

Possible signatures of higher dimension in thin accretion disk around brane world black hole

Ailin Liu,¹ Tong-Yu He,¹ Ming Liu,¹ Zhan-Wen Han,^{1,2} and Rong-Jia Yang^{a1,3,4,5,†}

¹*College of Physics Science and Technology,
Hebei University, Baoding 071002, China*

²*Yunnan Observatories, Chinese Academy of Sciences, Kunming 650216, China*

³*Hebei Key Lab of Optic-Electronic Information and Materials,
Hebei University, Baoding 071002, China*

⁴*National-Local Joint Engineering Laboratory of New Energy Photoelectric Devices,
Hebei University, Baoding 071002, China*

⁵*Key Laboratory of High-precision Computation and
Application of Quantum Field Theory of Hebei Province,
Hebei University, Baoding 071002, China*

Abstract

We probe deeply into the characteristics of thin accretion disk surrounding black hole within the brane world paradigm. We investigate how model parameters affect the physical properties of the disk. Our findings indicate that as the tidal charge parameter inherited from the higher dimension increases, the energy flux, the radiation temperature, the spectral cutoff frequency, the spectral luminosity, and the conversion efficiency of the disk all increase, but the radius of the innermost stable circular orbit decreases. Compared to cases of the Kerr and Schwarzschild black holes, the disk is hotter and more luminous for positive tidal charge parameter, while it is cooler and less luminous for negative tidal charge parameter, which suggests the potential for probing possible signatures of higher dimension.

^a Corresponding author

[†] yangrongjia@tsinghua.org.cn

I. INTRODUCTION

Black holes (BHs) have long been a focal point of research in both general relativity (GR) and higher-dimensional gravity theories. The brane world gravity has sparked new research interests in this field. The brane world theory is an extension of gravitational theory, proposing that our universe may be a brane or hypersurface embedded in a higher-dimensional spacetime, the physical fields (electro-magnetic, Yang-Mills etc.) are confined to the three brane, only gravity can freely propagate in both the brane and bulk spacetimes, with the gravitational self-couplings not significantly modified [1, 2]. These additional dimensions exist beyond the three spatial dimensions we typically observe and can only be detected through specific physical phenomena or experiments [3, 4]. In the development of this scenario, two brane models have been proposed: the thin brane model and the thick brane model. The well-known thin brane models, such as the Arkani-Hamed-Dimopoulos-Dvali (ADD) thin brane model and the Randall-Sundrum (RS) thin brane model, were primarily developed to address the hierarchy problem [1, 3, 4]. In [5], a detailed discussion and exposition of the exact static spherically symmetric vacuum solutions in the brane world model was provided for the first time. In [6, 7], the authors explored the precise steady-state solutions and axisymmetric solutions of BHs, validating the formation of a closed system in the effective gravitational field equations on the brane with the Kerr-Schild form of the metric, replacing the Kerr-Newman solutions from ordinary GR with tidal charge replacing the electric charge. The charged rotating BH (hereafter for short: BWBH) obtained in [7] has been studied extensively, see for example [8–20]. These studies involved exploring the nature and effects of extra dimensions and their connection and divergence from traditional four-dimensional gravitational theories. Here we will focus on the study of thin accretion disks around such a BWBH and probe possible signatures of higher dimensions in relative physical quantities.

The first comprehensive theory regarding accretion disks around BHs was established in [21], which was later extended to the mass accretion of a rotating BH in GR [22]. Further discussions on the characteristics of radiation energy flux on thin accretion disks are elaborated in [23] and [24]. Recently, Novikov-Thorne model has been applied to test various BHs, see for example [25–34]. These studies provide us with a more comprehensive model of BH behavior and contribute to a deeper understanding of the interaction and energy

conversion mechanisms between BHs and accretion disks. Physical properties of thin disk around BWBHs were also investigated in [8, 10], but the formula of the flux emanating from the accretion disk in their works is incorrect, and the related results obtained from it are also unreliable. So, it is necessary to reconsider this issue.

The structure of this paper is outlined as follows. In Sect. II, we will provide a concise overview of the BWBHs, followed by presenting the geodesic equations in the equatorial plane. Sect. III will delve into an exploration of the physical attributes of thin accretion disks surrounding BWBHs, focusing on the impact of tidal charge parameters on the energy flux, the radiation temperature, and the emission spectrum. Finally, a brief summary will be presented in the fourth section.

II. BRANE WORLD BLACK HOLES AND STABLE CIRCULAR ORBITS

In this section, we will briefly review the solutions of BWBHs and give the geodesic equations governing the motion of particles within the equatorial plane.

A. Brane world black holes

The Kerr-Schild form of metric is a mathematical assumption used to describe rotating BHs. This metric form offers certain simplifications and conveniences in mathematics, making it easier to study the spacetime structure around BHs. In the RS brane world theory, the Kerr-Schild form of metric was employed to describe rotating BHs localized on 3-branes [35]. In [7], the authors used this metric form and introduced an additional tidal charge parameter β to account for gravitational effects from extra dimensions. They derived the various components of the Kerr-Schild metric, including g_{tt} , $g_{t\phi}$, g_{rr} , $g_{\theta\theta}$ and $g_{\phi\phi}$. Subsequently, by applying the Boyer-Lindquist transformation, they obtained the metric expression describing rotating BHs. The metric expression are given as follows:

$$\begin{aligned} ds^2 &= -g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 \\ &= -\left(1 - \frac{2Mr - \beta}{\Sigma}\right)dt^2 - \frac{2a(2Mr - \beta)}{\Sigma}\sin^2\theta dtd\phi \\ &\quad + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2Mr - \beta}{\Sigma}a^2\sin^2\theta\right)\sin^2\theta d\phi^2, \end{aligned} \tag{1}$$

