Joint Liability Model with Adaptation to Climate Change

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Abstract

This paper extends the application of ESG score assessment methodologies from large corporations to individual farmers' production, within the context of climate change. Our proposal involves the integration of crucial agricultural sustainability variables into conventional personal credit evaluation frameworks, culminating in the formulation of a holistic sustainable credit rating referred to as the Environmental, Social, Economics (ESE) score. This ESE score is integrated into theoretical joint liability models, to gain valuable insights into optimal group sizes and individual-ESE score relationships. Additionally, we adopt a mean-variance utility function for farmers to effectively capture the risk associated with anticipated profits. Through a set of simulation exercises, the paper investigates the implications of incorporating ESE scores into credit evaluation systems, offering a nuanced comprehension of the repercussions under various climatic conditions.

Keywords: Climate Change, ESG, Joint Liability, Sustainable Agriculture, ESE, Mean-Variance Utility.

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1 Introduction

1.1 Credit Scoring System

Recently, there has been a growing interest worldwide in improving agricultural productivity, driven by concerns mainly over increasing food insecurity and population growth, in the face of climate change. One potential solution to these challenges is to provide better access to institutional credit, which includes microcredit, commercial banks, and agricultural banks, as pointed out by Römer and Mußhoff (2018). This improved access can help rural households to mitigate risks and obtain essential resources and technologies to modernize agricultural practices, as well as facilitate the development of farm/nonfarm linkages. Khandker and Koolwal (2016), for instance, argue that institutional credit enables rural households to achieve better access to technology and inputs, which can result in a modernized agricultural system. By reducing risk, institutional credit can improve the overall resilience of rural households, while simultaneously bolstering the rural economy. Through this approach, rural households can leverage institutional credit to access new markets and improve their overall productivity, creating thereby a more sustainable agricultural system that is better equipped to meet the demands of a fastly changing world in the face of global climate change.

Microfinance credit scoring has traditionally been found to be an effective tool for improving risk management in the microfinance sector. The use of credit scoring models has been shown to increase the efficiency of loan assessment in Bumacov et al. (2014), leading to increased productivity of loan officers. Additionally, Schreiner (2009) highlighted that credit scoring models are objectively able to reflect complex causal relationships, which improves the accuracy of credit risk assessments. The automated nature of credit scoring further assists loan officers in making lending decisions, leading to a reduction in credit risk and loan default. The use of credit scoring also leads to cost reductions in the microfinance sector. Van Gool et al. (2012) recommended incorporating credit scoring into the lending process as a means to integrate statistical and human best practices, thereby enhancing the precision of lending decisions. This claim is supported by evidence assembled in the credit-scoring literature that exists for developing countries in various regions of the world including Africa, Asia, Eastern Europe, and Latin America, as mentioned in Römer and Mußhoff (2018). In most of these regions, credit scoring models have been found to be suitable for determining credit risk or supporting loan officers in their decision-making processes.

1.2 Joint Liability Contracts

Joint liability is a common feature of many lending schemes, especially in the context of microfinance. In a joint liability scheme, multiple borrowers are held collectively responsible for repaying a loan. This can have important implications for borrowers, lenders, and even the broader communities. Joint liability has been shown to be effective in promoting repayment rates and in reducing default risk. For example, a study by Ghatak and Guinnane (1999) find that joint liability groups had significantly lower default rates than individual borrowers. Similarly, Khandker et al. (1998) find that group-based lending in Bangladesh increased the number of loans made to poor borrowers and reduced the cost of delivering credit. Microfinance has gained widespread recognition as a powerful instrument for poverty reduction and technology adoption by offering credit access to underprivileged households. Group lending is among the factors that have contributed to microfinance's success. As mentioned earlier, joint liability group lending has been promoted as a means of reducing default rates and enhancing community welfare by addressing four important drawbacks of conventional individual lending: adverse selection, moral hazard, auditing expenses, and enforcement of Loan Terms. Nevertheless, recent studies have raised doubts about the superiority of joint liability group lending over individual lending in terms of its impacts on borrowers and lenders. Madajewicz (2011), for example, compares joint liability group lending to individual lending in the presence of heterogeneity among borrowers. The authors show that joint liability group lending can be beneficial in terms of reducing adverse selection and moral hazard, but it may not be superior to individual lending in terms of efficiency and borrower welfare. Group loans may allow for larger loans for the poorest borrowers, but individual loans may be more beneficial for wealthier, credit-constrained borrowers. For these borrowers, individual loans may lead to larger businesses than group loans. In addition, the findings in Rezaei et al. (2017) show that group size can affect lending efficiency. Larger groups are more reliable in repaying loans for riskier projects, but the group size cannot become too large. Joint liability has a positive effect on borrowers' repayment amount and welfare compared to individual liability, provided that group members impose strong punishment strategies against any strategically defaulting member. Rezaei et al. (2017) also examined the possibility of relaxing some of the simplifying assumptions made in the basic model of joint liability. Under the climate change context, Castaing (2021) investigates the impact of risk-sharing networks on the adoption of weather shock management strategies in Burkina Faso; particularly it studies how the principle of forced solidarity may reduce efforts to adopt techniques that mitigate exposure to climate change. The study finds that a system based on mutual assistance between farmers may reduce their incentives to adopt risk-mitigating strategies, both incrementally and transformationally. The study highlights the need for complementary mechanisms, such as insurance, to help farmers reduce their vulnerability to climate change.

As the world becomes increasingly aware of the impact of climate change on the agricultural sector, there is a growing emphasis on the importance of *sustainable* agricultural practices. In particular, the concept of green agriculture and sustainable development has gained considerable attention in recent times with the development of agricultural technology. This emphasis on sustainability is particularly relevant in the context of agricultural projects, where the potential for sustainable practices can have a major impact on the future of these projects.

1.3 Sustainable Scoring System

Cumming et al. (2017) emphasized that sustainable agriculture practices contribute to the achievement of the Sustainable Development Goals (SDGs) set by the United Nations, which aim at addressing global challenges such as poverty, hunger, climate change, and environmental degradation. For example, sustainable agriculture practices help to conserve natural resources such as water, soil, and biodiversity, which are critical for human survival and well-being. Sustainable farming also helps to reduce greenhouse gas emissions and mitigate climate change by sequestering carbon in the soil and reducing the use of synthetic fertilizers and pesticides that contribute to emissions. Moreover, sustainable agriculture practices can help farmers increase their productivity, reduce their costs, and enhance their resilience to climate change, market volatility, and other risks. Sustainable farming practices can also improve the quality and safety of the food produced, reducing food insecurity and malnutrition in local and global populations, as argued by Calicioglu et al. (2019).

There is a growing interest among academics and practitioners to integrate the ESG score in the financial decision-making process to evaluate the risk profile of large companies. This trend has inevitably also been extended to the agricultural sector, which calls for sustainable agricultural practices and the use of sustainable metrics to evaluate agricultural activities. In their report, Rivera-Ferre (2008) discussed the future of the agriculture sector in the economy and provided a definition for sustainable agricultural practices. According to this report, sustainable agricultural practices are those that are environmentally, socially, and economically sustainable. Firstly, from an environmental standpoint, it avoids inefficient use of water resources, loss of biodiversity due to converting natural habitat into agricultural land, imprudent use of chemical compounds that negatively affect local and regional water and air quality, emissions of greenhouse gases that disrupt the global climate, and losses in soil health and fertility. Secondly, from an economic perspective, sustainable agriculture enhances the economic viability of the agricultural sector by improving agricultural productivity and profitability, advancing agricultural innovation, facilitating farmers' access to markets and credit, improving farmers' ability to manage risk, and reducing food losses throughout the supply chain. Lastly, from a social perspective, sustainable agriculture enhances farmers' well-being, respects farmers' rights, promotes equitable opportunities in rural communities, and benefits all of society by improving food supply system resilience and enhancing nutrition and health. Zhang et al. (2021) discussed the challenges of defining and measuring sustainable agriculture and the need for independent and transparent measurements of national sustainability in this field. They developed a Sustainable Agriculture Matrix (SAM) to quantify national performance indicators in agriculture and investigated the trade-offs and synergies based on historical data for most countries of the world. The SAM provides a useful starting point for evaluating progress, identifying priorities for improvement, and informing national policies and actions toward sustainable agriculture.

In recent years, the importance of incorporating sustainability factors into financial decision-making has gained traction. While credit metrics for corporations have been extensively studied, there remains a noticeable gap in the assessment of individual borrowers, particularly farmers seeking credit to finance their agricultural operations. To address this gap, this paper introduces a comprehensive scoring system known as an Environmental, Sustainability, and Economics (ESE), which considers not only traditional credit metrics, including economic factors, but importantly also environmental and social aspects that contribute to sustainable agricultural practices.

The integration of sustainability factors into credit evaluation processes is vital for both lenders and borrowers in the agricultural sector. For lenders, the ESE framework enables the assessment of a borrower's long-term financial viability, identification of risks and opportunities associated with sustainability practices, and ensures alignment with ESE principles. From the borrower's perspective, it demonstrates a commitment to sustainability, improves access to capital, and provides opportunities for cost savings through energy and resource efficiency. The ESE framework is particularly relevant for individual farmers, as it acknowledges the importance of balancing environmental and sustainability goals with economic viability. Farmers are more likely to adopt sustainable practices if they can see clear economic benefits from doing so. In the agricultural sector, adopting sustainable farming practices often requires initial investments, such as new equipment, technology, or training. However, if these practices lead to cost savings, increased productivity, or other economic advantages in the long run, farmers will be more inclined to adopt them.

