Greedy Capon Beamformer

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***Abstract*—We propose greedy Capon beamformer (GBF) for direction finding of narrow-band sources present in the array's viewing field. After defining the grid covering the location search space, the algorithm greedily builds the interference-plus-noise covariance matrix by identifying a high-power source on the grid using Capon's principle of maximizing the signal to interference plus noise ratio (SINR) while enforcing unit gain towards the signal of interest. An estimate of the power of the detected source is derived by exploiting the unit power constraint, which subsequently allows to update the noise covariance matrix by simple rank-1 matrix addition composed of outerproduct of the selected steering matrix with itself scaled by the signal power estimate. Our numerical examples demonstrate effectiveness of the proposed GCB in direction finding where it perform favourably compared to the state-of-the-art algorithms under a broad variety of settings. Furthermore, GCB estimates of direction-of-arrivals (DOAs) are very fast to compute.**

***Index Terms*—Beamforming, MVDR beamforming, direction finding, source localization, greedy pursuit**

I. Introduction

COMMON to most direction finding (DF) methods is the need to estimate the unknown array covariance matrix $\mathbf{\Sigma} = \mathrm{cov}(\mathbf{x}) \succ 0$ of the array output $\mathbf{x} \in \mathbb{C}^N$. For example, the commonly used conventional (delay-and-sum) beamformer [1] and the standard Capon beamformer (SCB) [2] require an estimate of $\mathbf{\Sigma}$ to measure the power of the beamformer output as a function of the direction-of-arrival (DOA). In addition, many high-resolution subspace-based DOA algorithms (such as MUSIC [3] or R(oot)-MUSIC [4]) compute the noise or signal subspaces from the eigenvectors of the array covariance matrix and exploit the fact that signal subspace eigenvectors and the array steering matrix span the same subspace. Array covariance matrix is conventionally estimated from the array snapshots via the sample covariance matrix (SCM). However, the SCM is poorly estimated when the snapshot size is small and adaptive beamformers (such as SCB) that are based on SCM are not computable when $N < L$. Sparse methods for DOA estimation [5] provide a remedy for these issues.

We assume that K narrowband sources are present in the array's viewing field. Let $\mathbf{x}$ denote the array output of N-sensors, and $\mathbf{a}(\theta) \in \mathbb{C}^N$ denote the array manifold, where θ denotes the generic location parameter in the location space Θ. For example, $\mathbf{a}(\theta) = (1, e^{-\jmath \cdot 1 \cdot \frac{2\pi d}{\lambda}\sin\theta}, \ldots, e^{-\jmath\cdot(N-1)\cdot\frac{2\pi d}{\lambda}\sin\theta})^\top$ for Uniform Linear Array (ULA), where λ is the wavelength, d is the element spacing between the sensors and $\theta \in \Theta = [-\pi/2, \pi/2)$ is the DOA in radians. For an arbitrary steering vector in the array manifold we use notation $\mathbf{a}$, dropping the dependency on θ. The output of a beamformer is defined by

$$y = \mathbf{w}^{\mathsf{H}}\mathbf{x},$$

where $\mathbf{w}$ is the beamformer weight vector that depends on θ through steering vector $\mathbf{a}$. Ideally, the beamformer weight $\mathbf{w} = \mathbf{w}(\theta)$ is chosen such that the beamformer will null the interferences and noise while allowing the signal of interest (SOI) at θ to pass undistorted. The objectives of beamforming can be formulated as [1], [6], [7]:

(A) to recover the signal of interest
(B) to estimate the locations of the K signals impinging on the array.

The latter objective is achieved by estimating the array output power distribution

$$P(\theta) = \mathbb{E}[|\mathbf{w}^{\mathsf{H}}\mathbf{x}|^2] = \mathbf{w}^{\mathsf{H}}\mathbf{\Sigma}\mathbf{w}$$

over a fine grid $\{\theta\}_{m=1}^M$ covering Θ, i.e., allowing θ (and hence $\mathbf{a}$ in the design of the beamformer weight $\mathbf{w}$) vary through the location space Θ. In this manner a spatial power spectrum can be constructed and locations of the signals can be found as peaks in the spectrum.

