

Generalized Step-Chirp Sequences With Flexible Bandwidth

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Abstract—Sequences with low aperiodic autocorrelation sidelobes have been extensively researched in literatures. With sufficiently low integrated sidelobe level (ISL), their power spectrums are asymptotically flat over the whole frequency domain. However, for the beam sweeping in the massive multi-input multi-output (MIMO) broadcast channels, the flat spectrum should be constrained in a passband with tunable bandwidth to achieve the flexible tradeoffs between the beamforming gain and the beam sweeping time. Motivated by this application, we construct a family of sequences termed the generalized step-chirp (GSC) sequence with a closed-form expression, where some parameters can be tuned to adjust the bandwidth flexibly. In addition to the application in beam sweeping, some GSC sequences are closely connected with Mow’s unified construction of sequences with perfect periodic autocorrelations, and may have a coarser phase resolution than the Mow sequence while their ISLs are comparable.

I. INTRODUCTION

Sequences with low aperiodic autocorrelation sidelobes are desirable in communications and radar engineering, e.g., some of the chirp-like sequences developed in [1]–[5]. With a low integrated sidelobe level (ISL), these sequences have quite flat spectrums [6], which can be utilized to achieve omnidirectional precoding in broadcast channels [7].

In 5G NR broadcast channels, the discrete Fourier transform (DFT) codebook is adopted for broadcasting common messages [8, Section 6.1.6.3] in the initial stage of communication. With the energy concentrated in the pointing direction, the maximum beamforming gain can be achieved by the DFT codebook. But for future wireless communication systems with massive number of antennas, the resultant beam would be too narrow, thus requiring many times of beam sweeping to cover the whole angular domain. In contrast, the chirp-like sequence-based omnidirectional beamforming spreads the energy in the whole angular domain, and thus avoids beam sweeping and improves the time efficiency. The omnidirectional beamforming, however, has no beamforming gain, and therefore may have insufficient range coverage for the millimeter-wave or terahertz-wave communication systems where a high beamforming gain is required for compensating the severe path loss.

To circumvent such a dilemma, it is desirable to achieve flexible tradeoffs between the beamforming gain and the beam sweeping time, as pursued by the 3GPP [9]. From the aspect of spectrum, we aim at designing sequences whose power

variation in the passband and power leakage in the stopband should be as small as possible, and the bandwidth of the passband should be flexibly tunable. Besides, their entries should have equal amplitudes for maximizing the energy efficiency of power amplifiers (PAs), and their phase resolutions should be coarse for the implementation using a low-cost phase shifter network (PSN).

Literatures on this topic include some numerical optimizations [10]–[12] and some schemes with closed-form solutions [9], [13]–[16]. Compared with the numerical optimizations, the schemes with closed-form solutions are easier for hardware implementation, but the bandwidth is less flexible except for the scheme in [15]. The sequence inferred from [15], referred to as the generalized chirp (GC) sequence in this paper, has flexible bandwidth the same as the numerical counterparts, and its spectrum in the passband is asymptotically flat [15]. Nevertheless, for the GC sequence, the phase resolution of the PSN is too fine to be cost-effective when the number of antennas is large, as shown in our simulations.

In recent years, polyphase sequences with low correlations and spectrally-null constraints were constructed in [17]–[20], whose N -point spectrums (with N being the sequence length) are ideally flat in the passbands and are ideally null in the stopbands. Nevertheless, the N -point spectrum is insufficient for beamforming because the user equipments (UEs) are distributed in a continuous angular range, rather than the N discrete directions. Besides, the passbands are interleaved with the stopbands [20] and the bandwidths are less flexible. Hence, they are still not suitable for beam sweeping.

To achieve flexible tradeoffs between the beamforming gain and the beam sweeping time, in this paper we construct a family of polyphase sequences with flexible bandwidth, termed as the generalized step-chirp (GSC) sequence. The GSC sequence enjoys a coarser phase resolution than the GC sequence. Besides, when the passband stretches over the whole frequency domain, the GSC sequence degenerates into a low-ISL sequence closely connected with the Mow sequence [5] with perfect periodic autocorrelation, and may require a coarser phase resolution than the Mow sequence.