Furthermore, the ESE framework helps lenders better assess the long-term financial viability of farmers who adopt sustainable practices. By considering the potential economic benefits of environmentally friendly and socially responsible farming practices, lenders can make more informed decisions about extending credit to borrowers, ultimately encouraging the adoption of sustainable practices across the agricultural sector. In summary, this paper stresses the critical link between environmental sustainability and economic viability for individual farmers through the ESE framework. This comprehensive approach ensures that sustainable practices are not only environmentally and socially beneficial, but also economically viable, which is essential to promote widespread adaptation measures and to sustain a long-term success.

Monitoring, reporting, and verification (MRV) is a system used to track and assess the effectiveness of actions taken to address environmental or social issues. It involves the collection and analysis of data to monitor progress toward specific goals or targets, reporting of that information to stakeholders, and verification of the results by an independent third party to ensure accuracy and credibility. The purpose of an MRV system is to provide transparent and reliable information that can be used to guide decision-making, assess the effectiveness of interventions, and demonstrate accountability to stakeholders. Examples of areas where MRV systems are commonly used include climate change mitigation, biodiversity conservation, and social impact assessment. MRV systems can be applied at different scales, from individual projects to national or international initiatives. While the MRV system and the Individual-ESE Score Mechanism appear to be two separately distinct tools, they are actually interconnected with each other. Specifically, the data collected through the MRV system can be used to update and refine the ESG scoring system, ensuring that it remains relevant and accurate over time. For example, if the MRV system shows that a particular Individual-ESE factor has a more significant impact than previously thought, the Individual-ESE Score Mechanism can be adjusted to reflect this new piece of information. By using data from the MRV system to update the Individual-ESE scoring system, lenders and borrowers can therefore make better-informed decisions and ensure that their joint liability contract is sustainable over a long-term period. Therefore, the MRV system and the Individual-ESE Score Mechanism can complement each other well and together they can enhance the effectiveness of joint liability contracts. Tang et al. (2018) listed limitations of the MRV system, including ambiguity in the legal status of relevant policies and regulations, unclear requirements for monitoring plans, the lack of consistency and harmonization in accounting and reporting guidelines, and the lack of information technology (IT); This calls for further research in this area.

1.4 Motivation and Outline

The motivation for this paper is to holistically evaluate a more comprehensive profile of individual farmer borrowers and provide a more accurate assessment of their creditworthiness. Creating an individual-ESE score for individual borrowers in a joint liability problem can help to create more sustainable joint liability contracts. The resulting individual-ESE score would be integrated into a joint liability model as an additional criterion for assessing creditworthiness. In this paper, we take a first but critical step in performing a theoretical exercise by linking the Individual-ESE score to our theoretical models and providing lenders and borrowers with optimal scores and corresponding optimal group sizes by maximizing the total utility of a group of borrowers. We start our analysis with the simplest case, which only contains two borrowers in the group. Later we generalize it to a group of n individual borrowers. It is important to incorporate the Individual-ESE Score Mechanism into our joint liability model, which can also benefit both lenders and borrowers. For lenders, this mechanism provides a better understanding of the borrower's risk profile, enabling them to make informed lending decisions and, in the process, ensure more sustainable joint liability contracts. It also provides information about the optimal group size for lending. For borrowers, the mechanism provides a clearer picture of their ranking relative to their peers, allowing them to identify strengths and weaknesses and work on improving ESE scores. It also provides information about the number of peers needed for a group borrowing purpose, helping borrowers form groups that are more likely to succeed and have improved access to credit. Incorporating the Individual-ESE Score Mechanism into credit evaluation processes can thus lead to better-informed lending decisions, improved access to credit, and sustainable development outcomes.

In the remaining parts of this paper, we list the steps and potential metrics for building an individual-ESE score mechanism in Section 2. Next, we introduce the model setup and the assumptions in Section 3, and derive a set of results from proposed theoretical models of a group size of two and then a group size of n in Section 4 and Section 5 respectively. In Section 6, we refine the utility function to a mean-variance form, which incorporates both the expected value of the return and risk simultaneously in the face of climate change. The resulting model does not have closed-form analytical expressions and, as a result, a set of simulation exercises is provided through a set of climate scenarios.

2 The Mechanism of Individual-ESE Scores

In order to uphold accountability for sustainable agriculture commitments and provide valuable insights for policymaking, conducting regular and transparent assessments is of paramount importance. However, there are a number of variations in the definitions of sustainable agriculture, and also a limited number of quantitative assessments of agricultural sustainability that currently are available, according to Agovino et al. (2018). While some experts view sustainable agriculture as a collection of management strategies, others see it as an ideology or a specific set of goals. Despite this, there is an increasing consensus that sustainable agriculture should be evaluated based on its environmental, economic, and social impacts, as emphasized by Zhang et al. (2021). Several frameworks and indicators have been developed. Existing sustainable agriculture indicators mainly assess the environmental impacts of agriculture production, and many have limited data availability. Although various organizations have called for monitoring agriculture globally, actual datasets to enable trend assessments are still currently lacking. The absence of a consistent quantification of agricultural sustainability across multiple dimensions impedes the successful identification of undesirable trade-offs and the development of win-win solutions. Given these challenges, relying on literature-based metrics is a reasonable starting point in developing a sustainable agricultural credit scoring system. Literature-based metrics are metrics that have been previously used and validated in other studies or publications and are therefore based on a larger sample size and a more diverse set of variables than any single data collection effort. Having literature-based metrics about agricultural credit scoring systems allows us to apply them when we obtain real data or use them to create a scoring system by using synthetic data. By compiling these synthetic metrics and adjusting them to fit the specific needs of the agricultural sector, we can create a credit scoring system that is more comprehensive and accurate than any single data collection effort.

The ultimate objective of this research is to eventually establish a comprehensive and sustainable agricultural credit scoring system at the individual level, which encompasses three critical aspects: Economic, Social, and Environmental. This system will be called an individual-ESE score. While traditional credit scoring systems focus mainly on borrowers' economic and financial status, our system will take into account a broader range of factors that contribute to sustainability. We recognize the importance of economic metrics and acknowledge that they are crucial to the classical credit scoring system. Therefore, we will use some of the commonly used economic metrics in the related literature as a foundation for our economic metrics. Presented below are synopses of selected academic papers that have conducted research on the agricultural credit scoring system. (1) Römer and Mußhoff (2018) investigated the use of weather data in agricultural credit scoring for microfinance institutions, finding that simple correlation analysis between weather events and loan repayment is insufficient for forecasting future repayment behavior. (2) Turvey (1991) reviewed the use of credit scoring for agricultural loans and its application to agricultural finance, identifying the importance of accurate data and the need for specialized credit scoring models for agricultural lending. (3) Lufburrow et al. (1984) developed a credit scoring model for farm loan pricing, utilizing statistical methods to determine the significant factors affecting loan repayment and default rates. (4) Limsombunchai et al. (2005) analyzed credit scoring for agricultural loans in Thailand, identifying the factors that affect credit scores and their impact on loan approval and interest rates. (5) Bumacov et al. (2014) examined the use of credit scoring in microfinance institutions, reporting that it can improve the outreach and efficiency of microfinance operations, but also highlighting the need for effective risk management strategies. (6) Mutamuliza et al. (2021) investigated the determinants of smallholder farmers' participation in microfinance markets in Rwanda, identifying factors such as gender, education, and access to information that can affect farmers' ability to access credit. We provide a summary of the key metrics employed in their credit scoring models as the foundations of economic metrics in Table 1.

Zhang et al. (2021) presents a variety of useful metrics for the environmental and social pillars. With additional information provided in Fuglie et al. (2016) and Movilla-Pateiro et al. (2021), we have compiled a comprehensive list of potential metrics for these pillars, along with detailed descriptions and methods of measurement. The metrics are shown in Table 2.

Metric	Description	Used as a Metric in the Scoring System
Age	Age of the applicant in years	(1), (4)
Gender	Male, Female, Others	(1), (6)
Education	Education level of the applicant	(6)
Collateral	Collateral in thousands	(1), (3), (4)
Assets	Assets in thousands	(1), (2), (3), (4), (5)
Debt	Debt to other banks in thousands	(1), (2), (3), (4), (5)
Deposit	Deposit in the bank account in thousands	(1), (2), (5)
Income	Monthly business and household income in thousands	(1), (2), (3), (4), (6)
Marital status	Single, Married, Divorced, Others	(1)
No. family members	Number of family members	(1)
Loan Size	Loan amount in thousands	(1), (4), (6)
Loan duration	Loan Duration in Months	(4)
Repayment ability	Projected net cash flow plus projected inventory divided by a total line of credit	(1), (2), (3)
Repayment history	The average of loan principal repaid divided by principal due over the past three years	(1), (3)
Tenure	The ratio of acres owned and farmed to total acres farmed	(3), (6)
Resident	1 if applicant is a resident; 0 otherwise	(1),
Working experience	Working experience in current profession in months	(1),
Region and Country	Region and Country of the Agricultural Business	(1), (2), (4), (6)
Distance	Distance to the microfinance institution in km	(6)

Table 1: A List of Economic Metrics Applied in Literature

Developing a comprehensive agricultural credit scoring system requires careful consideration of the various metrics that influence creditworthiness. Below is a scheme for generating a composite score based on multiple metrics, including how to assign weights to each metric. We aim at eventually developing a comprehensive ESE scoring system by either obtaining real data or generating synthetic data according to this scheme in the future.

1. Identify and gather relevant metrics: Begin by identifying the metrics that are most critical to assessing creditworthiness in agricultural lending, such as past repayment history, crop yields, farm management practices, and market conditions.

