

Travelling waves in an ensemble of excitable oscillators: the interplay of memristive coupling and noise

Ivan A. Korneev,¹ Ibadulla R. Ramazanov,¹ Andrei V. Slepnev,¹ Tatiana E. Vadivasova,¹ and Vladimir V. Semenov^{1, a)}

Institute of Physics, Saratov State University, Astrakhanskaya str. 83, 410012 Saratov, Russia

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Using methods of numerical simulation, we demonstrate the constructive role of memristive coupling in the context of the travelling waves formation and robustness in an ensemble of excitable oscillators described by the FitzHugh-Nagumo neuron model. First, the revealed aspects of the memristive coupling action are shown on an example of the deterministic model where the memristive properties of the coupling elements provide for achieving travelling waves at lower coupling strength as compared to non-adaptive diffusive coupling. In the presence of noise, the positive role of memristive coupling is manifested as significant increasing a noise intensity critical value corresponding to the noise-induced destruction of travelling waves as compared to classical diffusive interaction. In addition, we point out the second constructive factor, the Lévy noise whose properties provide for inducing travelling waves.

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Travelling waves of different nature and properties are observed in many physical, biological and chemical media and networks. Models exhibiting such structures are widely used for description of a broad spectrum of regular and chaotic spatio-temporal evolutionary processes from signal and energy transmission to climate change, spreads of epidemics, etc. To induce and suppress travelling waves and control their characteristics, one can vary the model parameters as well as apply regular or stochastic external forcing. One more approach for controlling travelling waves can be realized in networks of coupled oscillators: a control scheme based on tuning the coupling properties (for instance, the coupling strength, topology and adaptivity). In the current paper we address this issue in the context of interplay of adaptive coupling and noise perturbations. In addition, we extend a manifold of noise-induced effects observed in high-dimensional systems by Lévy-noise-induced travelling waves characterized by higher robustness in the presence of memristive coupling.

oscillators^{11–14}. Exhibited as spatially-periodic structures, fronts, backs and pulses, travelling waves are of a frequent occurrence in plasma physics^{15,16}, optics and electronics^{17–20}, hydrodynamics^{21–24}, chemistry^{25–27}, neurophysiology^{28–30}, as well as on the edge of ecology, population biology and epidemiology^{31–33}. Such diversity causes interest of specialists in nonlinear dynamics and complex systems focused on revealing the interdisciplinary, fundamental properties of travelling waves and controlling their characteristics and stability.

In the current paper, travelling waves are studied in an ensemble of stochastic excitable oscillators with local adaptive coupling. The features of coupling are associated with the properties of the coupling element, the memristor. The memristors have a wide range of practical applications, first of all, neuromorphic computing and the implementation of new generation memory elements^{34,35}. In the context of nonlinear dynamics, the memristor is interesting as an element whose intrinsic properties can essentially change the dynamics of oscillatory systems and are responsible for qualitatively new types of the behavior from bifurcations without parameters in single memristor-based oscillators with lines of equilibria^{36–40} to the Turing patterns⁴¹ and travelling waves in networks of memristive elements⁴². The presence of the memristor as a coupling element provides for the observation of initial-condition-dependent synchronization of regular⁴³ and chaotic⁴⁴ self-oscillators and wave processes in single-layer⁶ and multilayer⁷ networks. In the present study, we complement the results of paper⁶ addressing the impact of memristive coupling on travelling waves by the consideration of the stochastic dynamics.