In this letter, we propose greedy Capon beamformer (GCB) that greedily selects the next high-power source (not yet detected) using Capon beamforming principle and subsequently updates the interference-plus-noise covariance matrix. The approach is computationally light; it does not require computing the eigenvalue decomposition (EVD) as needed by MUSIC or R-MUSIC nor inverting the SCM as in SCB (i.e., $L > N$ is not required). There are similar sparsity and covariance based DOA estimation methods, e.g, [8]–[14] or review in [5], which are primarily iterative algorithms constructed using some optimization principles. Our simulation studies illustrate that the proposed GCB performs favourably against these state-of-the-art (SOTA) methods.

II. Capon beamformer

Let the array covariance matrix $\mathbf{\Sigma}$ be decomposed as

$$\mathbf{\Sigma} = \gamma \mathbf{a}\mathbf{a}^{\mathsf{H}} + \mathbf{Q} \tag{1}$$

where γ denotes the power of the SOI, $\mathbf{a}$ is the array steering vector of the SOI at θ, and $\mathbf{Q}$ is positive definite ($\mathbf{Q} \succ 0$) interference-plus-noise covariance matrix. It is assumed that the noise is spatially white and $K-1$ uncorrelated directional interfering signals are present. Then the matrix $\mathbf{Q}$ can be expressed in the form

$$\mathbf{Q} = \sum_{k=2}^{K} \gamma_k \mathbf{a}_k \mathbf{a}_k^{\mathsf{H}} + \sigma^2\mathbf{I}$$

where $\sigma^2\mathbf{I}$ is the noise covariance matrix (i.e., we assume spatially white noise), $\mathbf{a}_k = \mathbf{a}(\theta_k)$ is the steering vector of the directional interference signal at θ_k, and $\gamma_k > 0$ is the associated signal power ($k = 2, \ldots, K$). Without any loss of

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generality (w.l.o.g.) we will assume that all steering vectors are normalized such that $\|\mathbf{a}_k\|^2 = N$ holds.

Capon beamformer minimizes the array output power subject to the constraint that the SOI is passed undistorted:

$$\min_{\mathbf{w}} \mathbf{w}^{\mathsf{H}}\boldsymbol{\Sigma}\mathbf{w} \quad \text{subject to } \mathbf{w}^{\mathsf{H}}\mathbf{a} = 1, \tag{P1}$$

or equivalently, maximizing the signal to interference plus noise ratio (SINR) at its output while enforcing a unit gain towards the signal of interest:

$$\max_{\mathbf{w}} \frac{\gamma|\mathbf{w}^{\mathsf{H}}\mathbf{a}|^2}{\mathbf{w}^{\mathsf{H}}\mathbf{Q}\mathbf{w}} \quad \text{subject to } \mathbf{w}^{\mathsf{H}}\mathbf{a} = 1 \tag{P2}$$

where γ designates the power of the SOI. Due to the first formulation (P1), Capon beamformer is also often called minimum variance distortionless response (MVDR) beamformer. The solutions to power minimization problem (P1) and SINR maximization (P2) are equivalent and the optimum beamformer weight for both problems is given by

$$\mathbf{w}_{\text{opt}} = \frac{\boldsymbol{\Sigma}^{-1}\mathbf{a}}{\mathbf{a}^{\mathsf{H}}\boldsymbol{\Sigma}^{-1}\mathbf{a}} = \frac{\mathbf{Q}^{-1}\mathbf{a}}{\mathbf{a}^{\mathsf{H}}\mathbf{Q}^{-1}\mathbf{a}}. \tag{2}$$

The equivalence of the solutions can be easily verified using (1) and Sherman-Morrison formula for the inverse [15]. The optimal power and SINR are then

$$P_{\text{opt}} = \mathbf{w}_{\text{opt}}^{\mathsf{H}}\boldsymbol{\Sigma}\mathbf{w}_{\text{opt}} = \frac{1}{\mathbf{a}^{\mathsf{H}}\boldsymbol{\Sigma}^{-1}\mathbf{a}}, \tag{3}$$

$$\text{SINR}_{\text{opt}} = \frac{\gamma|\mathbf{w}_{\text{opt}}^{\mathsf{H}}\mathbf{a}|^2}{\mathbf{w}_{\text{opt}}^{\mathsf{H}}\mathbf{Q}\mathbf{w}_{\text{opt}}} = \gamma\mathbf{a}^{\mathsf{H}}\mathbf{Q}^{-1}\mathbf{a}. \tag{4}$$

The next lemma relates the power γ of the SOI to the optimum power and optimum SINR. This will be later used in proposed GBF estimate the power of the detected signal.

