Hadronic effects on Λ polarization in relativistic heavy ion collisions

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The Λ hyperon spin flip and non-flip cross sections are calculated in a simple hadronic model by including both the s-channel process involving the spin 3/2, positive parity $\Sigma^*(1358)$ resonance and the t-channel process via the exchange of a scalar σ meson. Although the t-channel process gives a thermally averaged cross section that is about a factor of 1.3 larger than that from the s-channel process, the Λ spin flip to non-flip cross sections is negligibly small in the t-channel compared to the constant value of 1/3.5 in the s-channel process. With these cross sections included in a schematic kinetic model, the effects of hadronic scatterings on the Λ spin polarization in Au-Au collisions at $\sqrt{s}_{NN} = 7.7$ GeV are studied. It is found that the Λ spin polarization only decreases by about 7% during the hadronic stage of these collisions, which thus justifies the assumption in theoretical studies that compare the Λ polarization calculated at the chemical freeze out to the measured one at the kinetic freeze out.

I. INTRODUCTION

Lambda (Λ) hyperons produced in non-central Au-Au collisions at the Relativistic Heavy ion Collider (RHIC) have been found by the STAR Collaboration [1] to be partially polarized in the direction perpendicular to the reaction plane, with a magnitude increasing with decreasing collision energy. Both hydrodynamic [2] and coarse-grained transport [3] models have been used to describe the measured data by assuming that Λ hyperons are in thermal equilibrium with the vorticity field present at the chemical freeze-out of the collisions. The experimental data can also be described by the non-equilibrium chiral kinetic approach [4], which takes into account the effect of the vorticity field on the equations of motion and scatterings of quarks and antiquarks, by assuming that Λ hyperons are formed from the coalescence of polarized s quarks with polarized u and d quarks at hadronization [5]. Besides the above Λ global polarization, the STAR Collaboration has also measured the Λ local polarization along the beam direction and its azimuthal angle dependence in the transverse plane of the collisions [6]. Although both hydrodynamic [2] and coarse-grained transport [3] models fail to describe the measured Λ local polarization by giving an opposite sign in the azimuthal angle dependence, the correct sign is obtained in the chiral kinetic approach [7] due to the redistribution of axial charges in the transverse plane. The latter result has been confirmed by a study based on a covariant angular-momentum-conserved chiral transport model [8]. This dynamic effect in the chiral kinetic approach can be included in the hydrodynamic approach by adding the effect from the thermal shear field [9–11], as in the study based on the 3+1 MUSIC hydrodynamic model [12] and the 3+1 viscous eHLLe and EVHO-QGP hydrodynamic models [13].

Since both Λ global and local spin polarizations decrease with temperature as the hadronic matter cools [14], the fact that the experimental data is consistent with the Λ spin polarization at the hadronization temperature indicates that it freezes out or decouples early from the hadronic matter. It was pointed out in Ref. [15] that this would be the case if the $\Lambda-\pi$ scattering is through the spin 3/2, parity positive $\Sigma^*(1358)$ resonance since the ratio of Λ spin non-flip to flip probabilities in this scattering is 3.5 based on a simple consideration of Λ spin and pion orbital angular momentum couplings in this scattering. A similar conclusion is reached in a recent study of the temperature dependence of the Λ spin relaxation time in a hot pion gas based also on the s-channel $\Sigma^*(1358)$ resonance approximation [16]. In the present paper, the Λ spin flip and non-flip cross sections by a pion are explicitly calculated not only for the s-channel process suggested in Ref. [15] but also for the t-channel process involving the exchange of the scalar σ meson. Including these cross sections in a schematic kinetic equation makes it possible to quantify the effect of hadronic scatterings on the Λ spin polarization as the hadronic matter expands from the chemical to kinetic freeze out. Results from the present study shows that a spin polarized Λ produced at the chemical freeze-out of heavy ion collisions in Au-Au collisions at $\sqrt{s_{NN}} = 7.7$ GeV only looses its spin polarization by about 7% after it decouples at the kinetic freeze-out. This result thus justifies the assumption used in the literature of comparing the calculated Λ spin polarization at chemical freeze-out to the measured value in experiments.

The paper is organized as follows. In Sec. II, the $\Lambda-\pi$ spin-dependent scattering cross sections are calculated. Their thermal averages are given in Sec. III, which is followed by the presentation of the schematic kinetic approach in Sec. IV. The time evolution of the temperature of the hadronic matter and the pion fugacity in Au-Au collisions at $\sqrt{s_{NN}} = 7.7$ GeV are given in Sec. V. Re-

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sults are then shown in Sec. VI, and conclusions are drawn in Sec. VII.

II. Λ SPIN-DEPENDENT SCATTERING CROSS SECTIONS WITH PION

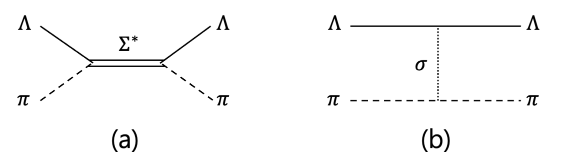

FIG. 1. Feynman diagrams for Λ scattering by pion in (a) s-channel and (b) t-channel processes with solid and dashed lines indicating Λ and π, respectively. The double solid line in s-channel and the dotted line in t-channel denote $\Sigma^*(1358)$ resonance and σ meson, respectively.