2. Determine the relative importance of each metric: Assign weights to each metric based on its perceived importance in predicting creditworthiness. This can be done through a combination of expert judgment and statistical analysis of historical data. Weights may also be assigned based on the level of risk exposure associated with each ESE factor, with higher weights assigned to metrics that pose a greater risk to a farmer's performance. Notable, the overall Thomson Reuters ESG Score is calculated by applying an automated, factual logic that determines the weight of each category. Each category consists of a different number of measures. The count of measures per category determines the weight of the respective category.

3. Normalize the metrics to ensure that each has the same scale, range, and units of measurement. This can be achieved through methods such as standardization or normalization.

4. Once the weights and normalized values of each metric have been determined, calculate the composite score by multiplying the weight of each metric by its normalized value and summing the products.

5. Validate the composite score by comparing it to actual default rates or repayment history. Refine the weights and metrics as needed based on the validation results.

6. Continuously monitor and update the credit scoring system by incorporating new data and adjusting weights and metrics as needed to improve the accuracy and predictive power of the model.

7. By following the above steps, a comprehensive agricultural credit scoring system can be developed that considers multiple metrics and assigns weights based on their relative importance in predicting creditworthiness.

Sector	Metric	How to Measure
E	Soil health	Measuring the quality and health of the soil, including factors such as nutrient levels, organic matter content, and erosion rates. Soil testing kits could be used to measure.
E	Water Consumption	Modelled evapotranspiration using climate data (adjusted for crop type and water availability). Water usage tracking software could be used as a tool to measure.
E	Water Quality	N & P concentration (measured in stream, or modeled using land cover for landscapes)
E	Habitat Conversion	Remotely sensed land use (% of study area covered by natural habitat)
E	Habitat Composition	Remotely sensed land use (no. of land cover classes, indicating habitat diversity)
E	Habitat Connectivity	Remotely sensed land use (calculated connectivity score)
E	Species Richness	Field samples (richness relative to reference natural landscape)
E	Species Abundance	Field samples (abundance relative to reference natural landscape, i.e. the most undisturbed nearby similar habitat)
E	Yield / Area	Harvested mass of crop per unit area over a 5-year rolling average, perhaps using geometric mean to penalize variation in yield
E	Use of pesticides and chemicals	Liters of pesticide or chemicals used per hectare
E	Carbon Footprint	The carbon footprint for different crop types are shown in Sah and Devakumar (2018). Measuring the amount of greenhouse gas emissions generated from the farmer's activities, including fertilizer use, transportation, and energy use.
S	Health & safety issues	Whether the applicant ever had safety issues while engaging in agricultural activities (0 if no, 1 if yes).
S	Insuring health	Whether the applicant has personal accident insurance or commercial insurance (0 if no, 1 if yes).
S	Labor practices	Measuring the fairness and safety of labor practices on the farm, including minimum wage compliance, safety regulations, and worker satisfaction.
S	Community engagement	Measuring the level of engagement and support the farmer provides to the local community, including economic benefits, charitable contributions, and community outreach.
S	Ethical business practices	Measuring the farmer's adherence to ethical business practices, including transparency, honesty, and accountability.
S	Compliance	Measuring the farmer's compliance with local and national regulations, including environmental, labor, and health and safety regulations.

Table 2: A List of ESG Metrics

3 A Framework for ESE-Joint Liability Model

The objective of the section is to examine a microcredit lending scenario in which a benevolent lender extends loans to a group of self-selected micro-entrepreneur borrowers who are jointly liable for repayment of the loan. Although each member of the group is given an individual loan, the group is collectively responsible for repaying the loans, and if the total loan is repaid, the entire group can qualify for a subsequent loan. The loans are then invested in a set of projects, which have an equal chance of success and can be either disjoint or correlated. It is assumed that the group members know the realized output of each project, while the lender does not. Both the lender and the borrowers have to make decisions in this scenario. The lender has to decide on the optimal contract structure, the degree of joint liability, and the level of punishment in the case of strategic default, while the borrowers have to decide whether to repay the loan or default.

One of the primary aims of this study is to determine the optimal group size that maximizes the benefit for the borrowers while ensuring that the lender breaks even. A larger group size can have a positive effect on the expected repayment amount, as more people are liable for repaying defaulted payments, thus ensuring a higher rate of repayment. However, a larger group size can also be a threat to repaying members, as they must repay all defaulting peers' repayments when everyone else in the group defaults on repayment. Therefore, the section proposes a method to determine the optimal group size that enables joint liability to achieve its maximum loan ceiling. Another objective of this section is to derive the optimal Individual-ESE Score. The importance of a scoring system for both lenders and borrowers cannot be overstated. For lending companies, having access to a specific scoring mechanism for individual borrowers can provide a better understanding of their creditworthiness and repayment ability. This information is essential for insurance and lending companies in determining the risk associated with lending to a particular borrower. Additionally, the optimal group size of borrowers can be determined using the scoring system, enabling the company to mitigate risk by lending to a diversified portfolio. For borrowers, having a scoring system can give them a clearer picture of their ranking and eligibility for loans. This information is critical for farmers looking for group borrowing purposes as it can help them determine the number of peers needed to secure a loan and improve their chances of obtaining funding. Therefore, both lenders and borrowers can benefit from the implementation of a comprehensive scoring system.

3.1 Model Assumptions

Prior to constructing the model, it is necessary to state the following assumptions:

1. The borrowers (who are the individual farmers) produce an agricultural product. For each borrower, the probability of high production (i.e., a successful project) is the same as e and the probability of low production (i.e., a failed project) is $1 - e$ respectively.

2. A higher individual-ESE score means the farmer has a stronger ability to succeed in his/her project due to the high-quality comprehensive evaluation. The probability of high yield is assumed to be linearly related to the borrower's ESE score; that is we assume that there is a linear relationship between e and ESE score: $e = k \cdot ESE + b$. The parameter k represents the weight or strength of the impact of the borrower's ESE score on the probability of high yield. A higher value of k indicates that a higher ESE score leads to a more significant positive impact on the probability of success. This aligns with the idea that a borrower's positive environmental, social, and economic practices contribute to project success. In other words, higher ESE scores suggest better alignment with responsible practices. The constant b represents the baseline or constant contribution of factors beyond an individual's control, such as climatic and geographical conditions. Even if a borrower has a lower ESE score, this constant term acknowledges that external factors can still influence the project's success. It's essential to account for these factors to avoid unfairly penalizing borrowers for circumstances beyond their control.

   The selection of the linear equation $e = k \cdot ESE + b$ is rooted in both the characteristics of the variables involved and a keen understanding of practical applicability. Notably, the non-negativity of k, b, and ESE ensures that the resulting probability, denoted as e, maintains its intrinsic non-negative nature. As ESE encompasses a continuous range from 0 to 100, it inherently aligns with this non-negative property. It is important to stress that the intuitive requirement for e to lie between 0 and 1, which is a fundamental property of a probability, is elegantly met by setting $b = 0$ and $k = \frac{1}{100}$. By adhering to these values, the probability of e adheres to the established probability range. In addition, the model accommodates real-world scenarios by allowing for a non-zero baseline influence ($b > 0$), often linked to exogenous factors. To this end, a modest value such as $b = 0.3$ can be accommodated without violating the probability constraints. Notably, this adjustment merely necessitates setting $k = \frac{0.7}{100}$, thereby guaranteeing that e remains within the bounds of a probability. While it's acknowledged that e cannot span the interval from 0 to 0.3, this limitation is justified by underscoring e as a representation of the probability of high yield, which is inherently a positive entity in practical settings. Hence, the rationale underlying the proposed equation effectively marries mathematical coherence with the nuanced demands of reality.

The exploration of using a sigmoid function to model e as $e = \frac{1}{1+\exp^{-ESE}}$ or $e = \frac{1}{1+\exp^{-(k \cdot ESE+b)}}$ is an intriguing avenue to consider. However, upon a closer examination, a critical observation emerged: under the assumption that k, b, and ESE are non-negative, the sigmoid-based approach resulted in a constraint where e precludes the values below 1/2. This deviation from the alignment with realistic scenarios and the entire spectrum of probability values was deemed undesirable. Thus, while the sigmoid formulation may possess merits in other contexts, its inherent restriction on precluding the probability values below 1/2 is at odd with the overarching goal of accurately representing the probability of high yield across its entire spectrum. As a result, the decision was made to retain the linear equation $e = k \cdot ESE + b$, which adheres to both the established criteria and delivers the sought-after flexibility in representing diverse probability values.

3. The amount borrowed by each borrower is denoted by L, and the amount for this borrower to be repaid is denoted by w. The unit selling price of the product is denoted by p, and the total income of high production is denoted by $p\overline{Y}$, while the total income of low production is denoted by $p\underline{Y}$.

4. High-yield borrowers repay their loans and low-yield borrowers pay back all the income they earn. As long as this group repays the total loans, this group can continue to get the subsequent loans. In the event that all borrowers are unsuccessful in their projects, the group will not be eligible for any future loans. This may result in a negative credit history for all members of the group, which could lead to further restrictions such as being unable to use public transportation or purchase high-cost items. Therefore, if all borrowers fail, they may have to use all of their earnings to avoid receiving negative credit reports, even though their total earnings will be less than the total repayments.

5. To increase the successful probability e, the borrower incurs additional costs such as purchasing green pesticides, advanced tools, etc. A convex disutility function, denoted by $C(e)$, is included in the specification of the total utility function.