where

$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad (2)$$

$$\Delta = r^2 + a^2 - 2Mr + \beta. \quad (3)$$

The symbol M represents the mass of the BH, a denotes the angular momentum, and β stands for the tidal charge parameter which is inherited from the higher dimensions and can be both positive and negative values. For positive β , Eq. (1) is similar to a Kerr-Newmann BH, while the case for negative β has no analogue in GR and thus provides a true signature of the additional spatial dimensions. When $\beta = 0$, the BH is simplified to the Kerr BH in GR. The event horizon is determined by the equation $\Delta = 0$, resulting

$$r_{\pm} = M \pm \sqrt{M^2 - a^2 - \beta}. \quad (4)$$

r_+ denoted as the largest root, representing the location of the outer event horizon. The structure of this event horizon is contingent upon the sign of the tidal charge. For the event horizon to exist, the condition $M^2 \geq a^2 + \beta$ must be satisfied, where equality corresponds to the limiting horizon.

B. Stable circular orbits

The timelike geodesics around compact astrophysical objects are very significant for theoretical as well as experimental point of view as they convey the information about the compact objects where very strong gravity takes place. Considering the geodesic of radial timelike particles in the equatorial plane ($\theta = \pi/2$) in which the accretion disk is usually assumed to lie, the Lagrangian takes the following form

$$\mathfrak{L} = -g_{tt}\dot{t}^2 + 2g_{t\phi}\dot{t}\dot{\phi} + g_{rr}\dot{r}^2 + g_{\phi\phi}\dot{\phi}^2, \quad (5)$$

where the dot means the derivative with respect to the affine parameter λ of the world line of the particle. From Euler-Lagrangian equation, we have

$$\begin{aligned} -g_{tt}\dot{t} + g_{t\phi}\dot{\phi} &= -E, \\ g_{t\phi}\dot{t} + g_{\phi\phi}\dot{\phi} &= L, \end{aligned} \quad (6)$$

where E and L are two constants of motion for particles which are known as the specific energy and angular momentum of the particles moving on circular orbits around the compact

objects. Solving the above two equations, we derive the orbiting particle's four-velocity components as

$$\dot{t} = \frac{Eg_{\phi\phi} + Lg_{t\phi}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}, \quad (7)$$

$$\dot{\phi} = -\frac{Eg_{t\phi} + Lg_{tt}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}. \quad (8)$$

The first integral of geodesics equation ($g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$) for time-like particles, yields

$$g_{rr}\dot{r}^2 = -1 + g_{tt}\dot{t}^2 - 2g_{t\phi}\dot{t}\dot{\phi} - g_{\phi\phi}\dot{\phi}^2. \quad (9)$$

Putting the values of $\dot{t}$ and $\dot{\phi}$, we have

$$g_{rr}\left(\frac{dr}{d\tau}\right)^2 = -1 + \frac{E^2g_{\phi\phi} + 2ELg_{t\phi} + L^2g_{tt}}{g_{t\phi}^2 + g_{tt}g_{\phi\phi}}, \quad (10)$$

from which we can define an effective potential as

$$V_{\text{eff}} = -1 + \frac{E^2g_{\phi\phi} + 2ELg_{t\phi} + L^2g_{tt}}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}. \quad (11)$$

Circular orbits in the equatorial plane are satisfied the conditions $V_{\text{eff}} = 0$, $V_{\text{eff},r} = 0$, and $V_{\text{eff},\theta} = 0$ [36–38]. Due to the reflection symmetry of the metric about the equatorial plane, particles located at $\theta = \frac{\pi}{2}$ naturally fulfill the condition $V_{\text{eff},\theta} = 0$. Utilizing these conditions, we derived the expressions for the specific energy E , the specific angular momentum L , and the specific angular velocity Ω for particles undergoing prototype orbital motion in the equatorial plane of a rotating Kerr-like BH spacetime.

$$\Omega = \frac{d\phi}{dt} = \frac{-g_{t\phi,r} + \sqrt{(g_{t\phi,r})^2 - g_{tt,r}g_{\phi\phi,r}}}{g_{\phi\phi,r}}, \quad (12)$$

$$E = -\frac{g_{tt} + g_{t\phi}\Omega}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2}}, \quad (13)$$

$$L = \frac{g_{t\phi} + g_{\phi\phi}\Omega}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2}}. \quad (14)$$

For test particles in the gravitational potential of a central celestial body, the innermost stable circular orbit (ISCO) is defined by the radius at which $V_{\text{eff},rr} = 0$. By satisfying this condition $V_{\text{eff},rr} = 0$, we can determine the marginally stable orbit around the central celestial body. We can express the effective potential as, $V_{\text{eff}}(r) \equiv -1 + f/g$, where $f \equiv E^2g_{\phi\phi} + 2ELg_{t\phi} + L^2g_{tt}$ and $g \equiv g_{t\phi}^2 - g_{tt}g_{\phi\phi}$. In addition, the constraint $g \neq 0$ is imposed. By setting $V_{\text{eff}}(r) = 0$, we can obtain $f = g$. The condition $V_{\text{eff},r}(r) = 0$ yields $f_{,r}g - fg_{,r} =$

0. Consequently, with these constraints, we can deduce $V_{\text{eff},rr}(r) = 0$, which provides the following important relationship

$$0 = (g_{t\phi}^2 - g_{tt}g_{\phi\phi})V_{\text{eff},rr} = E^2 g_{\phi\phi,rr} + 2ELg_{t\phi,rr} + L^2 g_{tt,rr} - (g_{t\phi}^2 - g_{tt}g_{\phi\phi})_{,rr}. \quad (15)$$

By substituting equations (12-14) into equation (15), we can solve for r and obtain the radius of the edge stable orbit. Additionally, we can also determine the edge stable orbit by investigating the condition of $dE/dr = 0$, which is exactly the method used here, as shown in the following equation:

$$Mr(6Mr - r^2 - 9\beta + 3a^2) + 4\beta(\beta - a^2) - 8a(Mr - \beta)^{3/2} = 0 \quad (16)$$

When $a = \beta = 0$, the edge stable orbital radius r_{isco} is equal to $6M$ for Schwarzschild BH. For $r < r_{\text{isco}}$, the equatorial circular orbits are unstable, thereby r_{isco} defines the inner edge of the thin accretion disk.