Since the hadronic matter produced in relativistic heavy ion collisions consists mostly of pions, only the spin-dependent scattering cross sections of Λ with a pion are considered in this Section. The two Feynman diagrams for the s- and t-channel processes in this scattering are shown in Fig. 1.

A. The s-channel process

For the s-channel $\Lambda - \pi$ scattering through the $\Sigma^*(1358)$ resonance shown in Fig. 1(a), the spin averaged cross section can be approximated by the Breit-Wigner formula,

$$\begin{aligned}\bar{\sigma}(s) &= \frac{2s_{\Sigma^*}+1}{(2s_\Lambda+1)(2s_\pi+1)}\frac{4\pi}{k^2}\frac{s\Gamma^2(s)}{(s-m_{\Sigma^*}^2)^2+s\Gamma^2(s)}\\ &= \frac{8\pi}{k^2}\frac{s\Gamma^2(s)}{(s-m_{\Sigma^*}^2)^2+s\Gamma^2(s)},\end{aligned}\qquad(1)$$

where $s_\Lambda = 1/2$, $s_\pi = 0$, and $s_{\Sigma^*} = 3/2$ are used for the spins of Λ, pion, and Σ^*, respectively. In Eq.(1), s is the square of total center-of-mass energy, k is the magnitude of the momentum of Λ and π in their center-of-mass frame, i.e.,

$$k = \sqrt{\frac{[s-(m_\Lambda+m_\pi)^2][s-(m_\Lambda-m_\pi)^2]}{4s}},\qquad(2)$$

$m_{\Sigma^*} = 1382$ MeV is the mass of Σ^* resonance, and $\Gamma(s)$ is the width of Σ^* if its mass is $\sqrt{s}$.

For a polarized Λ, its spin flip and non-flip scattering cross sections by a pion in the s-channel through the Σ^* resonance are then proportional to the spin averaged scattering cross section in Eq.(1) with their relative magnitude determined by the Clebsch-Gordan coefficients $\langle j_1m_1j_2m_2|JM\rangle$ for the coupling of the Λ spin state $|j_1,m_1\rangle$, the pion orbital angular momentum state $|j_1,m_2\rangle$ to the Σ^* spin state $|J,M\rangle$. For a Λ having an initial spin pointing along the z-direction, the cross section for its spin to remain pointing in this direction after the scattering with a pion is .

$$\begin{aligned}\sigma_{++} &= \frac{A}{3}\sum_{m,M}\Big|\Big\langle\frac{1}{2}\frac{1}{2}1m\Big|\frac{3}{2}M\Big\rangle\Big|^4\bar{\sigma}\\ &= \frac{A}{3}\Big[\Big|\Big\langle\frac{1}{2}\frac{1}{2}11\Big|\frac{3}{2}\frac{3}{2}\Big\rangle\Big|^4+\Big|\Big\langle\frac{1}{2}\frac{1}{2}10\Big|\frac{3}{2}\frac{1}{2}\Big\rangle\Big|^4\\ &\quad+\Big|\Big\langle\frac{1}{2}\frac{1}{2}1-1\Big|\frac{3}{2}-\frac{1}{2}\Big\rangle\Big|^4\Big]\bar{\sigma}\\ &= \frac{A}{3}\left[1+\left(\sqrt{\frac{2}{3}}\right)^4+\left(\sqrt{\frac{1}{3}}\right)^4\right]\bar{\sigma} = \frac{14A}{27}\bar{\sigma},\end{aligned}\qquad(3)$$

where the factor of 1/3 comes from averaging over the three components of the pion orbital angular momentum and A is a normalization constant. Similarly, the cross section for the Λ to have its spin pointing in the negative z-direction after scattering with the pion is

$$\begin{aligned}\sigma_{+-} &= \frac{A}{3}\sum_{m,M}\Big|\Big\langle\frac{1}{2}-\frac{1}{2}1m\Big|\frac{3}{2}M\Big\rangle\Big\langle\frac{3}{2}M\Big|\frac{1}{2}\frac{1}{2}1m\Big\rangle\Big|^2\bar{\sigma}\\ &= \frac{A}{3}\Big[\Big|\Big\langle\frac{1}{2}-\frac{1}{2}11\Big|\frac{3}{2}\frac{1}{2}\Big\rangle\Big\langle\frac{3}{2}\frac{1}{2}\Big|\frac{1}{2}\frac{1}{2}10\Big\rangle\Big|^2\\ &\quad+\Big|\Big\langle\frac{1}{2}-\frac{1}{2}10\Big|\frac{3}{2}-\frac{1}{2}\Big\rangle\Big\langle\frac{3}{2}-\frac{1}{2}\Big|\frac{1}{2}\frac{1}{2}1-1\Big\rangle\Big|^2\Big]\bar{\sigma}\\ &= \frac{A}{3}\left[\left(\sqrt{\frac{1}{3}}\sqrt{\frac{2}{3}}\right)^2+\left(\sqrt{\frac{2}{3}}\sqrt{\frac{1}{3}}\right)^2\right]\bar{\sigma} = \frac{4A}{27}\bar{\sigma}.\end{aligned}\qquad(4)$$