3.2 Joint Liability Contracts with a Group Size of Two

Under the joint liability agreement, each farmer's ex-ante expected profit π_i can be written as

$$\pi_i = e^2[p\overline{Y} - w] + e(1-e)[p\overline{Y} + p\underline{Y} - 2w] - C(e) \tag{3.1}$$

To clarify, both farmers have a chance of achieving a successful harvest $\overline{Y}$ with a probability of e^2, which gives them the profit of $p\overline{Y} - w$. The probability of one farmer defaulting is

$e(1-e)$, which means that farmer i would receive her own surplus from a successful harvest minus the other farmer's deficit, $p\overline{Y}+p\underline{Y}-2w$. If both producers fail at the same time, they will pay all of their incomes to the lenders, which means the profit is zero. The detailed profit distributions can be summarized in Table 3.

Case Description	**Probability**	**Profit=Earning-Payment**
The farmer succeeds, another farmer succeeds too.	e^2	$p\overline{Y}-w$
The farmer succeeds, another farmer fails.	$e(1-e)$	$p\overline{Y}+p\underline{Y}-2w$
The farmer fails, another farmer succeeds.	$(1-e)e$	$p\underline{Y}-p\underline{Y}=0$
The farmer fails, another farmer fails.	$(1-e)^2$	$p\underline{Y}-p\underline{Y}=0$

Table 3: Profit Distributions

Before setting the total objective (utility) function for the whole group, we first specify the following constraints for the optimization program.

1. If the group wants a subsequent loan from the lender, the group's total repayment amount must be affordable even in the worst-case scenario when everyone else's project has failed except his/hers:

$$\text{Constraint 1: } 2w \leqslant p\overline{Y}+p\underline{Y} \tag{3.2}$$

2. Strategic default in a joint liability contract occurs when one or more parties intentionally chooses or choose to default on their obligations, even though they have the ability to pay. This is often done to gain a strategic advantage, such as forcing the other parties to pay a greater share of the debt or renegotiating the terms of the contract. To avoid a strategic default for each successful borrower, the payoff of strategically defaulting cannot be larger than the payoff of repaying and being refinanced:

   (a) Assuming the other borrower also succeeds, this borrower will only be responsible for their own payment. In such a scenario, the earnings plus the discounted cost of the next loan would at least be equal to the earnings of strategically defaulting.

$$\text{Constraint 2: } p\overline{Y} \leqslant p\overline{Y}-w+\delta L \tag{3.3}$$

   (b) If the other borrower fails, then this borrower needs to pay for himself/herself as well as part of the other borrower's repayment amount. Now, the earnings plus the discounted cost of the next loan would also at least equal the earnings of strategically defaulting.

$$\text{Constraint 3: } p\overline{Y} \leqslant p\overline{Y} + p\underline{Y} - 2w + \delta L \tag{3.4}$$

3. The lender must be able to sustain the lending game over the periods or at least break even. Thus, the expected repayment amount of each borrower has to be at least as large as $L(1+\epsilon)$, where ϵ is a risk-free rate. Given that all constraints have been established under the presumption that the group will successfully repay the total loans, to ensure their eligibility for subsequent loan rounds, we have not factored in the scenario where all borrowers within the group experience failure on the LHS of Constraint 4.

$$\text{Constraint 4: } w\left[1-(1-e)^2\right] \geqslant L(1+\varepsilon) \tag{3.5}$$

Since $p\underline{Y} - w < 0$, it is easy to see that constraint 3 is tighter than constraint 2. Hence, we will only consider constraint 1, constraint 3, and constraint 4. Firstly, π_i and w have a strictly negative relationship as can be observed from Equation 3.1, then if we want to maximize π_i, w has to be set at a minimum value. From constraint 4, the minimum value of w should be set at

$$w = \frac{L(1+\varepsilon)}{1-(1-e)^2} \tag{3.6}$$

.

Next, we can substitute $w = \frac{L(1+\varepsilon)}{1-(1-e)^2}$ into constraints 1, and then it can be rewritten as:

$$2 \cdot \frac{L(1+\varepsilon)}{1-(1-e)^2} \leqslant p\overline{Y} + p\underline{Y}, \tag{3.7}$$

and the first loan ceiling bound can be expressed in Equation 3.8 as:

$$L_1 \leqslant \frac{p\overline{Y} + p\underline{Y}}{2(1+\varepsilon)}\left[1-(1-e)^2\right]. \tag{3.8}$$

After substituting $w = \frac{L(1+\varepsilon)}{1-(1-e)^2}$ into constraints 3, it can be rewritten as:

$$p\overline{Y} \leqslant p\overline{Y} + p\underline{Y} - 2\frac{L(1+\varepsilon)}{1-(1-e)^2} + \delta L, \tag{3.9}$$

$$\left(\frac{2(1+\varepsilon)}{1-(1-e)^2} - \delta\right) L \leqslant P\underline{Y}, \tag{3.10}$$

and thus we can obtain another bound of the loan ceiling as in Equation 3.11:

$$L_2 \leqslant \frac{p\underline{Y}}{\frac{2(1+\varepsilon)}{1-(1-e)^2} - \delta} \tag{3.11}$$

Now, we substitute $w = \frac{L(1+\varepsilon)}{1-(1-e)^2}$ into the objective function 3.1 and replace $e = k \cdot E + b$. The ex-ante expected profit function of an individual borrower (farmer) can be rearranged as

$$\begin{aligned}
&\max_E e^2(p\overline{Y} - w) + e(1-e)[p\overline{Y} + p\underline{Y} - 2w] - C(e) \\
&= e^2 p\overline{Y} + e(1-e)(p\overline{Y} + p\underline{Y}) - \left(2e - e^2\right) w - C(e) \\
&= e^2 p\overline{Y} + e(1-e)(p\overline{Y} + p\underline{Y}) - L(1+\varepsilon) - C(e) \\
&= (kE+b)^2 \cdot p\overline{Y} + (kE+b)(1-kE-b)(p\overline{Y} + p\underline{Y}) - L(1+\varepsilon) - C(e)
\end{aligned} \tag{3.12}$$

To simplify the problem further, we set $C(e) = \frac{1}{2}ce^2$ (where the cost parameter c is used to scale the total cost) as the disutility function. The disutility function $C(e) = \frac{1}{2}ce^2$ is a commonly used function in economics to represent the cost or disutility associated with effort or work. See e.g. Clark and Oswald (1996) and Kahneman et al. (1997). The function is quadratic, meaning that as effort increases, the cost or disutility of that effort increases at an increasing rate. In the context of an individual agricultural farmer, it is reasonable to use a disutility function that acknowledges that while some effort may be necessary and satisfying, there is a point at which additional effort becomes more and more unpleasant or burdensome. For instance, a farmer may be willing to put in extra effort to cultivate crops or tend to livestock, but as the required effort increases, the farmer may experience fatigue, stress, and reduced productivity, which can negatively impact their overall well-being. Furthermore, the quadratic form of the disutility function is also supported by empirical evidence on the relationship between effort and well-being. It is reasonable to assume that people tend to experience a phenomenon known as a diminishing marginal utility of effort, which implies that the additional unit of well-being gained by exerting more effort decreases as the level of effort increases.

3.3 Some Propositions

Now that we have listed the model assumptions and set up for our analysis, we can move on to exploring key propositions of this model. These propositions provide insights into the optimal individual-ESE scores, optimal group size, and loan ceiling constraints.

Proposition 1: Find the optimal individual-ESE score.

Finding the optimal ESE score for both lenders and borrowers in a joint liability problem is critical because it can help address the challenge of promoting sustainability in agricultural production while ensuring access to credit for small-scale farmers. By incorporating environmental and social factors into credit evaluation systems, it can incentivize farmers to adopt sustainable practices that can lead to positive environmental outcomes and long-term economic benefits. At the same time, lenders can mitigate the risks associated with lending to farmers by incorporating sustainability factors into their credit evaluation process.

To find the optimal individual-ESE score (noted in formulas as E) score, we take the first-order partial derivative w.r.t E of the ex-ante expected payoff function to obtain the following result:

$$\begin{aligned}\frac{\partial \pi}{\partial E} &= 2(kE+b)\cdot k(p\overline{Y}) + (k - 2(kE+b)\cdot k)\cdot[p\overline{Y} - p\underline{Y}] - k\cdot\frac{\partial C(e)}{\partial e} \\ &= kp\overline{Y} - [k - 2(kE+b)\cdot k]p\underline{Y} - kc\cdot(kE+b) = 0.\end{aligned} \tag{3.13}$$

After rearranging of terms, we can get the optimal individual-ESE score (E is between 0 and 100) as

$$E = \frac{p\overline{Y} + (2b-1)p\underline{Y} - cb}{ck - 2kp\underline{Y}} \tag{3.14}$$

Proposition 2: Reduce the two constraints about loan ceiling to only one constraint.

The two loan ceiling constraints (constraint 1 and constraint 3) arise from analyzing the success or failure of the other co-loaners assuming our own success, but most of the previous studies in this area have not explored the connection between these constraints. For instance, in Rezaei et al. (2017), they found that a feasible range of loan ceiling depends on the borrower's discount rate δ. Nevertheless, it is possible to reduce these two constraints to a single constraint in our model.

Recall the two constraints about the loaning ceiling in Equation 3.8 and Equation 3.11, named respectively as L_1 and L_2.

$$L_1 \leqslant \frac{p\overline{Y} + p\underline{Y}}{2(1+\varepsilon)}\left[1-(1-e)^2\right]$$

$$L_2 \leqslant \frac{P\underline{Y}}{\frac{2(1+\varepsilon)}{1-(1-e)^2} - \delta}$$

Next, we show that L_1 is always larger than L_2, which means that we can only use L_2 as the final loan ceiling bound. It simplifies the problem and potentially makes it easier to analyze and solve the model. In addition, L_2 takes δ into consideration, which may include more information and additional key factors. This conclusion can also be illustrated by Figure 1.