III. PHYSICAL PROPERTIES OF THIN ACCRETION DISKS

In this section, we will investigate the accretion process in thin disks around BWBHs. We will discuss in detail the effect of the tidal charge parameter β on the energy flux, the radiation temperature, the emission spectrum and the energy conversion efficiency, and so on.

We first briefly summarized the key physical characteristics of thin accretion disks according to the Novikov-Thorne model [21, 22]: (1) The spacetime surrounding the central mass object is static, axisymmetric, and asymptotically flat. (2) The self-gravity of the accretion disk is negligible, implying that its mass does not influence the background metric. (3) The accretion disk is geometrically thin, with its vertical scale h significantly smaller than its horizontal scale ($h \leq r$). (4) The radius of the marginally stable orbit dictates the inner edge of the disk and the particles move between the ISCO and the outer edge of the disk. (5) The accretion disk resides on the equatorial plane of the compact accreting object, orthogonal to the surface of the BH's spin. (6) It is assumed that the electromagnetic radiation emitted from the disk surface conforms to a black-body spectrum, which is influenced by the fluid dynamics and thermodynamic equilibrium conditions of the disk. (7) The mass accretion rate $\dot{M}_0$ remains constant over time due to the stable state of the accretion disk.

The physical constants and the representations of thin accretion disk used in this paper are as follows : $c = 2.997 \times 10^{10} \text{ cms}^{-1}$, $M_{\odot} = 1.989 \times 10^{33} \text{ g}$, $\dot{M}_0 = 2 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$, $1 \text{ yr} = 3.156 \times 10^7 \text{ s}$, $\sigma_{\text{SB}} = 5.67 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$, $h = 6.625 \times 10^{-27} \text{ erg s}$, $k_{\text{B}} = 1.38 \times 10^{-16} \text{ erg K}^{-1}$ and the mass of BH $M = 2 \times 10^6 M_{\odot}$, as used in [32].

A. The radiant energy flux

Considering accretion disks surrounding BWBHs, the equation governing the conservation of rest mass, energy, and angular momentum for disk particles allows us to determine the radiant energy flux at the disk surface as [22, 23]

$$F(r) = -\frac{\dot{M}_0 \Omega_{,r}}{4\pi \sqrt{-g} (E - \Omega L)^2} \int_{r_{\text{isco}}}^r (E - \Omega L) L_{,r} dr. \quad (17)$$

The mentioned equation is applicable solely in cylindrical coordinates. For spherical coordinates, we need to refer to [31]

$$F(r) = -\frac{\dot{M}_0 \Omega_{,r}}{4\pi \sqrt{-g/g_{\theta\theta}} (E - \Omega L)^2} \int_{r_{\text{isco}}}^r (E - \Omega L) L_{,r} dr \quad (18)$$

where $\dot{M}_0$ is the accretion rate of mass.

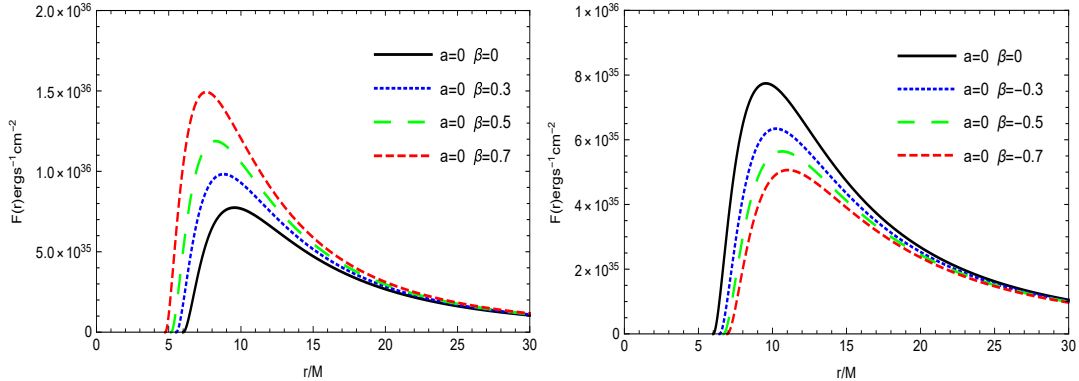


FIG. 1. The energy flux $F(r)$ of a disk around a non-rotating ($a = 0$) BWBH with different values of β .