The ratio of the spin non-flip to spin flip cross sections of a polarized Λ is thus $R = \sigma_{++}/\sigma_{+-} = 3.5$ as given in Ref. [15]. Requiring $\sigma_{++}+\sigma_{+-} = \frac{2A}{3}\bar{\sigma} = \bar{\sigma}$ gives $A = 3/2$, which leade to $\sigma_{++} = \frac{7}{9}\bar{\sigma}$ and $\sigma_{+-} = \frac{2}{9}\bar{\sigma}$. Shown in Fig. 2 by the solid line is the s-channel spin-averaged $\Lambda - \pi$ scattering cross section as a function of $\sqrt{s}-\sqrt{s_0}$ with $\sqrt{s_0} = m_\Lambda + m_\pi$, which is seen to peak at $\sqrt{s} = m_{\Sigma^*}$ as expected.

B. The t-channel process

The t-channel contribution to $\Lambda - \pi$ scattering shown by the Feynman diagram in Fig. 1(b) can be evaluated by using the following interaction Lagrangian densities,

$$\begin{aligned}\mathcal{L}_{\sigma\Lambda\Lambda} &= g_{\sigma\Lambda\Lambda}\bar{\Lambda}\Lambda\sigma,\\ \mathcal{L}_{\sigma\pi\pi} &= g_{\sigma\pi\pi}\sigma\pi\pi.\end{aligned}\qquad(5)$$

For the coupling constant $g_{\sigma\Lambda\Lambda}$, it has the value $g_{\sigma\Lambda\Lambda} = \frac{2}{3}g_{NN\sigma} = 7.07$ if one uses the $g_{\sigma NN}$ in Ref. [17] and the light quark counting rule for the relation between the couplings of light baryons and strange baryons to light mesons. The $g_{\sigma\pi\pi}$ can be determined from the σ meson decay width according to

$$\Gamma_\sigma = \frac{g_{\sigma\pi\pi}^2}{8\pi m_\sigma^2}\frac{\sqrt{m_\sigma^2-4m_\pi^2}}{2}.\qquad(6)$$

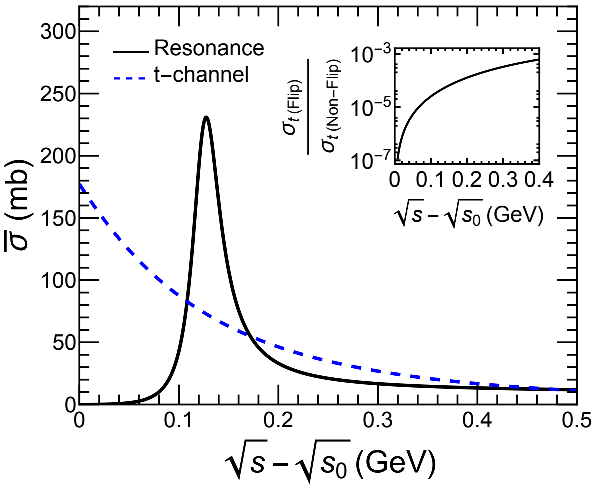

FIG. 2. Spin averaged $\Lambda + \pi \to \Lambda + \pi$ cross section for the s-channel (solid line) and the t-channel (dashed line) process as functions of $\sqrt{s}-\sqrt{s_0}$. Also shown in the inset is the ratio of spin flip to non-flip cross sections for the t-channel process.

Using $\Gamma_\sigma(s=m_\sigma^2) \approx 200$ MeV [18], the coupling constant $g_{\sigma\pi\pi}$ is found to have the value $g_{\sigma\pi\pi} = 2.37$ GeV.

For the t-channel process in $\Lambda(p_1)+\pi(p_2)\to\Lambda(p_3)+\pi(p_4)$ scattering, its invariant amplitude is

$$i\mathcal{M}_t = g_{\Lambda\Lambda\sigma} g_{\sigma\pi\pi} \bar{u}(p_3) \frac{i}{t - m_\sigma^2 + i m_\sigma \Gamma_\sigma} u(p_1), \tag{7}$$

where $t = (p_1 - p_3)^2 = -2p^2(1-\cos\theta)$ with p and θ being the magnitude of and angle between Λ initial and final momenta in their center-of-mass frame, respectively. The spin-averaged squared invariant amplitude for the t-channel process is then

$$\begin{aligned} |\overline{\mathcal{M}_t}|^2 &= \frac{g_{\Lambda\Lambda\sigma}^2 g_{\sigma\pi\pi}^2}{2} \frac{\mathrm{Tr}\left[(\not{p}_3 + m_\Lambda)(\not{p}_1 + m_\Lambda)\right]}{(t-m_\sigma^2)^2 + m_\sigma^2\Gamma_\sigma^2} \\ &= 2 g_{\Lambda\Lambda\sigma}^2 g_{\sigma\pi\pi}^2 \frac{p_1\cdot p_3 + m_\Lambda^2}{(t-m_\sigma^2)^2 + m_\sigma^2\Gamma_\sigma^2} \\ &= g_{\Lambda\Lambda\sigma}^2 g_{\sigma\pi\pi}^2 \frac{4m_\Lambda^2 - t}{(t-m_\sigma^2)^2 + m_\sigma^2\Gamma_\sigma^2}. \end{aligned} \tag{8}$$