The repeatedly pointed out similarity between the memristor behaviour and the functional peculiarities of neural cell synapses (for instance, see Refs.^{45–48}) has in-

I. INTRODUCTION

Travelling waves represent an interdisciplinary phenomenon uniting an incredibly broad variety of dynamical processes in deterministic and stochastic media^{1–5}, networks^{6–10} and delayed-feedback

^{a)}Electronic mail: semenov.v.v.ssu@gmail.com

spired us to consider an ensemble of excitable neurons coupled through the memristive coupling as a promising model for simulation effects in biological neural networks. Noise sources with various characteristics inevitably present in biological neural networks and significantly affect the neuron dynamics^{49,50}. One of the most common models to describe fluctuations in such systems is white or coloured Gaussian noise. In certain cases (for example, in the presence of abrupt stochastic impulses), stochastic processes with the Lévy distribution can model the dynamics of real biological neurons more accurately as compared to Gaussian noise^{51,52}. Motivated by the significance of Lévy processes in neural systems, we consider a network of excitable elements subject to the Lévy white noise. Thus, our research is focused on two factors affecting travelling waves: the presence of memristive coupling and noise including occasional high-amplitude impulses. This choice is dictated by an attempt to describe effects that can be potentially observed in biological neural networks and to reveal fundamental peculiarities of travelling waves in complex systems with adaptive coupling.

II. MODEL AND METHODS

The model under study is schematically illustrated in Fig. 1 (a). It represents an ensemble of the identical FitzHugh-Nagumo oscillators in the excitable regime. The oscillators interact through local memristive coupling. The model equations are:

$$\begin{cases} \frac{dx_i}{dt} = \frac{1}{\varepsilon} (x_i - y_i - x_i^3/3) \\ + s [M(z_{i-1})(x_{i-1} - x_i) + M(z_i)(x_{i+1} - x_i)], \\ \frac{dy_i}{dt} = 0.8x_i - y_i + 0.2 + \xi_i(t), \\ \frac{dz_i}{dt} = x_i - x_{i+1} - \delta z_i, \end{cases} \quad (1)$$

where x_i and y_i are the fast and slow dynamic variables which define the instantaneous state of the i -th oscillator ($i = 1, 2, \dots, N$, where N is the number of interacting oscillators). The parameter ε is usually assumed to be small, which corresponds to the relaxation behaviour of the single oscillator. Variables z_i determine the instantaneous states of the memristive coupling elements, whose conductivity is given by the function $M(z_i) = a + bz_i^2$ (the cubic memristor model where a and b are parameters). The memristor state equations $\dot{z}_i = x_i - x_{i+1} - \delta z_i$ contain the parameter δ which characterizes the memristor forgetting effect. The larger is the parameter δ ,

the shorter is the time range where the correlation between the initial and instantaneous states persists^{53–55}. To avoid the effects reported in paper⁶ and associated with a continuous dependence of the oscillatory dynamics characteristics on the initial conditions, case $\delta = 0$ (the ideal memristor case) is excluded from the consideration.

In the absence of noise, model (1) was considered in Ref.⁶ where the aspects of memristive coupling are described in more detail. In the current paper, we study the coupled FitzHugh-Nagumo oscillators in the excitable regime subject to Lévy noise. Statistically independent additive Lévy noise sources $\xi_i(t)$ in Eqs. (1) are defined as the formal derivatives of the Lévy stable motion. Lévy noise is characterized by four parameters: a stability index $\alpha \in (0 : 2]$, a skewness (asymmetry) parameter $\beta \in [-1 : 1]$, a parameter μ (is assumed to be zero) being a mean value of the Lévy noise when $1 \leq \alpha \leq 2$ and a scale parameter σ . Parameter $D = \sigma^\alpha$ is introduced as the noise intensity. The characteristic function of noise sources $\xi_i(t)$ takes the form^{56–58}:

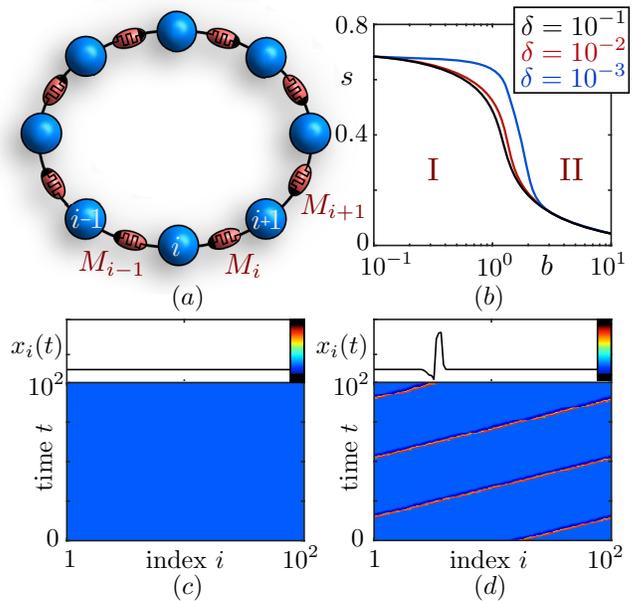


FIG. 1. (a) Schematic representation of an ensemble of the memristively coupled (through elements M_i) FitzHugh-Nagumo neurons (see Eqs. (1)); (b) Map of regimes on the plane (b, s) characterising the deterministic system, $\xi_i(t) \equiv 0$. The only regime of the collective dynamics in area I is the quiescent steady state regime (panel (c)). This regime coexists with travelling waves (panel (d)) in area II; (c)-(d) Space-time plots $x_i(t)$ illustrating the ensemble dynamics. The upper insets show the ensemble state at the last moment $t = 10^2$. System parameters are $a = 1$, $\varepsilon = 0.01$.

$$\phi(k) = \int_{-\infty}^{+\infty} \exp(ikx) L_{\alpha,\beta}(\xi, \sigma, \mu) dx = \begin{cases} \exp \left[-\sigma^\alpha |k|^\alpha \left(1 - i\beta \operatorname{sgn}(k) \tan \frac{\pi\alpha}{2} \right) \right], & \text{for } \alpha \neq 1, \\ \exp \left[-\sigma |k| \left(1 + i\beta \frac{2}{\pi} \operatorname{sgn}(k) \ln |k| \right) \right], & \text{for } \alpha = 1, \end{cases} \quad (2)$$

where $L_{\alpha,\beta}(\xi, \sigma, \mu)$ is the probability density function. To generate random sequences ξ_i corresponding to char-

acteristic function (2), the Janicki-Weron algorithm is used^{56,59}:

$$\begin{aligned} \xi_i &= \sigma S_{\alpha,\beta} \times \frac{\sin(\alpha(V + B_{\alpha,\beta}))}{(\cos(V))^{1/\alpha}} \times \left(\frac{\cos(V - \alpha(V + B_{\alpha,\beta}))}{W} \right)^{\frac{1-\alpha}{\alpha}}, & \text{for } \alpha \neq 1, \\ \xi_i &= \frac{2\sigma}{\pi} \left[\left(\frac{\pi}{2} + \beta V \right) \tan(V) - \beta \ln \left(\frac{\frac{\pi}{2} W \cos(V)}{\frac{\pi}{2} + \beta V} \right) \right], & \text{for } \alpha = 1, \end{aligned} \quad (3)$$

where $B_{\alpha,\beta} = \left(\arctan \left(\beta \tan \left(\frac{\pi\alpha}{2} \right) \right) \right) / \alpha$, $S_{\alpha,\beta} = \left(1 + \beta^2 \tan^2 \left(\frac{\pi\alpha}{2} \right) \right)^{1/2\alpha}$, V is a random variable uniformly distributed on $\left(-\frac{\pi}{2} : \frac{\pi}{2} \right)$, W is an exponential random variable with mean 1 (variables W and V are statistically independent). The same numerical procedure was used in Ref.⁶⁰ addressing the issue of Lévy noise-induced coherence resonance in the single FitzHugh-Nagumo oscillator in the excitable regime. In case $\alpha = 2$, signals $\xi_i(t)$ represent independent sources of white Gaussian noise. In case $\alpha < 2$, the noise distribution is non-Gaussian and contains long heavy tails associated with random impulses of high amplitude.