Lemma 1. *The power γ of the SOI at steering vector $\mathbf{a}$ is given by*

$$\gamma = \mathbf{w}_{\text{opt}}^{\mathsf{H}}(\boldsymbol{\Sigma} - \mathbf{Q})\mathbf{w}_{\text{opt}} = P_{\text{opt}} - \frac{1}{\mathbf{a}^{\mathsf{H}}\mathbf{Q}^{-1}\mathbf{a}} \tag{5}$$

where P_{opt} is the optimum beamformer power (3).

Proof. Due to unit gain constraint one has that $\mathbf{w}_{\text{opt}}^{\mathsf{H}}\mathbf{a} = 1$ and since $\gamma\mathbf{a}\mathbf{a}^{\mathsf{H}} = \boldsymbol{\Sigma} - \mathbf{Q}$ from (1), we may write

$$\begin{aligned}
\gamma = \gamma|\mathbf{w}_{\text{opt}}^{\mathsf{H}}\mathbf{a}|^2 &= \mathbf{w}_{\text{opt}}^{\mathsf{H}}(\gamma\mathbf{a}\mathbf{a}^{\mathsf{H}})\mathbf{w}_{\text{opt}} && (6)\\
&= \mathbf{w}_{\text{opt}}^{\mathsf{H}}(\boldsymbol{\Sigma} - \mathbf{Q})\mathbf{w}_{\text{opt}} && (7)\\
&= \mathbf{w}_{\text{opt}}^{\mathsf{H}}\boldsymbol{\Sigma}\mathbf{w}_{\text{opt}} - \mathbf{w}_{\text{opt}}^{\mathsf{H}}\mathbf{Q}\mathbf{w}_{\text{opt}} \\
&= P_{\text{opt}} - \frac{1}{\mathbf{w}_{\text{opt}}^{\mathsf{H}}\mathbf{Q}^{-1}\mathbf{w}_{\text{opt}}}, && (8)
\end{aligned}$$

where the last identity follows by recalling (3) and noting that $\mathbf{w}_{\text{opt}}^{\mathsf{H}}\mathbf{Q}\mathbf{w}_{\text{opt}} = (\mathbf{w}_{\text{opt}}^{\mathsf{H}}\mathbf{Q}^{-1}\mathbf{w}_{\text{opt}})^{-1} = (\text{SINR}_{\text{opt}}/\gamma)^{-1}$, when the latter form of $\mathbf{w}_{\text{opt}}$ in (2) is invoked. □

So far we have discussed the ideal situation, so assuming that θ of SOI, and hence the steering vector $\mathbf{a}$ and the covariance matrix $\mathbf{Q}$ are known exactly. These imply also the complete knowledge of array covariance matrix $\boldsymbol{\Sigma}$. Objective (B) requires access to $\boldsymbol{\Sigma}$ which is unavailable in practise, and commonly estimated by the SCM $\hat{\boldsymbol{\Sigma}}$, defined by

$$\hat{\boldsymbol{\Sigma}} = \frac{1}{L}\sum_{l=1}^{L}\mathbf{x}_l\mathbf{x}_l^{\mathsf{H}} = L^{-1}\mathbf{X}\mathbf{X}^{\mathsf{H}},$$