To take into account the finite size of hadrons, form factors are introduced at the $\sigma\Lambda\Lambda$ and $\sigma\pi\pi$ vertices. Using the monopole form factor of the form

$$F(t) = \frac{\Lambda^2 + t}{\Lambda^2 + m_\sigma^2}, \tag{9}$$

with the value $\Lambda = 1.8$ GeV as in Ref. [19] for meson-meson scatterings, the spin-averaged differential t-channel cross section can then be calculated from

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\overline{\mathcal{M}_t}|^2 F^4(t). \tag{10}$$

The total spin-averaged t-channel cross section after integrating the scattering angles is shown in Fig. 2 as a function of $\sqrt{s}-\sqrt{s_0}$ by the dashed line. Its value is larger than that of the s-channel cross section except at energy near the Σ^* resonance.

Although it is possible to calculate the Λ spin flip and non-flip cross sections in the t-channel process by using the Λ spin projection operator, the explicit spin-dependent spinors of the Λ are used in the present study. In this approach, the spin-dependent amplitude is proportional to the scalar product of Λ spinors with initial spin direction s_i and final spin direction s_f, i.e.,

$$\begin{aligned} \bar{u}_{s_f}(p_3) u_{s_i}(p_1) &= (E_\Lambda + m_\Lambda)\left(\chi_{s_f}^\dagger, \chi_{s_f}^\dagger \tfrac{\mathbf{p}_3\cdot\boldsymbol{\sigma}}{E_\Lambda + m}\right) \\ &\quad \times \begin{pmatrix} \chi_{s_i} \\ -\frac{\mathbf{p}_1\cdot\boldsymbol{\sigma}}{E_\Lambda + m_\Lambda}\chi_{s_i} \end{pmatrix} \\ &= (E_\Lambda + m_\Lambda)\left\{\left[1 - \frac{\mathbf{p}_3\cdot\mathbf{p}_1}{(E_\Lambda + m_\Lambda)^2}\right]\chi_{s_f}^\dagger \chi_{s_i} \right. \\ &\quad \left. - i\frac{\mathbf{p}_3\times\mathbf{p}_1}{(E_\Lambda + m_\Lambda)^2}\cdot\chi_{s_f}^\dagger \boldsymbol{\sigma}\chi_{s_i}\right\}. \end{aligned} \tag{11}$$

The Λ spin flip and non-flip amplitudes can be explicitly evaluated from Eq.(11) by taking the $x-z$ plane to be the scattering plane with the Λ initial momentum $\mathbf{p}_1 = p\hat{z}$ and final momentum $\mathbf{p}_3 = p(\sin\theta\hat{x} + \cos\theta\hat{z})$, which leads to $\mathbf{p}_3\times\mathbf{p}_1 = -p^2\sin\theta\hat{y}$ and thus $\mathbf{p}_3\times\mathbf{p}_1\cdot\chi_{s_f}^\dagger\boldsymbol{\sigma}\chi_{s_i} = -p^2\sin\theta\chi_{s_f}^\dagger\sigma_y\chi_{s_i}$. Along a direction characterized by the polar angles (θ_s, ϕ_s), the Λ spin up and down states are

$$\chi_+ = \begin{pmatrix} \cos\frac{\theta_s}{2} \\ e^{i\phi_s}\sin\frac{\theta_s}{2} \end{pmatrix}, \; \chi_- = \begin{pmatrix} \sin\frac{\theta_s}{2} \\ -e^{i\phi_s}\cos\frac{\theta_s}{2} \end{pmatrix}. \tag{12}$$

Using these expressions and $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, one has

$$\begin{aligned} \chi_+^\dagger \sigma_y \chi_+ &= \sin\phi_s \sin\theta_s, \\ \chi_-^\dagger \sigma_y \chi_+ &= -\cos\theta_s \sin\phi_s - i\cos\phi_s, \end{aligned} \tag{13}$$

which lead to

$$\begin{aligned} \bar{u}_+(p_3) u_+(p_1) &= (E_\Lambda + m_\Lambda)\left[1 - \frac{p^2\cos\theta}{(E_\Lambda + m_\Lambda)^2} \right. \\ &\quad \left. + i\frac{p^2\sin\theta}{(E+m)^2}\sin\phi_s\sin\theta_s\right], \\ \bar{u}_-(p_3) u_+(p_1) &= \frac{p^2\sin\theta}{E_\Lambda + m_\Lambda}(\cos\phi_s - i\cos\theta_s\sin\phi_s). \end{aligned} \tag{14}$$

