Wakefield regeneration in a plasma accelerator

J. P. Farmer*

Max-Planck-Institut für Physik, München, Germany

G. Zevi Della Porta

CERN, Genève, Switzerland and
Max-Planck-Institut für Physik, München, Germany

Plasma wakefields offer high acceleration gradients, orders of magnitude larger than conventional RF accelerators. However, the achievable luminosity remains relatively low, typically limited by the plasma recovery time and the charge which can be accelerated per shot. In this work, we show that a train of drive bunches can be harnessed to accelerate multiple witness bunches in a single shot. We demonstrate that periodically loading the wakefields removes the limit on the energy transfer from the drive beam to the plasma, which allows the luminosity to be increased. Proof-of-concept simulations for the AWAKE scheme are carried out to demonstrate the technique, achieving a doubling of the accelerated charge while exploiting only a fraction of the drive train.

Plasma-wakefield acceleration can achieve accelerating gradients orders of magnitude larger than conventional RF accelerating systems. Typically, a short driver, either a laser pulse or charged particle bunch, is used to excite a plasma wave. A trailing witness bunch with the correct delay will be accelerated [1, 2]. Over recent years, huge progress has been made in terms of the attainable beam quality [3]. However, in order for wakefield accelerators to be competitive with RF systems, significant development is still required to address the key issue of luminosity [4].

Short drivers ($L_D \sim 1/k_p = c/\omega_p$, with c the speed of light and ω_p the plasma frequency) are typically preferred due to the relative simplicity of the scheme. Periodic drivers were first considered for laser-driven schemes before the availability of femtosecond lasers, achieved either though a laser-beat [1] or though the self-modulation of a long laser pulse [5, 6]. More recently, the AWAKE experiment at CERN has demonstrated a proton-driven scheme, again exploiting self-modulation due to the lack of a suitably short driver [7, 8]. Periodic laser drivers have recently become the subject of renewed focus due to the higher efficiency with which such beams can be generated [9]. However, the accelerating gradient in these resonantly driven schemes is limited by saturation of the wakefield amplitude due to plasma nonlinearities [10, 11].

The luminosity of any accelerator is limited by the repetition rate and the charge which can be accelerated. The repetition rate for plasma wakefield accelerators is typically determined by the driver, how long the plasma wave takes to decay, as well as limitations introduced by heating [4]. The accelerated charge is limited by beamloading as the plasma reacts to the witness bunch [12]. Too much charge will overload the wakefields, leading to either low particle energy or high energy spread [3], with the limit determined by the plasma density and wakefield amplitude. Transverse beamloading can occur for an electron witness due to ion motion [13], while for positrons it occurs due to the plasma electron response and represents an intrinsic limitation of plasma-based acceleration [14]. Since transverse beamloading increases the slice energy spread of a beam, it may set a limit on the maximum witness current.

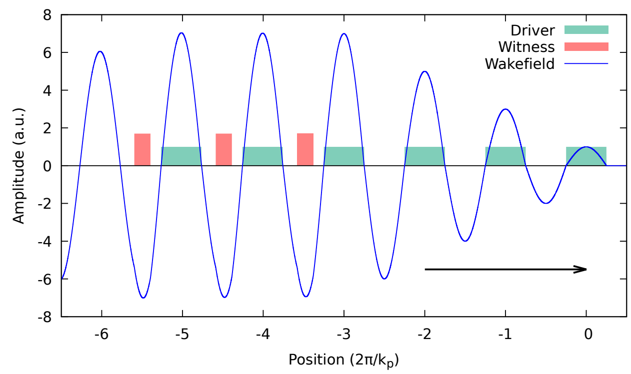

FIG. 1. A train of drive bunches (green), propagating to the right, excites a plasma wakefield (blue). By correctly loading the wakefields, a train of witness bunches can be accelerated to the same energy.

