta| \forall \eta \in \mathcal{C}$, $\int F dP < \infty$, then

$$\sup_{\eta \in \mathcal{C}} \left| \frac{1}{n} \sum_{i=1}^n f_\eta(\mathbf{x}_i) - \int f_\eta(\mathbf{x}) dP(\mathbf{x}) \right| \rightarrow 0 \text{ a.s.}$$

For fixed x , $g(x, \theta, \xi) = \{\theta + (1 - 2\theta)\mathbb{1}_{[x < \xi]}\} |x - \xi|$ is continuous in (ξ, θ) , because $\xi \rightarrow x \Rightarrow g(x, \theta, \xi) \rightarrow 0$ regardless of whether ξ comes from above

or from below. Therefore, for fixed $\mathbf{y} = (x_1, \dots, x_p, c)$,

$$U(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, \mathbf{y}) = \sum_{j=1}^p \lambda_j g(x_j, \theta_j, \xi_{cj}) - \sum_{j=1}^p \log [\lambda_j (\theta_j (1 - \theta_j))]$$

is continuous. It can be bounded by a Q -integrable function. Let $\infty > \xi^+ > \max_{k \in \{1, \dots, K\}, \theta \in \{\kappa, 1-\kappa\}} |q_k(\theta)|$, $q_k(\theta)$ being the θ -quantile of Q_k . Then,

$$\begin{aligned} U(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, \mathbf{y}) &\leq U^+(\mathbf{y}) = \sum_{j=1}^p \lambda^+ (|x_j| + \xi^+) - \sum_{j=1}^p \log \left[\frac{\kappa^2}{2} \right], \\ \int U^+(\mathbf{y}) dQ(\mathbf{y}) &< \infty \text{ because of A1.} \end{aligned}$$

Therefore,

$$\sup_{(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}) \in \mathcal{C}} |V_n(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, \tilde{\mathbf{Y}}_n) - V(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}, Q)| \rightarrow 0 \text{ a.s.} \quad (10)$$

The existence of a Q -integrable envelope of U together with continuity of U imply the continuity of V as function of $(\boldsymbol{\lambda}, \boldsymbol{\theta}, \boldsymbol{\xi}) \in \mathcal{C}$. This and A2 imply $T_n(\tilde{\mathbf{Y}}_n) \rightarrow T(Q)$ a.s., because otherwise with probability > 0 a subsequence of $T_n(\tilde{\mathbf{Y}}_n)$ can converge to $(\boldsymbol{\lambda}^*, \boldsymbol{\theta}^*, \boldsymbol{\xi}^*) \neq T(Q)$ but still $\in \mathcal{C}$ and with $V(\boldsymbol{\lambda}^*, \boldsymbol{\theta}^*, \boldsymbol{\xi}^*, Q) = V(T(Q), Q)$, with contradiction to A2.

4 Experiments

4.1 Simulation study

We evaluated the performance of the VWQC in a simulation study under four scenarios with two populations:

1. observations generated independently from a multivariate distribution with Student's t_3 marginals;
2. observations generated as in the previous scenario, then each variable $W_j (j = 1, \dots, p)$ was subsequently transformed according to $W_j \mapsto \log(|W_j|)$, to induce asymmetry;
3. observations generated from a multivariate distribution with exponential marginals;

4. observations generated from a multivariate Gaussian distribution, but the p variables W_j were split into five balanced blocks to which we applied different transformations: (i) $W_j \mapsto W_j$; (ii) $W_j \mapsto \exp(W_j)$; (iii) $W_j \mapsto \log(|W_j|)$; (iv) $W_j \mapsto W_j^2$; (v) $W_j \mapsto |W_j|^{0.5}$.

The two populations differed by a location shift equal to +0.5 for Y_j with respect to X_j , $j = 1, \dots, p$, for the first scenario, ± 0.4 for the second scenario (+0.4 for half of the variables, -0.4 for the other half) and +0.2 for the third and the fourth scenarios. For each scenario, we evaluated combinations of overall sample sizes $n \in \{50, 100, 500\}$ (with $n/2$ observations in each class), dimensions $p \in \{10, 50, 100, 500\}$ and percentages of relevant variables for classification in $\{10\%, 50\%, 100\%\}$. Furthermore, we considered both uncorrelated and correlated variables. In the second case, the variance-covariance matrix was defined as in (Farcomeni et al., 2022) by using $\Sigma = \mathbf{A}^T \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_p) \mathbf{A}$, with $\sigma_j = \sigma_j^*/\max(\sigma_1^*, \sigma_2^*, \dots, \sigma_p^*)$; $\sigma_j^* = (p+2-j)^{1.1+\frac{0.8}{p-1}j}$, and \mathbf{A} being an orthogonal matrix obtained from the QR decomposition of a $p \times p$ matrix \mathbf{B} defined so that $\text{vec}(\mathbf{B}) = (1, 1 + \frac{1}{p^2-1}, \dots, 1 + \frac{p^2-2}{p^2-1}, 2)$. The corresponding pairwise correlations are on the interval $(-0.25, 0.39)$ when $p = 10$, $(-0.42, 0.62)$ when $p = 50$, $(-0.48, 0.67)$ when $p = 100$, $(-0.63, 0.80)$ when $p = 500$. This resulted in a total of 288 different simulation settings. Data generation under each setting was replicated 100 times. Observations in the training and test sets were generated in the same way. Variables were standardised to unit within-class pooled variance in scenario 4, because their scales seem incompatible, whereas in datasets like those from the other scenarios the reasons against standardization given in Hennig and Viroli (2016, Section 4.1) may apply (imagining that such data arise in the real world).

We compared the proposed quantile classifier with that of ten other classifiers in terms of the misclassification rate on the test data: the directional quantile classifier (DQC) (Farcomeni et al., 2022), the centroid classifier (Centroid) (Tibshirani et al., 2002), the median classifier (Median) (Hall et al., 2009), the original quantile classifier (OQC) (Hennig and Viroli, 2016), the ensemble quantile classifier (EQC) (Lai and McLeod, 2020), Fisher's linear discriminant analysis (LDA), the k-nearest neighbour classifier (KNN) (Cover and Hart, 1967), penalised logistic regression (PLR) (Park and Hastie, 2008), the support vector machines (SVM) (Cortes and Vapnik, 1995; Wang et al., 2008), the naive Bayes classifier (Bayes) Hand and Yu (2001) and random forest (RF) (Ho, 1995). Tuning parameters for PLR, KNN, and SVM

where selected using cross-validation. The Galton correction was used on the OQC to reduce skewness, and the optimal quantile was selected by minimising the error rate on the training set. All analyses were carried out in R version 4.1.1 (R Core Team, 2021) using the following packages: **Qtools** for the directional quantile classifier; **quantileDA** for the centroid, median and original quantile classifiers, **eqc** for the ensemble quantile classifier, **MASS** for linear discriminant analysis, the package **class** for k-nearest neighbour classifier, **stepPlr** for penalised logistic regression, **e1071** for support vector machines and the naive Bayes classifier, and **randomForest** for the random forest classifier. Except where indicated above, default tuning of the functions was used.

We computed the relative performance of each classifier with respect to the misclassification rates of the VWQC. More specifically, we evaluated the misclassification rate of each classifier as the error rate minus the VWQC error rate divided by the average error rate in the given setting. The distributions over the 100 simulated data sets of these rescaled results for different choices of n , p , percentage of relevant variables and dependence/independence is represented in the boxplots of Figure 1.

The misclassification rates averaged over 100 replications for all simulations are reported in the Appendix in Tables 3 to 10. The results indicate that the performance of the VWQC improves as the sample size and the number of (relevant) variables increase: the average misclassification rate decreases and the corresponding standard errors go to zero, in agreement with the theoretical results. The VWQC outperforms its competitors with just a few exceptions. In particular, in the symmetric scenario (t), the ensemble quantile classifier and - not surprisingly - the median classifier slightly tend to prevail, as well as the random forest in the scenario with differently distributed variables (ddv). In the latter case, however, the gap between our classifier and all the other competitors is substantial. The potential of our method is highlighted the most by the exponential scenario (exp), where even the first quartiles of the misclassification rates of any classifier relative to the VWQC are positive.

4.2 Application to real data sets

We applied VWQC to a collection of seven real data sets representative of a broad range of scenarios in terms of number of instances, attributes, and classes, as described in Table 1. With respect to the simulation study,

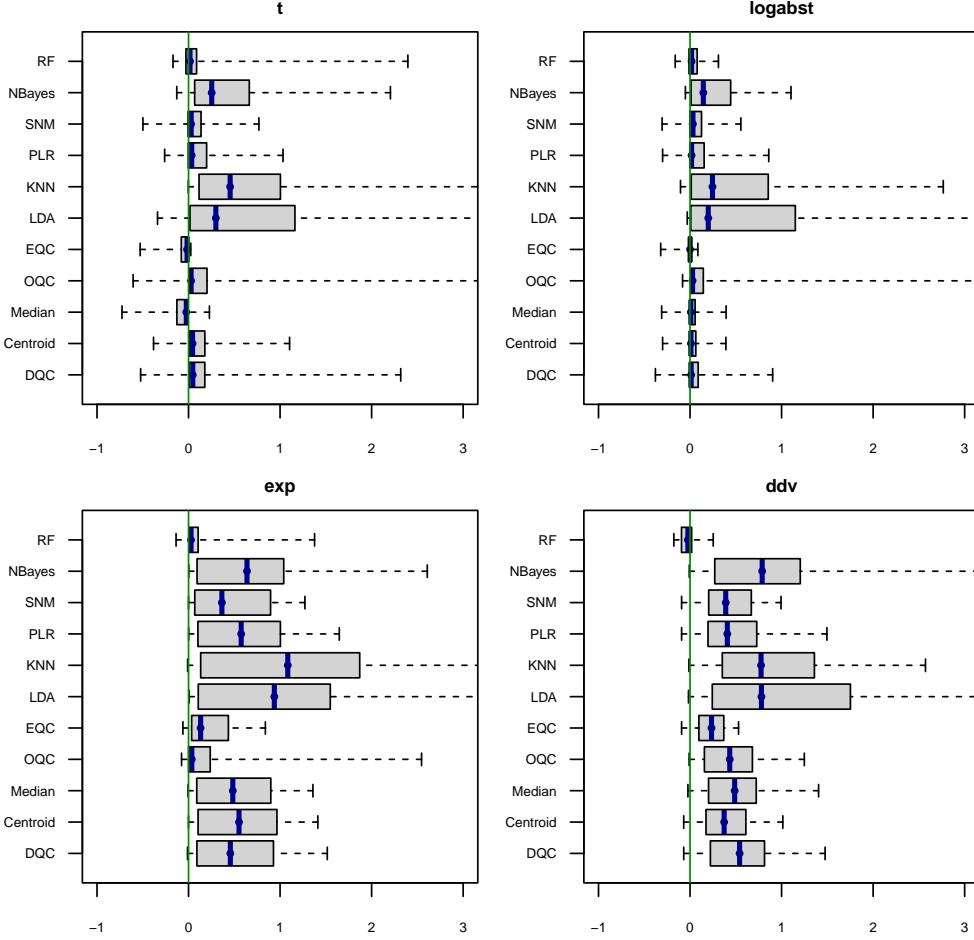


Figure 1: Performance of the classifiers relative to the variable-wise quantile classifier. The four panels show the distributions of the misclassification rates over simulated data sets for variables generated from: multivariate distributions with Student's t_3 marginals (t), multivariate asymmetric distributions through logarithmic (logabst), exponential distributions (exp), and different distributions (ddv).

penalized logistic regression was not included among the competitors due to the presence of cases with $K > 2$; the nearest shrunken centroid (NSC) (Tibshirani et al., 2002) was considered instead, and applied using the R package **pamr**.

According to the resulting misclassification rates reported in Table 2,

Table 1: Short description of the data sets used for the analysis.

Name	n	p	k	source	reference
Bankrupt	66	2	2	R (MixGHD)	Altman (1968)
Chiro	202	17	5	available upon request	Atchley and Martin (1971)
Iris	150	4	3	UCI machine learning repository, R datasets	Fisher (1936)
Music	306	20	5	musical-style-recognition.net	Van Kranenburg and Backer (2005)
Sonar	208	60	2	UCI machine learning repository, R (MixGHD)	Gorman and Sejnowski (1988)
Thyroid	215	5	3	UCI machine learning repository	Quinlan (1986)
Wine	178	13	3	UCI machine learning repository, R(HDclassif)	Aeberhard et al. (1993)

Table 2: Ten-fold cross-validated misclassification rates (%) for the seven data sets, with standard errors in parentheses. For each data set, the best performances are highlighted in bold. In case of ties, those associated to the lowest standard error are selected.