With above results, one obtains the following expression for the squared Λ spin non-flip and flip amplitudes after including the coupling constants from the vertices and the propagator of exchanged σ meson,

$$|\mathcal{M}_{t++}|^2 = \frac{g^2_{\Lambda\Lambda\sigma}g^2_{\sigma\pi\pi}}{(t-m_\sigma^2)^2+m_\sigma^2\Gamma_\sigma^2} \times(E_\Lambda+m_\Lambda)^2\left[\left(1-\frac{p^2\cos\theta}{(E_\Lambda+m_\Lambda)^2}\right)^2 + \frac{p^4\sin^2\theta}{(E+m)^4}\sin^2\theta_s\sin^2\phi_s\right], \quad (15)$$

$$|\mathcal{M}_{t+-}|^2 = \frac{g^2_{\Lambda\Lambda\sigma}g^2_{\sigma\pi\pi}}{(t-m_\sigma^2)^2+m_\sigma^2\Gamma_\sigma^2}\frac{p^4\sin^2\theta}{(E_\Lambda+m_\Lambda)^2} \times(\cos^2\theta_s\sin\phi_s^2+\cos^2\phi_s). \quad (16)$$

It can be easily shown from Eqs.(8), (15), and (16) that $|\mathcal{M}_{t++}|^2+|\mathcal{M}_{t+-}|^2=|\overline{\mathcal{M}_t}|^2$ as expected.

Eq.(16) shows that, with the Λ having initial momentum in the z-direction and final momentum in the $x-z$ plane, it has the largest spin-flip cross section if it is initially longitudinally polarized along the z-direction ($\theta_s=0$) or transversely polarized along the x-direction ($\theta_s=\pi/2$ with $\phi_s=0$). In these cases, Eqs.(15) and (16) become

$$|\mathcal{M}_{t++}|^2 = \frac{g^2_{\Lambda\Lambda\sigma}g^2_{\sigma\pi\pi}}{(t-m_\sigma^2)^2+m_\sigma^2\Gamma_\sigma^2} \times(E_\Lambda+m_\Lambda)^2\left[1-\frac{p^2\cos\theta}{(E_\Lambda+m_\Lambda)^2}\right]^2, \quad (17)$$

$$|\mathcal{M}_{t+-}|^2 = \frac{g^2_{\Lambda\Lambda\sigma}g^2_{\sigma\pi\pi}}{(t-m_\sigma^2)^2+m_\sigma^2\Gamma_\sigma^2}\frac{p^4\sin^2\theta}{(E_\Lambda+m_\Lambda)^2}. \quad (18)$$

The ratio of spin flip to non-flip cross sections calculated from Eq.(10) using Eqs.(17) and (18) in Eq.(10) and after integrating the scattering angle is shown in the inset of Fig. 2, which is seen to be negligibly small even for the cases that the polarized Λ has the largest spin-flip cross section. This result is not surprising because the flip of Λ spin due to the exchange of a scalar meson is a relativistic effect, which is small because of the small Λ velocity in the hadronic matter as a result of its large mass.

III. THERMALLY AVERAGED CROSS SECTIONS

For Λ scattering in a thermalized hadronic matter, the time evolution of its spin polarization in the kinetic approach depends on the thermally averaged $\Lambda-\pi$ scattering cross sections,

$$\langle\sigma_{ab\to cd}v_{ab}\rangle = \frac{\int d^3\mathbf{p}_a d^3\mathbf{p}_b f_a(\mathbf{p}_a)f_b(\mathbf{p}_b)\sigma_{ab\to cd}v_{ab}}{\int d^3\mathbf{p}_a d^3\mathbf{p}_b f_a(\mathbf{p}_a)f_b(\mathbf{p}_b)}. \quad (19)$$

In the above, $f_i(\mathbf{p}_i)$ is the Boltzmann momentum distribution of particle species $i=a,b$, i.e., $f_i(\mathbf{p}_i)=g_ie^{-\sqrt{\mathbf{p}_i^2+m_i^2}/T}$ with m_i being the particle mass, and v_{ab} is the relative velocity between particles a and b. Figure 3 shows the thermally averaged s- and t-channel spin-averaged scattering cross sections of Λ by a pion. It is seen that the one from the t-channel process is about a factor of 1.3 larger than that from the s-channel process at all temperature of the hadronic matter, indicating that the average invariant mass of a Λ and π pair in the hadronic matter is below the Σ^* mass. Also shown in the inset the ratio of thermally averaged Λ spin flip to non-flip cross sections, which is again negligibly small as in the case for their spin-averaged cross sections.