Numerical simulations were carried out by the integration of model equations (1) using the Heun method⁶¹ with the time step $\Delta t = 10^{-3}$ or smaller. It is important to note that numerical modelling of equations including α -stable stochastic process with finite time step implies the normalization of the noise term by $\Delta t^{1/\alpha}$ (see also^{62,63}). The boundary conditions are chosen to be periodic: $x_{i\pm N} = x_i$, $y_{i\pm N} = y_i$ and $z_{i\pm N} = z_i$. Sufficiently long time of transient processes are discarded to observe the established natural and stationary dynamics. The exploration is carried out by the analysis of plotted instantaneous spatial profiles and spatio-temporal diagrams.

III. DETERMINISTIC DYNAMICS

First, model (1) is considered in the absence of noise, $\xi_i(t) \equiv 0$ to reveal the impact of memristive properties of coupling on travelling waves in the deterministic system. In such a case, one can induce travelling waves in the system by starting simulation from $x_i(0) = \sin(2\pi i/N)$, $y_i(0) = \cos(2\pi i/N)$, if the coupling strength s is sufficiently high. Then continuous decreasing parameter s allows to reveal the area of travelling

wave existence⁶⁴. Result of applying such method for different values of the memristor parameter b is depicted in Fig. 1 (b) as a map of regimes on the plane (b, s) which contains two areas. The only kind of collective dynamics in area I is a completely quiescent steady state regime corresponding to the realization $x_i(t) \approx -1.076$ and $y_i(t) \approx -0.661$ (coordinates of a stable steady state in the phase space of the single oscillator). In contrast, the coexistence of the quiescent regime and travelling waves is realized in area II. As can be seen in Fig. 1 (b), increasing the parameter b being responsible for manifestation of memristive properties allows to achieve travelling waves at lower values of the coupling strength. Changing the parameter δ being responsible for the memristor forgetting effect allows to shift the boundary between the areas of the existence and the absence of travelling waves, but the revealed effect persists: memristor-based adaptive coupling provides for achieving travelling waves at lower coupling strength.

IV. IMPACT OF NOISE

A. Gaussian noise

We begin studying the impact of fluctuations from the case of white Gaussian noise when fixing noise parameters $\alpha = 2$, $\beta = 0$, $\mu = 0$ and varying σ . In the absence of memristive properties (at $b = 0$), increasing the noise intensity results in suppression of travelling waves as demonstrated in Fig. 2 (a1) and reduces the dynamics to the slightly fluctuating quiescent regime. Further increasing the noise level induces spontaneous local spiking activity which extends along the ensemble due to the action of coupling [Fig. 2 (a2)]. When the noise intensity growth continues, spiking activity becomes more regular (the occurrence of coherence resonance) and spatially coherent (the manifestation of synchronization of noise-