with L denoting the number of snapshots, $\mathbf{x}_l$ representing the l^{th} snapshot and $\mathbf{X} = (\mathbf{x}_1 \cdots \mathbf{x}_L) \in \mathbb{C}^{N\times L}$ designating the snapshot data matrix. Consider a grid $\{\theta\}_{m=1}^{M}, \theta_1 < \ldots < \theta_M$, of location parameters covering Θ, and let $\{\mathbf{a}_m\}_{m=1}^{M}$ denote the corresponding steering vectors. Let $\mathbf{A} = (\mathbf{a}_1 \cdots \mathbf{a}_M)$ denote the associated steering matrix and $\boldsymbol{\theta} = (\theta, \ldots, \theta_M)^{\top}$ the vector of location parameters. The *standard Capon beamformer (SCB)* computes an estimate of the spatial power for SOI at θ_i using

$$\hat{P}_{\text{SCB},i} = \frac{1}{\mathbf{a}_i^{\mathsf{H}}\hat{\boldsymbol{\Sigma}}^{-1}\mathbf{a}_i}, \ i = 1, \ldots, M. \tag{9}$$

which is empirical (sample based) estimate of (3). Typically, it is assumed that the grid resolution is fine enough so that the true location parameters of the sources lie on or are in close proximity to the grid. Then, given the knowledge that K sources are present in the array output, Capon beamformer chooses the DOA estimates as the K largest peaks of the estimated spectrum. SCB algorithm proceeds as follows:

Input: $\hat{\boldsymbol{\Sigma}}$, K, $\mathbf{A}$, $\boldsymbol{\theta}$.

1) Compute the beamformer output powers $\hat{P}_{\text{SCB},i}$ in (9).
2) Identify the indices of the K peaks in the spatial spectrum:
$$\mathcal{M} = \mathsf{peaks}_K(\hat{P}_{\text{SCB},1}, \ldots, \hat{P}_{\text{SCB},M})$$
where $\mathcal{M} \subset \{1, \ldots, M\}$ with $|\mathcal{M}| = K$.

Output: $\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_{\mathcal{M}}$, the vector of K DOA estimates.

We use $\mathbf{a}_{\mathcal{M}}$ to denote a K-vector consisting of components of an M-vector $\mathbf{a}$ corresponding to indices in set $\mathcal{M} \subset \{1, \ldots, M\}$ with $|\mathcal{M}| = K < M$.

III. The Greedy Capon beamformer

Our greedy Capon beamformer (GBF) for direction finding proceeds as follows.

Initialize: Set $\mathbf{Q} = [\text{tr}(\hat{\boldsymbol{\Sigma}})/N] \cdot \mathbf{I}$, $\mathcal{M} = \emptyset$.

Main iteration: for $k = 1, \ldots, K$, iterate the steps below

1) Compute the beamformer output power estimates
$$\hat{P}_i = \mathbf{w}_i^{\mathsf{H}}\hat{\boldsymbol{\Sigma}}\mathbf{w}_i, \ i = 1, \ldots, M,$$
where $\mathbf{w}_i = \mathbf{Q}^{-1}\mathbf{a}_i/(\mathbf{a}_i^{\mathsf{H}}\mathbf{Q}^{-1}\mathbf{a}_i)$ signifies the beamformer weight for location θ_i.
2) Identify the indices of the k largest peaks in the spatial spectrum:
$$\mathcal{K} = \mathsf{peaks}_k(\hat{P}_1, \ldots, \hat{P}_M),$$
i.e., $\mathcal{K} \subset \{1, \ldots, M\}$ with $|\mathcal{K}| = k$.
3) Choose the index that has the least coherence with steering vectors that have been picked up so far:
$$i_k = \arg\min_{i\in\mathcal{K}}(\max_{j\in\mathcal{M}}|\mathbf{a}_i^{\mathsf{H}}\mathbf{a}_j|).$$
Thus i_k is the index from set $\mathcal{K}$ representing the steering vector $\mathbf{a}_{i_k}$ that is the least coherent to steering vectors $\mathbf{a}_j$, $j \in \mathcal{M}$, that has already been chosen.