In this Letter, we discuss the use of a periodic driver to accelerate a train of witness bunches. Although similar schemes have previously been proposed [15], the key benefit of this technique appears to have been entirely overlooked: periodically loading and replenishing the wakefields avoids the plasma nonlinearities which lead to saturation. This removes the constraint on the average power transfer from the driver to the plasma, avoiding the impact of nonlinear frequency shifts and even wavebreaking. Proof-of-concept simulations based on the AWAKE experiment at CERN are used to show that this allows the accelerated charge per drive train to be significantly increased, allowing for higher luminosity. The potential application and impact of this technique is then discussed.

A simple schematic of the generalized scheme is shown in Fig. 1, with a train of drive bunches and the linear plasma response superimposed. The limitations of linear theory will be discussed below. The accelerating field behind the first bunch is larger than the decelerating field acting on the bunch, allowing the per-particle witness energy to exceed that of the driver. The ratio between the accelerating and decelerating fields is known as the transformer ratio, and can reach up to two for a symmetric driver [16]. Each drive bunch excites a wakefield, with the bunches separated by $2\pi/k_p$ such that the wakes sum coherently. This configuration leads to high accelerating fields, which scale with the number of drive bunches. However, the decelerating field acting on each drive bunch also increases along the drive train, so the transformer ratio is asymptotic to unity. In schemes where acceleration is limited by depletion of the driver, this results in a lower efficiency [17]. Despite this, the periodic driver still offers advantages where short drivers do not exist, or where long drive trains can be generated more efficiently [9].

A witness bunch, shown in red in Fig. 1, may be injected into the wakefield such that it will be accelerated. For bunches with the same parity, one may consider the witness as being out of phase with the drive train, such that the wake it generates destructively interferes with that excited by the driver. The plasma wake is then suppressed while the witness gains energy.

If the witness bunch is injected within the drive train, the following drive bunch will be in phase with the depleted wakefield, allowing the wakefield to be regenerated. In this way, a train of witness bunches may be accelerated. In order to increase the luminosity of the accelerator, each witness bunch must be accelerated to the same energy. This can be achieved by choosing the witness bunch such that it loads the wakefield to the level before the preceding drive bunch. In this case, the system becomes periodic, with each subsequent drive bunch re-exciting the wake to its previous amplitude, so that all witness bunches gain the same energy. If the wakefields are not correctly loaded, each witness bunch will be accelerated to a different energy. Such beams may have applications as probes, or be attractive as candidates for compression.

The linear increase of the wakefield amplitude with the number of drive bunches breaks down when the plasma response becomes nonlinear. As the wakefield amplitude increases, its period increases, such that the drive train no longer resonantly drives the wakefield [10]. Since the plasma wave has a finite radial extent, the wakefield period has a radial dependence, leading to a "bowing" of the wakefields [18], which can further reduce the on-axis accelerating field [11]. This dephasing between the drive train and the wakefields it drives results in saturation of the accelerating field. In this limit, adding more drive bunches will reduce the efficiency of the system. Even in the absence of nonlinear effects, the longitudinal profile of the drive bunches may reduce the transformer ratio as the wakefields grow [19].

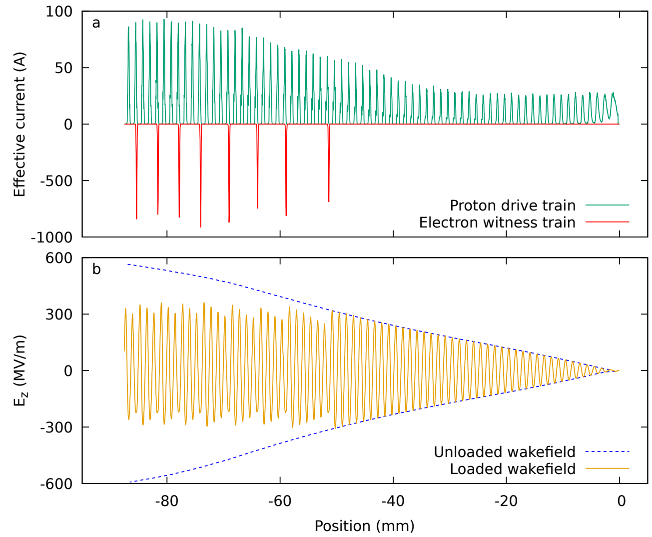

FIG. 2. a) The proton drive train (green) and electron witness train (red) 10 m into the plasma. The long proton beam has self-modulated due to its interaction with the plasma, while the train of eight electron bunches has just been injected. b) The average wakefields over the 1 m acceleration length (10 - 11 m) for the unloaded (blue dashed line, envelope only) and loaded (yellow line) cases.