Notations: $\lfloor \cdot \rfloor$ stands for taking the floor value. \mathbb{Z}^+ represents the set of positive integers, $\mathbb{Z}_n = \{0, 1, \dots, n-1\}$. $\omega_N = e^{j\frac{2\pi}{N}}$. $\|\cdot\|$ is the Frobenius norm. For $x, y, s, t \in \mathbb{R}$, $x \equiv y \pmod s$ stands for that $x - y$ is an integer multiple of

$s; x = y \bmod [s, t]$ means that $x - y$ is an integer multiple of $|s - t|$ and $y \in [s, t]$; $x = y \bmod s$ is equivalent to $x = y \bmod [0, s)$.

II. PRELIMINARIES

In this section, we review two kinds of passive beamformings for the common message broadcasting: the conventional beam sweeping based on the DFT codebook [8, Section 6.1.6.3] in Section II-A and the omnidirectional beamforming based on the chirp-like sequence [5] in Section II-B.

A. DFT Codebook-based Beam Sweeping

Consider a uniform linear array (ULA) of N isotropic antennas with half wavelength spacing. Given a beamforming vector $\mathbf{a} = [a_0, a_1, \dots, a_{N-1}]$ with $|a_n| = \frac{1}{\sqrt{N}}, n \in \mathbb{Z}_N$, the radiated power at azimuth angle θ and elevation angle φ is

$$y(u) = \left| \sum_{n=0}^{N-1} a_n e^{-j\pi n u} \right|^2 \quad (1)$$

where $u = \cos \varphi \cos \theta$. Note that $-1 \leq u \leq 1$, hence $y(u)$ is essentially the power spectrum of the sequence \mathbf{a} .

A DFT codeword is $\mathbf{d}(u_0) = [d_0, d_1, \dots, d_{N-1}]$ with $d_n = \frac{1}{\sqrt{N}} e^{j\pi n u_0}, n \in \mathbb{Z}_N$, where u_0 is the beam direction in the u -domain. Let $\Delta u \triangleq u - u_0$, then the radiated power is

$$y(u) = \frac{1}{N} \left| \sum_{n=0}^{N-1} e^{-j\pi n \Delta u} \right|^2 = \begin{cases} \left| \frac{\sin(\frac{\pi N}{2} \Delta u)}{\sqrt{N} \sin(\frac{\pi}{2} \Delta u)} \right|^2, & \Delta u \neq 0 \\ N, & \Delta u = 0 \end{cases} \quad (2)$$

By (2), the maximum beamforming gain (the ratio of the maximum received power to the average received power) N can be achieved if $u = u_0$, and for a sufficiently large N ,

$$\lim_{N \rightarrow \infty} \frac{y(u_0 \pm \frac{1}{N})}{y(u_0)} = \left| \lim_{N \rightarrow \infty} \frac{1}{N \sin(\frac{\pi}{2N})} \right|^2 = \frac{4}{\pi^2} \approx 0.4, \quad (3)$$

i.e., $u_0 \pm \frac{1}{N}$ is closed to the half-power points of the beam. Hence the DFT codeword $\mathbf{d}(u_0)$ is designed to cover $[u_0 - \frac{1}{N}, u_0 + \frac{1}{N}]$. Then a DFT codebook $\{\mathbf{d}(u_0) \mid u_0 \in \mathcal{T}\}$ with $\mathcal{T} = \{\frac{2i+1}{N} - 1 \mid i \in \mathbb{Z}_N\}$ is adopted to sweep the beam over the whole space for broadcasting common message, as illustrated by Fig. 1 (a). The beam sweeping would consume too many time slots if N is large.

B. Chirp-like Sequence-based Omnidirectional Beamforming

In contrast to the beam sweeping that requires many time slots, the omnidirectional beamforming aims at broadcasting messages using only one time slot, which can be achieved by designing a sequence with a flat power spectrum.