Proof:

If $L_1 \leqslant L_2$, then

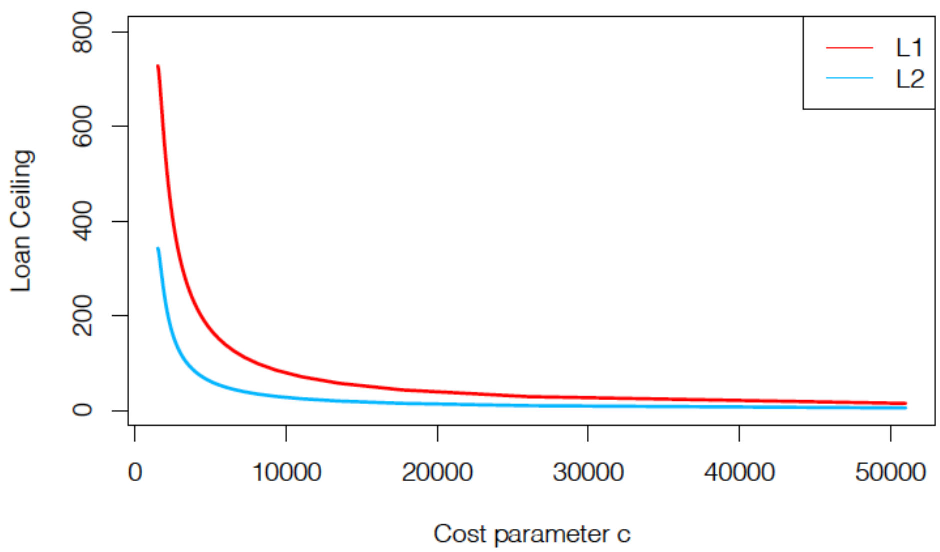

Figure 1: The Relationship between Two Loan Ceiling Constraints

$$\frac{p\overline{Y}+p\underline{Y}}{2(1+\varepsilon)}\left[1-(1-e)^2\right] \leqslant \frac{p\underline{Y}}{\frac{2(1+\varepsilon)}{1-(1-e)^2}-\delta} \tag{3.15}$$

After rearranging the terms, we get:

$$\delta \geqslant \frac{\overline{Y}\cdot 2(1+\varepsilon)}{[1-(1-e)^2]\,(\overline{Y}+\underline{Y})} > 1 \tag{3.16}$$

From the above results, if $L_1 \leqslant L_2$, the discount factor δ is always greater than 1, which leads to a contradiction. So, we conclude that $L_1 > L_2$.

Proposition 3: A higher individual-ESE score leads to a higher loan ceiling.

Next, we examine the relationship between the ESE score and the loan ceiling. Intuitively, a borrower with a high-quality ESE score should be eligible for a higher loan amount, and therefore the loan ceiling should also be higher.

Since L_1 is always greater than L_2, we will use L_2 to determine the loan ceiling, which defines the maximum value of L_2 to be the loan ceiling.

$$\begin{aligned} L_2 &= \frac{p\underline{Y}}{\frac{2(1+\varepsilon)}{1-(1-e)^2}-\delta} \\ &= \frac{p\underline{Y}\,[1-(1-e)^2]}{2(1+\varepsilon)-\delta\,[1-(1-e)^2]} \\ &= \frac{p\underline{Y}\,(2e-e^2)}{2(1+\varepsilon)-\delta\,(2e-e^2)} \end{aligned} \tag{3.17}$$

Then, we take a partial derivative of L_2 w.r.t. E to get:

$$\begin{aligned}\frac{\partial L_2}{\partial E} &= \frac{\partial L_2}{\partial e}\cdot\frac{\partial e}{\partial E}\\ &= \frac{p\underline{Y}(2-2e)\cdot[2(1+\varepsilon)-\delta\,(2e-e^2)] - p\underline{Y}\,(2e-e^2)\,(-\delta)\,(2-2e)}{[2(1+\varepsilon)-\delta\,(2e-e^2)]^2}\times k\end{aligned}\tag{3.18}$$

Obviously, the denominator is greater than 0. The numerator is also greater than 0, which can be illustrated by writing out the following result:

$$\begin{aligned}\text{Numerator } &= kp\underline{Y}(2-2e)\times\left[2(1+\varepsilon)-\delta\left(2e-e^2\right)+\delta\left(2e-e^2\right)\right]\\ &= kp\underline{Y}(2-2e)[2(1+\varepsilon)]>0\end{aligned}\tag{3.19}$$

A borrower with a higher optimal ESE score, which means he/she has better overall performance, will be perceived as less risky by the lender. As a result, the borrower will be offered a larger loan ceiling, which is the maximum amount of loan that a lender is willing to provide to a borrower. In turn, if a borrower wants to obtain a larger loan amount, he/she may be motivated to improve the ESE performance.

3.4 Joint Liability Contracts with a Group Size of n

In this sub-section, we present a model which formalizes the assumption that the group of borrowers is collectively responsible for the entire group loan (i.e., nw). The expected utility of a borrower i who adopts a repayment strategy during any period when they receive financing is specified as

$$\begin{aligned}\max_E\ \pi_i &= \sum_{k=0}^{n-1}\begin{pmatrix} n-1\\ k\end{pmatrix}\cdot\left[e^{n-k}(1-e)^k\right]\cdot\left[p\overline{Y}-w-\frac{k(w-p\underline{Y})}{n-k}\right]-C(e)\\ &= e(p\overline{Y}-w)-(w-p\underline{Y})\cdot\sum_{k=0}^{n-1}\begin{pmatrix} n-1\\ k\end{pmatrix}\cdot\left[e^{n-k}(1-e)^k\right]\cdot\frac{k}{n-k}-C(e)\\ &= e(p\overline{Y}-w)-(w-p\underline{Y})\cdot((1-e)-(1-e)^n)-C(e)\\ &= ep\overline{Y}-w(1-(1-e)^n)+p\underline{Y}\,((1-e)-(1-e)^n)-C(e)\\ &= (kE+b)p\overline{Y}-w\,(1-(1-kE-b)^n)+p\underline{Y}\,[(1-kE-b)-(1-kE-b)^n]-C(e)\end{aligned}\tag{3.20}$$

Constraints similar to those in the case of a group size of two are as follows:

1. There is at least one borrower who is successful and capable of paying the entire group's repayment amount.

$$nw\leqslant p\overline{Y}+(n-1)p\underline{Y}\tag{3.21}$$

2. For each successful borrower, the payoff of strategically defaulting cannot be larger than the payoff of repaying and being refinanced.

$$p\overline{Y} \leqslant p\overline{Y} + (n-1)p\underline{Y} - nw + \delta L \tag{3.22}$$

3. The lender must be able to sustain the lending game over the periods or at least break even. Thus, the expected repayment amount of each borrower has to be at least as large as $L(1+\varepsilon)$, where ε is the risk-free rate. Still, given that all constraints have been established under the presumption that the group will successfully repay the total loans, in order to ensure their eligibility for subsequent loan rounds, we have not factored in the scenario where all borrowers within the group experience failure on the LHS of this constraint.

$$w\left[1-(1-e)^n\right] \geqslant L(1+\varepsilon) \tag{3.23}$$

Similarly, we can get $w \geqslant \frac{L(1+\varepsilon)}{1-(1-e)^n}$. Thus, the value of w should be set as $\frac{L(1+\varepsilon)}{1-(1-e)^n}$ due to the negative relationship between w and π_i.

After substituting $w = \frac{L(1+\varepsilon)}{1-(1-e)^n}$ in the objective function (Equation 3.20), the ex-ante expected payoff function for the borrower i in a size n joint liability group is represented as

$$\max_E \pi_i = (kE+b)p\overline{Y} - L(1+\varepsilon) + p\underline{Y}\left[(1-kE-b)-(1-kE-b)^n\right] - C(e) \tag{3.24}$$

Proposition 4: The optimal individual-ESE score and group size n have a negative relationship.

A higher ESE score indicates a lower default risk, which means that the borrower is more creditworthy and can obtain a loan with a smaller group size. On the other hand, each member of the group is jointly responsible for the repayment of the loan, and if one member defaults, the other members will have to repay the outstanding amount. Thus, a low-quality borrower may choose to loan with a larger group to reduce the probability of default and minimize the potential burden on any one individual. Next, we verify this intuition based on the following derivations.