The key benefit of the novel scheme proposed in this work is that it allows the limitation of saturation to be completely avoided. Indeed, if the plasma response were fully linear, the same total charge could be accelerated to the same energy as a single witness bunch after the entire drive train. However, saturation will always play a role in resonantly driven wakefield accelerators if large gradients are desired. Even in the limit where the drive train is tailored to compensate the nonlinear lengthening of the wakefield period [20, 21], the wakefield amplitude is still constrained by wavebreaking [22]. The plasma response at high wakefield amplitude therefore limits the energy transfer from the driver to the plasma. Periodically loading and regenerating the wake avoids this limitation. Since the spacing between bunches in a train is much shorter than the distance between trains of bunches and the time for the plasma to recover, this allows a higher average power transfer from the drive beam to the plasma, and so to the witness bunches.

In addition to avoiding the limitations of the plasma response to the driver, splitting the witness bunch into a train will reduce transverse beamloading. This scheme therefore has the potential to accelerate a high average current of positrons.

To demonstrate the potential of wakefield regeneration for real applications, we consider the case of AWAKE. A proton drive beam self-modulates in plasma, with the resulting train of microbunches resonantly driving a plasma wakefield to high amplitudes. These wakefields are harnessed to accelerate a witness bunch of electrons. In the forthcoming Run 2c [23], two plasma stages will be used, with self-modulation of the beam occurring in the first stage. The witness bunch will then be injected into the second plasma stage, allowing controlled acceleration. The AWAKE scheme is an ideal candidate for wakefield regeneration, as the proton drive train is already sufficiently long for wakefield saturation to occur. The European Strategy for Particle Physics identified techniques to improve the luminosity of the AWAKE scheme as a key area for future research [24]. Altering the length or repetition rate (on the order of 0.1 Hz) of the proton driver would require significant development of the SPS facility [25], so the ability to instead accelerate more witness charge per drive beam would allow this challenge to be met within the timeline of the AWAKE project.

Simulations were carried out using the quasistatic particle-in-cell code LCODE [26, 27]. The 400 GeV CERN SPS proton beam, with a total population of 3×10^{11}, an RMS length of 7.5 cm, a radius of 175 µm and a normalised emittance of 2.9 µm, is propagated through plasma. The beam is cut at a position 7.5 cm (one sigma) ahead of its centre, equivalent to seeding the plasma wakefields with a relativistic ionization front [28], used to ensure a reproducible bunch train. The plasma has a density of $7 \times 10^{14}\,\mathrm{cm}^{-3}$, with a 3% density step after 1.25 m to avoid the decay of the wakefields after self-modulation [29].

The proton beam after 10 m propagation is shown in 2a, in the form of the effective current, $I_{\mathrm{eff}} = \int_{\Delta t} q\,\mathrm{K}_0(k_p r)\,\mathrm{d}t/\Delta t$, with q the particle charge, r the distance from the axis, and K_0 the zeroth-order modified Bessel function of the second kind. This gives a measure of how strongly each beam slice drives plasma wakefields [12]. As can be seen, the drive beam has been modulated due to its interaction with the plasma. The envelope of the unloaded wakefields are shown in Fig. 2b. The sub-linear scaling of the wakefield amplitude with the number of bunches shows the influence of the nonlinear plasma response, leading to saturation.

Since the wakefields which drive self-modulation grow along the proton beam, the microbunches are nonuniform. The idealised, fully periodic case illustrated in Fig. 1 is therefore not possible. Nevertheless, monoenergetic acceleration of the witness bunch train can be achieved by tailoring the position and charge of the individual witness bunches.