Definition 1: For a length- N complex sequence \mathbf{a} with $\|\mathbf{a}\| = 1$, its aperiodic autocorrelation is defined as

$$R_a(\tau) \triangleq \sum_{n=0}^{N-1} a_n \bar{a}_{n-\tau}, \quad 1 - N \leq \tau \leq N - 1, \quad (4)$$

where $a_n = 0$ if $n < 0$ or $n \geq N$, and the overbar represents the complex conjugation.

The power spectrum of \mathbf{a} is

$$y(u) = \sum_{\tau=1-N}^{N-1} R_a(\tau) e^{-j\pi u \tau}, \quad (5)$$

and the variance of the power spectrum is

$$\begin{aligned} & \frac{1}{2} \int_{-1}^1 (y(u) - 1)^2 du \\ &= \frac{1}{2} \sum_{\tau_1 \neq 0} \sum_{\tau_2 \neq 0} R_a(\tau_1) R_a^*(\tau_2) \int_{-1}^1 e^{-j\pi u(\tau_1 - \tau_2)} du \\ &= \sum_{\tau \neq 0} |R_a(\tau)|^2 \triangleq ISL_a \end{aligned} \quad (6)$$

where ISL_a is the integrated sidelobe level (ISL) of $R_a(\tau)$. Hence for omnidirectional beamforming, the sequence's ISL should be small, e.g., some of the chirp-like sequences [1]–[5]. As a unified construction of sequences with perfect periodic autocorrelation, the Mow sequence family [5] where some sequences have low ISL, is given below for ease of reference.

Definition 2: [5] The Mow sequence is a kind of sequences of length $N = sm^2$ with s being the square-free part of N , whose entries are $\frac{1}{\sqrt{N}} \omega_N^{m \xi_n}, n \in \mathbb{Z}_N$ with

$$\xi_{km+l} = mc(s)\alpha(l)k^2 + \beta(l)k + f_l(0), \quad l \in \mathbb{Z}_m, k \in \mathbb{Z}_{sm} \quad (7)$$

where

$$c(s) = \begin{cases} \frac{1}{2}, & \text{for } s \text{ even} \\ 1, & \text{for } s \text{ odd} \end{cases}, \quad (8)$$

$\alpha(l) \in \{1, 2, \dots, s-1\}$ is any function with $\gcd(\alpha(l), s) = 1, \forall l \in \mathbb{Z}_m$, and $\beta(l) \in \mathbb{Z}_{sm}$ is any function such that $\beta(l) \bmod m$ is a permutation of \mathbb{Z}_m , and $f_l(0), \forall l \in \mathbb{Z}_m$ are any rational numbers.

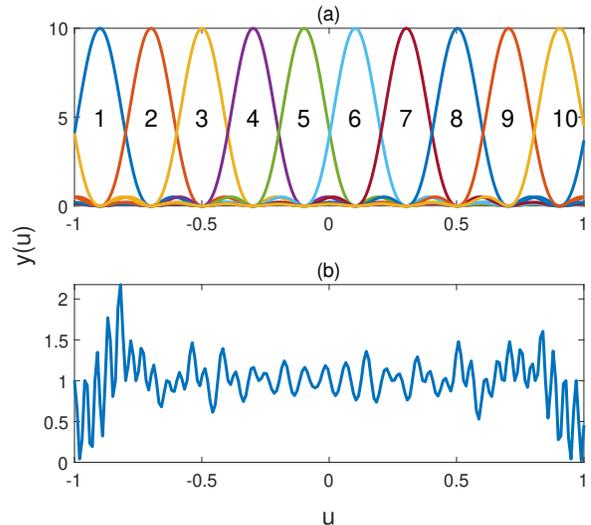


Fig. 1. The power spectra of two kinds of beamformings. (a): the DFT codebook-based beam sweeping; (b) the chirp-like sequence-based omnidirectional beamforming.