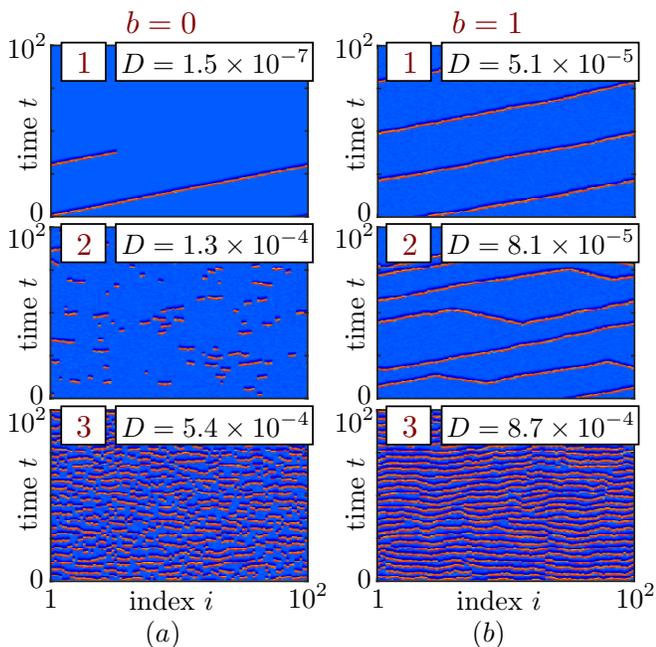


FIG. 2. Transformation of travelling waves when increasing the intensity D of white Gaussian noise in the absence ($b = 0$, panel (a)) and in the presence ($b = 1$, panel (b)) of the coupling element memristive properties. System parameters are $s = 0.7$, $a = 1$, $\varepsilon = 0.01$, $\delta = 10^{-2}$. Noise impact parameters are: $\alpha = 2$, $\beta = 0$, $\mu = 0$ (white Gaussian noise), the noise intensity is introduced as $D = \sigma^\alpha$.

induced oscillations) as depicted in Fig. 2 (a3).

When the coupling elements express the memristive properties (at $b > 0$), travelling waves become much more robust. To illustrate this fact, panel (b1) in Fig. 2 contains a travelling wave which persists at the noise intensity $D = 5.1 \times 10^{-5}$. This value is 340 times higher than the critical value of the noise intensity corresponding to the noise-induced travelling wave destruction observed when the memristive properties of coupling are not expressed (compare Fig. 2 (a1) and Fig. 2 (b1)). Moreover, further growth of the noise intensity does not reduce travelling waves to the quiescent dynamics, but transforms this regime to wandering wave motion including the interaction of the initial wave with new ones spontaneously induced by noise as depicted in Fig. 2 (b2). Noise of larger intensity induces synchronization of the noise-induced oscillations [Fig. 2 (b3)]. Despite the space-time plot in Fig. 2 (b3) is obtained at higher noise-intensity as compared to Fig. 2 (a3), the effect of synchronization is more pronounced which indicates the constructive role of memristive coupling in the context of both travelling waves and the phenomenon of synchronization.

B. Lévy noise

In case $\alpha < 2$, the noise impact is characterised by the appearance of high-amplitude impulses [Fig. 3 (a)].

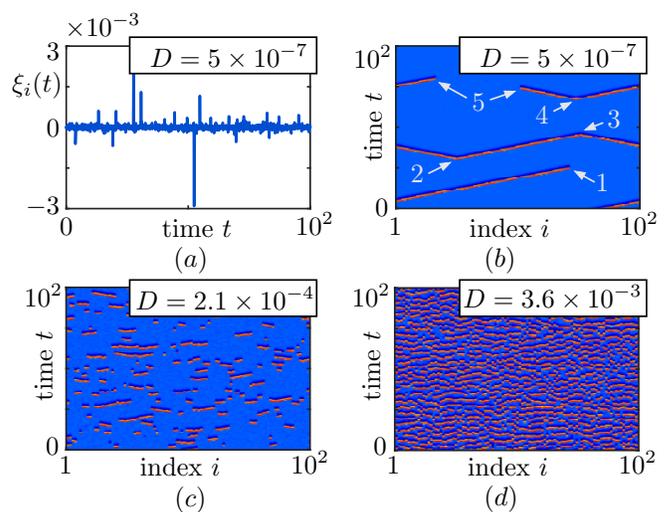


FIG. 3. (a) Particular realization of Lévy noise signal $\xi_i(t)$ at $\alpha = 1.8$, $\beta = 0$, $\mu = 0$, $D = 5 \times 10^{-7}$. (b)-(d) Transformation of travelling waves when increasing the intensity D of Lévy noise in the absence ($b = 0$) of the coupling element memristive properties. System parameters are $s = 0.7$, $a = 1$, $\varepsilon = 0.01$, $\delta = 10^{-2}$. Noise impact parameters are $\alpha = 1.8$, $\beta = 0$, $\mu = 0$.