	Bankrupt	Chiro	Iris	Music	Sonar	Thyroid	Wine
VWQC	3.1 (6.5)	2.5 (3.5)	4.0 (5.6)	32.9 (8.6)	22.4 (12.1)	6.0 (5.3)	2.3 (3.0)
LDA	8.1 (8.6)	1.5 (2.3)	2.0 (4.5)	24.1 (6.7)	25.4 (8.0)	8.9 (3.5)	1.1 (2.4)
NSC	8.1 (8.6)	6.5 (4.1)	6.7 (4.4)	38.5 (5.0)	25.4 (9.1)	16.7 (4.5)	4.5 (3.7)
KNN	4.8 (7.7)	6.9 (2.5)	2.7 (3.4)	39.6 (6.0)	20.1 (7.4)	5.2 (4.1)	23.5 (10.2)
SVM	4.3 (7.1)	1.5 (2.4)	4.0 (6.4)	24.1 (7.4)	15.3 (5.6)	4.7 (3.9)	2.2 (3.8)
nBayes	6.2 (8.0)	1.0 (2.1)	4.0 (4.7)	29.7 (4.7)	22.9 (11.4)	2.8 (3.3)	2.8 (3.8)
QC	6.4 (8.3)	8.9 (6.0)	7.3 (4.9)	43.4 (6.0)	30.2 (8.3)	9.3 (4.5)	18.0 (6.3)
Centroid	9.3 (8.2)	12.9 (6.3)	7.3 (6.6)	47.6 (7.3)	31.7 (5.7)	13.5 (7.4)	26.4 (9.3)
Median	3.3 (7.0)	9.0 (4.6)	8.0 (5.3)	42.1 (5.5)	28.3 (9.9)	11.2 (7.3)	26.9 (9.1)
EQC	6.2 (8.0)	4.0 (4.6)	5.3 (4.2)	34.0 (7.0)	22.5 (9.2)	4.6 (4.3)	3.4 (2.9)
RF	3.1 (6.5)	1.0 (2.1)	4.7 (5.5)	29.0 (7.2)	13.4 (9.7)	3.7 (2.9)	1.7 (2.7)
DQC	3.3 (7.0)	12.9 (4.2)	7.3 (4.9)	55.8 (3.7)	33.0 (9.2)	11.2 (6.7)	26.3 (8.3)

the VWQC provided good performances, ranking always above the median and thus highlighting again the adaptability of the classifier. Furthermore, an optimal outcome is obtained for the “bankrupt” data set, tied with the random forest classifier. On the contrary, the highest misclassification rates for this data set are provided by the centroid classifier, NSC, and LDA. Although the latter ranks below the median in another case (“thyroid”), it shows also some highs, ranking first in three data sets (“iris”, “music”, “wine”), hence being ranked among the most successful competitors for this study, together with the random forest.

5 Conclusion

It can be expected that in most high-dimensional classification problems different variables have different distributional shapes within classes. The

original quantile classifier by Hennig and Viroli (2016) is based on just a single percentage value for computing the quantiles used for classification of all variables. Although this often works well and a single parameter is easy to optimise, it can be expected that using different percentage parameters for different variables may use the information in the data more efficiently. This requires a rescaling of the quantile-based discrepancy, and because of this, we introduce variable-wise scaling parameters on top of the percentages. The resulting variable-wise quantile classifier is still relatively simple to compute, and in many situations it can improve upon the original quantile classifier. Its performance, while not universally optimal, also can compete well with other existing popular classifiers.

Although the VWQC can be extended to more than two classes in a straightforward manner, we constrained ourselves to two classes in the simulation study. Generally, there are lots of options and possibilities for such studies so that certain limitations of this kind are inevitable. Most of the considered real data sets come with more than two classes.

Interesting further work could be done by exploring possibilities for a theory for high-dimensional settings, i.e., allowing $p \rightarrow \infty$, and for finite sample performance guarantees.

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A Simulation study – misclassification rates

Table 3: Misclassification rates (with standard deviations) averaged over 100 replications in brackets for twelve classifiers in the t scenario with uncorrelated variables.

Sample size n	50	100	500	50	100	500	50	100	500	
Relevant variables	10%				50%				100%	
<i>p = 10</i>										
VWQC	0.436 (0.048)	0.455 (0.033)	0.426 (0.025)	0.436 (0.046)	0.442 (0.038)	0.433 (0.028)	0.443 (0.044)	0.450 (0.036)	0.426 (0.028)	
DQC	0.443 (0.046)	0.458 (0.031)	0.467 (0.020)	0.440 (0.045)	0.457 (0.032)	0.471 (0.022)	0.443 (0.044)	0.455 (0.034)	0.462 (0.021)	
Centroid	0.439 (0.049)	0.453 (0.037)	0.445 (0.023)	0.447 (0.037)	0.455 (0.038)	0.445 (0.023)	0.439 (0.042)	0.453 (0.036)	0.443 (0.025)	
Median	0.437 (0.047)	0.453 (0.034)	0.422 (0.025)	0.440 (0.045)	0.441 (0.042)	0.429 (0.025)	0.438 (0.045)	0.422 (0.043)	0.422 (0.025)	
CQC	0.445 (0.045)	0.457 (0.036)	0.435 (0.028)	0.448 (0.044)	0.449 (0.037)	0.438 (0.026)	0.442 (0.047)	0.454 (0.037)	0.435 (0.028)	
EQC	0.441 (0.047)	0.451 (0.034)	0.427 (0.026)	0.436 (0.049)	0.443 (0.040)	0.432 (0.028)	0.439 (0.046)	0.448 (0.037)	0.424 (0.027)	
LDA	0.442 (0.050)	0.452 (0.034)	0.445 (0.023)	0.440 (0.045)	0.450 (0.038)	0.443 (0.024)	0.435 (0.049)	0.455 (0.037)	0.441 (0.026)	
KNN	0.446 (0.042)	0.456 (0.035)	0.475 (0.019)	0.443 (0.044)	0.465 (0.025)	0.482 (0.014)	0.442 (0.046)	0.451 (0.033)	0.479 (0.014)	
PLR	0.442 (0.049)	0.452 (0.032)	0.444 (0.024)	0.437 (0.041)	0.451 (0.036)	0.441 (0.024)	0.438 (0.049)	0.453 (0.037)	0.440 (0.026)	
SVM	0.437 (0.046)	0.457 (0.034)	0.452 (0.022)	0.448 (0.041)	0.456 (0.035)	0.445 (0.024)	0.443 (0.046)	0.455 (0.035)	0.453 (0.021)	
Bayes	0.442 (0.049)	0.456 (0.034)	0.460 (0.020)	0.445 (0.043)	0.455 (0.031)	0.463 (0.021)	0.444 (0.048)	0.463 (0.031)	0.462 (0.022)	
RF	0.434 (0.048)	0.459 (0.034)	0.447 (0.023)	0.434 (0.043)	0.452 (0.031)	0.452 (0.024)	0.438 (0.046)	0.454 (0.035)	0.448 (0.023)	
<i>p = 50</i>										
VWQC	0.422 (0.050)	0.418 (0.047)	0.366 (0.024)	0.292 (0.067)	0.234 (0.048)	0.186 (0.019)	0.166 (0.059)	0.132 (0.041)	0.097 (0.014)	
DQC	0.425 (0.056)	0.447 (0.042)	0.439 (0.022)	0.306 (0.064)	0.273 (0.048)	0.265 (0.024)	0.173 (0.059)	0.149 (0.041)	0.141 (0.015)	
Centroid	0.435 (0.050)	0.433 (0.041)	0.394 (0.025)	0.326 (0.075)	0.280 (0.048)	0.235 (0.025)	0.179 (0.057)	0.152 (0.042)	0.152 (0.016)	
Median	0.422 (0.058)	0.413 (0.048)	0.364 (0.022)	0.264 (0.068)	0.215 (0.040)	0.183 (0.018)	0.135 (0.050)	0.107 (0.032)	0.094 (0.014)	
CQC	0.428 (0.055)	0.435 (0.047)	0.374 (0.027)	0.310 (0.090)	0.238 (0.050)	0.191 (0.021)	0.195 (0.087)	0.127 (0.045)	0.097 (0.015)	
EQC	0.424 (0.055)	0.423 (0.044)	0.366 (0.023)	0.273 (0.072)	0.230 (0.042)	0.187 (0.032)	0.149 (0.051)	0.097 (0.039)	0.097 (0.015)	
LDA	0.439 (0.042)	0.448 (0.037)	0.397 (0.026)	0.422 (0.056)	0.332 (0.051)	0.249 (0.023)	0.400 (0.065)	0.254 (0.047)	0.164 (0.017)	
KNN	0.431 (0.045)	0.447 (0.039)	0.457 (0.023)	0.374 (0.064)	0.356 (0.051)	0.320 (0.023)	0.302 (0.073)	0.261 (0.047)	0.212 (0.022)	
PLR	0.437 (0.050)	0.447 (0.039)	0.395 (0.024)	0.339 (0.076)	0.314 (0.052)	0.250 (0.020)	0.243 (0.070)	0.212 (0.043)	0.168 (0.017)	
SVM	0.432 (0.048)	0.439 (0.041)	0.396 (0.026)	0.322 (0.076)	0.290 (0.046)	0.242 (0.021)	0.218 (0.064)	0.197 (0.041)	0.157 (0.017)	
Bayes	0.444 (0.042)	0.452 (0.038)	0.441 (0.022)	0.372 (0.068)	0.356 (0.046)	0.298 (0.024)	0.286 (0.067)	0.255 (0.040)	0.195 (0.021)	
RF	0.419 (0.055)	0.412 (0.050)	0.359 (0.021)	0.281 (0.066)	0.247 (0.048)	0.200 (0.019)	0.187 (0.056)	0.160 (0.035)	0.120 (0.015)	
<i>p = 100</i>										
VWQC	0.413 (0.060)	0.398 (0.048)	0.318 (0.024)	0.230 (0.073)	0.165 (0.041)	0.102 (0.014)	0.096 (0.046)	0.060 (0.030)	0.032 (0.009)	
DQC	0.433 (0.046)	0.428 (0.045)	0.417 (0.024)	0.355 (0.024)	0.263 (0.060)	0.217 (0.044)	0.162 (0.016)	0.136 (0.050)	0.102 (0.011)	
Centroid	0.425 (0.055)	0.417 (0.041)	0.388 (0.024)	0.316 (0.024)	0.177 (0.055)	0.142 (0.037)	0.101 (0.014)	0.062 (0.031)	0.030 (0.008)	
Median	0.400 (0.059)	0.388 (0.052)	0.324 (0.023)	0.303 (0.101)	0.172 (0.053)	0.106 (0.013)	0.123 (0.022)	0.076 (0.042)	0.033 (0.008)	
CQC	0.422 (0.054)	0.414 (0.053)	0.398 (0.048)	0.319 (0.026)	0.197 (0.061)	0.156 (0.040)	0.103 (0.014)	0.071 (0.037)	0.053 (0.025)	
EQC	0.410 (0.058)	0.454 (0.032)	0.370 (0.025)	0.372 (0.075)	0.438 (0.043)	0.187 (0.020)	0.273 (0.069)	0.395 (0.063)	0.099 (0.012)	
LDA	0.388 (0.047)	0.457 (0.052)	0.451 (0.024)	0.356 (0.052)	0.330 (0.057)	0.272 (0.028)	0.208 (0.062)	0.145 (0.018)	0.000 (0.000)	
KNN	0.427 (0.056)	0.436 (0.044)	0.370 (0.026)	0.285 (0.065)	0.257 (0.047)	0.197 (0.021)	0.149 (0.052)	0.128 (0.035)	0.111 (0.016)	
PLR	0.432 (0.053)	0.431 (0.047)	0.363 (0.025)	0.276 (0.066)	0.233 (0.044)	0.173 (0.019)	0.140 (0.052)	0.123 (0.032)	0.091 (0.012)	
SVM	0.441 (0.045)	0.446 (0.038)	0.438 (0.022)	0.345 (0.063)	0.315 (0.048)	0.267 (0.021)	0.218 (0.061)	0.188 (0.044)	0.148 (0.015)	
Bayes	0.402 (0.064)	0.389 (0.046)	0.305 (0.024)	0.223 (0.057)	0.181 (0.041)	0.125 (0.014)	0.125 (0.049)	0.094 (0.034)	0.053 (0.009)	
<i>p = 500</i>										
VWQC	0.342 (0.076)	0.274 (0.046)	0.152 (0.018)	0.059 (0.045)	0.020 (0.022)	0.004 (0.004)	0.012 (0.022)	0.006 (0.011)	0.001 (0.002)	
DQC	0.369 (0.065)	0.344 (0.044)	0.325 (0.022)	0.063 (0.039)	0.038 (0.020)	0.028 (0.008)	0.003 (0.008)	0.002 (0.005)	0.001 (0.002)	
Centroid	0.374 (0.059)	0.333 (0.048)	0.222 (0.020)	0.093 (0.053)	0.049 (0.022)	0.019 (0.006)	0.017 (0.033)	0.004 (0.006)	0.002 (0.002)	
Median	0.321 (0.066)	0.250 (0.043)	0.145 (0.016)	0.022 (0.022)	0.009 (0.009)	0.002 (0.002)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	
CQC	0.445 (0.040)	0.446 (0.046)	0.156 (0.022)	0.386 (0.064)	0.356 (0.086)	0.008 (0.006)	0.299 (0.058)	0.283 (0.079)	0.005 (0.006)	
EQC	0.318 (0.059)	0.264 (0.049)	0.147 (0.019)	0.028 (0.029)	0.011 (0.010)	0.003 (0.003)	0.001 (0.003)	0.000 (0.001)	0.000 (0.000)	
LDA	0.433 (0.046)	0.426 (0.044)	0.467 (0.021)	0.327 (0.082)	0.243 (0.047)	0.439 (0.033)	0.229 (0.073)	0.134 (0.042)	0.391 (0.039)	
KNN	0.437 (0.048)	0.433 (0.042)	0.403 (0.026)	0.288 (0.059)	0.233 (0.064)	0.137 (0.031)	0.141 (0.076)	0.068 (0.043)	0.025 (0.014)	
PLR	0.379 (0.064)	0.335 (0.046)	0.264 (0.023)	0.088 (0.043)	0.035 (0.022)	0.029 (0.008)	0.011 (0.015)	0.004 (0.006)	0.003 (0.002)	
SVM	0.365 (0.062)	0.332 (0.048)	0.230 (0.022)	0.090 (0.038)	0.058 (0.023)	0.026 (0.008)	0.012 (0.015)	0.004 (0.009)	0.004 (0.003)	
Bayes	0.415 (0.060)	0.417 (0.050)	0.412 (0.023)	0.200 (0.057)	0.181 (0.034)	0.165 (0.021)	0.062 (0.037)	0.047 (0.021)	0.045 (0.009)	
RF	0.332 (0.066)	0.264 (0.050)	0.146 (0.016)	0.082 (0.042)	0.046 (0.021)	0.014 (0.006)	0.023 (0.021)	0.010 (0.011)	0.002 (0.002)	