The beam sweeping based on the DFT codebook and the omnidirectional beamforming based on the Mow sequence are compared in Fig. 1. For the DFT codebook, $N = 10$; for the Mow sequence, $N = 50$, $s = 2$, $m = 5$, $c(s) = \frac{1}{2}$, $\alpha(l) = 1$, $\beta(l) = l - 25$, $f_i(0) = -9.5l$. The DFT codebook in Fig. 1 (a) achieves the maximum beamforming gain but requires 10 times of beam sweeping, while the Mow sequence in Fig. 1 (b) can broadcast messages in one time slot but has no beamforming gain.

III. GENERALIZED STEP-CHIRP SEQUENCE

To achieve flexible tradeoffs between the beamforming gain and the beam sweeping time, in Section III-A, we construct a family of polyphase sequences with tunable bandwidth, termed as the generalized step-chirp (GSC) sequence; in Section III-B, we discuss the relationships between the GSC sequence, the DFT codebook, the generalized chirp (GC) sequence inferred from [15] and the Mow sequence [5].

A. Construction of Generalized Step-chirp Sequence

Consider a step-chirp signal as follows:

$$c(t) = e^{j2\pi \int_0^t f(\tau) d\tau}, \quad 0 \leq t \leq T, \quad (9)$$

where $f(t)$ is a step approximation of linear frequency modulation (LFM):

$$f(t) = a(\lfloor t \rfloor + b), \quad 0 \leq t \leq T \quad (10)$$

for $\forall a > 0, \forall b \in \mathbb{R}$. An LFM $f'(t) = t$ and its step approximation $f(t)$ with $a = 1, b = \frac{1}{2}$, and $T = 10$ are illustrated by Fig. 2. The bandwidth of the step-chirp signal is aT approximately. Besides, we require the Nyquist sampling number $aT^2 \geq 1$.

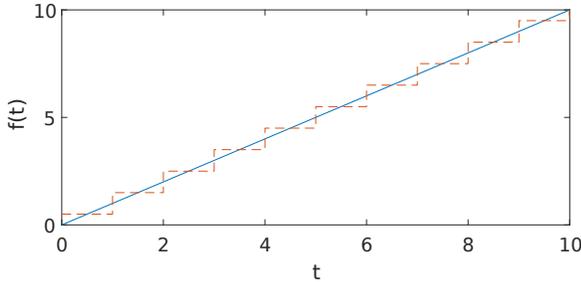


Fig. 2. Step approximation of LFM, $a = 1, b = \frac{1}{2}$, and $T = 10$.

Now sample $c(t)$ in (9) at rate $m \triangleq aT/\gamma$ with $0 < \gamma \leq 1$, where m is assumed to be an integer via setting a properly. We then obtain N samples

$$c(t) \big|_{t=\frac{n}{m}} = e^{j\phi_n}, \quad n \in \mathbb{Z}_N \quad (11)$$

with

$$N = mT = aT^2/\gamma = m^2\gamma/a. \quad (12)$$

Because $aT^2 \geq 1$, we have $\gamma = \frac{aT^2}{N} \geq \frac{1}{N}$.

Factor $n \in \mathbb{Z}_N$ into

$$n = km + l, \quad k \triangleq \lfloor n/m \rfloor, \quad l \in \mathbb{Z}_m. \quad (13)$$

Direct calculations show that

$$\phi_n = 2\pi \frac{ak(k-1+2b)}{2} + 2\pi \frac{a(k+b)l}{m}. \quad (14)$$

Note from (12) that $a = \frac{m^2\gamma}{N}$; thus,

$$\phi_n = \frac{2\pi}{N} m\gamma \left(\frac{k(k-1)}{2} m + kl + bn \right). \quad (15)$$

Besides, the Fourier transform of $c(t)$ can be derived to be a weighted summation of T sinc functions:

$$C(f) = \sum_{i=0}^{T-1} e^{j\pi[ai^2+2(ab-f)i+ab-f]} \text{sinc}[f - a(i+b)]. \quad (16)$$