Take the first-order partial derivative of the expected payoff w.r.t E to get the optimal individual-ESE score for a borrower:

$$\frac{\partial \pi}{\partial E} = kp\overline{Y} + p\underline{Y}k\left[n(1-kE-b)^{n-1}-1\right] - k\frac{\partial c(e)}{\partial e} = 0 \tag{3.25}$$

Since it is hard to find an explicitly analytical form of n and E, we adopt an Implicit function derivation. Let $E(n)$ replace E and take the partial derivative w.r.t. n on both sides of $kp\overline{Y} + p\underline{Y}\left[n(1-kE-b)^{n-1}-1\right] - kc(kE+b) = 0$. Then we can get the following result:

$$p\underline{Y}\left\{(1-kE(n)-b)^{n-1} + n \cdot \frac{\partial\left[e^{(n-1)\log(1-kE(n)-b)}\right]}{\partial n}\right\} - C\left[k\frac{\partial E(n)}{\partial n}\right] = 0 \tag{3.26}$$

where,

$$\begin{aligned}\frac{\partial\left[e^{(n-1)\log(1-kE(n)-b)}\right]}{\partial n} &= e^{(n-1)\log(1-kE(n)-b)} \cdot \left[\frac{(n-1)\cdot(-k)\frac{\partial E(n)}{\partial n}}{1-k\cdot E(n)-b} + \log(1-kE(n)-b)\right] \\ &= (1-k(n)-b)^{n-1} \cdot \left[\frac{(n-1)k}{1-kE(n)-b} \cdot \left(-\frac{\partial E(n)}{\partial n}\right) + \log(1-kE(n)-b)\right]\end{aligned} \tag{3.27}$$

After reorganizing the formula, we get the partial derivative of $E(n)$ w.r.t n as

$$\frac{\partial E(n)}{\partial n} = \frac{p\underline{Y}\left\{(1-e)^{n-1}[1+n\cdot\log(1-e)]\right\}}{p\underline{Y}\left[n\cdot(1-e)^{n-1}\cdot\frac{(n-1)k}{1-e}\right] + ck} \tag{3.28}$$

Obviously, the denominator in Equation 3.28 is greater than 0. When $n \geqslant 1$ and $e \in (0,1)$, then $n\log(1-e) < -1$, and $(1-e)^{n-1}[1+n\cdot\log(1-e)] < 0$. Thus, the numerator is less than 0. As a result, $\frac{\partial E(n)}{\partial n} < 0$, which proves the optimal individual-ESE score and group size n have a negative relationship.

In light of the negative correlation between optimal individual-ESE scores and group size, it is recommended that insurance companies prioritize lenders with higher ESE scores when the group size is smaller to mitigate potential risks. Borrowers with high scores are typically perceived to exhibit positive economic, environmental, and social performance, which leads to higher expectations for sustained success and repayment probability over the long term. Despite the relatively small group size, lenders may still have heightened confidence in the ability of high-scoring borrowers to demonstrate honesty and success, and to exhibit willingness to repay their loans. As a result, lenders may consider reducing group size requirements for high-scoring borrower groups. Conversely, as group size increases, the requirement for lenders' ESE scores may potentially decrease, as a larger group may possess a greater risk mitigation capacity and an increased likelihood of successfully repaying the group loan. Consequently, lenders may opt to reduce the score requirements in such scenarios.

Proposition 5: The optimal individual-ESE score converges to a constant level when the group size reaches a large number.

The Equation 3.28 tells us that $\lim_{n\to\infty} \frac{\partial E(n)}{\partial n} = 0$, which means that the optimal individual-ESE score converges to a constant level when the group size reaches a certain large number. To find the limit optimal score value, we still need to recall the first-order condition of the ex-ante expected payoff, which is

$$\frac{\partial \pi}{\partial E} = kp\overline{Y} + p\underline{Y}k\left[n(1-kE-b)^{n-1} - 1\right] - k\frac{\partial C(e)}{\partial e}. \tag{3.29}$$

Similarly, we set $C(e) = \frac{1}{2}ce^2$, then

$$\frac{\partial C(e)}{\partial e} = ce = c(kE+b).$$

When $n \to \infty$, $n(1-kE-b)^{n-1} \to 0$, then Equation 3.29 can approach to $kp(\overline{Y} - \underline{Y}) - kc \cdot (kE+b)$. Let $kp(\overline{Y} - \underline{Y}) - kc \cdot (kE+b) = 0$, the limit of the optimal ESE score will be expressed as a constant:

$$E \to \left[\frac{p(\overline{Y} - \underline{Y})}{c} - b\right]/k \tag{3.30}$$

To illustrate this proposition, we provide a numerical example, which is shown in Figure 2. Set the unit price $p = 1$, the successful probability $e = k \times E + b = 1/100 \times E + 0$, high yield amount $\overline{Y} = 1000$, low yield amount $\underline{Y} = 500$, and perform a simulation under the different group size $n \in [1, 100]$.

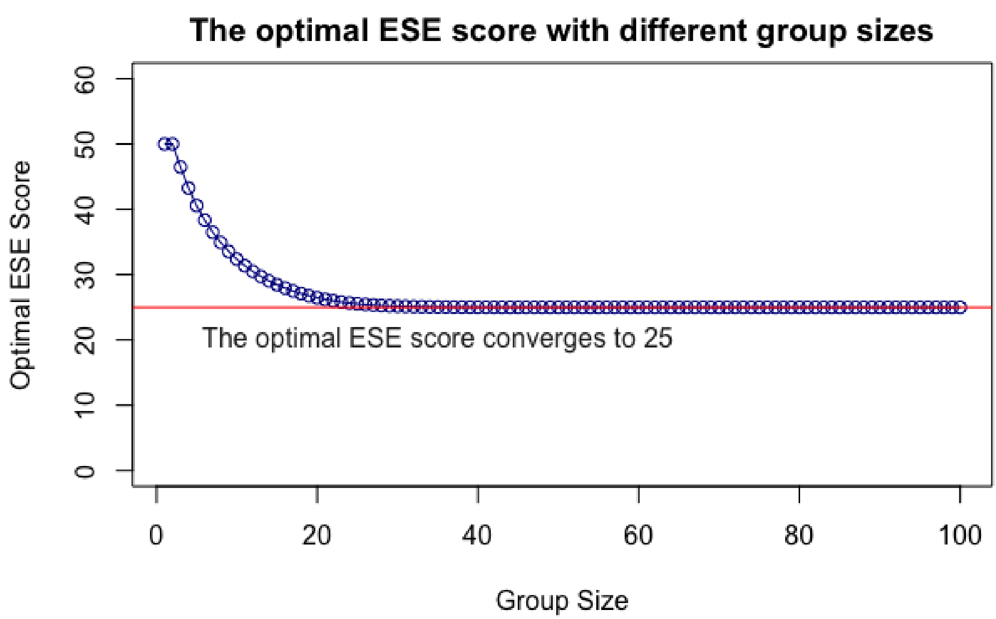

Figure 2: Simulation Results of A Numerical Example

In situations where the group size is small, insurance companies may seek to attract borrowers with high ESE scores in order to mitigate risk. However, as the group size expands, the marginal benefit of an individual effort on group outcomes diminishes, resulting in a reduction of an individual farmer's optimal effort level. This can create incentives for farmers to contribute less effort, leading to a 'free-rider' problem and decreased profits for the group. Therefore, the threshold value of the optimal ESE score can serve as a reference point for both lenders and borrowers, enabling them to avoid an inefficient resource allocation and a free-rider problem.

4 Joint Liability Contracts under a Mean-Variance Utility Framework

4.1 Crop yields under climate change

Climate change refers to the long-term alterations in the Earth's climate system, including changes in temperature, precipitation patterns, and sea levels, which are primarily caused by human activities. These changes are having major impacts on the agriculture sector of the economy, which is heavily dependent on weather patterns, soil fertility, water resources, and pest and disease pressures. Weather patterns are becoming more extreme and unpredictable due to climate change. This can cause droughts, floods, and heat waves, which can all negatively impact crop yields. For example, droughts can cause soil to become dry and hard, reducing plant growth and leading to low yields, while floods can wash away topsoil and damage crops. Extreme heat can also reduce yields by causing plants to wilt or preventing pollination. Soil fertility is also being affected by climate change. Higher temperatures and changes in rainfall patterns can lead to soil erosion, nutrient depletion, and soil compaction. Soil erosion can remove valuable topsoil, which is necessary for plant growth, while nutrient depletion can reduce the number of nutrients available to plants, leading to lower yields. Soil compaction can make it difficult for plant roots to penetrate the soil, reducing their ability to absorb water and nutrients. Water resources are also being impacted by climate change. Changes in precipitation patterns can cause droughts or floods, which can reduce water availability for crops. In addition, warmer temperatures can increase water demand, causing crops to require more water for growth. This can put a strain on already limited water resources, leading to competition between different water users, including farmers. Pest and disease pressures are also being affected by climate change. Changes in temperature and precipitation can alter the behavior of pests and disease-causing organisms, leading to outbreaks or changes in distribution. This can lead to increased crop damage and lower yields.

The Actuaries Climate Index (ACI) is an innovative climate monitoring tool that provides a comprehensive assessment of changes in extreme weather conditions and sea level rise in the United States and Canada. The ACI is a joint initiative of the American Academy of Actuaries and the Canadian Institute of Actuaries, designed to provide policymakers, business leaders, and other stakeholders with a reliable source of information on climate trends and their potential impacts. The ACI is based on the analysis of six climate indicators, including high and low temperatures, heavy precipitation, drought, and sea level rise. The index measures changes in these indicators over time and across regions, providing a clear picture of how the climate is changing in different parts of North America. One potential application of the ACI is to use it as a tool for predicting agricultural yields. Climate change can have significant impacts on agriculture, and the ACI can provide valuable information on how extreme weather events and other climate factors are likely to affect crop yields in different regions. Pan et al. (2022) used regression analysis to investigate the relationship between the ACI and crop yield and found that the ACI is a significant predictor of crop yield and (re)insurance ratemaking. Jiang and Weng (2019) examine the impact of climate change risk on agriculture-related stocks in Canada and the United States. To measure climate change risk, they use the ACI as a proxy and find that ACI trends have an adverse effect on agricultural production. They then assess the correlation between ACI trends and corporate profitability of agriculture-related companies, finding that companies in regions with higher climate change risk have worse financial performance. Based on these findings, they construct a risk-adjusted stock trading strategy that adjusts to climate change risk and find that the ACI is predictable on stock returns, indicating market inefficiency towards climate change risk. Their study contributes to the literature by using ACI as a reliable proxy for climate change risk and providing evidence of the mispricing of risk in the stock market.