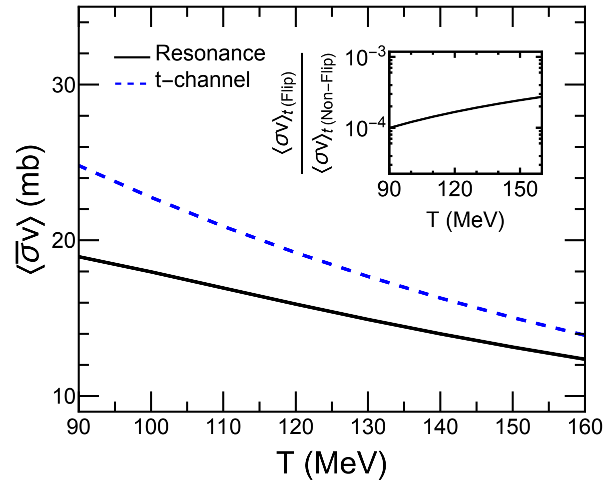

FIG. 3. Thermally averaged $\Lambda-\pi$ scattering cross section in s-channel (solid line) and t-channel (dashed line) as functions of temperature. Also shown in the inset is the ratio of thermally averaged Λ spin flip to non-flip cross sections as a function of temperature.

IV. KINETIC EQUATIONS FOR Λ SPIN EVOLUTION

In the kinetic approach, the numbers of spin-up and spin-down Λ hyperons, denoted by N_{Λ_+} and N_{Λ_-}, respectively, change with time according to the kinetic equations,

$$\frac{dN_{\Lambda_+}}{d\tau} = -\langle\sigma_{\Lambda_+\pi\to\Lambda_-\pi}v\rangle z_\pi n_\pi^{(0)}N_{\Lambda_+} + \langle\sigma_{\Lambda_-\pi\to\Lambda_+\pi}v\rangle z_\pi n_\pi^{(0)}N_{\Lambda_-}, \quad (20)$$

$$\frac{dN_{\Lambda_-}}{d\tau} = \langle\sigma_{\Lambda_+\pi\to\Lambda_-\pi}v\rangle z_\pi n_\pi^{(0)}N_{\Lambda_+} - \langle\sigma_{\Lambda_-\pi\to\Lambda_+\pi}v\rangle z_\pi n_\pi^{(0)}N_{\Lambda_-}. \quad (21)$$

In the above, $n_\pi^{(0)}$ is the density of thermally equilibrated pions, given by $n_\pi^{(0)}=g_\pi\int\frac{d^3\mathbf{p}}{(2\pi)^3}f_\pi(p)$ with $g_\pi=3$ due to its isospin degeneracy, and z_π is its fugacity to take into account the fact that the effective pion number, which includes both free pions and those from resonance decays, remains unchanged during the expansion and cooling of the hadronic matter according to the statistical hadronization model [20].

Using the relation $\langle\sigma_{\Lambda_+\pi\to\Lambda_-\pi}v\rangle = \langle\sigma_{\Lambda_-\pi\to\Lambda_+\pi}v\rangle \equiv \langle\sigma v\rangle$, the number difference between spin-up and spin-down Λ hyperons, $\Delta N(\tau) = N_{\Lambda_+}(\tau) - N_{\Lambda_-}(\tau)$, then satisfies the equation,

$$\frac{d\Delta N}{d\tau} = -2\langle\sigma v\rangle z_\pi n_\pi^{(0)}\Delta N, \tag{22}$$

which can be solved to give

$$\Delta N(\tau) = \Delta N_0 e^{-2\int\langle\sigma v\rangle z_\pi n_\pi^{(0)}d\tau}, \tag{23}$$

where ΔN_0 denotes the initial number difference between spin up and spin down Λ hyperons.

V. TIME EVOLUTION OF TEMPERATURE AND PION FUGACITY IN HADRONIC MATTER

Since the largest Λ polarization reported by the STAR Collaboration at beam energy scan experiments at RHIC is from Au-Au central 20-50% collisions at $\sqrt{s}_{NN} = 7.7$ GeV [1], the hadronic scattering effects on Λ polarization in these collisions are considered in the present study. According to the statistical hadronization model based on the grand canonical ensemble fit to the experimentally measured yields of produced particles in 30-40% collision centrality of these collisions Ref. [21], the extracted chemical freeze-out temperature, baryon chemical potential, and strangeness chemical potential are about $T_{\rm ch} = 146$ MeV, $\mu_B = 376$ MeV, and $\mu_S = 88$ MeV, respectively. Also, using the blast-wave model to fit the transverse momentum spectra of measured particles, a kinetic freeze-out temperature of $T_{\rm k} = 129$ MeV is found in Ref. [21]. The small charged chemical potential given in Ref. [21] is, however, neglected in the present study.

$\sqrt{s}_{NN}$ (GeV)	$T_{\rm ch}$ (MeV)	$T_{\rm k}$ (MeV)	$\tau_{\rm ch}$ (fm/c)	$\tau_{\rm k}$ (fm/c)
7.7	146	129	5.13	6.46

TABLE I. Parameter values in Eq.(24) for the time evolution of the temperature of the hadronic matter produced in Au-Au collisions at $\sqrt{s_{NN}} = 7.7$ GeV.