4) Update the set $\mathcal{M} \leftarrow \mathcal{M} \cup \{i_k\}$ of chosen indices.
5) Estimate the signal power of chosen source as $\gamma_k = \hat{P}_{i_k} - (\mathbf{a}_{i_k}^{\mathsf{H}} \mathbf{Q}^{-1} \mathbf{a}_{i_k})^{-1}$, i.e., using equation (5) of Lemma 1.
6) Update the interference and noise covariance matrix as $\mathbf{Q} \leftarrow \mathbf{Q} + \gamma_k \mathbf{a}_{i_k} \mathbf{a}_{i_k}^{\mathsf{H}}$. Such rank-1 update allows to compute the inverse covariance matrix $\mathbf{Q}^{-1}$ efficiently using the Sherman-Morrison [16] formula[1], yielding

$$\mathbf{Q}^{-1} \leftarrow \mathbf{Q}^{-1} - \frac{\gamma_k \mathbf{\Sigma}^{-1} \mathbf{a}_{i_k} \mathbf{a}_{i_k}^{\mathsf{H}} \mathbf{\Sigma}^{-1}}{1 + \gamma_k \mathbf{a}_{i_k}^{\mathsf{H}} \mathbf{\Sigma}^{-1} \mathbf{a}_{i_k}} \tag{10}$$

After main iteration loop terminates (i.e., after K iterations), one obtains the index set $\mathcal{M}$, which then identifies source locations as $\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_{\mathcal{M}}$ as in SCB algorithm outlined in Section II. The pseudo code is tabulated in algorithm 1.

Algorithm 1: **G**reedy **C**apon's **B**eamformer for DF

Input : $\hat{\mathbf{\Sigma}} = L^{-1}\mathbf{X}\mathbf{X}^{\mathsf{H}}$, K, $\mathbf{A}$, $\boldsymbol{\theta}$
Initialize: $\mathbf{Q}^{-1} = [N/\operatorname{tr}(\hat{\mathbf{\Sigma}})]\mathbf{I}$, $\mathcal{M} = \emptyset$
for $k = 1, \dots, K$ **do**

$$\hat{P}_i = \frac{\mathbf{a}_i^{\mathsf{H}} \mathbf{Q}^{-1} \hat{\mathbf{\Sigma}} \mathbf{Q}^{-1} \mathbf{a}_i}{(\mathbf{a}_i^{\mathsf{H}} \mathbf{Q}^{-1} \mathbf{a}_i)^2}, \ i = 1, \dots, M.$$

$$\mathcal{K} = \mathsf{peaks}_k(\hat{P}_1, \dots, \hat{P}_M)$$

$$\mathcal{M} \leftarrow \mathcal{M} \cup \{i_k\} \text{ with } i_k = \arg\min_{i \in \mathcal{K}} (\max_{j \in \mathcal{M}} |\mathbf{a}_i^{\mathsf{H}} \mathbf{a}_j|).$$

$$\gamma_k = \hat{P}_{i_k} - (\mathbf{a}_{i_k}^{\mathsf{H}} \mathbf{Q}^{-1} \mathbf{a}_{i_k})^{-1}$$

$$\mathbf{Q}^{-1} \leftarrow \mathbf{Q}^{-1} - \frac{\gamma_k \mathbf{Q}^{-1} \mathbf{a}_{i_k} \mathbf{a}_{i_k}^{\mathsf{H}} \mathbf{Q}^{-1}}{1 + \gamma_k \mathbf{a}_{i_k}^{\mathsf{H}} \mathbf{Q}^{-1} \mathbf{a}_{i_k}}$$

Output : $\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}_{\mathcal{M}}$, the vector of K DOA estimates