Figures 1, 2, and 3 depict the energy flux on the disk surface for different values of the parameters. From figure 1, it is evident that as the tidal charge β increases with a fixed rotation parameter a , the energy flux also increases, while the radius of the ISCO decreases. Figures 2, and 3 show that as the rotation speed a increases, the influence of the tidal charge

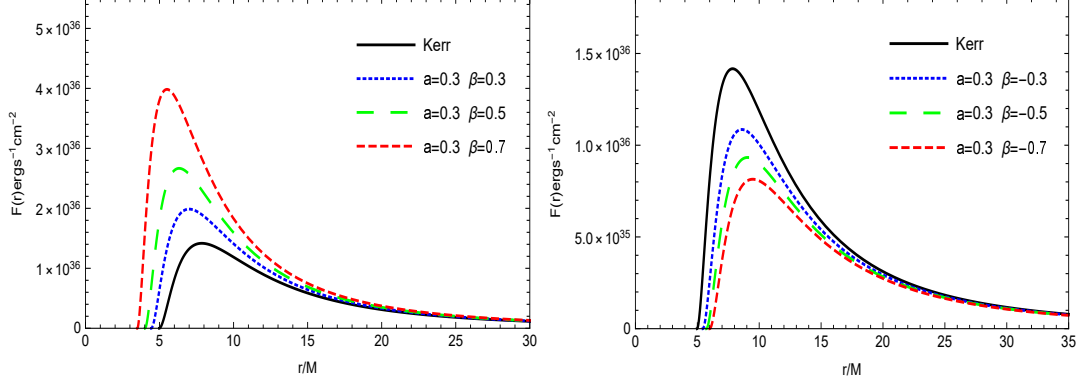


FIG. 2. The energy flux $F(r)$ of a disk around a rotating ($a = 0.3$) BWBH with different values of β .

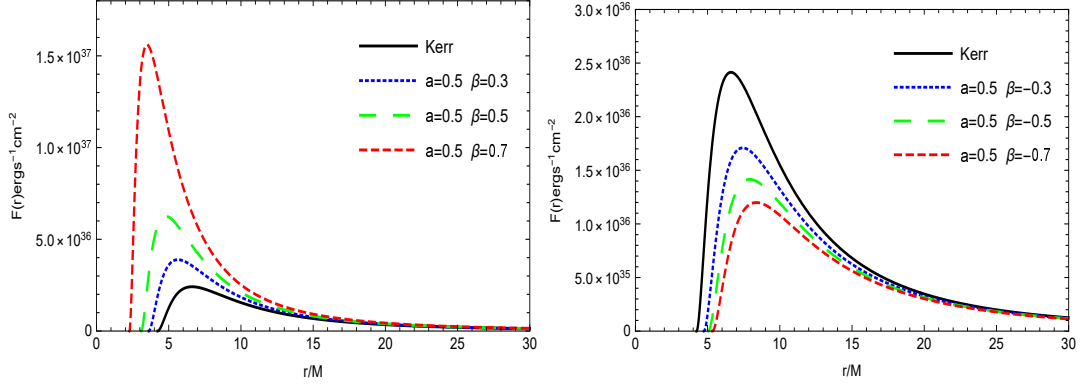


FIG. 3. The energy flux $F(r)$ of a disk around a rotating ($a = 0.5$) BWBH with different values of β .

β becomes more pronounced. The energy flux values increase more rapidly as β increases at equal intervals. On the left panel of figures 1, 2, and 3, it can be observed that for positive β , the energy flux exceeds that of the Kerr BH ($a \neq 0, \beta = 0$) and the Schwarzschild BH ($a = 0, \beta = 0$). Conversely, on the right panel of figures 1, 2, and 3, for negative β , the energy flux is lower compared to the Kerr and Schwarzschild BHs. Furthermore, it is notable that as the β value increases, the radius of the maximum flux gradually shifts towards lower r/M , indicating that most of the radiation originates from within the accretion disk.

B. The radiation temperature

Now we explore the radiation temperature $T(r)$ of disk. In the context of a steady-state thin disk model, accreting matter is typically assumed to be in thermal equilibrium, implying that the radiation emitted by the disk can be considered as perfect black-body radiation. The disk's radiation temperature $T(r)$ is related to the energy flux $F(r)$ through the Stefan-Boltzmann law, $F(r) = \sigma_{\text{SB}} T^4(r)$, where σ_{SB} is the Stefan-Boltzmann constant.

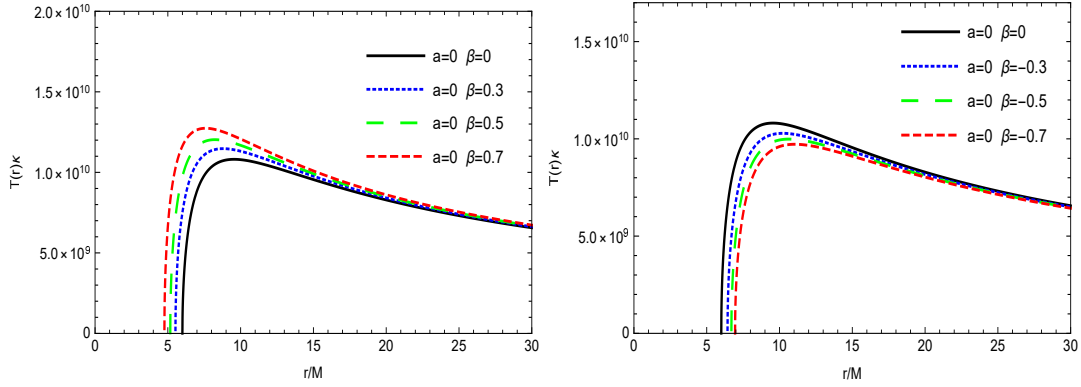


FIG. 4. The radiation temperature $T(r)$ of a disk around a non-rotating ($a = 0$) BWBH with different values of β .