4.2 The optimal ESE score with different risk aversion parameters

When borrowers consider potential projects, they want to know not only what the most likely outcome is, but also how likely it is that they will earn a high return or experience a significant loss. The variability of the profit, or the range of possible returns, is an important consideration. If the potential variability of returns is too high or the risk of significant losses is too large, the borrower may decide not to invest in the venture. As far as I am aware, the current literature on joint liability problems only focuses on the expected return or expected profit. This means that they concentrate solely on the central point of an individual's profit without taking into account the distribution of potential profits or the variability of these returns. However, focusing only on the expected return can be limiting, as it does not account for the risks. Therefore, we following incorporate the risk (variance) associated with expected profit into our utility function and conduct further analysis of mean-variance utility in the context of joint liability problems. The Mean-Variance Utility function for an individual farmer can be incorporated in his/her ex-ante expected profit function in Equation 4.1, where P (a random variable) represents the expected profit and γ represents the risk aversion parameter of this farmer.

$$
\begin{aligned}
\max_{E} \pi_i &= \mathbb{E}(P) - \frac{\gamma}{2}\mathrm{Var}(P) - C(e) \\
&= e^2(p\overline{Y} - w) + e(1-e)[p\overline{Y} + p\underline{Y} - 2w] - C(e) \\
&- \frac{\gamma}{2}\left\{e^2(p\overline{Y} - w)^2 + e(1-e)(p\overline{Y} + p\underline{Y} - 2w)^2\right\} \\
&- \frac{\gamma}{2}\left\{- \left[e^2(p\overline{Y} - w) + e(1-e)(p\overline{Y} + p\underline{Y} - 2w)\right]^2\right\} \\
&= e[p\overline{Y} + p\underline{Y} - 2w] - e^2(p\underline{Y} - w) - C(e) \\
&- \frac{\gamma}{2}\left\{\left(e^2 - e^4\right)(p\overline{Y} - w)^2 + \left[e - 2e^2 + 2e^3 - e^4\right](p\overline{Y} + p\underline{Y} - 2w)^2\right\} \\
&+ \frac{\gamma}{2}\left\{2\left(e^3 - e^4\right)(p\overline{Y} - w)(p\overline{Y} + p\underline{Y} - 2w)\right\}
\end{aligned} \tag{4.1}
$$

In order to derive the required optimal individual-ESE score, it is needed to take the first-order partial derivative of the ex-ante profit w.r.t the ESE score.

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial E} =& k(p\overline{Y} + p\underline{Y} - 2w) - 2ek(p\underline{Y} - w) - k\frac{\partial C(e)}{\partial e} \\
&- \frac{\gamma k}{2}\left\{\left(2e - 4e^3\right)(p\overline{Y} - w)^2 + \left(1 - 4e + 6e^2 - 4e^3\right)(p\overline{Y} + p\underline{Y} - 2w)^2\right. \\
&\quad \left. -2\left(3e^2 - 4e^3\right)(p\overline{Y} - w)(p\overline{Y} + p\underline{Y} - 2w)\right\} \\
=& 0
\end{aligned} \tag{4.2}
$$

It is hard to derive the analytical solution of E score; so we instead use simulation studies to find some insightful patterns.

Recall that the probability of achieving success in a project is mathematically represented as $e = k \times ESE + b$, wherein the parameters k and b are predetermined. Subsequently, the probability may be disaggregated into two constituent components. Specifically, the product of k and ESE represents the contribution of personal impact towards project success, whereas b signifies the constant contribution of exogenous factors beyond individual control. The value of k is utilized to calibrate the impact of ESE on project success, while b pertains to the influence of exogenous factors, including climatic and geographical conditions. To determine the value of b, the Actuaries Climate Index (ACI) for each region can be utilized as a reference. It is commonly understood that favorable climatic conditions have a positive effect on crop yields, which, in turn, increases the probability of project success. Conversely, unfavorable climatic conditions have an adverse impact on the likelihood of success. Hence, we delve deeper into the impact of varying climatic conditions caused by climate change on ESE score requirements. By changing the farmer's risk aversion from 0 to 1 under different cost parameters, a series of simulations are conducted for a high success probability project under good climatic conditions (where $b = 0.7$, see in Figure 3), a neutral probability project under neutral climatic conditions (where $b = 0.5$, see in Figure 4), and a relatively low probability project under bad climatic conditions (where $b = 0.3$, see in Figure 5). After observing and comparing these three figures, we conclude our analysis with three findings below. In general, any factor that can increase the success probability of the project and the repayment probability can directly or indirectly reduce the requirements of the ESE score.

1. By examining Figure 3, Figure 4, and Figure 5, we notice that as the probability of success decreases, the ESE score requirement continues to increase if the control cost remains constant. That is, the probability of a project's success is inversely proportional to the required ESE score. This is because a higher success rate leads to an increased farmers' ability to repay their loans, thus lowering the risk of default for the insurance company. As a result, the insurance company can lower its ESE score requirements when lending to farmers.

2. If we focus on any one of the three following figures, it is easy to find that as the risk aversion increases, the required ESE score will decrease monotonically. That is because an increase in farmers' risk aversion score leads to more cautious contract behavior, reducing the probability of strategic default and increasing the probability of repayment. In other words, a higher degree of risk aversion among farmers implies a lower level of risk for the insurance company, leading to a lower ESE score requirement.

3. Still considering any one of the three figures, an increase in the cost parameter will result in a higher requirement for the ESE score. An increase in the cost parameter raises the production expenditure of farmers and burdens their economic resources, thus reducing their profits. Therefore, to avoid the high default probability associated with high costs, the corresponding ESE score requirement should increase. Hence, as the cost parameter increases, the insurance company should raise its ESE score requirement to reduce the risk of default among farmers.

Finally, we are investigating how changes in crop yields can impact the required score for ESE score. To do this, we vary the values of yields and observe the effect on the required ESE score. Specifically, we consider two scenarios: a 'Low Yield Case' where $\overline{Y} = 1,000$

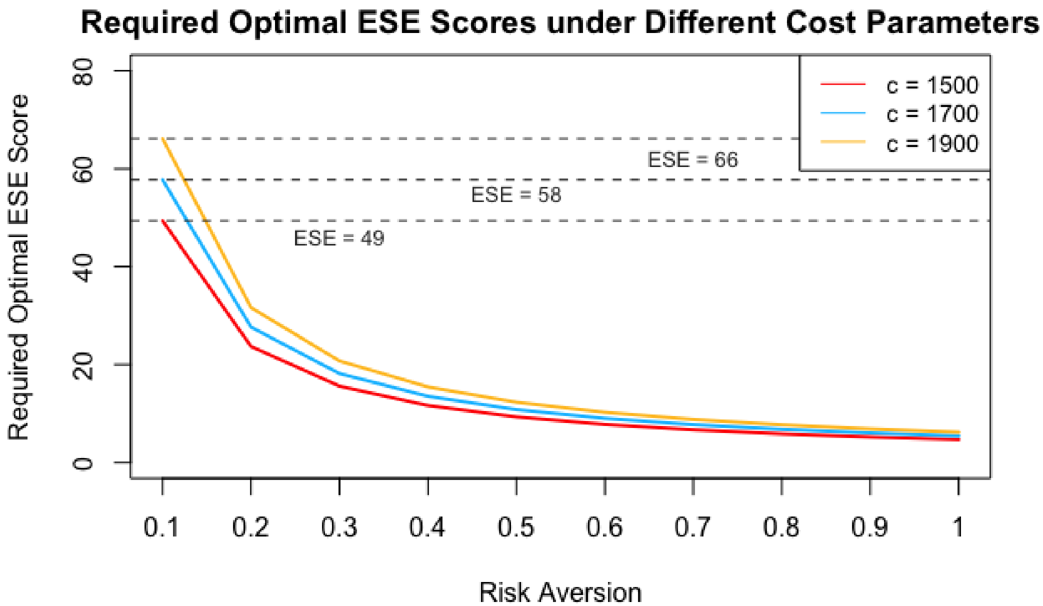

Figure 3: High Successful Probability (Good Climatic Conditions)

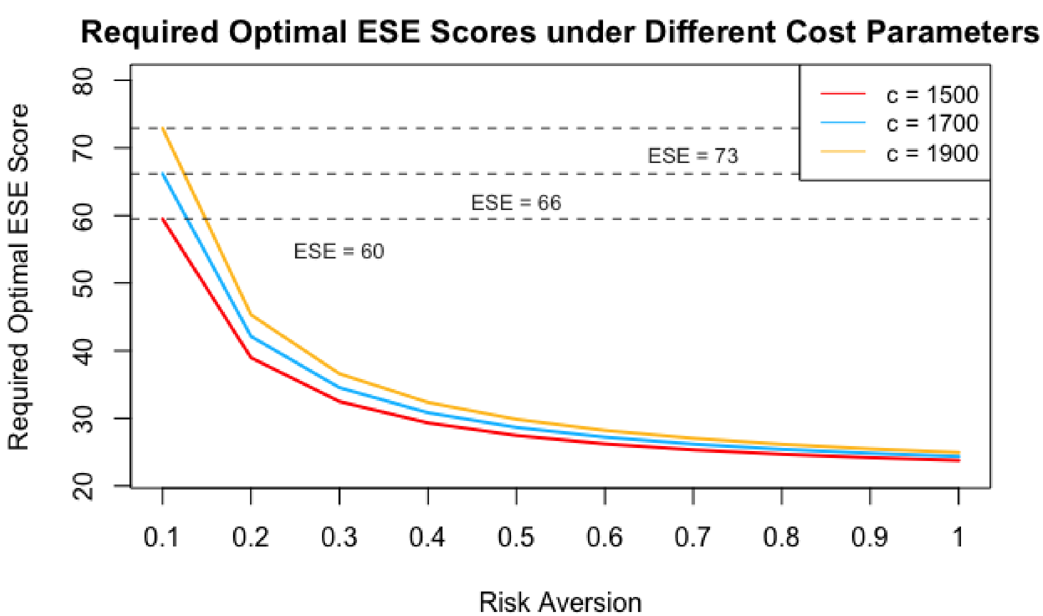

Figure 4: Neutral Successful Probability (Neutral Climatic Conditions)