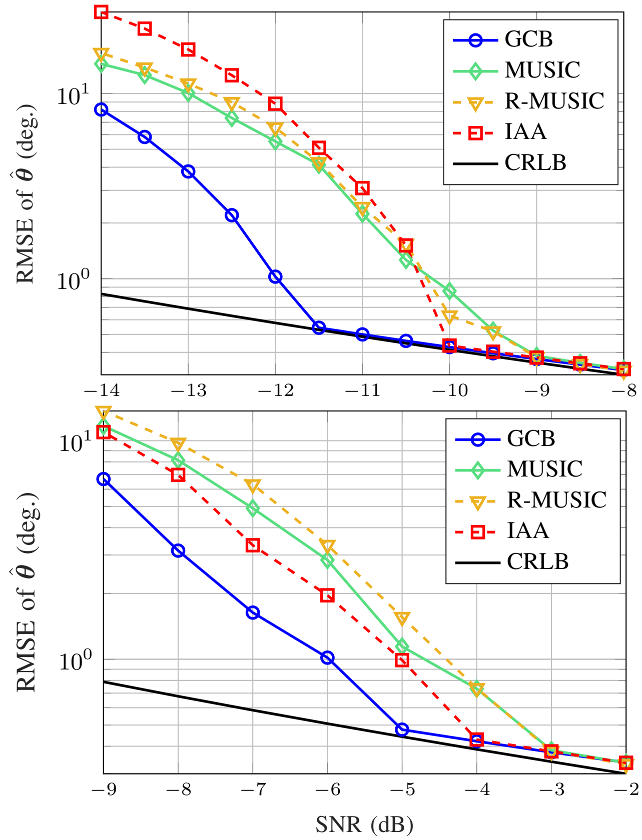

Fig. 1: RMSE versus SNR when $L = 125$ (top panel) and $L = 25$ (bottom panel); $N = 20$, $M = 1801$ ($\Delta\theta = 0.1^o$), $\boldsymbol{\theta} = (-30.1^o, -20.02^o, -10.02^o, 3.02^o)$.

IV. Simulation study

Our simulation set-up is as follows. The array is ULA with half a wavelength inter-element spacing and number of sensors is $N = 20$. The grid size is $M = 1801$, and spacing is uniform, thus providing angular resolution $\Delta\theta = 0.1^o$. The number of Monte-Carlo (MC) trials is 5000. We assume that K independent source signals impinge on the array, where the SNR of k^{th} source is defined as $\text{SNR}_k = \gamma_k/\sigma^2$, where $\gamma_k = \mathbb{E}[|s_k|^2]$ is the signal power and σ^2 is the variance of spatially white noise term. The array SNR is defined as average of source SNR-s as SNR (dB) $= \frac{10}{K}\sum_{k=1}^{K} \log_{10} \text{SNR}_k$.

We compare the proposed GCB method against the Cramér-Rao lower bound (CRLB), MUSIC [3], R-MUSIC [4], and IAA(-APES) [10, Table 2], All methods, except R-MUSIC, use a grid of DOA angles specifying the possible angular resolution of the method. SCB was not included in the study as it performed much worse than MUSIC and R-MUSIC. IAA is sparsity-based iterative approach, and we set 500 as the maximum number of iterations for IAA. We selected IAA to our comparison as it was generally performing the best among other similar sparse iterative approaches and it is relatively fast to compute. In our set-up we have $K = 4$ sources at DOAs $\theta_1 = -30.1^o$, $\theta_2 = -20.02^o$, $\theta_3 = -10.02^o$ and $\theta_4 = 3.02^o$. Note that all, except the 1[st] source, is off the predefined grid. The SNR of the last 3 sources are -1, -2 and -5 dB relative to the 1[st] source. We then calculated the DOA estimate $\hat{\boldsymbol{\theta}} = (\hat{\theta}_1, \dots, \hat{\theta}_K)^\top$ for each MC trial, and report the empirical root mean squared error (RMSE) $\|\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}\|$ averaged over all MC trials. The K sources are following complex circular Gaussian distribution, $s_k \sim \mathcal{CN}(0, \gamma_k)$, $k = 1, \dots, K$, unless otherwise noted.

RMSE versus SNR. Figure 1 displays the performance when $L = 25$ (bottom panel) and $L = 125$ (top panel) and SNR varies. The proposed GCB exhibits the best performance across all SNR levels and sample lengths. At low sample size ($L = 25$), the second-best performing methods is IAA but its performance deterioritates and is similar to MUSIC and R-MUSIC when $L = 125$. At very low SNR, the proposed GCB has superior performance. For example, at SNR of -12 dB and $L = 125$, GCB has one order of magnitude smaller RMSE (in degrees) than the state-of-the-art method IAA.