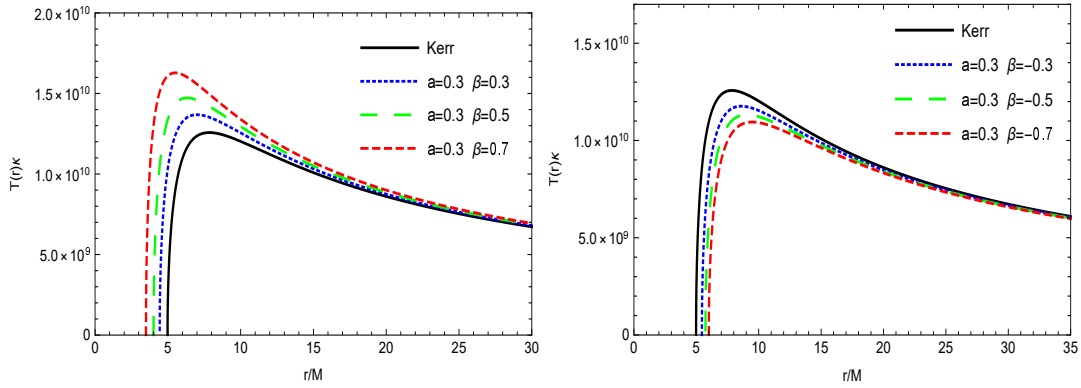


FIG. 5. The radiation temperature $T(r)$ of a disk around a rotating ($a = 0.3$) BWBH with different values of β .

Figures 4, 5 and 6 depict the radiation temperature on the disk surface for different values of the parameters. Similar to case of the energy flux, with increasing tidal charge β , the radiation temperature increases and the maximum values shift closer to the inner edge of

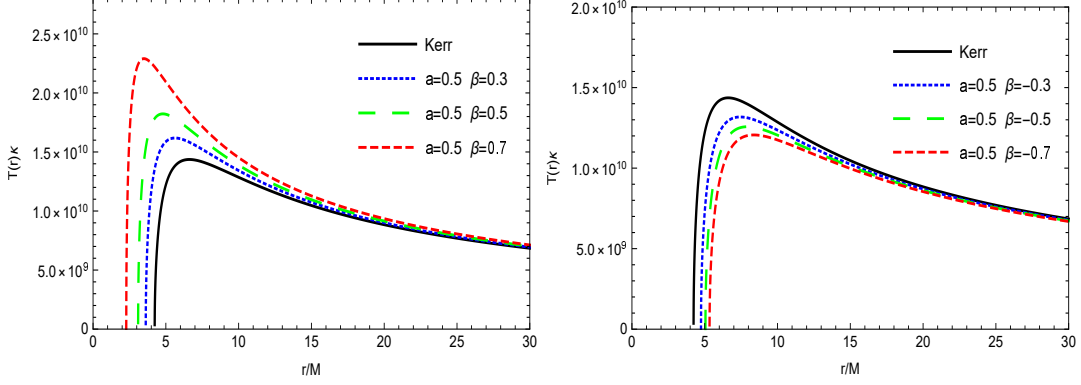


FIG. 6. The radiation temperature $T(r)$ of a disk around a rotating ($a = 0.5$) BWBH with different values of β .

the disk. On the left side of figures 4, 5, and 6, it can be observed that for positive β , the disk around a BWBH is hotter than that around a Kerr ($a \neq 0, \beta = 0$) or a Schwarzschild ($a = 0, \beta = 0$) BH. Conversely, on the right side of figures 4, 5, and 6, for negative β , the disk is cooler compared to the cases of Kerr and Schwarzschild BHs.

C. The observed luminosity

The luminosity $L(v)$ of a thin accretion disk surrounding a BH is inferred from the redshifted black-body spectrum, as discussed in [39].

$$L(v) = 4\pi d^2 I(v) = \frac{8\pi h \cos \gamma}{c^2} \int_{r_i}^{r_f} \int_0^{2\pi} \frac{v_e^3 r d\phi dr}{e^{\frac{h v_e}{k_B T}} - 1}, \quad (19)$$

where d is the distance from the center of the disk, $I(v)$ represents the heat flux radiated by the disk, h is the Planck constant, k_B is the Boltzmann constant and γ is the inclination of the disk (taking $\gamma=0$ here). The symbols r_f and r_i denote the outer and inner radii of the disk edge, respectively. We assumed that the flux on the disk surface diminishes at r_f , hence we set $r_i = r_{\text{ischo}}$ and $r_f \rightarrow \infty$. The emitted frequency is given by $v_e = v(1+z)$. The redshift factor z can be written as

$$1 + z = \frac{1 + \Omega r \sin \phi \sin \gamma}{\sqrt{-g_{tt} - 2\Omega g_{t\phi} - \Omega^2 g_{\phi\phi}}}. \quad (20)$$

Here the bending effect of light is ignored[40, 41].

As shown in figures 7 and 8, we can clearly see the spectral energy distributions of accretion disks around BWBHs. Similar to the energy flux and radiation temperature, as

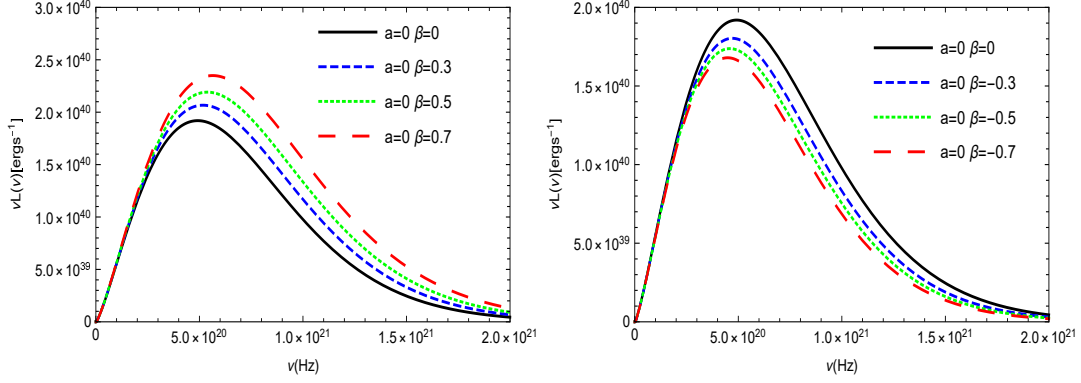


FIG. 7. The emission spectrum $vL(v)$ of disk around a non-rotating BWBH ($a = 0$) with different values of β .