A train of eight witness bunches is injected after 10 m. Each bunch has a Gaussian profile with an RMS length of 60 µm, an initial energy of 150 MeV, and a normalised emittance of 2 µm. The effective current of the witness bunch train at injection is shown in 2a, and the corresponding loaded wakefields shown in 2b. Rather than inject into adjacent accelerating buckets of the wakefield, a witness bunch is injected every few plasma periods. This reduces the size of the optimization problem, with the additional benefit that higher-charge witness bunches will drive their own plasma blowout, allowing emittance control [30]. The effective current of the driver increases along the length of the witness beam, so later witness bunches are injected more closely together.

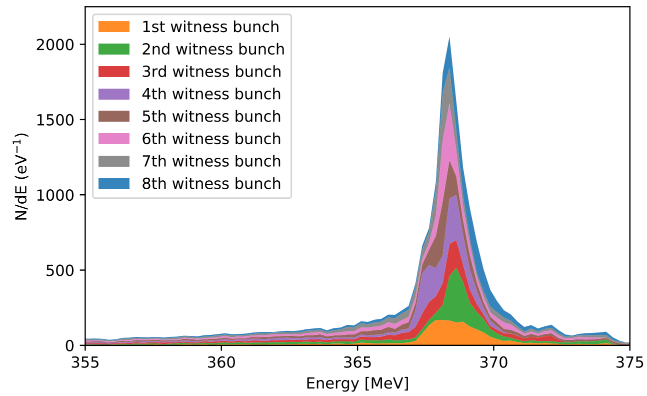

FIG. 3. The energy spectrum of the 912 pC witness train in Fig. 2 after 1 m of acceleration. The contribution of the eight witness bunches is shown. 56% of the accelerated charge is within a ±0.5 % energy range.

The first witness bunch has a charge of 100 pC, with the charge of subsequent bunches adjusted such that the loaded wakefields give the same energy gain for all bunches. The energy gain after 1 m of acceleration, chosen to facilitate a rapid optimization, is shown in Fig. 3. A total witness charge of 912 pC is injected and accelerated to 368 MeV, with 56% of the accelerated charge within a ±0.5 % energy range. The optimal witness train has some dependence on the acceleration length due to the slow evolution of the driver, and so acceleration over longer distances would require the optimization be repeated. The density step was chosen to maximize the length of the resulting drive train, and so the acceleration gradient could likely be improved through further optimization of the SMI stage. Injecting later in the drive beam or injecting a lower witness charge would also improve the energy gain.

The benefit of the wakefield-regeneration scheme is immediately apparent from Fig. 2b. After the wakefield is loaded by a witness bunch, its amplitude recovers as subsequent drive bunches replenish the wake. However, the growth rate during this recovery is larger than the growth rate of the unloaded wake. This is precisely due to the onset of saturation at larger wakefield amplitude. In the absence of loading, the wakefields gradually dephase with the drive train, and the growth is reduced. By loading the wakefields, this frequency shift is reduced, and so the drive train remains in phase with the wakefields.

The unloaded wakefields do continue to grow after the chosen injection point, albeit at a lower rate. Injecting a single witness bunch at a later position would therefore

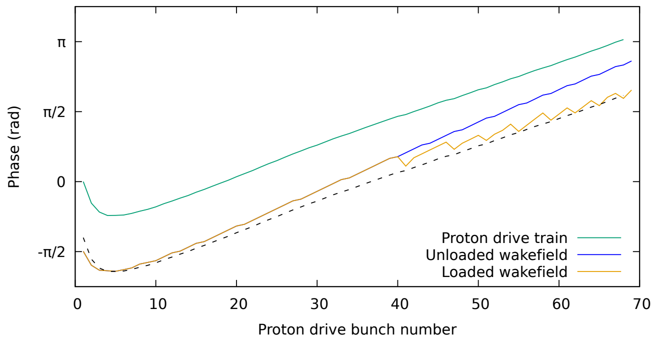

FIG. 4. Phase of the proton drive bunch train (green) and the wakefields it excites, after 1 m acceleration. Both the unloaded (blue) and loaded (yellow) wakefields are shown. The drive train phase is calculated from the bunch centroids, wakefields as the peaks of the electric field, both taken relative to the first microbunch. The black dashed line shows a constant phase offset from the drive train. When the wakefields are loaded, the phase difference between the drive train and wakefields remains near constant.