RMSE versus sample size L. Figure 2 displays the RMSE w.r.t. to sample size L at SNR -9 (top panel) dB and -7 dB (bottom panel). At SNR of -7 dB, IAA and GCB attain same performance, while GCB is superior to IAA when SNR decreases. As L increases, MUSIC and R-MUSIC catch up with sparsity based GCB and IAA methods, and for higher SNR and sample sizes, they perform better than GCB or IAA.

[1]The Sherman-Morrison formula states that $(\mathbf{A} + \mathbf{u}\mathbf{v}^{\mathsf{H}})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{u}\mathbf{v}^{\mathsf{H}}\mathbf{A}^{-1}/(1 + \mathbf{v}^{\mathsf{H}}\mathbf{A}^{-1}\mathbf{u})$

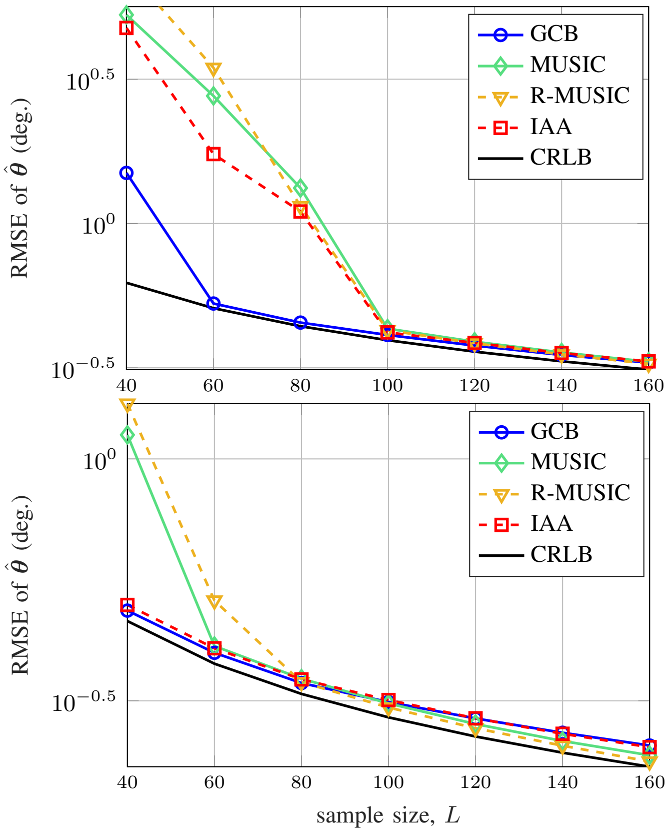

Fig. 2: RMSE versus L when SNR is -9 dB (top panel) and -7 dB (bottom panel); $N = 20$, $M = 1801$ ($\Delta\theta = 0.1^o$), $\boldsymbol{\theta} = (-30.1^o, -20.02^o, -10.02^o, 3.02^o)$.