At $f_0 \triangleq a(b - \frac{1}{2})$, the value of the left-most sinc function ($i = 0$) is $\text{sinc}(-\frac{a}{2})$; at $f_1 \triangleq a(b - \frac{1}{2} + T)$, the value of the right-most sinc function ($i = T - 1$) is $\text{sinc}(\frac{a}{2})$. Hence the interval $(-\infty, f_0) \cup (f_1, +\infty)$ can be regarded as the stopband since most of the sinc functions have attenuated to a low level. Because the bandwidth of the step-chirp signal is aT approximately, the interval $[f_0, f_1]$ can be regarded as the passband of $C(f)$. Note that the analog bandwidth aT is scaled to be the digital bandwidth $2\pi\gamma$ by over-sampling, hence the passband of the sample sequence is $[\omega_0, \omega_0 + 2\pi\gamma]$ where

$$\omega_0 = \frac{2\pi\gamma}{aT} f_0 = \frac{2\pi}{N} m\gamma \left(b - \frac{1}{2} \right). \quad (17)$$

The above arguments established the following theorem.

Theorem 1: The GSC sequence is a family of polyphase sequences with entries $\frac{1}{\sqrt{N}} \omega_N^{m\zeta_n}$, $n \in \mathbb{Z}_N$, where

$$\zeta_n = \gamma \left(\frac{k(k-1)}{2} m + kl + bn \right), \quad (18)$$

$$k = \lfloor n/m \rfloor, \quad l = n - km,$$

with parameter set $\{N, \gamma, m, b \mid N \in \mathbb{Z}^+, m|N, \frac{1}{N} \leq \gamma \leq 1, b \in \mathbb{R}\}$. The passband of the power spectrum of the GSC sequence is

$$[\omega_0, \omega_0 + 2\pi\gamma] \quad (19)$$

where $\omega_0 = \frac{2\pi}{N} m\gamma (b - \frac{1}{2})$. For beam sweeping, the beam is pointed at u_0 to cover $[u_0 - \gamma, u_0 + \gamma]$, where

$$u_0 = \frac{2}{N} m\gamma \left(b - \frac{1}{2} \right) + \gamma \bmod [-1, 1). \quad (20)$$

The bandwidth of the GSC sequence can be flexibly adjusted by tuning the parameter γ , thus achieving the flexible tradeoffs between the beamforming gain and the beam sweeping time, as shown in Simulations.

B. Relationships Between the GSC sequence and Other Sequences

The relationships between the GSC sequence, the DFT codebook, the GC sequence and the Mow sequence family are illustrated by Fig. 3, as explained below.

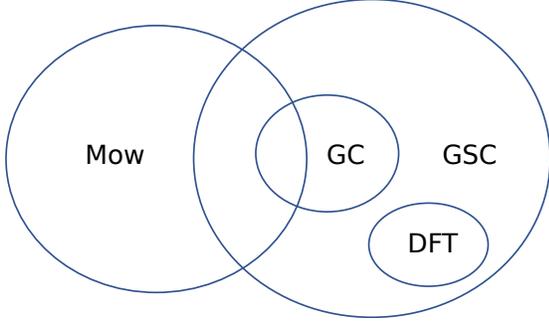


Fig. 3. The relationships between the Mow sequence, the GC sequence, the DFT codebook and the GSC sequence.

1) *GSC Sequence and DFT Codebook*: In Theorem 1, let $m = N$ and $\gamma = \frac{1}{N}$. Then we have $k = 0$ and $n = l$ for $\forall n \in \mathbb{Z}_N$. This degenerated GSC sequence has entries

$$\frac{1}{\sqrt{N}} e^{j \frac{2\pi}{N} m \zeta_n} = \frac{1}{\sqrt{N}} e^{j \frac{2\pi}{N} b n}, \quad n \in \mathbb{Z}_N, \quad (21)$$

and the passband is

$$\left[\frac{2\pi}{N} \left(b - \frac{1}{2} \right), \frac{2\pi}{N} \left(b + \frac{1}{2} \right) \right]. \quad (22)$$

According to Section II-A, this is exactly a DFT codeword pointing at $u_0 = \frac{2b}{N} \bmod [-1, 1)$. Hence the GSC sequence encompasses the DFT codebook and thus may be backward-compatible with the current industrial standard.