allow a higher charge to be accelerated. The unloaded wakefield amplitude at the position of the eighth witness bunch is 1.9 times that at the position of the first. The accelerating gradient is $E_z = E_{z0}\,(1 - \eta_{\rm load})$, where E_{z0} is the unloaded wakefield amplitude and $\eta_{\rm load}$ is the fractional beamloading, 29% for the first witness bunch. Noting that the bunch charge $Q \sim \eta_{\rm load} E_z$ we see that a single witness bunch with a charge of 400 pC could be injected at this later position and reach the same energy. The injection of eight witness bunches with a combined charge of 912 pC, exploiting the enhanced growth rate of the loaded wake, represents a doubling of the charge accelerated in a single shot. We note that only a fraction of the drive beam is used in these simulations, with the eighth witness bunch approximately at the centre of the drive beam. Further gains could readily be achieved by adding more witness bunches, exploiting more of the proton beam.

The dephasing between the driver and wakefields is further explored in Fig. 4, which shows the phase of the microbunch train and the wakefield it drives. The periodicity of the microbunch train is slightly longer than the resonant plasma frequency due to the growth of the self-modulation instability [31, 32], and the weakly nonlinear plasma response during the self-modulation growth. The phase of the wakefields follows the same general trend, although the relative phase of the driver and the unloaded wakefields gradually varies along the length of the drive train. It is this dephasing which leads to the reduction in the wakefield growth. For the case of the loaded wakefield, the phase difference is kept at a roughly constant level. This allows the drive train to resonantly excite the wakefields along its entire length, as demonstrated in Fig. 2b by the enhanced wakefield growth after each witness bunch.

The simulation results show that a train of witness bunches allows more energy to be transferred from the driver to the witness. However, the practical implementation of such a beam may prove challenging. The electron source envisioned for AWAKE Run 2c incorporates an S-band RF photo injector [33], which would prevent the injection of witness bunches with a spacing of a few millimetres. One solution, albeit a costly one, would be to incorporate multiple such electron guns into the experimental setup. Alternative injection schemes, such as laser-foil injection [34, 35], or laser-plasma injection [36], would potentially allow the creation of a suitable train of witness bunches by using a train of laser pulses.

The optimization itself is relatively straightforward, as the charge and position of each subsequent witness bunch is essentially tuned independently until they reach the energy of the preceding bunches. Since the optimization must be redone for each witness bunch, but the optimization itself will be similar, this would be a good candidate for machine learning techniques.

As discussed above, the requirement for a nonuniform witness train is a direct consequence of the drive beam in the AWAKE scheme. The use of driver made up of uniform bunches would allow the use of a uniform witness train, significantly easing the constraints on tuning. For the SPS proton beam, such a drive train could potentially be achieved using a dielectric bunching structure [37]. Laser schemes such as the plasma-modulated plasma accelerator [9] or electro-optic frequency combs [38, 39] could also provide a suitable driver. Periodic electron drivers could be generated using a train of laser pulses on a photocathode [40], or by modulating an electron beam using a mask [41], a terahertz cavity [42], or inverse-FEL bunching [43].

In summary, we propose a new scheme of plasma wakefield acceleration, where a train of witness microbunches are accelerated in the wake driven by a train of drive bunches. Key to this concept is that periodically loading the wakefields prevents the dephasing which occurs in resonantly driven wakefield accelerators as the plasma response becomes nonlinear. The energy transfer from the driver to the plasma can therefore be significantly increased, allowing a larger total witness charge to be accelerated per drive train, increasing the luminosity. Proof-of-concept simulations for the AWAKE experiment demonstrate the acceleration of a 912 pC witness bunch train, approximately twice that which could be accelerated in a single bunch. Further gains are readily available through the addition of more witness bunches. This method is generally applicable to resonantly driven wakefield accelerators, and so could equally be applied to other schemes such as the plasma-modulated plasma accelerator [9]. The scheme also permits a high average witness current while keeping the peak witness current low, desirable for plasma-based positron acceleration.