Table 4: Misclassification rates (with standard deviations) averaged over 100 replications in brackets for twelve classifiers in the t scenario with correlated variables.

Sample size n	50	100	500	50	100	500	50	100	500									
Relevant variables	10%				50%				100%									
<i>p = 10</i>																		
<i>p = 50</i>																		
VWQC	0.429 (0.049)	0.438 (0.042)	0.414 (0.031)	0.439 (0.043)	0.436 (0.042)	0.411 (0.027)	0.434 (0.050)	0.443 (0.042)	0.412 (0.026)									
DQC	0.441 (0.047)	0.451 (0.035)	0.455 (0.024)	0.442 (0.026)	0.456 (0.035)	0.457 (0.020)	0.437 (0.048)	0.450 (0.034)	0.454 (0.025)									
Centroid	0.441 (0.046)	0.443 (0.037)	0.424 (0.026)	0.443 (0.043)	0.449 (0.042)	0.424 (0.023)	0.438 (0.053)	0.439 (0.042)	0.427 (0.026)									
Median	0.435 (0.051)	0.438 (0.041)	0.408 (0.028)	0.437 (0.047)	0.438 (0.047)	0.411 (0.047)	0.432 (0.049)	0.435 (0.044)	0.411 (0.027)									
CQC	0.438 (0.049)	0.447 (0.039)	0.420 (0.029)	0.445 (0.039)	0.442 (0.040)	0.423 (0.029)	0.435 (0.049)	0.445 (0.044)	0.419 (0.027)									
EQC	0.440 (0.048)	0.440 (0.042)	0.411 (0.030)	0.448 (0.044)	0.432 (0.042)	0.412 (0.026)	0.431 (0.048)	0.440 (0.042)	0.411 (0.028)									
LDA	0.433 (0.053)	0.434 (0.042)	0.409 (0.026)	0.439 (0.047)	0.438 (0.044)	0.412 (0.025)	0.434 (0.052)	0.436 (0.046)	0.416 (0.025)									
KNN	0.445 (0.042)	0.459 (0.029)	0.466 (0.020)	0.444 (0.040)	0.445 (0.034)	0.471 (0.018)	0.445 (0.040)	0.453 (0.035)	0.475 (0.016)									
PLR	0.431 (0.049)	0.434 (0.040)	0.408 (0.026)	0.440 (0.049)	0.441 (0.045)	0.410 (0.024)	0.432 (0.054)	0.437 (0.045)	0.414 (0.024)									
SVM	0.443 (0.051)	0.452 (0.034)	0.438 (0.025)	0.447 (0.047)	0.447 (0.045)	0.419 (0.049)	0.432 (0.054)	0.440 (0.042)	0.417 (0.025)									
Bayes	0.434 (0.048)	0.456 (0.030)	0.447 (0.024)	0.444 (0.048)	0.453 (0.032)	0.449 (0.022)	0.436 (0.049)	0.448 (0.041)	0.448 (0.024)									
RF	0.426 (0.050)	0.446 (0.037)	0.428 (0.029)	0.446 (0.042)	0.442 (0.043)	0.431 (0.025)	0.434 (0.052)	0.444 (0.043)	0.427 (0.024)									
<i>p = 100</i>																		
VWQC	0.402 (0.066)	0.385 (0.052)	0.322 (0.024)	0.263 (0.068)	0.186 (0.049)	0.134 (0.014)	0.111 (0.045)	0.089 (0.033)	0.075 (0.014)									
DQC	0.418 (0.054)	0.421 (0.049)	0.420 (0.022)	0.225 (0.061)	0.202 (0.035)	0.186 (0.018)	0.077 (0.037)	0.060 (0.021)	0.054 (0.011)									
Centroid	0.419 (0.063)	0.410 (0.055)	0.352 (0.022)	0.243 (0.063)	0.201 (0.036)	0.157 (0.017)	0.122 (0.045)	0.087 (0.030)	0.061 (0.012)									
Median	0.406 (0.064)	0.378 (0.052)	0.325 (0.024)	0.203 (0.063)	0.160 (0.041)	0.128 (0.015)	0.067 (0.037)	0.053 (0.021)	0.041 (0.010)									
CQC	0.415 (0.055)	0.400 (0.057)	0.332 (0.024)	0.251 (0.082)	0.186 (0.048)	0.135 (0.017)	0.138 (0.081)	0.070 (0.030)	0.045 (0.011)									
EQC	0.393 (0.066)	0.395 (0.057)	0.321 (0.023)	0.217 (0.067)	0.177 (0.043)	0.132 (0.016)	0.076 (0.044)	0.049 (0.012)	0.049 (0.012)									
LDA	0.440 (0.044)	0.402 (0.053)	0.318 (0.022)	0.400 (0.072)	0.254 (0.048)	0.156 (0.016)	0.344 (0.080)	0.139 (0.039)	0.063 (0.012)									
KNN	0.441 (0.045)	0.457 (0.030)	0.443 (0.020)	0.311 (0.067)	0.280 (0.045)	0.246 (0.019)	0.195 (0.064)	0.162 (0.043)	0.108 (0.015)									
PLR	0.402 (0.058)	0.399 (0.052)	0.320 (0.022)	0.245 (0.061)	0.212 (0.040)	0.160 (0.017)	0.199 (0.050)	0.167 (0.026)	0.077 (0.014)									
SVM	0.400 (0.061)	0.441 (0.044)	0.325 (0.022)	0.248 (0.060)	0.199 (0.041)	0.156 (0.017)	0.095 (0.043)	0.083 (0.022)	0.062 (0.012)									
Bayes	0.431 (0.053)	0.436 (0.043)	0.400 (0.025)	0.340 (0.068)	0.297 (0.043)	0.224 (0.021)	0.138 (0.048)	0.128 (0.037)	0.099 (0.014)									
RF	0.387 (0.064)	0.365 (0.050)	0.310 (0.022)	0.243 (0.062)	0.191 (0.039)	0.139 (0.018)	0.120 (0.047)	0.118 (0.038)	0.108 (0.015)									
<i>p = 500</i>																		
VWQC	0.356 (0.072)	0.308 (0.050)	0.230 (0.019)	0.151 (0.060)	0.090 (0.031)	0.042 (0.010)	0.035 (0.029)	0.029 (0.024)	0.017 (0.006)									
DQC	0.399 (0.056)	0.370 (0.049)	0.281 (0.023)	0.152 (0.051)	0.111 (0.037)	0.065 (0.028)	0.045 (0.031)	0.022 (0.014)	0.010 (0.004)									
Centroid	0.399 (0.075)	0.308 (0.055)	0.237 (0.021)	0.107 (0.047)	0.067 (0.027)	0.038 (0.008)	0.010 (0.014)	0.006 (0.010)	0.004 (0.003)									
Median	0.348 (0.075)	0.308 (0.055)	0.238 (0.021)	0.107 (0.047)	0.067 (0.027)	0.038 (0.008)	0.010 (0.014)	0.006 (0.010)	0.004 (0.003)									
CQC	0.401 (0.063)	0.360 (0.059)	0.245 (0.020)	0.235 (0.097)	0.096 (0.050)	0.043 (0.011)	0.177 (0.106)	0.040 (0.032)	0.006 (0.005)									
EQC	0.344 (0.074)	0.302 (0.049)	0.227 (0.019)	0.113 (0.051)	0.075 (0.029)	0.040 (0.010)	0.019 (0.019)	0.015 (0.015)	0.007 (0.005)									
LDA	0.402 (0.057)	0.439 (0.049)	0.221 (0.019)	0.302 (0.072)	0.392 (0.069)	0.057 (0.012)	0.122 (0.051)	0.331 (0.077)	0.011 (0.004)									
KNN	0.424 (0.056)	0.445 (0.040)	0.409 (0.027)	0.247 (0.075)	0.162 (0.047)	0.106 (0.037)	0.064 (0.012)	0.038 (0.028)	0.021 (0.013)									
PLR	0.386 (0.066)	0.349 (0.054)	0.236 (0.018)	0.143 (0.047)	0.106 (0.037)	0.052 (0.010)	0.029 (0.022)	0.015 (0.014)	0.008 (0.003)									
SVM	0.358 (0.068)	0.302 (0.052)	0.228 (0.019)	0.140 (0.051)	0.096 (0.036)	0.137 (0.016)	0.057 (0.035)	0.044 (0.023)	0.034 (0.009)									
Bayes	0.404 (0.057)	0.399 (0.047)	0.348 (0.023)	0.248 (0.062)	0.212 (0.044)	0.100 (0.029)	0.053 (0.037)	0.069 (0.028)	0.061 (0.011)									
RF	0.326 (0.064)	0.266 (0.054)	0.209 (0.016)	0.142 (0.053)	0.100 (0.029)	0.053 (0.012)	0.069 (0.037)	0.069 (0.028)	0.061 (0.011)									

Table 5: Misclassification rates (with standard deviations) averaged over 100 replications in brackets for twelve classifiers in the logabst scenario with uncorrelated variables.