2) *GSC Sequence and GC Sequence*: When $m = 1$, we have from (18) that $\frac{1}{\sqrt{N}} \omega_N^{m \zeta_n} = \frac{1}{\sqrt{N}} \omega_N^{\gamma \frac{n(n+2b-1)}{2}}$, which is the GC sequence inferred from over-sampling a chirp signal [15]. Therefore, the GSC sequence is also a generalization of the GC sequence. The phase resolution of a sequence with phases in $\{\frac{2\pi p}{P} | p \in \mathbb{Z}_P\}$ is $\frac{2\pi}{P}$. Note that the parameter m can be tuned for coarser phase resolution, e.g., suppose $b \in \mathbb{Z}$ and γ is a rational number of form $\frac{p}{q}$ with p, q coprime, then the phase resolutions are

$$R_{gsc} = \frac{2\pi}{Nq/\gcd(Nq, mp)}, \quad R_{gc} = \frac{2\pi}{Nq/\gcd(Nq, p)}, \quad (23)$$

from which we have $R_{gc} \leq R_{gsc} \leq mR_{gc}$, e.g., if $p = 1$, then $R_{gsc} = mR_{gc}$.

3) *GSC Sequence and Mow Sequence*: Set $\gamma = 1$ (i.e., the Nyquist sampling), and we obtain another kind of degenerated GSC sequence with entries $\frac{1}{\sqrt{N}} \omega_N^{m \zeta_n}$, $n \in \mathbb{Z}_N$, where

$$\zeta_n = \frac{k(k-1)}{2} m + kl + bn, \quad (24)$$

$$k = \lfloor n/m \rfloor, \quad l = n - km,$$

with parameter set $\{N, m, b | N \in \mathbb{Z}^+, m|N, b \in \mathbb{R}\}$, which is related to the Mow sequence as shown below.

Proposition 1: With the following two constraints on the parameter m and b in (24), respectively, the degenerated GSC sequence in (24) is a special case of the Mow sequence in (7):

- 1) m is the square part of N , i.e., $N = sm^2$.
- 2) $2b$ is odd if s is even or b is an integer if s is odd.

Proof: First note that with the first constraint, the sequence length in (24) is sm^2 , which is the same as the Mow sequence.

Second, if s is even and $2b$ is odd, then (24) is a special case of (7) with $c(s) = \frac{1}{2}$, $\alpha(l) = 1$, $\beta(l) = \frac{2b-1}{2}m + l$, $f_i(0) = bl$ and $\frac{2b-1}{2}$ is an integer such that $\beta(l) \equiv l \bmod m$ is a permutation of \mathbb{Z}_m .

If s is odd and b is an integer, then denote $s = 2d - 1$ for some $d \in \mathbb{Z}^+$. Since $k(k+2b-1)$ is an even number, it holds that

$$dk(k+2b-1) = \frac{k(k+2b-1)}{2}(s+1) \equiv \frac{k(k+2b-1)}{2} \bmod s. \quad (25)$$

Rewrite (24) as

$$\zeta_{km+l} = \frac{k(k+2b-1)}{2}m + kl + bl. \quad (26)$$

It follows from (26) and (25) that

$$\zeta_{km+l} \equiv dk(k+2b-1)m + kl + bl \bmod sm, \quad (27)$$

which is a special case of (7) with $c(s) = 1$, $\alpha(l) = d$ [one may verify that $\gcd(d, 2d-1) = 1$], $\beta(l) = (2b-1)dm + l$, $f_i(0) = bl$. ■

Indeed, one may relax the constraints in Proposition 1 to improve the phase resolution of the degenerated GSC sequence in (24). The phase resolution of the Mow sequence in (7) with $f_i(0)$ being an integer is

$$R_{mow} = \begin{cases} \frac{\pi}{N/m}, & s \text{ is even, } m \text{ is odd} \\ \frac{2\pi}{N/m}, & \text{otherwise} \end{cases}, \quad (28)$$

and the phase resolution of the GSC sequence with $\gamma = 1, b \in \mathbb{Z}^+$ is $R_{gsc} = \frac{2\pi}{N/m}$. If m is larger than the square part of N , then the phase resolution of the GSC sequence would be coarser than the Mow sequence as shown in Simulations.