These impulses can both suppress and induce travelling waves. If the coupling does not exhibit the memristive properties ($b = 0$) and the noise intensity is low, one can observe both effects on the same spatio-temporal diagram (see points 1-5 in Fig. 3 (b)). In particular, the initial wave process in Fig. 3 (b) is suppressed by noise in point 1. However, two new waves travelling to the left and to the right are induced by noise in point 2 and collide in point 3. Then a new pair of waves is induced in point 4 and destroyed in points 5 due to the action of noise.

Similarly to the case of Gaussian noise, travelling waves are much more robust against the Lévy noise in the presence of memristive coupling. In particular, high-amplitude noise impulses provide for appearance of new travelling waves (see points * in Fig. 4 (a),(b)), whereas spontaneous suppression of waves is not observed even for high noise intensities (see Fig. 4 (c) where all the noise-induced waves disappear through the collision). Further noise intensity growth transforms the stochastic dynamics into the regime of synchronized spiking activity [Fig. 4 (d)].

V. CONCLUSIONS

In the current paper, we have demonstrated a constructive role of memristive coupling in the context of travelling waves observed in an ensemble of deterministic excitable oscillators as well as in the presence of noise. In the deterministic case, the memristive properties of the coupling elements provide for realization of travelling waves at sufficiently lower coupling strength as compared

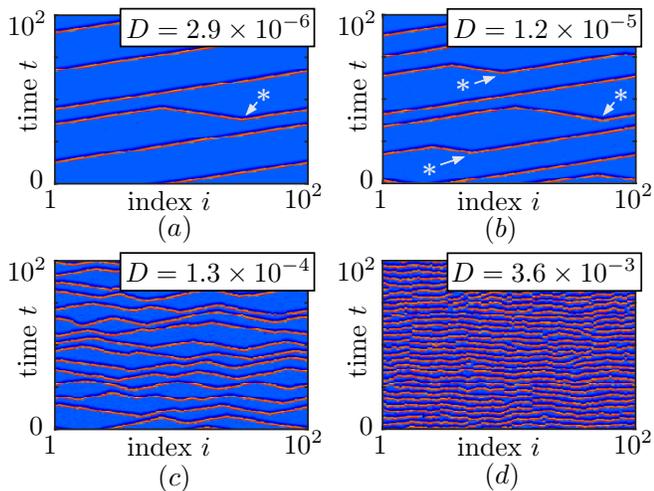


FIG. 4. Transformation of travelling waves when increasing the intensity D of Lévy noise in the presence ($b = 1$) of the coupling element memristive properties. System parameters are $s = 0.7$, $a = 1$, $\varepsilon = 0.01$, $\delta = 10^{-2}$. Noise impact parameters are $\alpha = 1.8$, $\beta = 0$, $\mu = 0$.

to the classical diffusive interaction of fixed intensity. If travelling waves are exhibited by coupled stochastic oscillators, memristive coupling becomes a factor supporting travelling waves such that the noise-induced destruction of travelling waves is observed at higher noise intensities in comparison with diffusive coupling not exhibiting memristive properties.

The second intriguing effect consists in the action of noise. Depending on the properties of the additive stochastic impact, it can both suppress and induce travelling waves. Noise-induced travelling waves can be easily obtained if the stochastic forcing includes high-amplitude impulses. We used the Lévy noise model to visualize this fact.

For sufficiently high noise level, the regime of travelling waves is transformed into the regime of synchronized noise-induced spiking activity being a result of coherence resonance. In our paper, we illustrate this by means of space-time diagrams. However, the revealed evolution into the regime of synchronization requires additional detailed analysis which is an issue for further study. In addition, the presented results can be extended in the following by the consideration of another adaptive coupling models to formulate a generalized conclusion on the impact of synaptic plasticity being an intrinsic property of biological neural networks on effects associated with wave propagation and synchronization in such networks.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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