Sample size n	50	100	500	50	100	500	50	100	500									
Relevant variables	10%				50%				100%									
$p = 10$																		
$p = 50$																		
$p = 100$																		
VWQC	0.442 (0.047)	0.459 (0.035)	0.441 (0.027)	0.446 (0.044)	0.449 (0.037)	0.439 (0.023)	0.441 (0.043)	0.444 (0.041)	0.438 (0.025)									
DQC	0.439 (0.049)	0.456 (0.031)	0.467 (0.024)	0.451 (0.043)	0.450 (0.035)	0.464 (0.019)	0.447 (0.043)	0.454 (0.029)	0.462 (0.020)									
Centroid	0.439 (0.042)	0.450 (0.040)	0.447 (0.024)	0.449 (0.047)	0.449 (0.037)	0.444 (0.021)	0.439 (0.049)	0.455 (0.034)	0.444 (0.024)									
Median	0.439 (0.048)	0.453 (0.037)	0.443 (0.027)	0.443 (0.040)	0.448 (0.042)	0.441 (0.022)	0.449 (0.036)	0.453 (0.036)	0.443 (0.026)									
CQC	0.445 (0.046)	0.455 (0.036)	0.446 (0.029)	0.441 (0.046)	0.447 (0.039)	0.446 (0.026)	0.445 (0.046)	0.453 (0.037)	0.442 (0.027)									
EQC	0.437 (0.046)	0.449 (0.039)	0.438 (0.027)	0.441 (0.049)	0.455 (0.037)	0.436 (0.021)	0.451 (0.038)	0.451 (0.037)	0.438 (0.020)									
LDA	0.436 (0.048)	0.450 (0.039)	0.447 (0.023)	0.441 (0.048)	0.445 (0.038)	0.445 (0.021)	0.455 (0.045)	0.455 (0.033)	0.444 (0.025)									
KNN	0.437 (0.042)	0.464 (0.028)	0.480 (0.015)	0.442 (0.041)	0.487 (0.038)	0.473 (0.019)	0.441 (0.041)	0.464 (0.033)	0.479 (0.014)									
PLR	0.432 (0.050)	0.449 (0.042)	0.447 (0.024)	0.442 (0.048)	0.449 (0.037)	0.444 (0.021)	0.443 (0.046)	0.445 (0.034)	0.445 (0.025)									
SVM	0.437 (0.043)	0.459 (0.030)	0.467 (0.020)	0.445 (0.043)	0.457 (0.035)	0.446 (0.022)	0.447 (0.050)	0.461 (0.027)	0.444 (0.025)									
Bayes	0.439 (0.046)	0.462 (0.026)	0.463 (0.022)	0.447 (0.045)	0.456 (0.034)	0.465 (0.021)	0.449 (0.043)	0.456 (0.034)	0.461 (0.022)									
RF	0.437 (0.050)	0.451 (0.038)	0.457 (0.022)	0.444 (0.044)	0.450 (0.043)	0.454 (0.022)	0.440 (0.041)	0.451 (0.035)	0.455 (0.022)									
$p = 50$																		
VWQC	0.445 (0.042)	0.460 (0.028)	0.482 (0.014)	0.448 (0.041)	0.461 (0.032)	0.481 (0.014)	0.197 (0.062)	0.151 (0.039)	0.107 (0.014)									
DQC	0.442 (0.046)	0.462 (0.029)	0.482 (0.013)	0.447 (0.036)	0.459 (0.026)	0.483 (0.013)	0.174 (0.056)	0.132 (0.034)	0.121 (0.013)									
Centroid	0.445 (0.044)	0.461 (0.027)	0.484 (0.012)	0.448 (0.044)	0.463 (0.029)	0.482 (0.014)	0.187 (0.056)	0.130 (0.038)	0.130 (0.014)									
Median	0.442 (0.044)	0.457 (0.030)	0.482 (0.013)	0.444 (0.045)	0.457 (0.031)	0.485 (0.011)	0.191 (0.057)	0.152 (0.036)	0.131 (0.015)									
CQC	0.445 (0.047)	0.460 (0.030)	0.482 (0.014)	0.444 (0.048)	0.459 (0.032)	0.461 (0.022)	0.227 (0.077)	0.162 (0.046)	0.110 (0.015)									
EQC	0.449 (0.039)	0.458 (0.030)	0.481 (0.014)	0.439 (0.042)	0.482 (0.031)	0.482 (0.014)	0.193 (0.056)	0.110 (0.038)	0.110 (0.016)									
LDA	0.442 (0.046)	0.462 (0.029)	0.481 (0.014)	0.443 (0.040)	0.463 (0.029)	0.482 (0.014)	0.395 (0.070)	0.237 (0.043)	0.142 (0.016)									
KNN	0.447 (0.045)	0.470 (0.026)	0.485 (0.011)	0.439 (0.048)	0.455 (0.033)	0.453 (0.019)	0.284 (0.069)	0.240 (0.048)	0.197 (0.019)									
PLR	0.447 (0.037)	0.459 (0.031)	0.482 (0.014)	0.449 (0.042)	0.463 (0.026)	0.481 (0.014)	0.218 (0.063)	0.197 (0.041)	0.147 (0.017)									
SVM	0.445 (0.045)	0.462 (0.028)	0.484 (0.013)	0.439 (0.038)	0.462 (0.026)	0.408 (0.023)	0.212 (0.062)	0.173 (0.037)	0.137 (0.015)									
Bayes	0.442 (0.047)	0.462 (0.032)	0.481 (0.015)	0.447 (0.039)	0.453 (0.028)	0.466 (0.021)	0.284 (0.065)	0.223 (0.043)	0.141 (0.018)									
RF	0.434 (0.044)	0.461 (0.030)	0.482 (0.013)	0.445 (0.044)	0.456 (0.033)	0.420 (0.024)	0.209 (0.054)	0.162 (0.039)	0.121 (0.016)									
$p = 100$																		
VWQC	0.416 (0.062)	0.397 (0.049)	0.330 (0.020)	0.237 (0.069)	0.183 (0.043)	0.112 (0.015)	0.113 (0.047)	0.074 (0.032)	0.041 (0.009)									
DQC	0.427 (0.057)	0.410 (0.046)	0.406 (0.024)	0.233 (0.064)	0.189 (0.046)	0.168 (0.018)	0.091 (0.047)	0.065 (0.027)	0.049 (0.010)									
Centroid	0.422 (0.054)	0.406 (0.050)	0.345 (0.023)	0.227 (0.065)	0.185 (0.043)	0.136 (0.015)	0.108 (0.049)	0.079 (0.029)	0.055 (0.010)									
Median	0.419 (0.058)	0.410 (0.047)	0.346 (0.024)	0.237 (0.067)	0.191 (0.041)	0.136 (0.015)	0.104 (0.043)	0.079 (0.029)	0.056 (0.010)									
CQC	0.426 (0.058)	0.419 (0.044)	0.335 (0.023)	0.229 (0.065)	0.197 (0.046)	0.117 (0.014)	0.088 (0.045)	0.071 (0.029)	0.043 (0.010)									
EQC	0.416 (0.065)	0.405 (0.045)	0.333 (0.024)	0.229 (0.062)	0.190 (0.046)	0.113 (0.013)	0.106 (0.045)	0.071 (0.029)	0.043 (0.009)									
LDA	0.355 (0.046)	0.456 (0.035)	0.361 (0.023)	0.355 (0.069)	0.423 (0.049)	0.162 (0.017)	0.275 (0.076)	0.404 (0.060)	0.077 (0.012)									
KNN	0.433 (0.049)	0.443 (0.039)	0.454 (0.018)	0.358 (0.065)	0.301 (0.051)	0.253 (0.023)	0.219 (0.062)	0.121 (0.044)	0.121 (0.017)									
PLR	0.421 (0.055)	0.414 (0.045)	0.359 (0.026)	0.250 (0.066)	0.218 (0.049)	0.173 (0.018)	0.126 (0.052)	0.101 (0.034)	0.084 (0.013)									
SVM	0.428 (0.057)	0.414 (0.047)	0.350 (0.023)	0.254 (0.064)	0.196 (0.044)	0.144 (0.015)	0.120 (0.051)	0.087 (0.031)	0.077 (0.012)									
Bayes	0.430 (0.048)	0.442 (0.039)	0.407 (0.022)	0.314 (0.076)	0.286 (0.042)	0.180 (0.017)	0.210 (0.069)	0.144 (0.037)	0.070 (0.012)									
RF	0.412 (0.058)	0.388 (0.049)	0.311 (0.019)	0.232 (0.058)	0.188 (0.040)	0.123 (0.014)	0.135 (0.051)	0.093 (0.033)	0.055 (0.013)									
$p = 500$																		
VWQC	0.352 (0.072)	0.292 (0.047)	0.167 (0.020)	0.056 (0.037)	0.029 (0.022)	0.004 (0.003)	0.015 (0.029)	0.005 (0.010)	0.000 (0.000)									
DQC	0.356 (0.068)	0.334 (0.046)	0.308 (0.022)	0.048 (0.032)	0.028 (0.016)	0.014 (0.005)	0.000 (0.003)	0.000 (0.002)	0.000 (0.001)									
Centroid	0.337 (0.059)	0.293 (0.045)	0.187 (0.020)	0.050 (0.031)	0.025 (0.013)	0.007 (0.003)	0.003 (0.008)	0.001 (0.003)	0.000 (0.002)									
Median	0.355 (0.066)	0.298 (0.049)	0.194 (0.019)	0.052 (0.035)	0.026 (0.015)	0.007 (0.004)	0.003 (0.008)	0.001 (0.002)	0.000 (0.002)									
CQC	0.431 (0.047)	0.405 (0.053)	0.176 (0.022)	0.266 (0.067)	0.221 (0.036)	0.008 (0.007)	0.123 (0.052)	0.086 (0.030)	0.004 (0.005)									
EQC	0.343 (0.064)	0.297 (0.050)	0.168 (0.021)	0.046 (0.036)	0.024 (0.017)	0.004 (0.003)	0.003 (0.007)	0.000 (0.002)	0.000 (0.000)									
LDA	0.429 (0.053)	0.413 (0.050)	0.468 (0.022)	0.293 (0.070)	0.214 (0.041)	0.417 (0.039)	0.193 (0.067)	0.111 (0.034)	0.381 (0.049)									
KNN	0.413 (0.048)	0.425 (0.045)	0.393 (0.025)	0.197 (0.064)	0.145 (0.043)	0.080 (0.019)	0.045 (0.041)	0.039 (0.023)	0.010 (0.005)									
PLR	0.345 (0.058)	0.301 (0.047)	0.227 (0.022)	0.051 (0.031)	0.031 (0.016)	0.012 (0.005)	0.003 (0.008)	0.001 (0.003)	0.000 (0.001)									
SVM	0.348 (0.063)	0.299 (0.047)	0.194 (0.018)	0.054 (0.032)	0.032 (0.016)	0.010 (0.004)	0.003 (0.009)	0.000 (0.001)	0.000 (0.001)									
Bayes	0.418 (0.056)	0.395 (0.047)	0.314 (0.021)	0.170 (0.049)	0.110 (0.029)	0.034 (0.027)	0.012 (0.011)	0.002 (0.002)	0.002 (0.001)									
RF	0.343 (0.066)	0.271 (0.046)	0.150 (0.020)	0.089 (0.045)	0.052 (0.028)	0.027 (0.022)	0.010 (0.010)	0.001 (0.002)	0.001 (0.001)									

Table 6: Misclassification rates (with standard deviations) averaged over 100 replications in brackets for twelve classifiers in the logabst scenario with correlated variables.