IV. SIMULATIONS

This section presents simulation examples to verify the capability of the GSC sequence in making flexible tradeoffs between the beamforming gain and the beam sweeping time, and its advantages over the GC sequence and the Mow sequence in terms of the phase resolution and the spectrum.

A. Tradeoffs Between the Beamforming Gain and the Beam Sweeping Time

To show the flexibility of the GSC sequence for beam sweeping, we simulate and show in Fig. 4 the beam patterns of the GSC sequences of length $N = 120$ with $(\gamma, m) \in \{(\frac{1}{2}, 15), (\frac{1}{5}, 24), (\frac{1}{7}, 30), (\frac{1}{13}, 40)\}$. The parameter b is chosen so that the beam direction u_0 in (20) runs through $\{(2i-1)\gamma - 1 | i = 1, 2, \dots, \frac{1}{\gamma}\}$ for the contiguous coverage of $[-1, 1)$. Fig. 4 (a) illustrates 2 times of beam sweeping with 2x beamforming gain while Fig. 4 (d) represents 13 times of beam sweeping with 13x beamforming gain. In summary, by adjusting γ and b to control the bandwidth and the beam direction, flexible tradeoffs between the beamforming gain and

the beam sweeping time can be achieved for efficient beam sweeping. We want to emphasize that the y-axis is in the linear scale. Thus, the power fluctuation in the passband is less than 3dB.

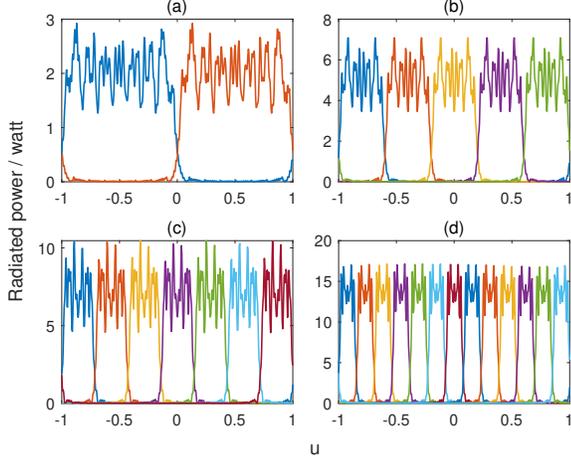


Fig. 4. Flexible tradeoffs between the beamforming gain and the beam sweeping time. (a): $\gamma = \frac{1}{2}$; (b): $\gamma = \frac{1}{5}$; (c): $\gamma = \frac{1}{7}$; (d): $\gamma = \frac{1}{13}$.

B. Phase Resolution and Spectrum

For a GSC sequence $\mathbf{g} = [g_0, g_1, \dots, g_{N-1}]$, the normalized root mean square error (NRMSE) of passband is defined as

$$\sqrt{\frac{1}{|\mathcal{I}_p|} \sum_{i \in \mathcal{I}_p} \left(\gamma \left| \sum_{n=0}^{N-1} g_n e^{-j \frac{2\pi}{N'} in} \right|^2 - 1 \right)^2} \quad (29)$$

where N' is the DFT length and $\mathcal{I}_p \subset \mathbb{Z}_{N'}$ is the set of passband indices. Here we set $N' = 4N$. And the stopband leakage ratio is defined as

$$\frac{1}{N'} \sum_{i \in \mathcal{I}_s} \left| \sum_{n=0}^{N-1} g_n e^{-j \frac{2\pi}{N'} in} \right|^2 \quad (30)$$

where $\mathcal{I}_s = \mathbb{Z}_{N'} \setminus \mathcal{I}_p$ is the set of the stopband indices.