For the time evolution of the temperature of the hadronic matter from $T_{\rm ch} = 146$ MeV to $T_{\rm k} = 129$ MeV, it can be obtained from the result in Ref. [22], which is based on the AMPT model simulation of Au-Au collisions at $\sqrt{s}_{NN}$=7.7 GeV, by parameterizing it as follows,

$$T(\tau) = T_{\rm ch} - (T_{\rm ch} - T_{\rm k})\left(\frac{\tau - \tau_{\rm ch}}{\tau_{\rm k} - \tau_{\rm ch}}\right)^{0.9}. \tag{24}$$

In the above, $\tau_{\rm ch}$ and $\tau_{\rm k}$ are the chemical freeze-out time and the kinetic freeze-out time, respectively. Values of these parameters are given in TABLE I. The time evolution of the hadronic matter temperature in Au-Au collisions at $\sqrt{s}_{NN} = 7.7$ GeV is shown by the solid line in the upper panel of Fig. 4.

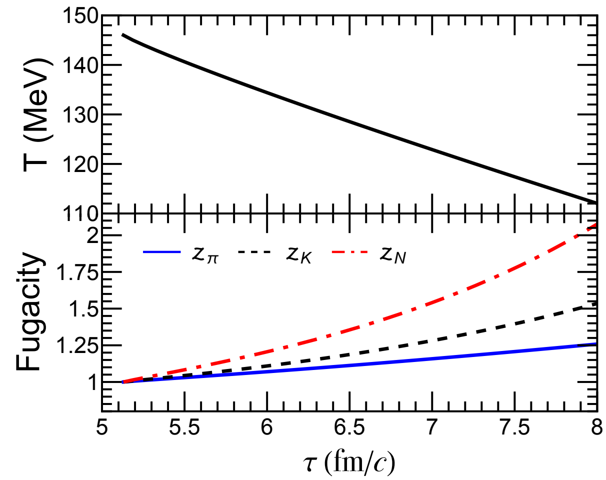

FIG. 4. Time evolution of temperature (upper panel) and pion (solid line), kaon (dashed line) and nucleon (dash-dotted line fugacities (lower panel) of the hadronic matter in Au+Au collisions at $\sqrt{s}_{NN} = 7.7$ GeV.

For the pion fugacity z_π in Eqs.(20) and (21), it can be determined together with the kaon fugacity z_k, and nucleon fugacity z_N by requiring the constancy of the effective pion, kaon, and nucleon numbers during the expansion of the hadronic matter under the condition of constant entropy per hadron [22] as described in detail in Ref. [23]. Specifically, one first expresses the effective pion, kaon, and nucleon density at temperature T in terms of the thermally equilibrated hadron densities $n_i^{(0)} = g_i\int\frac{d^3\mathbf{p}}{(2\pi)^3}f_i(\mathbf{p})$, where g_i is the spin and isospin degeneracy of hadron species i. and the pion, kaon, and nucleon fugacities as follows,

$$\begin{aligned} n_\pi^{\rm eff}(T) &= z_\pi n_\pi^{(0)} + z_\pi^2 n_\rho^{(0)} + z_\pi z_K n_{K^*}^{(0)} + z_\pi z_N n_\Delta^{(0)} + \cdots, \\ n_K^{\rm eff}(T) &= z_K n_K^{(0)} + z_\pi z_K n_{K^*}^{(0)} + z_\pi^2 z_K n_{K_1}^{(0)} + z_K^2 n_\phi^{(0)} \\ &\quad + \cdots, \\ n_N^{\rm eff}(T) &= z_N n_N^{(0)} + z_\pi z_N n_\Delta^{(0)} + \cdots, \end{aligned} \tag{25}$$

where $\cdots$ denotes the contribution from strong decays of other resonances, which includes all particles of masses up to 1.7 GeV for mesons and 2 GeV for baryons in the particle data book. In obtaining the above equations, one has used the relations $z_\rho = z_\pi^2$, $z_{K^*} = z_\pi z_K$, $z_{K_1} = z_\pi^2 z_K$, $z_\Delta = z_\pi z_N$, etc. because of expected chemical equilibrium between π and ρ, among K^*, K and π, among Δ, N and π, etc. as a result of the large cross sections for the reactions $\pi\pi \leftrightarrow \rho$, $K^* \leftrightarrow K\pi$, $\Delta \leftrightarrow N\pi$ etc. One then calculates the entropy density and total particle density according to $s(T) = -\sum_i g_i\int\frac{d^3\mathbf{p}}{(2\pi)^3}(z_if_i)\ln(z_if_i)$ and $n(T) = \sum_i z_in_i^{(0)}$. Requiring $n_{\pi,K,N}^{\rm eff}(T)V(T) =$

$n^{\mathrm{eff}}_{\pi,K,N}(T_{\mathrm{ch}})V(T_{\mathrm{ch}})$ and $s(T)/n(T) = s(T_{\mathrm{ch}})/n(T_{\mathrm{ch}})$ and with unity pion, kaon and nucleon fugacities at T_{ch}, these four equations can be solved to give the pion, kaon and nucleon fugacities as well as the volume ratio $V(T)/V(T_{\mathrm{ch}})$ at a given T. Shown in the lower panel of Fig. 4 are the pion fugacity z_π (solid line), the kaon fugacity z_k (dashed line) and the nucleon fugacity z_N (dash-dotted line) as functions of time. One notes that the constant entropy per hadron in this study is 6.31.