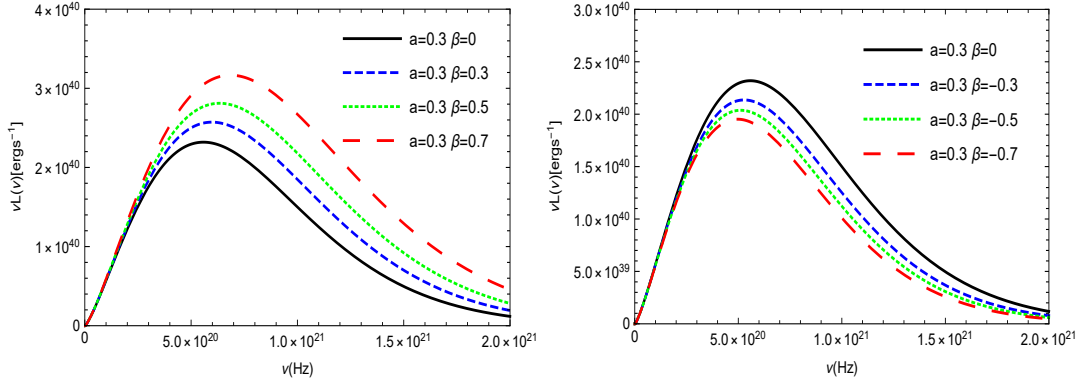


FIG. 8. The emission spectrum $vL(v)$ of disk around a rotating BWBH ($a = 0.3$) with different values of β .

seen from the left side of figures 7 and 8, for positive tidal charge, the spectral energy of disk around a BWBH is higher (more luminous) than that around a Kerr ($a \neq 0$, $\beta = 0$) or a Schwarzschild BH ($a = 0$, $\beta = 0$). Conversely, for negative β , the spectral energy of disk around a BWBH is lower (less luminous), compared to the cases of Kerr and Schwarzschild BH.

D. The radiative efficiency and some important values

During mass accretion around a BH, radiation efficiency is a key feature that describes the ability of the central object to convert its resting mass into outward radiating energy. In general, this conversion efficiency can be expressed by measuring the ratio of two rates:

a	β	$\bar{r}_{\text{isco}}$	$F_{\text{max}}(\bar{r})\text{ergs}^{-1}\text{cm}^{-2}$	$T_{\text{max}}(\bar{r})\text{K}$	$\nu_{\text{crit}}\text{Hz}$	$\nu L(\nu)_{\text{max}}\text{ergs}^{-1}$	ϵ
0	-0.5	6.69	5.64×10^{35}	9.99×10^9	4.59×10^{20}	1.74×10^{40}	0.052
	-0.3	6.43	6.34×10^{35}	1.03×10^{10}	4.71×10^{20}	1.80×10^{40}	0.054
	0	6	7.74×10^{35}	1.08×10^{10}	4.92×10^{20}	1.92×10^{40}	0.057
	0.3	5.52	9.82×10^{35}	1.15×10^{10}	5.17×10^{20}	2.07×10^{40}	0.062
	0.5	5.17	1.19×10^{36}	1.20×10^{10}	5.38×10^{20}	2.19×10^{40}	0.065
0.3	-0.5	5.73	9.33×10^{35}	1.13×10^{10}	5.11×10^{20}	2.04×10^{40}	0.061
	-0.3	5.45	1.09×10^{36}	1.18×10^{10}	5.28×10^{20}	2.14×10^{40}	0.064
	0	4.98	1.42×10^{36}	1.26×10^{10}	5.58×10^{20}	2.32×10^{40}	0.069
	0.3	4.44	1.99×10^{36}	1.37×10^{10}	5.98×10^{20}	2.57×10^{40}	0.077
	0.5	4.02	2.67×10^{36}	1.47×10^{10}	6.34×10^{20}	2.81×10^{40}	0.084
0.5	-0.5	5.05	1.42×10^{36}	1.26×10^{10}	5.58×10^{20}	2.32×10^{40}	0.069
	-0.3	4.74	1.71×10^{36}	1.38×10^{10}	5.80×10^{20}	2.46×10^{40}	0.074
	0	4.23	2.41×10^{36}	1.43×10^{10}	6.21×10^{20}	2.73×10^{40}	0.082
	0.3	3.62	3.89×10^{36}	1.62×10^{10}	6.82×10^{20}	3.16×10^{40}	0.095
	0.5	3.09	6.26×10^{36}	1.80×10^{10}	7.43×10^{20}	3.63×10^{40}	0.110

TABLE I. The maximum values of the energy flux $F_{\text{max}}(\bar{r})$, the disk temperature $T_{\text{max}}(\bar{r})$, and $\nu L(\nu)_{\text{max}}$ for a thin accretion disk surrounding a BWBH. The dimensionless ISCO radius, the cut-off frequency ν_{crit} , and the Novikov-Thorne efficiency ϵ are also presented.

the rate of radiation energy escaping from the disk surface to infinity and the rate of mass energy transfer of the central dense object during mass accretion [22, 23]. If all the emitted photons can successfully escape to infinity, then the efficiency ϵ will be determined by the specific energy of the particle at the edge stable orbit r_{isco} [22, 23].