Compared with the GC sequence and the Mow sequence, the GSC sequence with a proper parameter m may have a coarser phase resolution and a comparable spectrum or even flatter.

1) *GSC Sequence versus GC sequence*: Fig. 5 shows the impact of the parameter m on the spectrum and the phase resolution of a GSC sequence, where $N = 50$, $\gamma = \frac{1}{2}$, $b = 1$. Compared with the GC sequence, i.e., $m = 1$, the GSC sequence with $m = 10$ has smaller passband NRMSE and stopband leakage ratio as shown in Fig. 5 (a), and the phase resolution of the proposed GSC sequence is 10 times coarser as shown in Fig. 5 (b) and Fig. 5 (c).

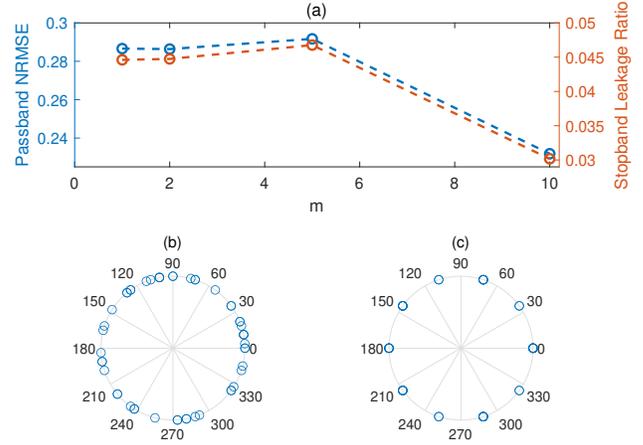


Fig. 5. The improvement of phase resolution and spectrum of the GSC sequence against the GC sequence. (a): the passband NRMSE and the stopband leakage ratio of a length-50 GSC sequence for different m ; (b): the phases of the GC sequence corresponding to $m = 1$ in (a); (c): the phases of the GSC sequence corresponding to $m = 10$ in (a).

2) *GSC sequence versus Mow sequence*: Fig. 6 shows the ISL of a GSC sequence of length $N = 462$ for different parameters m , with $\gamma = 1$ and $b = \frac{1}{2}$. Note that the square part of $N = 462$ is $m = 1$, thus the point with $m = 1$ in Fig. 6 corresponds to a Mow sequence, which can be verified by simulation to have exactly the minimum ISL among all the 55440 Mow sequences of length 462 with a phase resolution $\frac{\pi}{462}$ [5, Theorem 5]. Remarkably, the ISL for $m = 1$ is 0.0297 and the ISL for $m = 21$ is 0.0307, which means a reduction of phase resolution by a factor of 21 but with a negligible increase of ISL, i.e., a comparably flat spectrum.

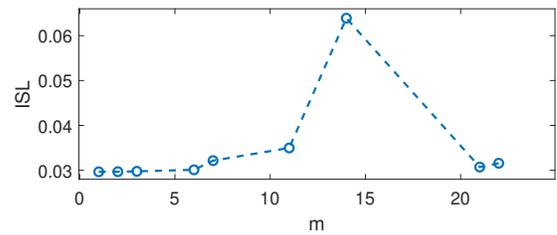


Fig. 6. The ISL of a length-462 GSC sequence for different m .

V. CONCLUSIONS

In this paper, we construct the generalized step-chirp (GSC) sequence, which can achieve flexible tradeoffs between the beamforming gain and the beam sweeping time for the common message broadcasting in massive MIMO systems. The GSC sequence has a coarser phase resolution than the generalized chirp (GC) sequence, which facilitates its implementation with a low-cost phase shifter network (PSN). Besides, the GSC sequence may have coarser phase resolution than the Mow sequence with a negligible increase of the integrated sidelobe level (ISL).

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