Sample size n	50	100	500	50	100	500	50	100	500	
Relevant variables	10%				50%				100%	
$p = 10$										
VWQC	0.431 (0.056)	0.445 (0.037)	0.440 (0.025)	0.439 (0.045)	0.450 (0.040)	0.459 (0.032)	0.453 (0.026)	0.435 (0.055)	0.447 (0.038)	0.440 (0.023)
DQC	0.449 (0.041)	0.450 (0.034)	0.469 (0.018)	0.436 (0.046)	0.453 (0.033)	0.447 (0.024)	0.433 (0.050)	0.436 (0.051)	0.450 (0.039)	0.467 (0.021)
Centroid	0.445 (0.045)	0.445 (0.038)	0.448 (0.023)	0.437 (0.047)	0.453 (0.033)	0.447 (0.024)	0.433 (0.050)	0.449 (0.036)	0.449 (0.024)	
Median	0.438 (0.037)	0.439 (0.026)	0.448 (0.042)	0.448 (0.047)	0.448 (0.036)	0.442 (0.026)	0.439 (0.045)	0.439 (0.045)	0.443 (0.025)	
CQC	0.440 (0.046)	0.453 (0.040)	0.444 (0.026)	0.443 (0.039)	0.443 (0.039)	0.445 (0.025)	0.432 (0.049)	0.455 (0.034)	0.445 (0.027)	
EQC	0.441 (0.042)	0.445 (0.041)	0.439 (0.026)	0.448 (0.043)	0.442 (0.038)	0.436 (0.027)	0.430 (0.052)	0.447 (0.037)	0.438 (0.026)	
LDA	0.436 (0.046)	0.447 (0.038)	0.447 (0.022)	0.439 (0.041)	0.442 (0.033)	0.448 (0.022)	0.434 (0.052)	0.451 (0.035)	0.449 (0.023)	
KNN	0.441 (0.048)	0.457 (0.030)	0.478 (0.016)	0.445 (0.044)	0.451 (0.034)	0.477 (0.017)	0.437 (0.051)	0.448 (0.036)	0.479 (0.015)	
PLR	0.444 (0.048)	0.447 (0.038)	0.447 (0.022)	0.432 (0.042)	0.453 (0.035)	0.447 (0.022)	0.434 (0.056)	0.450 (0.037)	0.448 (0.023)	
SVM	0.446 (0.045)	0.444 (0.039)	0.459 (0.022)	0.438 (0.050)	0.457 (0.033)	0.454 (0.024)	0.429 (0.048)	0.450 (0.036)	0.447 (0.024)	
Bayes	0.447 (0.042)	0.449 (0.032)	0.460 (0.022)	0.435 (0.045)	0.456 (0.032)	0.463 (0.023)	0.438 (0.047)	0.453 (0.035)	0.463 (0.020)	
RF	0.441 (0.049)	0.448 (0.034)	0.455 (0.023)	0.439 (0.043)	0.449 (0.037)	0.451 (0.026)	0.426 (0.057)	0.447 (0.035)	0.453 (0.022)	
$p = 50$										
VWQC	0.444 (0.044)	0.461 (0.025)	0.481 (0.014)	0.444 (0.043)	0.458 (0.033)	0.482 (0.014)	0.168 (0.062)	0.120 (0.038)	0.091 (0.013)	
DQC	0.439 (0.046)	0.463 (0.028)	0.483 (0.012)	0.442 (0.043)	0.464 (0.028)	0.479 (0.015)	0.161 (0.055)	0.126 (0.030)	0.116 (0.016)	
Centroid	0.438 (0.042)	0.455 (0.032)	0.482 (0.014)	0.442 (0.042)	0.461 (0.030)	0.482 (0.013)	0.179 (0.058)	0.122 (0.017)	0.122 (0.017)	
Median	0.442 (0.041)	0.465 (0.029)	0.482 (0.013)	0.442 (0.040)	0.458 (0.032)	0.483 (0.014)	0.179 (0.058)	0.149 (0.035)	0.123 (0.015)	
CQC	0.442 (0.048)	0.464 (0.025)	0.479 (0.016)	0.445 (0.043)	0.458 (0.032)	0.445 (0.030)	0.191 (0.071)	0.145 (0.048)	0.093 (0.015)	
EQC	0.447 (0.040)	0.457 (0.030)	0.483 (0.014)	0.440 (0.045)	0.461 (0.030)	0.483 (0.012)	0.167 (0.058)	0.093 (0.015)	0.093 (0.015)	
LDA	0.454 (0.030)	0.459 (0.032)	0.482 (0.013)	0.431 (0.046)	0.467 (0.029)	0.481 (0.013)	0.386 (0.075)	0.226 (0.055)	0.136 (0.018)	
KNN	0.447 (0.038)	0.466 (0.027)	0.482 (0.013)	0.435 (0.042)	0.453 (0.034)	0.434 (0.025)	0.318 (0.060)	0.223 (0.045)	0.190 (0.018)	
PLR	0.441 (0.043)	0.460 (0.031)	0.482 (0.013)	0.440 (0.042)	0.463 (0.030)	0.481 (0.013)	0.201 (0.066)	0.187 (0.042)	0.139 (0.018)	
SVM	0.441 (0.047)	0.456 (0.030)	0.482 (0.012)	0.444 (0.042)	0.453 (0.036)	0.407 (0.024)	0.198 (0.063)	0.170 (0.040)	0.128 (0.017)	
Bayes	0.442 (0.048)	0.457 (0.031)	0.479 (0.016)	0.446 (0.044)	0.455 (0.032)	0.459 (0.022)	0.248 (0.064)	0.199 (0.043)	0.117 (0.016)	
RF	0.442 (0.046)	0.469 (0.025)	0.478 (0.017)	0.452 (0.038)	0.451 (0.036)	0.408 (0.023)	0.179 (0.060)	0.140 (0.033)	0.100 (0.015)	
$p = 100$										
VWQC	0.416 (0.060)	0.378 (0.054)	0.303 (0.021)	0.215 (0.066)	0.153 (0.041)	0.101 (0.012)	0.081 (0.044)	0.058 (0.027)	0.029 (0.007)	
DQC	0.426 (0.056)	0.420 (0.051)	0.406 (0.023)	0.227 (0.065)	0.188 (0.039)	0.122 (0.016)	0.054 (0.038)	0.054 (0.026)	0.046 (0.010)	
Centroid	0.420 (0.056)	0.398 (0.052)	0.336 (0.021)	0.220 (0.057)	0.176 (0.044)	0.133 (0.014)	0.096 (0.041)	0.069 (0.027)	0.050 (0.010)	
Median	0.416 (0.054)	0.403 (0.054)	0.337 (0.022)	0.225 (0.065)	0.181 (0.038)	0.129 (0.014)	0.086 (0.040)	0.071 (0.025)	0.050 (0.009)	
CQC	0.421 (0.050)	0.406 (0.053)	0.313 (0.025)	0.274 (0.082)	0.178 (0.048)	0.104 (0.014)	0.082 (0.086)	0.057 (0.044)	0.031 (0.009)	
EQC	0.418 (0.050)	0.388 (0.056)	0.309 (0.025)	0.208 (0.071)	0.165 (0.043)	0.102 (0.015)	0.081 (0.037)	0.057 (0.025)	0.031 (0.009)	
LDA	0.355 (0.049)	0.456 (0.033)	0.350 (0.022)	0.347 (0.066)	0.425 (0.047)	0.158 (0.017)	0.251 (0.073)	0.391 (0.057)	0.072 (0.011)	
KNN	0.437 (0.046)	0.449 (0.036)	0.384 (0.027)	0.324 (0.059)	0.307 (0.047)	0.244 (0.023)	0.222 (0.053)	0.155 (0.046)	0.121 (0.018)	
PLR	0.412 (0.051)	0.412 (0.051)	0.350 (0.024)	0.235 (0.066)	0.209 (0.046)	0.166 (0.017)	0.107 (0.040)	0.091 (0.036)	0.075 (0.012)	
SVM	0.416 (0.054)	0.454 (0.037)	0.340 (0.021)	0.236 (0.061)	0.188 (0.045)	0.149 (0.015)	0.118 (0.046)	0.082 (0.033)	0.054 (0.010)	
Bayes	0.435 (0.052)	0.440 (0.045)	0.387 (0.021)	0.312 (0.067)	0.270 (0.044)	0.162 (0.017)	0.175 (0.059)	0.120 (0.036)	0.053 (0.010)	
RF	0.405 (0.062)	0.357 (0.059)	0.279 (0.023)	0.212 (0.057)	0.167 (0.041)	0.109 (0.015)	0.108 (0.050)	0.076 (0.030)	0.039 (0.010)	
$p = 500$										
VWQC	0.293 (0.068)	0.229 (0.049)	0.108 (0.015)	0.035 (0.034)	0.014 (0.018)	0.002 (0.002)	0.011 (0.025)	0.003 (0.010)	0.000 (0.000)	
DQC	0.366 (0.066)	0.324 (0.046)	0.301 (0.024)	0.042 (0.028)	0.020 (0.013)	0.011 (0.004)	0.000 (0.004)	0.000 (0.001)	0.000 (0.000)	
Centroid	0.353 (0.068)	0.285 (0.045)	0.170 (0.017)	0.041 (0.027)	0.018 (0.014)	0.005 (0.003)	0.003 (0.007)	0.001 (0.002)	0.000 (0.001)	
Median	0.346 (0.070)	0.282 (0.050)	0.174 (0.018)	0.045 (0.028)	0.016 (0.013)	0.005 (0.003)	0.002 (0.006)	0.000 (0.002)	0.000 (0.001)	
CQC	0.399 (0.054)	0.354 (0.057)	0.123 (0.022)	0.197 (0.065)	0.133 (0.042)	0.006 (0.005)	0.062 (0.036)	0.029 (0.018)	0.003 (0.003)	
EQC	0.324 (0.071)	0.246 (0.042)	0.111 (0.016)	0.030 (0.025)	0.009 (0.009)	0.002 (0.002)	0.001 (0.003)	0.000 (0.000)	0.000 (0.000)	
LDA	0.431 (0.048)	0.405 (0.048)	0.462 (0.027)	0.291 (0.062)	0.186 (0.047)	0.418 (0.037)	0.185 (0.066)	0.101 (0.040)	0.383 (0.052)	
KNN	0.418 (0.066)	0.402 (0.050)	0.374 (0.024)	0.154 (0.072)	0.121 (0.052)	0.067 (0.019)	0.107 (0.050)	0.043 (0.025)	0.007 (0.004)	
PLR	0.348 (0.067)	0.286 (0.046)	0.211 (0.022)	0.047 (0.029)	0.023 (0.015)	0.009 (0.004)	0.003 (0.007)	0.000 (0.002)	0.000 (0.001)	
SVM	0.343 (0.065)	0.281 (0.045)	0.174 (0.017)	0.048 (0.028)	0.007 (0.014)	0.007 (0.004)	0.000 (0.006)	0.000 (0.002)	0.000 (0.001)	
Bayes	0.403 (0.060)	0.363 (0.049)	0.264 (0.020)	0.138 (0.050)	0.071 (0.028)	0.016 (0.006)	0.027 (0.026)	0.005 (0.008)	0.001 (0.001)	
RF	0.312 (0.067)	0.215 (0.046)	0.087 (0.013)	0.060 (0.034)	0.027 (0.017)	0.005 (0.003)	0.016 (0.019)	0.004 (0.008)	0.000 (0.001)	

Table 7: Misclassification rates (with standard deviations) averaged over 100 replications in brackets for twelve classifiers in the exp scenario with uncorrelated variables.