The authors would like to thank Eduardo Granados, who first proposed this topic, and Konstantin Lotov and Patric Muggli for fruitful discussions.

---

* j.farmer@cern.ch

[1] T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979).

[2] P. Chen, J. M. Dawson, R. W. Huff, and T. Katsouleas, Phys. Rev. Lett. **54**, 693 (1985).

[3] C. A. Lindstrøm, J. M. Garland, S. Schröder, L. Boulton, G. Boyle, J. Chappell, R. D'Arcy, P. Gonzalez, A. Knetsch, V. Libov, G. Loisch, A. Martinez de la Ossa, P. Niknejadi, K. Põder, L. Schaper, B. Schmidt, B. Sheeran, S. Wesch, J. Wood, and J. Osterhoff, Phys. Rev. Lett. **126**, 014801 (2021).

[4] R. D'Arcy, J. Chappell, J. Beinortaite, S. Diederichs, G. Boyle, B. Foster, M. J. Garland, P. G. Caminal, C. A. Lindstrøm, G. Loisch, S. Schreiber, S. Schröder, R. J. Shalloo, M. Thévenet, S. Wesch, M. Wing, and J. Osterhoff, Nature **603**, 58 (2022).

[5] N. E. Andreev, L. M. Gorbunov, V. I. Kirsanov, A. A. Pogosova, and R. R. Ramazashvili, JETP Lett. **55**, 571 (1992).

[6] J. Krall, A. Ting, E. Esarey, and P. Sprangle, Phys. Rev. E **48**, 2157 (1993).

[7] A. Caldwell and K. V. Lotov, Phys. Plasmas **18**, 103101 (2011).

[8] AWAKE Collaboration, Phys. Rev. Lett. **122**, 054802 (2019).

[9] O. Jakobsson, S. M. Hooker, and R. Walczak, Phys. Rev. Lett. **127**, 184801 (2021).

[10] M. N. Rosenbluth and C. S. Liu, Phys. Rev. Lett. **29**, 701 (1972).

[11] K. V. Lotov, Phys. Plasmas **20**, 083119 (2013).

[12] T. Katsouleas, S. Wilks, P. Chen, J. M. Dawson, and J. J. Su, Part. Accel. **22**, 81 (1987).

[13] J. B. Rosenzweig, A. M. Cook, A. Scott, M. C. Thompson, and R. B. Yoder, Phys. Rev. Lett. **95**, 195002 (2005).

[14] G. J. Cao, C. A. Lindstrøm, E. Adli, S. Corde, and S. Gessner, Phys. Rev. Accel. Beams **27**, 034801 (2024).

[15] S. van der Meer, *Improving the power efficiency of the plasma wakefield accelerator*, Tech. Rep. (CERN, Geneva, 1985).

[16] R. D. Ruth, A. W. Chao, P. L. Morton, and P. B. Wilson, Part. Accel. **17**, 171 (1985).

[17] R. Maeda, T. Katsouleas, P. Muggli, C. Joshi, W. B. Mori, and W. Quillinan, Phys. Rev. ST Accel. Beams **7**, 111301 (2004).

[18] E. Esarey, P. Sprangle, J. Krall, and A. Ting, Plasma Science, IEEE Transactions on **24**, 252 (1996).

[19] J. P. Farmer, R. Martorelli, and A. Pukhov, Phys. Plasmas **22**, 123113 (2015).

[20] D. Umstadter, E. Esarey, and J. Kim, Phys. Rev. Lett. **72**, 1224 (1994).

[21] J. J. van de Wetering, S. M. Hooker, and R. Walczak, Phys. Rev. E **109**, 025206 (2024).

[22] J. M. Dawson, Phys. Rev. **113**, 383 (1959).

[23] E, Gschwendtner *et al.* (AWAKE Collaboration), Symmetry **14**, 1680 (2002).

[24] N. Mounet, ed., *European Strategy for Particle Physics - Accelerator R&D Roadmap* (CERN Yellow Reports: Monographs, 2022).

[25] J. P. Farmer, A. Caldwell, and A. Pukhov, Preliminary investigation of a Higgs factory based on proton-driven plasma wakefield acceleration (2024), arXiv:2401.14765 [physics.acc-ph].