$$\epsilon = 1 - E_{\text{isco}} \quad (21)$$

In table I, we present the maximum values of the energy flux $F_{\text{max}}(\bar{r})$, the radiation temperature $T_{\text{max}}(\bar{r})$, and $\nu L(\nu)_{\text{max}}$ for a thin accretion disk around a BWBH, along with the dimensionless ISCO radius, the critical cutoff frequency ν , and the Novikov-Thorne efficiency ϵ . It is observed that these values increase with an increase in the tidal charge

β . Specifically, for negative β , these values are smaller compared to the Schwarzschild and Kerr BH; conversely, for positive β , the trend is reversed. The ISCO radius of disk around a BWBH decreases with increasing of tidal charge β . Furthermore, it is clear that as the rotation parameter a increases, the difference in the flux maximum between the rotating BWBH and the Kerr BH ($a \neq 0, \beta = 0$) also increases. For increasing tidal charge β with fixed a , the radiative efficiency increases; and for increasing a with fixed tidal charge, the radiative efficiency increases faster.

IV. CONCLUSIONS

This study employed the Novikov-Thorne model to investigate the characteristics of thin accretion disks surrounding BHs in the brane world scenario. The equation for the ISCO is solved numerically. The analysis revealed a decrease in the ISCO radius of thin disks around BWBHs as the tidal charge β increases. The impact of β on the energy flux, the radiation temperature, the luminosity spectrum, and the energy conversion efficiency of the thin disk is explored under specific values of the rotation parameter a . The findings indicate a positive correlation between β and these quantities. Furthermore, it is observed that under identical rotation parameters and positive β , the ISCO radius of BWBHs is smaller than that of Kerr BHs. Conversely, when β is negative, the ISCO radius of BWBHs surpasses that of Kerr BHs.

Notably, the observable characteristics of the thin disk, including the maximum values of $F_{\max}(r)$, $T_{\max}(r)$, and $vL(v)_{\max}$, demonstrate that the thin accretion disk around a BWBH is hotter and brighter for positive β compared to Schwarzschild and Kerr BH in GR. Conversely, it appears colder and dimmer for negative β .

Given the significant differences in energy flux, the radiation temperature, the luminosity spectrum, and the conversion efficiency between BWBHs and those in GR, these findings suggest the potential for distinguishing between standard GR and brane world gravity based on observable characteristics.

[1] L. Randall and R. Sundrum, "A Large mass hierarchy from a small extra dimension," *Phys. Rev. Lett.*, vol. 83, pp. 3370–3373, 1999.

- [2] L. Randall and R. Sundrum, “An Alternative to compactification,” *Phys. Rev. Lett.*, vol. 83, pp. 4690–4693, 1999.
- [3] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, “The Hierarchy problem and new dimensions at a millimeter,” *Phys. Lett. B*, vol. 429, pp. 263–272, 1998.
- [4] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, “New dimensions at a millimeter to a Fermi and superstrings at a TeV,” *Phys. Lett. B*, vol. 436, pp. 257–263, 1998.
- [5] N. Dadhich, R. Maartens, P. Papadopoulos, and V. Rezanian, “Black holes on the brane,” *Phys. Lett. B*, vol. 487, pp. 1–6, 2000.
- [6] K. A. Bronnikov, V. N. Melnikov, and H. Dehnen, “On a general class of brane world black holes,” *Phys. Rev. D*, vol. 68, p. 024025, 2003.
- [7] A. N. Aliev and A. E. Gümrükçüoğlu, “Charged rotating black holes on a 3-brane,” *Physical Review D*, vol. 71, May 2005.
- [8] C. S. J. Pun, Z. Kovacs, and T. Harko, “Thin accretion disks onto brane world black holes,” *Phys. Rev. D*, vol. 78, p. 084015, 2008.
- [9] L. Amarilla and E. F. Eiroa, “Shadow of a rotating braneworld black hole,” *Phys. Rev. D*, vol. 85, p. 064019, 2012.
- [10] I. Banerjee, S. Chakraborty, and S. SenGupta, “Decoding signatures of extra dimensions and estimating spin of quasars from the continuum spectrum,” *Phys. Rev. D*, vol. 100, no. 4, p. 044045, 2019.
- [11] N. Tsukamoto, “Black hole shadow in an asymptotically-flat, stationary, and axisymmetric spacetime: The Kerr-Newman and rotating regular black holes,” *Phys. Rev. D*, vol. 97, no. 6, p. 064021, 2018.
- [12] S. Vagnozzi and L. Visinelli, “Hunting for extra dimensions in the shadow of M87*,” *Phys. Rev. D*, vol. 100, no. 2, p. 024020, 2019.
- [13] U. Nucamendi, R. Becerril, and P. Sheoran, “Bounds on spinning particles in their innermost stable circular orbits around rotating braneworld black hole,” *Eur. Phys. J. C*, vol. 80, no. 1, p. 35, 2020.
- [14] Y. Hou, M. Guo, and B. Chen, “Revisiting the shadow of braneworld black holes,” *Phys. Rev. D*, vol. 104, no. 2, p. 024001, 2021.
- [15] A. K. Mishra, A. Ghosh, and S. Chakraborty, “Constraining extra dimensions using observations of black hole quasi-normal modes,” *Eur. Phys. J. C*, vol. 82, no. 9, p. 820, 2022.