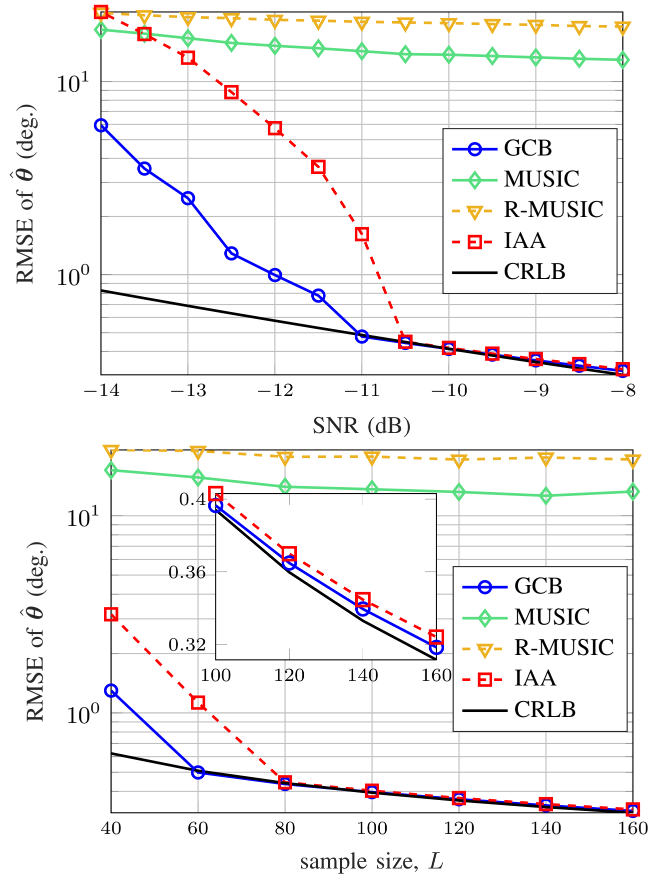

Fig. 3: Correlated constant modulus sources scenario (source 1 and 4 are coherent). Top: RMSE versus SNR when $L = 125$. Bottom: RMSE versus L when SNR is -9dB. $N = 20$, $M = 1801$ ($\Delta\theta = 0.1^o$), $\boldsymbol{\theta} = (-30.1^o, -20.02^o, -10.02^o, 3.02^o)$.

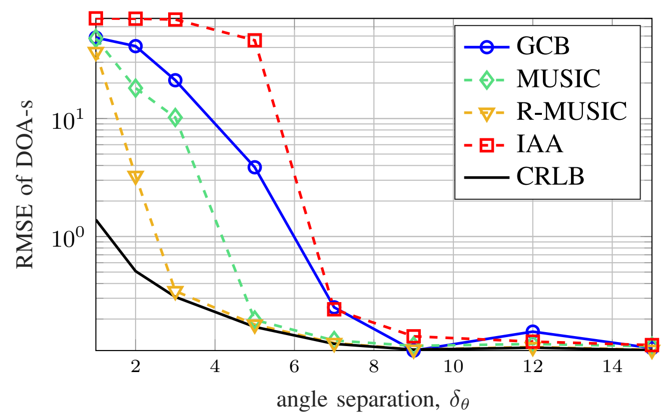

Fig. 4: RMSE versus $\delta_\theta = \theta_2 - \theta_1$ in two source scenario when SNR is -3 dB, $N = 20$, $\theta_1 = -30.02$, $M = 1801$.

Coherent sources scenario. We generate non-Gaussian sources with constant modulus which is more common setting in communications applications. We generate k^{th} source as $s_k = \gamma_k \exp(\jmath \vartheta_k)$ where the phases ϑ_k, $k = 1, \ldots, K$ are independently and uniformly distributed in $[0, 2\pi)$ while power γ_k of the sources are as earlier. We set $\vartheta_1 = \vartheta_4$, i.e., source 1 and source 4 have identical phases, and are thus fully coherent. We repeated previous simulation studies and display the RMSE versus SNR when $L = 125$ and RMSE versus sample size L when SNR is -9 dB in Figure 3. When comparing Figure 3 to top panel plots of Figure 2 and Figure 1, we can observe that the proposed GCB (as well as IAA) is robust to assumption of uncorrelated sources. This phenomenon is common to many sparsity based covariance learning methods (see [17] for theoretical explanation). Again GCB performs the best for all SNR levels and all sample sizes L. MUSIC and R-MUSIC are not able to localize the 4 sources due to coherence.

RMSE versus angle separation: We consider two sources with varying angle separation δ_θ. The DOA of the 1^{st} source is $\theta_1 = -30.02$ (off-grid) and the other source $\theta_2 = \theta_1 + \delta_\theta$, while SNR $= -3$ dB and $L = 125$. Figure 4 illustrates that the sparse methods, GCB and IAA, are not able to identify close-by sources as well as MUSIC an R-MUSIC when $\delta_\theta \leq 6^o$. GCB has better performance over IAA when $\delta_\theta \leq 7^o$ but perform on par otherwise.

V. Concluding remarks

We proposed greedy Capon beamformer that greedily selects a high-power source using Capon SINR maximization principle and spatial power spectrum. The power of the selected source is estimated and subsequently the covariance matrix is updated. The method is computationally light and performed favourably against some SOTA methods.

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