[26] K. V. Lotov, Phys. Rev. ST Accel. Beams **6**, 061301 (2003).

[27] *See* `https://lcode.info` *for code description and manual.*

[28] F. Batsch, P. Muggli, *et al.* (AWAKE Collaboration), Phys. Rev. Lett. **126**, 164802 (2021).

[29] K. V. Lotov, Phys. Plasmas **22**, 103110 (2015), https://doi.org/10.1063/1.4933129.

[30] V. K. B. Olsen, E. Adli, and P. Muggli, Phys. Rev. Accel. Beams **21**, 011301 (2018).

[31] A. Pukhov, N. Kumar, T. Tückmantel, A. Upadhyay, K. Lotov, P. Muggli, V. Khudik, C. Siemon, and G. Shvets, Phys. Rev. Lett. **107**, 145003 (2011).

[32] C. B. Schroeder, C. Benedetti, E. Esarey, F. J. Grüner, and W. P. Leemans, Phys. Rev. Lett. **107**, 145002 (2011).

[33] K. Pepitone, S. Doebert, R. Apsimon, J. Bauche, M. Bernardini, C. Bracco, G. Burt, A. Chauchet, E. Chevallay, N. Chritin, S. Curt, H. Damerau, M. D. Kelisani, C. Delory, V. Fedosseev, F. Friebel, F. Galleazzi, I. Gorgisyan, E. Gschwendtner, J. Hansen, L. Jensen, F. Keeble, L. Maricalva, S. Mazzoni, G. McMonagle, O. Mete, A. Pardons, C. Pasquino, V. Verzilov, J. Schmidt, L. Soby, B. Williamson, E. Yamakawa, S. Pitman, and J. Mitchell, Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment **909**, 102 (2018), 3rd European Advanced Accelerator Concepts workshop (EAAC2017).

[34] V. Khudiakov and A. Pukhov, Phys. Rev. E **105**, 035201 (2022).

[35] T. C. Wilson, J. Farmer, A. Pukhov, Z.-M. Sheng, and B. Hidding (2024), in preparation.

[36] D. F. G. Minenna *et al.*, JACoW **IPAC2023**, TUPA089 (2023).

[37] A. Petrenko and I. Sheinman, in *Proc. of Russian Particle Accelerator Conference (RuPAC'16), St. Petersburg, Russia, November 21-25, 2016*, Russian Particle Accelerator Conference No. 25 (JACoW, Geneva, Switzerland, 2017) pp. 303–306, doi:10.18429/JACoW-RuPAC2016-TUPSA041.

[38] H. Ye, F. Leroi, L. Pontagnier, G. Santarelli, J. Boullet, and E. Cormier, Opt. Express **30**, 10605 (2022).

[39] R. Yang, J. Wu, H. Yang, H. Fu, L. Yu, X. Xing, Y. Dong, P. Hu, and J. Tan, Nanomaterials **12**, 10.3390/nano12213907 (2022).

[40] G. Andonian, N. Burger, N. Cook, S. Doran, T. Hodgetts, S. Kim, G. Ha, W. Liu, W. Lynn, N. Majernik, J. Power, A. Pronikov, J. Rosenzweig, and E. Wisniewski, Instruments **8**, 10.3390/instruments8020028 (2024).

[41] P. Muggli, V. Yakimenko, M. Babzien, E. Kallos, and K. P. Kusche, Phys. Rev. Lett. **101**, 054801 (2008).

[42] M. T. Hibberd, A. L. Healy, D. S. Lake, V. Georgiadis, E. J. H. Smith, O. J. Finlay, T. H. Pacey, J. K. Jones, Y. Saveliev, D. A. Walsh, E. W. Snedden, R. B. Appleby, G. Burt, D. M. Graham, and S. P. Jamison, Nature Photonics **14**, 755 (2020).

[43] P. Manwani, N. Majernik, M. Yadav, C. Hansel, and J. B. Rosenzweig, Phys. Rev. Accel. Beams **24**, 051302 (2021).