Sample size n	50	100	500	50	100	500	50	100	500									
Relevant variables	10%				50%				100%									
$p = 10$																		
$p = 50$																		
$p = 100$																		
VWQC	0.432 (0.051)	0.435 (0.046)	0.420 (0.020)	0.444 (0.043)	0.436 (0.045)	0.420 (0.021)	0.441 (0.045)	0.436 (0.041)	0.422 (0.017)									
DQC	0.450 (0.036)	0.456 (0.030)	0.476 (0.017)	0.439 (0.046)	0.457 (0.031)	0.473 (0.018)	0.444 (0.039)	0.466 (0.026)	0.476 (0.017)									
Centroid	0.444 (0.042)	0.460 (0.030)	0.468 (0.019)	0.446 (0.041)	0.459 (0.030)	0.468 (0.021)	0.441 (0.048)	0.462 (0.029)	0.472 (0.021)									
Median	0.449 (0.045)	0.461 (0.030)	0.467 (0.022)	0.441 (0.043)	0.458 (0.029)	0.466 (0.023)	0.441 (0.047)	0.459 (0.030)	0.470 (0.019)									
CQC	0.440 (0.043)	0.448 (0.036)	0.435 (0.027)	0.446 (0.040)	0.445 (0.041)	0.433 (0.025)	0.437 (0.045)	0.454 (0.035)	0.437 (0.025)									
EQC	0.443 (0.042)	0.461 (0.032)	0.453 (0.025)	0.433 (0.046)	0.449 (0.036)	0.454 (0.024)	0.448 (0.048)	0.457 (0.035)	0.453 (0.024)									
LDA	0.447 (0.039)	0.462 (0.030)	0.469 (0.019)	0.448 (0.043)	0.454 (0.029)	0.468 (0.021)	0.444 (0.044)	0.458 (0.032)	0.471 (0.020)									
KNN	0.450 (0.034)	0.457 (0.034)	0.482 (0.013)	0.452 (0.033)	0.461 (0.031)	0.480 (0.015)	0.443 (0.041)	0.466 (0.024)	0.481 (0.014)									
PLR	0.449 (0.042)	0.462 (0.029)	0.468 (0.019)	0.444 (0.044)	0.455 (0.032)	0.468 (0.021)	0.442 (0.047)	0.459 (0.034)	0.471 (0.020)									
SVM	0.452 (0.035)	0.457 (0.031)	0.468 (0.018)	0.446 (0.040)	0.456 (0.034)	0.476 (0.019)	0.441 (0.046)	0.467 (0.022)	0.473 (0.018)									
Bayes	0.445 (0.045)	0.457 (0.031)	0.464 (0.018)	0.445 (0.044)	0.461 (0.029)	0.461 (0.023)	0.444 (0.046)	0.452 (0.032)	0.467 (0.020)									
RF	0.449 (0.040)	0.438 (0.042)	0.437 (0.021)	0.437 (0.040)	0.437 (0.036)	0.445 (0.025)	0.438 (0.050)	0.447 (0.022)	0.444 (0.022)									
$p = 500$																		
VWQC	0.383 (0.066)	0.336 (0.050)	0.224 (0.020)	0.186 (0.066)	0.066 (0.033)	0.012 (0.005)	0.091 (0.048)	0.008 (0.012)	0.001 (0.001)									
DQC	0.447 (0.050)	0.455 (0.034)	0.463 (0.021)	0.405 (0.060)	0.382 (0.043)	0.340 (0.021)	0.322 (0.068)	0.274 (0.046)	0.241 (0.019)									
Centroid	0.439 (0.049)	0.447 (0.022)	0.411 (0.058)	0.383 (0.051)	0.333 (0.021)	0.309 (0.049)	0.258 (0.020)	0.258 (0.020)	0.258 (0.022)									
Median	0.437 (0.048)	0.445 (0.034)	0.447 (0.025)	0.410 (0.055)	0.380 (0.025)	0.326 (0.020)	0.336 (0.071)	0.287 (0.050)	0.249 (0.022)									
CQC	0.423 (0.062)	0.363 (0.070)	0.224 (0.021)	0.286 (0.096)	0.093 (0.050)	0.011 (0.005)	0.171 (0.086)	0.027 (0.037)	0.001 (0.001)									
EQC	0.429 (0.058)	0.441 (0.043)	0.398 (0.030)	0.335 (0.069)	0.284 (0.055)	0.217 (0.018)	0.170 (0.062)	0.131 (0.040)	0.131 (0.016)									
LDA	0.443 (0.045)	0.456 (0.035)	0.450 (0.023)	0.439 (0.050)	0.407 (0.053)	0.340 (0.020)	0.421 (0.054)	0.362 (0.055)	0.269 (0.019)									
KNN	0.443 (0.044)	0.460 (0.029)	0.479 (0.014)	0.432 (0.049)	0.445 (0.041)	0.424 (0.024)	0.418 (0.060)	0.389 (0.052)	0.382 (0.024)									
PLR	0.440 (0.045)	0.453 (0.034)	0.450 (0.023)	0.411 (0.060)	0.399 (0.053)	0.341 (0.021)	0.365 (0.070)	0.269 (0.019)	0.269 (0.017)									
SVM	0.439 (0.044)	0.455 (0.035)	0.448 (0.022)	0.404 (0.057)	0.385 (0.054)	0.313 (0.021)	0.343 (0.065)	0.288 (0.050)	0.201 (0.017)									
Bayes	0.441 (0.044)	0.452 (0.035)	0.431 (0.022)	0.424 (0.057)	0.389 (0.047)	0.263 (0.021)	0.366 (0.062)	0.310 (0.045)	0.184 (0.019)									
RF	0.414 (0.060)	0.355 (0.053)	0.212 (0.019)	0.239 (0.066)	0.107 (0.036)	0.016 (0.006)	0.130 (0.052)	0.039 (0.021)	0.003 (0.003)									
$p = 100$																		
VWQC	0.333 (0.075)	0.265 (0.047)	0.104 (0.015)	0.136 (0.058)	0.019 (0.016)	0.001 (0.002)	0.054 (0.043)	0.001 (0.004)	0.000 (0.001)									
DQC	0.439 (0.043)	0.450 (0.041)	0.449 (0.039)	0.426 (0.024)	0.365 (0.060)	0.347 (0.051)	0.275 (0.020)	0.281 (0.066)	0.249 (0.042)									
Centroid	0.438 (0.044)	0.449 (0.039)	0.424 (0.020)	0.354 (0.068)	0.339 (0.054)	0.263 (0.020)	0.260 (0.065)	0.223 (0.038)	0.168 (0.019)									
Median	0.437 (0.046)	0.448 (0.038)	0.424 (0.020)	0.354 (0.068)	0.362 (0.041)	0.000 (0.001)	0.132 (0.067)	0.089 (0.033)	0.055 (0.010)									
CQC	0.429 (0.058)	0.320 (0.091)	0.102 (0.016)	0.277 (0.070)	0.198 (0.046)	0.138 (0.016)	0.134 (0.057)	0.089 (0.033)	0.055 (0.010)									
EQC	0.425 (0.057)	0.417 (0.045)	0.352 (0.027)	0.416 (0.062)	0.452 (0.037)	0.296 (0.024)	0.367 (0.071)	0.438 (0.040)	0.207 (0.021)									
LDA	0.446 (0.039)	0.459 (0.031)	0.431 (0.023)	0.420 (0.052)	0.443 (0.037)	0.371 (0.023)	0.302 (0.023)	0.290 (0.066)	0.278 (0.052)									
KNN	0.440 (0.043)	0.450 (0.034)	0.451 (0.015)	0.431 (0.025)	0.374 (0.064)	0.371 (0.050)	0.255 (0.023)	0.287 (0.063)	0.259 (0.044)									
PLR	0.432 (0.052)	0.450 (0.035)	0.421 (0.022)	0.364 (0.070)	0.354 (0.048)	0.221 (0.018)	0.321 (0.065)	0.283 (0.043)	0.149 (0.018)									
SVM	0.436 (0.046)	0.447 (0.033)	0.429 (0.022)	0.418 (0.021)	0.407 (0.054)	0.337 (0.044)	0.221 (0.024)	0.002 (0.002)	0.018 (0.014)									
Bayes	0.446 (0.041)	0.452 (0.032)	0.418 (0.021)	0.417 (0.052)	0.349 (0.024)	0.049 (0.024)	0.002 (0.002)	0.079 (0.038)	0.000 (0.001)									
$p = 500$																		
VWQC	0.220 (0.063)	0.098 (0.039)	0.001 (0.002)	0.043 (0.038)	0.001 (0.005)	0.000 (0.000)	0.002 (0.010)	0.000 (0.001)	0.000 (0.002)									
DQC	0.420 (0.051)	0.424 (0.045)	0.393 (0.022)	0.218 (0.066)	0.157 (0.039)	0.107 (0.014)	0.058 (0.035)	0.029 (0.016)	0.014 (0.005)									
Centroid	0.427 (0.046)	0.414 (0.050)	0.341 (0.023)	0.228 (0.067)	0.179 (0.044)	0.094 (0.012)	0.101 (0.043)	0.056 (0.025)	0.021 (0.006)									
Median	0.423 (0.058)	0.408 (0.045)	0.336 (0.021)	0.224 (0.060)	0.167 (0.040)	0.083 (0.012)	0.074 (0.038)	0.044 (0.019)	0.017 (0.006)									
CQC	0.355 (0.068)	0.202 (0.064)	0.001 (0.002)	0.171 (0.061)	0.031 (0.030)	0.000 (0.000)	0.136 (0.057)	0.010 (0.012)	0.000 (0.000)									
EQC	0.378 (0.067)	0.304 (0.051)	0.195 (0.019)	0.071 (0.039)	0.031 (0.017)	0.009 (0.004)	0.008 (0.014)	0.002 (0.004)	0.000 (0.001)									
LDA	0.441 (0.043)	0.447 (0.036)	0.479 (0.015)	0.412 (0.063)	0.452 (0.028)	0.345 (0.050)	0.344 (0.064)	0.258 (0.053)	0.436 (0.034)									
KNN	0.441 (0.040)	0.464 (0.031)	0.475 (0.016)	0.371 (0.057)	0.343 (0.059)	0.303 (0.039)	0.263 (0.078)	0.268 (0.048)	0.200 (0.022)									
PLR	0.430 (0.049)	0.421 (0.046)	0.374 (0.022)	0.231 (0.066)	0.192 (0.042)	0.125 (0.015)	0.106 (0.047)	0.063 (0.025)	0.038 (0.009)									
SVM	0.427 (0.045)	0.416 (0.048)	0.342 (0.023)	0.241 (0.064)	0.180 (0.013)	0.088 (0.013)	0.058 (0.022)	0.010 (0.004)	0.010 (0.004)									
Bayes	0.440 (0.042)	0.429 (0.039)	0.353 (0.020)	0.323 (0.063)	0.278 (0.041)	0.135 (0.015)	0.261 (0.065)	0.219 (0.043)	0.062 (0.014)									
RF	0.312 (0.066)	0.120 (0.037)	0.002 (0.002)	0.061 (0.038)	0.002 (0.005)	0.000 (0.000)	0.027 (0.025)	0.000 (0.000)	0.000 (0.000)									

Table 8: Misclassification rates (with standard deviations) averaged over 100 replications in brackets for twelve classifiers in the exp scenario with correlated variables.

Sample size n	50	100	500	50	100	500	50	100	500									
Relevant variables	10%				50%				100%									
$p = 10$																		
$p = 50$																		
VWQC	0.433 (0.052)	0.442 (0.036)	0.431 (0.018)	0.431 (0.048)	0.442 (0.038)	0.434 (0.022)	0.440 (0.052)	0.445 (0.037)	0.432 (0.021)									
DQC	0.445 (0.045)	0.456 (0.033)	0.476 (0.017)	0.436 (0.045)	0.457 (0.034)	0.472 (0.019)	0.450 (0.037)	0.458 (0.033)	0.471 (0.019)									
Centroid	0.442 (0.044)	0.459 (0.034)	0.468 (0.020)	0.443 (0.041)	0.453 (0.033)	0.462 (0.020)	0.441 (0.045)	0.454 (0.034)	0.462 (0.021)									
Median	0.436 (0.045)	0.460 (0.034)	0.468 (0.020)	0.444 (0.045)	0.458 (0.035)	0.464 (0.023)	0.446 (0.046)	0.454 (0.034)	0.463 (0.023)									
CQC	0.441 (0.047)	0.451 (0.038)	0.438 (0.023)	0.438 (0.051)	0.453 (0.036)	0.440 (0.022)	0.442 (0.047)	0.450 (0.040)	0.441 (0.024)									
EQC	0.442 (0.043)	0.455 (0.033)	0.451 (0.021)	0.439 (0.048)	0.448 (0.040)	0.450 (0.024)	0.446 (0.046)	0.452 (0.034)	0.447 (0.023)									
LDA	0.446 (0.040)	0.456 (0.033)	0.461 (0.022)	0.437 (0.050)	0.455 (0.034)	0.458 (0.022)	0.444 (0.039)	0.453 (0.036)	0.457 (0.022)									
KNN	0.444 (0.046)	0.463 (0.032)	0.481 (0.016)	0.448 (0.040)	0.460 (0.032)	0.479 (0.016)	0.436 (0.045)	0.458 (0.032)	0.479 (0.014)									
PLR	0.443 (0.038)	0.456 (0.034)	0.460 (0.021)	0.438 (0.049)	0.455 (0.035)	0.458 (0.022)	0.448 (0.039)	0.452 (0.034)	0.456 (0.022)									
SVM	0.448 (0.042)	0.460 (0.031)	0.446 (0.020)	0.446 (0.053)	0.456 (0.035)	0.465 (0.023)	0.446 (0.036)	0.453 (0.036)	0.466 (0.020)									
Bayes	0.440 (0.040)	0.455 (0.033)	0.463 (0.021)	0.437 (0.042)	0.457 (0.031)	0.463 (0.023)	0.443 (0.043)	0.459 (0.030)	0.459 (0.022)									
RF	0.436 (0.048)	0.450 (0.037)	0.445 (0.023)	0.434 (0.053)	0.444 (0.039)	0.445 (0.022)	0.435 (0.051)	0.441 (0.038)	0.446 (0.022)									
$p = 100$																		
VWQC	0.416 (0.058)	0.368 (0.056)	0.288 (0.025)	0.194 (0.065)	0.088 (0.038)	0.025 (0.007)	0.104 (0.059)	0.042 (0.022)	0.011 (0.005)									
DQC	0.434 (0.045)	0.449 (0.037)	0.456 (0.021)	0.365 (0.072)	0.340 (0.049)	0.301 (0.023)	0.230 (0.068)	0.194 (0.040)	0.165 (0.018)									
Centroid	0.441 (0.042)	0.443 (0.038)	0.429 (0.024)	0.370 (0.063)	0.346 (0.047)	0.289 (0.024)	0.237 (0.044)	0.186 (0.019)	0.186 (0.019)									
Median	0.442 (0.046)	0.445 (0.040)	0.425 (0.025)	0.364 (0.054)	0.347 (0.046)	0.246 (0.069)	0.222 (0.045)	0.183 (0.019)	0.183 (0.019)									
CQC	0.431 (0.048)	0.386 (0.059)	0.287 (0.023)	0.298 (0.088)	0.121 (0.051)	0.018 (0.007)	0.166 (0.068)	0.065 (0.038)	0.005 (0.003)									
EQC	0.440 (0.044)	0.424 (0.047)	0.378 (0.024)	0.314 (0.071)	0.269 (0.053)	0.188 (0.018)	0.146 (0.062)	0.103 (0.016)	0.103 (0.016)									
LDA	0.443 (0.047)	0.446 (0.040)	0.416 (0.025)	0.419 (0.057)	0.358 (0.050)	0.285 (0.021)	0.417 (0.064)	0.282 (0.055)	0.187 (0.018)									
KNN	0.437 (0.043)	0.453 (0.034)	0.479 (0.016)	0.409 (0.057)	0.416 (0.047)	0.417 (0.021)	0.378 (0.069)	0.372 (0.058)	0.295 (0.028)									
PLR	0.442 (0.043)	0.440 (0.040)	0.416 (0.025)	0.377 (0.068)	0.341 (0.044)	0.284 (0.021)	0.276 (0.070)	0.276 (0.042)	0.191 (0.019)									
SVM	0.440 (0.046)	0.442 (0.041)	0.418 (0.024)	0.379 (0.068)	0.335 (0.044)	0.259 (0.022)	0.246 (0.062)	0.195 (0.045)	0.128 (0.015)									
Bayes	0.438 (0.045)	0.453 (0.033)	0.410 (0.025)	0.394 (0.072)	0.358 (0.051)	0.224 (0.019)	0.286 (0.067)	0.227 (0.042)	0.139 (0.015)									
RF	0.422 (0.055)	0.370 (0.052)	0.273 (0.020)	0.229 (0.071)	0.121 (0.040)	0.023 (0.007)	0.142 (0.054)	0.076 (0.035)	0.013 (0.007)									
$p = 500$																		
VWQC	0.381 (0.064)	0.344 (0.049)	0.225 (0.021)	0.121 (0.055)	0.045 (0.026)	0.004 (0.003)	0.057 (0.035)	0.013 (0.011)	0.001 (0.002)									
DQC	0.436 (0.047)	0.440 (0.042)	0.433 (0.023)	0.281 (0.070)	0.276 (0.046)	0.216 (0.021)	0.133 (0.055)	0.094 (0.031)	0.067 (0.011)									
Centroid	0.430 (0.045)	0.437 (0.044)	0.390 (0.025)	0.296 (0.068)	0.288 (0.047)	0.198 (0.021)	0.197 (0.065)	0.145 (0.037)	0.085 (0.014)									
Median	0.434 (0.055)	0.425 (0.047)	0.384 (0.025)	0.287 (0.076)	0.277 (0.047)	0.197 (0.019)	0.164 (0.053)	0.123 (0.034)	0.083 (0.012)									
CQC	0.419 (0.065)	0.378 (0.056)	0.200 (0.022)	0.213 (0.075)	0.085 (0.049)	0.002 (0.002)	0.120 (0.067)	0.028 (0.028)	0.001 (0.001)									
EQC	0.414 (0.064)	0.398 (0.049)	0.315 (0.024)	0.212 (0.075)	0.171 (0.053)	0.090 (0.011)	0.094 (0.045)	0.059 (0.025)	0.030 (0.009)									
LDA	0.332 (0.051)	0.451 (0.032)	0.362 (0.023)	0.376 (0.079)	0.439 (0.042)	0.191 (0.020)	0.272 (0.061)	0.403 (0.062)	0.091 (0.016)									
KNN	0.440 (0.044)	0.463 (0.025)	0.462 (0.022)	0.417 (0.052)	0.370 (0.046)	0.380 (0.023)	0.314 (0.063)	0.233 (0.029)	0.190 (0.017)									
PLR	0.434 (0.050)	0.433 (0.048)	0.365 (0.026)	0.289 (0.070)	0.285 (0.049)	0.212 (0.022)	0.187 (0.056)	0.163 (0.039)	0.109 (0.017)									
SVM	0.429 (0.050)	0.465 (0.027)	0.359 (0.022)	0.293 (0.068)	0.272 (0.043)	0.157 (0.018)	0.143 (0.057)	0.110 (0.032)	0.048 (0.011)									
Bayes	0.428 (0.051)	0.437 (0.041)	0.373 (0.021)	0.345 (0.062)	0.305 (0.046)	0.161 (0.015)	0.211 (0.058)	0.155 (0.039)	0.079 (0.013)									
RF	0.390 (0.077)	0.323 (0.047)	0.179 (0.018)	0.149 (0.062)	0.067 (0.029)	0.005 (0.004)	0.100 (0.032)	0.058 (0.032)	0.013 (0.008)									