VI. RESULTS

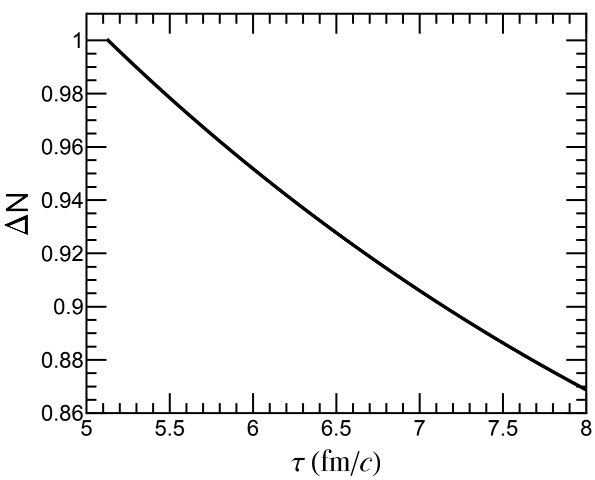

FIG. 5. Time evolution of the normalized number difference between spin up and spin down Λ hyperons in Au-Au collisions at $\sqrt{s}_{NN} = 7.7$ GeV.

The solid line in Fig. 5 shows the time evolution of the normalized number difference between spin up and spin down Λ hyperons, which is defined by taking $\Delta N_0 = 1$, in Au-Au collisions at $\sqrt{s}_{NN} = 7.7$ GeV. It is seen that its value decreases to 0.93 or is reduced by 7% at the end of hadronic evolution after 1.33 fm/c, which justifies the neglect of hadronic scatterings on the Λ spin polarization assumed in the literature. The hadronic effect on Λ polarization remains small even one doubles the lifetime of the hadronic phase in these collisions to 2.66 fm/c as the value of ΔN is now 0.88 or is reduced by 12% in this case.

VII. CONCLUSIONS

The effects of hadronic scatterings on the Λ spin polarization in Au-Au collisions at $\sqrt{s}_{NN} = 7.7$ GeV have been studied in a schematic kinetic approach by using thermally averaged Λ spin flip and non-flip cross sections. These cross sections are calculated by including contributions from both the s-channel process through the Σ^* resonance and the t-channel process via the exchange a scalar σ meson. Although the thermally averaged cross sections due to the t-channel process is about a factor of 1.3 larger than those of the s-channel process, the ratio of the spin flip to non-flip cross sections is negligibly small compared to that from the s-channel process, which has a constant value of 1/3.5. Because of the latter small value, the Λ spin polarization is found to decrease by only about 7% during the hadronic stage of these collisions. This result thus justifies the assumption in theoretical studies of Λ polarization that compare its value calculated at the chemical freeze out to the measured one at the kinetic freeze out.

The present study has neglected the interference between the s-channel and t-channel processes. Because of the negligible spin flip to non-flip cross sections in the t-channel process, including the interference terms is not expected to affect the results and conclusion from the present study. Also, the effect of Λ scattering by nucleons is not considered in the present study. The $\Lambda - N$ spin flip to non-flip scattering due to the σ meson exchange is also likely to be small as in $\Lambda - \pi$ scattering because of the small Λ velocity, which makes the relativistic spin flip effect negligible. Including $\Lambda - N$ scattering through the σ meson exchange is also not expected to affect the conclusion of present study. However, $\Lambda - \pi$ and $\Lambda - N$ scatterings can also go through the exchange of an ω meson. For $\Lambda - \pi$ scattering via the ω-exchange, its contribution to Λ spin flip cross section is expected to be suppressed as it involves creating a massive ρ meson in the final state. Although the contribution to Λ spin flip cross section due to $\Lambda - N$ scattering via the ω exchange may not be small, its effect on Λ spin polarization could be small compared to the $s-$channel $\Lambda - \pi$ scattering through the Σ^* resonance because the smaller nucleon than pion numbers in Au-Au collisions at $\sqrt{s}_{NN} = 7.7$ GeV [1].

ACKNOWLEDGEMENTS

This work was supported by the Korea National Research Foundation under Grant No. 2023R1A2C300302311, 2023K2A9A1A0609492411 (S.H.L.) and the U.S. Department of Energy under Award No. DE-SC0015266 (C.M.K.). S. H. Lee also acknowledges the support from the Samsung Science and Technology Foundation under Project No. SSTF-BA1901-04. H. Sung thanks the Cyclotron Institute of Texas A&M University for its hospitality during her stay as a visiting scholar supported by the Korean Women Graduate Research in the US Program of the National Research Foundation of Korea through funds from the Korean Ministry of Science and ICT (2022K1A3A1A12097807).

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