- [16] M. Shahzadi, M. Kološ, Z. Stuchlík, and Y. Habib, “Testing alternative theories of gravity by fitting the hot-spot data of Sgr A*,” *Eur. Phys. J. C*, vol. 82, no. 5, p. 407, 2022.
- [17] S.-W. Wei, H.-M. Wang, Y.-P. Zhang, and Y.-X. Liu, “Effects of tidal charge on magnetic reconnection and energy extraction from spinning braneworld black hole,” *JCAP*, vol. 04, no. 04, p. 050, 2022.
- [18] S. Vagnozzi *et al.*, “Horizon-scale tests of gravity theories and fundamental physics from the Event Horizon Telescope image of Sagittarius A,” *Class. Quant. Grav.*, vol. 40, no. 16, p. 165007, 2023.
- [19] S. Roy, P. Sarkar, S. Sau, and S. SenGupta, “Exploring axions through the photon ring of a spherically symmetric black hole,” *JCAP*, vol. 11, p. 099, 2023.
- [20] T. Zi, “Extreme mass-ratio inspiral as a probe of extra dimensions: The case of spinning massive object,” *Phys. Lett. B*, vol. 850, p. 138538, 2024.
- [21] N. I. Shakura and R. A. Sunyaev, “Black holes in binary systems. Observational appearance,” *Astron. Astrophys.*, vol. 24, pp. 337–355, Jan. 1973.
- [22] I. D. Novikov and K. S. Thorne, “Astrophysics and black holes,” in *Les Houches Summer School of Theoretical Physics: Black Holes*, pp. 343–550, 1973.
- [23] D. N. Page and K. S. Thorne, “Disk-Accretion onto a Black Hole. Time-Averaged Structure of Accretion Disk,” *Astrophys. J.*, vol. 191, pp. 499–506, July 1974.
- [24] K. S. Thorne, “Disk accretion onto a black hole. 2. Evolution of the hole,” *Astrophys. J.*, vol. 191, pp. 507–520, 1974.
- [25] T. Zhu, Q. Wu, M. Jamil, and K. Jusufi, “Shadows and deflection angle of charged and slowly rotating black holes in Einstein-Æther theory,” *Phys. Rev. D*, vol. 100, no. 4, p. 044055, 2019.
- [26] M. Heydari-Fard, M. Heydari-Fard, and H. R. Sepangi, “Thin accretion disks around rotating black holes in 4D Einstein-Gauss-Bonnet gravity,” *Eur. Phys. J. C*, vol. 81, no. 5, p. 473, 2021.
- [27] Z. Zhang, S. Chen, X. Qin, and J. Jing, “Polarized image of a Schwarzschild black hole with a thin accretion disk as photon couples to Weyl tensor,” *Eur. Phys. J. C*, vol. 81, no. 11, p. 991, 2021.
- [28] R. K. Karimov, R. N. Izmailov, A. Bhattacharya, and K. K. Nandi, “Accretion disks around the Gibbons–Maeda–Garfinkle–Horowitz–Strominger charged black holes,” *Eur. Phys. J. C*, vol. 78, no. 9, p. 788, 2018.

- [29] S. Kazempour, Y.-C. Zou, and A. R. Akbarieh, “Analysis of accretion disk around a black hole in dRGT massive gravity,” *Eur. Phys. J. C*, vol. 82, no. 3, p. 190, 2022.
- [30] G. Gyulchev, P. Nedkova, T. Vetsov, and S. Yazadjiev, “Image of the thin accretion disk around compact objects in the Einstein–Gauss–Bonnet gravity,” *Eur. Phys. J. C*, vol. 81, no. 10, p. 885, 2021.
- [31] L. G. Collodel, D. D. Doneva, and S. S. Yazadjiev, “Circular Orbit Structure and Thin Accretion Disks around Kerr Black Holes with Scalar Hair,” *Astrophys. J.*, vol. 910, no. 1, p. 52, 2021.
- [32] T.-Y. He, Z. Cai, and R.-J. Yang, “Thin accretion disks around a black hole in Einstein–Aether-scalar theory,” *Eur. Phys. J. C*, vol. 82, no. 11, p. 1067, 2022.
- [33] H. Feng and R.-J. Yang, “Thin accretion disk around Kerr–Sen black hole in Einstein–Maxwell–dilaton–axion gravity,” 3 2024.
- [34] S. Shahidi, T. Harko, and Z. Kovács, “Distinguishing Brans–Dicke–Kerr type naked singularities and black holes with their thin disk electromagnetic radiation properties,” *Eur. Phys. J. C*, vol. 80, no. 2, p. 162, 2020.
- [35] N. Dadhich, R. Maartens, P. Papadopoulos, and V. Rezanian, “Black holes on the brane,” *Physics Letters B*, vol. 487, p. 1–6, Aug. 2000.
- [36] J. R. Gair, C. Li, and I. Mandel, “Observable properties of orbits in exact bumpy spacetimes,” *Physical Review D*, vol. 77, Jan. 2008.
- [37] C. Bambi and E. Barausse, “Constraining the quadrupole moment of stellar-mass black-hole candidates with the continuum fitting method,” *Astrophys. J.*, vol. 731, p. 121, 2011.
- [38] C. Bambi and E. Barausse, “The Final stages of accretion onto non-Kerr compact objects,” *Phys. Rev. D*, vol. 84, p. 084034, 2011.
- [39] D. F. Torres, “Accretion disc onto a static nonbaryonic compact object,” *Nucl. Phys. B*, vol. 626, pp. 377–394, 2002.
- [40] J. P. Luminet, “Image of a spherical black hole with thin accretion disk,” *Astron. Astrophys.*, vol. 75, pp. 228–235, 1979.
- [41] S. Bhattacharyya, R. Misra, and A. V. Thampan, “General relativistic spectra of accretion disks around rotating neutron stars,” *Astrophys. J.*, vol. 550, p. 841, 2001.