Table 9: Misclassification rates (with standard deviations) averaged over 100 replications in brackets for twelve classifiers in the ddv scenario with uncorrelated variables.

Sample size n	50	100	500	50	100	500	50	100	500	
Relevant variables	10%				50%				100%	
$p = 10$										
VWQC	0.431 (0.045)	0.459 (0.032)	0.468 (0.021)	0.357 (0.067)	0.318 (0.050)	0.301 (0.022)	0.261 (0.067)	0.228 (0.048)	0.204 (0.021)	
DQC	0.444 (0.043)	0.455 (0.034)	0.472 (0.019)	0.415 (0.055)	0.419 (0.047)	0.404 (0.023)	0.374 (0.066)	0.359 (0.049)	0.339 (0.019)	
Centroid	0.443 (0.047)	0.452 (0.037)	0.469 (0.020)	0.411 (0.062)	0.404 (0.049)	0.384 (0.025)	0.370 (0.072)	0.352 (0.047)	0.330 (0.022)	
Median	0.445 (0.046)	0.451 (0.038)	0.471 (0.022)	0.415 (0.054)	0.405 (0.050)	0.392 (0.023)	0.380 (0.064)	0.373 (0.049)	0.351 (0.022)	
CQC	0.445 (0.042)	0.455 (0.038)	0.474 (0.019)	0.408 (0.061)	0.381 (0.039)	0.345 (0.046)	0.341 (0.078)	0.287 (0.077)	0.258 (0.069)	
EQC	0.441 (0.041)	0.456 (0.036)	0.474 (0.019)	0.397 (0.058)	0.400 (0.055)	0.358 (0.026)	0.349 (0.073)	0.315 (0.058)	0.286 (0.024)	
LDA	0.439 (0.043)	0.454 (0.035)	0.468 (0.020)	0.413 (0.058)	0.408 (0.046)	0.383 (0.026)	0.376 (0.070)	0.357 (0.049)	0.332 (0.022)	
KNN	0.447 (0.046)	0.454 (0.032)	0.481 (0.013)	0.432 (0.051)	0.434 (0.035)	0.432 (0.024)	0.412 (0.059)	0.408 (0.050)	0.379 (0.022)	
PLR	0.441 (0.042)	0.455 (0.033)	0.468 (0.021)	0.416 (0.057)	0.409 (0.047)	0.383 (0.025)	0.374 (0.066)	0.359 (0.048)	0.332 (0.022)	
SVM	0.440 (0.040)	0.460 (0.033)	0.480 (0.015)	0.413 (0.065)	0.415 (0.050)	0.388 (0.024)	0.375 (0.063)	0.357 (0.047)	0.344 (0.022)	
Bayes	0.449 (0.043)	0.456 (0.032)	0.480 (0.014)	0.425 (0.057)	0.426 (0.050)	0.375 (0.024)	0.395 (0.062)	0.378 (0.058)	0.299 (0.022)	
RF	0.441 (0.041)	0.456 (0.033)	0.477 (0.017)	0.371 (0.071)	0.329 (0.046)	0.277 (0.020)	0.270 (0.064)	0.214 (0.046)	0.168 (0.019)	
$p = 50$										
VWQC	0.387 (0.060)	0.371 (0.047)	0.323 (0.022)	0.170 (0.065)	0.128 (0.037)	0.081 (0.013)	0.080 (0.048)	0.046 (0.022)	0.022 (0.008)	
DQC	0.443 (0.043)	0.451 (0.037)	0.453 (0.022)	0.358 (0.064)	0.336 (0.048)	0.308 (0.021)	0.247 (0.061)	0.223 (0.046)	0.196 (0.018)	
Centroid	0.436 (0.048)	0.438 (0.044)	0.407 (0.023)	0.340 (0.067)	0.340 (0.050)	0.249 (0.021)	0.204 (0.062)	0.163 (0.017)	0.163 (0.012)	
Median	0.435 (0.041)	0.443 (0.039)	0.418 (0.025)	0.354 (0.064)	0.323 (0.048)	0.280 (0.019)	0.265 (0.061)	0.232 (0.046)	0.198 (0.018)	
CQC	0.433 (0.052)	0.438 (0.037)	0.397 (0.034)	0.336 (0.074)	0.279 (0.055)	0.211 (0.041)	0.241 (0.077)	0.188 (0.051)	0.122 (0.030)	
EQC	0.426 (0.053)	0.427 (0.041)	0.378 (0.026)	0.315 (0.071)	0.249 (0.062)	0.192 (0.038)	0.168 (0.062)	0.155 (0.036)	0.098 (0.014)	
LDA	0.446 (0.040)	0.444 (0.040)	0.410 (0.027)	0.425 (0.055)	0.349 (0.051)	0.259 (0.021)	0.399 (0.064)	0.272 (0.052)	0.174 (0.016)	
KNN	0.443 (0.045)	0.455 (0.032)	0.472 (0.018)	0.389 (0.059)	0.395 (0.057)	0.351 (0.023)	0.306 (0.065)	0.292 (0.053)	0.248 (0.022)	
PLR	0.439 (0.044)	0.443 (0.040)	0.410 (0.026)	0.357 (0.074)	0.361 (0.059)	0.261 (0.021)	0.267 (0.067)	0.232 (0.049)	0.177 (0.017)	
SVM	0.438 (0.047)	0.441 (0.040)	0.420 (0.027)	0.349 (0.069)	0.313 (0.051)	0.258 (0.021)	0.244 (0.067)	0.230 (0.048)	0.166 (0.016)	
Bayes	0.438 (0.042)	0.454 (0.035)	0.437 (0.021)	0.413 (0.059)	0.384 (0.050)	0.271 (0.022)	0.344 (0.066)	0.288 (0.050)	0.162 (0.017)	
RF	0.402 (0.068)	0.350 (0.052)	0.285 (0.021)	0.201 (0.063)	0.103 (0.036)	0.048 (0.010)	0.084 (0.038)	0.032 (0.019)	0.008 (0.004)	
$p = 100$										
VWQC	0.343 (0.069)	0.309 (0.048)	0.238 (0.020)	0.113 (0.055)	0.065 (0.027)	0.026 (0.008)	0.034 (0.034)	0.014 (0.012)	0.003 (0.003)	
DQC	0.427 (0.060)	0.444 (0.037)	0.440 (0.025)	0.320 (0.062)	0.280 (0.042)	0.245 (0.021)	0.167 (0.056)	0.138 (0.032)	0.112 (0.013)	
Centroid	0.429 (0.047)	0.426 (0.043)	0.373 (0.022)	0.290 (0.059)	0.235 (0.044)	0.172 (0.016)	0.156 (0.057)	0.119 (0.031)	0.082 (0.012)	
Median	0.425 (0.051)	0.433 (0.038)	0.390 (0.023)	0.318 (0.072)	0.262 (0.043)	0.213 (0.018)	0.184 (0.057)	0.155 (0.037)	0.114 (0.014)	
CQC	0.427 (0.051)	0.433 (0.046)	0.367 (0.028)	0.231 (0.062)	0.231 (0.057)	0.154 (0.014)	0.098 (0.044)	0.058 (0.027)	0.035 (0.009)	
EQC	0.422 (0.060)	0.422 (0.048)	0.325 (0.026)	0.232 (0.068)	0.169 (0.044)	0.107 (0.014)	0.098 (0.044)	0.058 (0.027)	0.035 (0.009)	
LDA	0.43 (0.043)	0.453 (0.033)	0.384 (0.023)	0.377 (0.074)	0.431 (0.045)	0.197 (0.019)	0.310 (0.068)	0.409 (0.050)	0.105 (0.014)	
KNN	0.441 (0.040)	0.453 (0.035)	0.462 (0.019)	0.385 (0.069)	0.336 (0.046)	0.298 (0.024)	0.258 (0.072)	0.239 (0.045)	0.182 (0.022)	
PLR	0.425 (0.054)	0.434 (0.043)	0.385 (0.022)	0.316 (0.069)	0.257 (0.046)	0.204 (0.020)	0.171 (0.061)	0.145 (0.036)	0.132 (0.018)	
SVM	0.426 (0.053)	0.433 (0.042)	0.380 (0.024)	0.310 (0.070)	0.243 (0.041)	0.180 (0.016)	0.173 (0.063)	0.143 (0.035)	0.086 (0.012)	
Bayes	0.435 (0.047)	0.454 (0.033)	0.424 (0.021)	0.393 (0.063)	0.348 (0.052)	0.223 (0.021)	0.304 (0.068)	0.239 (0.048)	0.119 (0.016)	
RF	0.361 (0.072)	0.277 (0.050)	0.180 (0.017)	0.125 (0.054)	0.008 (0.023)	0.042 (0.023)	0.008 (0.004)	0.044 (0.029)	0.001 (0.009)	
$p = 500$										
VWQC	0.201 (0.058)	0.139 (0.038)	0.052 (0.009)	0.023 (0.026)	0.004 (0.009)	0.000 (0.002)	0.002 (0.011)	0.000 (0.002)	0.000 (0.001)	
DQC	0.406 (0.055)	0.390 (0.050)	0.365 (0.022)	0.146 (0.052)	0.093 (0.028)	0.064 (0.011)	0.022 (0.021)	0.008 (0.009)	0.004 (0.003)	
Centroid	0.384 (0.055)	0.338 (0.048)	0.229 (0.021)	0.114 (0.041)	0.054 (0.024)	0.018 (0.006)	0.018 (0.020)	0.005 (0.012)	0.001 (0.001)	
Median	0.397 (0.062)	0.358 (0.054)	0.270 (0.024)	0.133 (0.048)	0.081 (0.029)	0.037 (0.008)	0.021 (0.020)	0.012 (0.012)	0.003 (0.003)	
CQC	0.398 (0.062)	0.336 (0.053)	0.234 (0.026)	0.161 (0.056)	0.083 (0.032)	0.014 (0.007)	0.052 (0.039)	0.019 (0.015)	0.003 (0.003)	
EQC	0.357 (0.062)	0.288 (0.057)	0.155 (0.019)	0.050 (0.041)	0.015 (0.013)	0.003 (0.003)	0.004 (0.009)	0.001 (0.002)	0.000 (0.000)	
LDA	0.437 (0.048)	0.433 (0.050)	0.467 (0.020)	0.350 (0.069)	0.261 (0.050)	0.432 (0.034)	0.261 (0.075)	0.156 (0.043)	0.396 (0.045)	
KNN	0.431 (0.048)	0.425 (0.047)	0.411 (0.024)	0.246 (0.068)	0.271 (0.049)	0.141 (0.027)	0.114 (0.052)	0.070 (0.028)	0.035 (0.011)	
PLR	0.389 (0.054)	0.347 (0.052)	0.270 (0.023)	0.118 (0.043)	0.060 (0.022)	0.027 (0.006)	0.020 (0.020)	0.007 (0.008)	0.002 (0.002)	
SVM	0.382 (0.052)	0.350 (0.051)	0.242 (0.021)	0.118 (0.046)	0.061 (0.023)	0.023 (0.006)	0.020 (0.021)	0.006 (0.007)	0.002 (0.002)	
Bayes	0.430 (0.049)	0.436 (0.044)	0.383 (0.023)	0.311 (0.071)	0.260 (0.052)	0.139 (0.017)	0.192 (0.062)	0.130 (0.033)	0.042 (0.010)	
RF	0.259 (0.073)	0.081 (0.028)	0.144 (0.017)	0.017 (0.020)	0.001 (0.003)	0.000 (0.000)	0.004 (0.010)	0.000 (0.001)	0.000 (0.000)	

Table 10: Misclassification rates (with standard deviations) averaged over 100 replications in brackets for twelve classifiers in the t scenario with correlated variables.