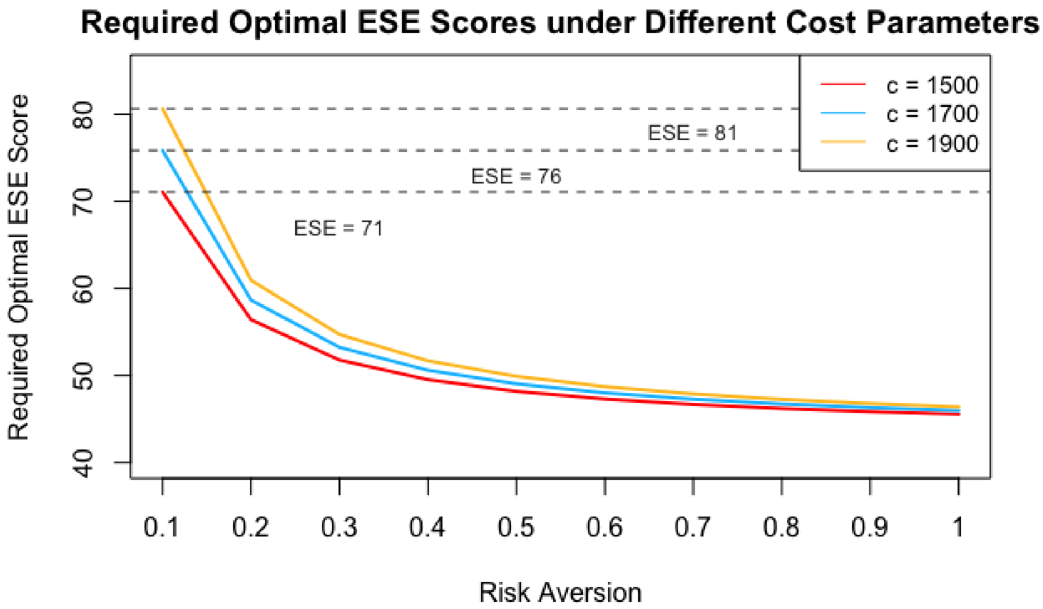

Figure 5: Low Successful Probability (Bad Climatic Conditions)

and $\underline{Y} = 500$, and a 'High Yield Case' where $\overline{Y} = 600$ and $\underline{Y} = 300$". The results of our simulation in Figure 6 show that there is a negative correlation between crop yields and the required ESE score, meaning that as crop yields increase, the required ESE score decreases. Conversely, as crop yields decrease, the required ESE score increases. When a borrower has lower yields, this can be an indication of lower income and potentially higher default risk. In other words, the higher requirement for borrowers with lower yields is a reflection of the lender's risk management strategy. By setting stricter requirements for borrowers with lower yields, the lender is attempting to minimize their exposure to financial risk and ensure that borrowers are able to repay their loans on time and in full.

5 Conclusions

In this paper, we incorporate the individual-ESE score into a joint liability model for evaluating the sustainability and creditworthiness of individual farmers acting as borrowers in the model. To achieve this objective, we initially only considered the expected profit of an individual borrower resulting from the specification of his/her utility function. Our primary objective was to determine the optimal group size that maximizes the benefit for borrowers while ensuring that the lender breaks even. Additionally, we sought to derive the optimal Individual-ESE Score, which can provide several benefits to both lending companies and farmers as borrowers, such as a better understanding of the borrower's risk profile, informa-

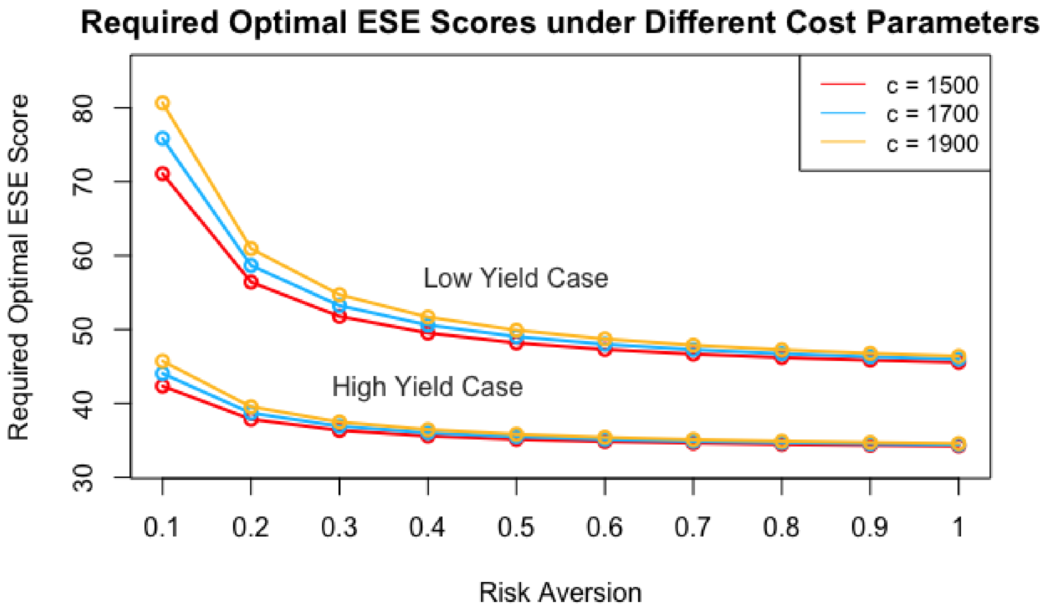

Figure 6: High Yields v.s. Low Yields

tion about the optimal group size for lending, and improved access to credit. The inclusion of this score in credit evaluation processes can lead to better-informed lending decisions and sustainable development outcomes. We listed five propositions. In this paper, proposition 1 focused on finding the optimal individual-ESE score. Proposition 2 aimed at reducing the two constraints about the loan ceiling to only one constraint. Proposition 3 demonstrated that a higher individual-ESE score leads to a higher loan ceiling. Proposition 4 revealed that the optimal individual-ESE score and group size n have a negative relationship. Finally, Proposition 5 showed that the optimal individual-ESE score converges to a constant level when the group size reaches a large number.

Furthermore, our study has also incorporated the risk associated with expected profit into the utility function of the farmers and analyzed the resulting mean-variance utility framework in the context of joint liability problems. The study additionally disentangled the probability of achieving success in a project into two constituent components, namely the impact of personal factors (represented by $k \times ESE$) and exogenous factors beyond individual farmer's control (represented by b), such as climatic and geographical conditions. Our findings reveal that the probability of a project's success is inversely proportional to the required ESE score. This implies that a higher likelihood of project success leads to increased farmers' ability to repay their loans, thereby lowering the risk of default for the insurance company. As a result, the insurance company can lower its ESE score requirements when lending to high-quality farmers. We have also found that an increase in farmers' risk aversion score leads to more cautious contract behavior, reducing the probability of strategic default and increasing the probability of repayment. Hence, as the risk aversion score increases, the corresponding ESE score requirement decreases. Finally, an increase in the cost parameter raises the production expenditure of farmers and burdens their economic resources, reducing their profits. Therefore, to avoid the high default probability associated with high costs, the corresponding ESE score requirement should increase.

Subsequently, we proceed to explore the effects of climate change on the required ESE score in order to ensure a successful project implementation. Our simulation results demonstrate that a higher likelihood of project success, driven by high yields resulting from favorable climatic conditions, leads to a lower ESE score requirement in a joint liability contract. This suggests that incorporating climate scenarios into the evaluation of creditworthiness can result in better-informed lending decisions and ultimately contribute to more favorable sustainable development outcomes.

As an avenue for future research, this study paves the way for exploring partial joint liability – a legal framework distributing shared responsibility among parties for specific obligations. Unlike complete joint liability, partial joint liability assigns a proportionate debt share to each party. Moreover, the concept of flexible joint liability introduces innovation. Participants begin with collaborative lending similar to standard joint liability but, in strategic default scenarios, opt for a punitive phase rather than immediate maximum penalties. After a defined period (T as previously mentioned), defaulters are reintegrated, offering a path for forgiveness and adherence to the flexible joint liability contract. The diverse joint liability arrangements highlight the need for a thorough and separate analysis. Last but not least, our review of the literature has revealed that while traditional credit metrics concentrate on financial and economic factors, assessing sustainability requires taking into account environmental and social factors as well. Therefore, we have suggested the concept of an individual-ESE score that integrates economic, social, and environmental considerations to offer a more all-encompassing appraisal of the farming sector. It follows that another avenue for future research is to eventually develop a credit scoring system that addresses the unique requirements of the agricultural sector by applying literature-based metrics for agricultural credit scoring systems to the construction of the scores. By compiling and adjusting these metrics, we eventually aim at creating a comprehensive credit scoring system with adaptation to climate change for the agricultural sector that addresses limitations of the ESG scores when they are applied to the agricultural sector.

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