Sample size n	50	100	500	50	100	500	50	100	500									
Relevant variables	10%				50%				100%									
$p = 10$																		
$p = 50$																		
VWQC	0.433 (0.048)	0.458 (0.033)	0.469 (0.019)	0.333 (0.066)	0.395 (0.053)	0.278 (0.017)	0.222 (0.058)	0.193 (0.043)	0.175 (0.020)									
DQC	0.443 (0.040)	0.456 (0.033)	0.474 (0.019)	0.410 (0.060)	0.415 (0.046)	0.387 (0.022)	0.351 (0.068)	0.336 (0.049)	0.323 (0.022)									
Centroid	0.441 (0.042)	0.453 (0.036)	0.464 (0.021)	0.396 (0.059)	0.394 (0.052)	0.363 (0.020)	0.344 (0.068)	0.328 (0.048)	0.311 (0.024)									
Median	0.439 (0.048)	0.454 (0.035)	0.470 (0.020)	0.401 (0.064)	0.405 (0.050)	0.370 (0.021)	0.362 (0.069)	0.348 (0.050)	0.329 (0.022)									
CQC	0.438 (0.042)	0.455 (0.037)	0.472 (0.019)	0.387 (0.072)	0.354 (0.069)	0.320 (0.044)	0.289 (0.081)	0.241 (0.071)	0.209 (0.061)									
EQC	0.433 (0.046)	0.461 (0.031)	0.472 (0.018)	0.370 (0.075)	0.366 (0.060)	0.329 (0.021)	0.302 (0.067)	0.264 (0.053)	0.235 (0.020)									
LDA	0.443 (0.041)	0.450 (0.035)	0.464 (0.021)	0.395 (0.060)	0.399 (0.047)	0.362 (0.020)	0.359 (0.065)	0.335 (0.048)	0.314 (0.024)									
KNN	0.444 (0.040)	0.454 (0.035)	0.481 (0.013)	0.422 (0.051)	0.428 (0.046)	0.446 (0.024)	0.408 (0.058)	0.390 (0.053)	0.351 (0.022)									
PLR	0.443 (0.041)	0.452 (0.036)	0.465 (0.021)	0.396 (0.062)	0.398 (0.047)	0.362 (0.021)	0.357 (0.063)	0.335 (0.049)	0.314 (0.024)									
SVM	0.449 (0.037)	0.455 (0.036)	0.484 (0.011)	0.427 (0.046)	0.397 (0.051)	0.365 (0.023)	0.347 (0.068)	0.333 (0.047)	0.315 (0.020)									
Bayes	0.442 (0.039)	0.461 (0.030)	0.479 (0.015)	0.412 (0.058)	0.407 (0.044)	0.344 (0.023)	0.363 (0.065)	0.330 (0.052)	0.259 (0.021)									
RF	0.440 (0.043)	0.462 (0.036)	0.475 (0.017)	0.337 (0.083)	0.296 (0.054)	0.256 (0.019)	0.233 (0.065)	0.173 (0.043)	0.139 (0.017)									
$p = 50$																		
VWQC	0.384 (0.067)	0.357 (0.049)	0.305 (0.020)	0.148 (0.054)	0.105 (0.037)	0.067 (0.011)	0.052 (0.038)	0.026 (0.017)	0.013 (0.005)									
DQC	0.443 (0.045)	0.442 (0.037)	0.445 (0.024)	0.318 (0.068)	0.300 (0.046)	0.271 (0.019)	0.171 (0.054)	0.143 (0.036)	0.130 (0.016)									
Centroid	0.435 (0.044)	0.427 (0.043)	0.400 (0.027)	0.276 (0.057)	0.240 (0.048)	0.196 (0.018)	0.153 (0.050)	0.103 (0.035)	0.103 (0.014)									
Median	0.435 (0.047)	0.437 (0.037)	0.400 (0.025)	0.295 (0.050)	0.271 (0.050)	0.229 (0.018)	0.171 (0.052)	0.149 (0.038)	0.130 (0.014)									
CQC	0.435 (0.050)	0.434 (0.047)	0.381 (0.028)	0.295 (0.069)	0.239 (0.049)	0.170 (0.030)	0.169 (0.067)	0.117 (0.041)	0.079 (0.020)									
EQC	0.428 (0.052)	0.421 (0.052)	0.360 (0.026)	0.246 (0.069)	0.189 (0.046)	0.138 (0.016)	0.099 (0.049)	0.062 (0.026)	0.044 (0.009)									
LDA	0.442 (0.046)	0.440 (0.041)	0.389 (0.025)	0.423 (0.060)	0.296 (0.049)	0.205 (0.018)	0.392 (0.076)	0.209 (0.051)	0.110 (0.016)									
KNN	0.438 (0.045)	0.439 (0.037)	0.466 (0.019)	0.366 (0.070)	0.328 (0.047)	0.315 (0.021)	0.218 (0.056)	0.214 (0.051)	0.154 (0.018)									
PLR	0.429 (0.045)	0.436 (0.043)	0.389 (0.025)	0.307 (0.065)	0.296 (0.055)	0.206 (0.018)	0.176 (0.060)	0.116 (0.040)	0.116 (0.017)									
SVM	0.438 (0.049)	0.435 (0.046)	0.388 (0.027)	0.299 (0.065)	0.258 (0.049)	0.203 (0.019)	0.172 (0.054)	0.132 (0.037)	0.101 (0.014)									
Bayes	0.448 (0.042)	0.449 (0.038)	0.424 (0.024)	0.365 (0.064)	0.329 (0.042)	0.209 (0.018)	0.234 (0.062)	0.176 (0.040)	0.086 (0.014)									
RF	0.388 (0.067)	0.342 (0.052)	0.270 (0.020)	0.149 (0.044)	0.075 (0.029)	0.037 (0.008)	0.052 (0.038)	0.019 (0.015)	0.004 (0.003)									
$p = 100$																		
VWQC	0.336 (0.068)	0.285 (0.053)	0.226 (0.021)	0.090 (0.048)	0.052 (0.024)	0.019 (0.007)	0.017 (0.024)	0.006 (0.010)	0.002 (0.002)									
DQC	0.426 (0.054)	0.420 (0.050)	0.293 (0.026)	0.253 (0.040)	0.253 (0.040)	0.194 (0.015)	0.061 (0.034)	0.043 (0.019)	0.050 (0.009)									
Centroid	0.412 (0.053)	0.392 (0.057)	0.338 (0.022)	0.203 (0.057)	0.159 (0.035)	0.110 (0.015)	0.061 (0.034)	0.043 (0.019)	0.034 (0.008)									
Median	0.422 (0.053)	0.412 (0.045)	0.363 (0.021)	0.231 (0.061)	0.190 (0.037)	0.147 (0.018)	0.080 (0.043)	0.058 (0.023)	0.052 (0.009)									
CQC	0.433 (0.048)	0.418 (0.048)	0.351 (0.028)	0.245 (0.068)	0.171 (0.046)	0.112 (0.016)	0.043 (0.043)	0.030 (0.010)	0.027 (0.005)									
EQC	0.419 (0.050)	0.393 (0.060)	0.300 (0.024)	0.180 (0.065)	0.116 (0.037)	0.068 (0.012)	0.038 (0.031)	0.018 (0.014)	0.011 (0.005)									
LDA	0.439 (0.047)	0.454 (0.035)	0.350 (0.025)	0.338 (0.067)	0.422 (0.050)	0.135 (0.014)	0.204 (0.072)	0.364 (0.066)	0.044 (0.010)									
KNN	0.431 (0.047)	0.426 (0.037)	0.449 (0.021)	0.302 (0.062)	0.302 (0.060)	0.222 (0.022)	0.150 (0.060)	0.097 (0.037)	0.082 (0.015)									
PLR	0.423 (0.046)	0.407 (0.052)	0.352 (0.024)	0.243 (0.069)	0.184 (0.039)	0.143 (0.016)	0.089 (0.042)	0.062 (0.025)	0.045 (0.010)									
SVM	0.422 (0.048)	0.399 (0.051)	0.351 (0.023)	0.224 (0.065)	0.200 (0.045)	0.115 (0.014)	0.077 (0.039)	0.048 (0.022)	0.035 (0.008)									
Bayes	0.432 (0.045)	0.446 (0.031)	0.401 (0.024)	0.334 (0.074)	0.284 (0.044)	0.152 (0.018)	0.168 (0.052)	0.110 (0.028)	0.047 (0.010)									
RF	0.357 (0.067)	0.261 (0.047)	0.170 (0.019)	0.086 (0.047)	0.026 (0.018)	0.007 (0.004)	0.024 (0.026)	0.004 (0.006)	0.000 (0.001)									
$p = 500$																		
VWQC	0.187 (0.056)	0.117 (0.034)	0.047 (0.011)	0.008 (0.018)	0.001 (0.004)	0.000 (0.002)	0.001 (0.006)	0.000 (0.000)	0.000 (0.001)									
DQC	0.366 (0.060)	0.339 (0.042)	0.323 (0.019)	0.051 (0.035)	0.028 (0.017)	0.016 (0.006)	0.000 (0.002)	0.000 (0.001)	0.000 (0.000)									
Centroid	0.334 (0.063)	0.271 (0.047)	0.158 (0.017)	0.029 (0.024)	0.010 (0.010)	0.002 (0.002)	0.000 (0.002)	0.000 (0.001)	0.000 (0.000)									
Median	0.353 (0.072)	0.302 (0.049)	0.203 (0.018)	0.045 (0.033)	0.025 (0.015)	0.009 (0.005)	0.001 (0.003)	0.001 (0.002)	0.000 (0.001)									
CQC	0.381 (0.060)	0.314 (0.054)	0.176 (0.022)	0.108 (0.051)	0.052 (0.024)	0.006 (0.005)	0.004 (0.008)	0.001 (0.002)	0.000 (0.000)									
EQC	0.326 (0.066)	0.245 (0.048)	0.118 (0.016)	0.013 (0.015)	0.005 (0.007)	0.001 (0.003)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)									
LDA	0.430 (0.047)	0.395 (0.049)	0.465 (0.022)	0.288 (0.077)	0.179 (0.048)	0.070 (0.043)	0.040 (0.045)	0.025 (0.016)	0.044 (0.006)									
KNN	0.388 (0.057)	0.436 (0.042)	0.352 (0.029)	0.132 (0.067)	0.093 (0.040)	0.041 (0.012)	0.004 (0.009)	0.002 (0.005)	0.002 (0.002)									
PLR	0.336 (0.064)	0.283 (0.043)	0.195 (0.021)	0.034 (0.024)	0.014 (0.012)	0.004 (0.003)	0.000 (0.000)	0.000 (0.001)	0.000 (0.000)									
SVM	0.341 (0.061)	0.285 (0.045)	0.169 (0.017)	0.035 (0.026)	0.012 (0.011)	0.004 (0.003)	0.000 (0.000)	0.000 (0.001)	0.000 (0.000)									
Bayes	0.430 (0.050)	0.411 (0.048)	0.321 (0.022)	0.211 (0.063)	0.138 (0.043)	0.043 (0.009)	0.022 (0.024)	0.005 (0.008)	0.001 (0.001)									
RF	0.228 (0.070)	0.070 (0.024)	0.111 (0.014)	0.009 (0.004)	0.000 (0.000)	0.003 (0.008)	0.000 (0.000)	0.000 (0.002)	0.000 (0.000)									