Nucleation transitions in polycontextural networks towards consensus

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Abstract

Recently, we proposed polycontextural networks as a model of evolving systems of interacting beliefs. Here, we present an analysis of the phase transition as well as the scaling properties. The model contains interacting agents that strive for consensus, each with only subjective perception. Depending on a parameter that governs how responsive the agents are to changing their belief systems the model exhibits a phase transition that mediates between an active phase where the agents constantly change their beliefs and a frozen phase, where almost no changes appear. We observe the build-up of convention-aligned clusters only in the intermediate regime of diverging susceptibility. Here, we analyze in detail the behavior of polycontextural networks close to this transition. We provide an analytical estimate of the critical point and show that the scaling properties and the space-time structure of these clusters show self-similar behavior. Our results not only contribute to a better understanding of the emergence of consensus in systems of distributed beliefs but also show that polycontextural networks are models, motivated by social systems, where susceptibility – the sensitivity to change own beliefs – drives the growth of consensus clusters.

Keywords: Polycontextural Logic, Nucleation Transition, Worldviews

1 Introduction

An abrupt change in the properties of a system upon a small change in an external condition characterizes a phase transition. Such transitions occur in different contexts [1]: If water is heated above 100°C a sudden transition occurs and it starts to boil. Likewise, certain materials lose their magnetic properties, if they are heated above the Curie point [2]. Besides these transitions, statistical physics also knows the concept of *geometric* phase transitions, e.g. in percolation theory [3] or random-graph theory [4]: At a critical fraction of added links a network of disconnected clusters merges into a large, system-wide cluster. However, phase transitions are not limited to the physical world, and they have applications in the social and behavioral sciences as well [5]: They can be observed in negotiation and opinion dynamics [6–8], collective decision-making [9] and models of social influence [10]. Also, the formation of social groups bears similarities to phase transitions [11].

An important aspect of phase transition phenomena is that the observed macroscopic property is the result of a large number of microscopic interactions. In social systems, these interactions require agents to perceive and interpret transmitted signals in the same way. However, as we know from human communication research as well as social psychology, interacting persons often interpret facts differently because they have different beliefs [12–15]. Here, belief systems refer to a "set of predispositions within an individual to perceive, construe, and interpret stimuli or events in a consistent manner" [16]. Our individual belief system hence provides ways of "construing or dimensionalizing relevant aspects of the world" [16].

In our initial publication [17], we used the term *worldview* instead of *belief system*. The distinction between both terms in the scientific literature is rather blurred and, depending on the scientific discipline, they are interchangeably used to describe a set of beliefs that we use to describe and make sense of reality [16, 18–20]. The term *worldview* (in German *Weltanschauung*) has been introduced by the philosopher Kant and has since also been used in other scientific disciplines like theology [21–23], psychology [19], cultural sciences [24] and sociology [25]. Here, worldview usually refers to a self-contained and comprehensive framework of mutually compatible beliefs. In contrast, belief systems can refer to an interrelated subset of beliefs that, only when coherently clustered, are recognizable as generalized worldviews [25]. While the philosophical implications of the model played a greater role in our initial publication, the concept of beliefs seems more appropriate for mathematical consideration [26, 27], and thus also for this paper.

The mechanisms that lead to consensus have been analyzed by different groups and in various models. However, these models largely presuppose a shared understanding of facts and hence assume that perceived beliefs represent objective truth. The models hence neglect any subjectivity and possible *external dissonance* [26], although their importance is backed by social influence studies. To answer whether and how people with different belief systems can reach a consensus, we proposed *polycontextrual networks* [17], a model of interacting agents that strive for consensus, however, each with only a subjective perception of the world. Depending on a parameter that governs how responsive the agents are to changing their beliefs, we observed a phase transition between a system with permanent changes of the beliefs and a frozen state where no belief changes can be observed. A build-up of aligned clusters was only observable around the transition point.

In this paper, we analyze the space-time structure, as well as the scaling properties of the belief-aligned clusters that appear in our polycontextural network model. We find that our model shows scale invariance in the spatial as well as in the time domain. Additionally, we propose a mechanistic explanation of our model's dynamics close to the critical point. Since polycontextural networks serve as a generic model for the build-up of shared beliefs, our detailed analysis of the behavior close to the phase transition helps to better understand phenomena like opinion polarization and the stabilization of opinion communities.

The remainder of the manuscript is as follows. In the next section, we recall the definition as well as the basic characteristics of the polycontextural networks model and introduce the threshold parameter q. In section 3 we derive analytical estimates for the upper and lower bounds of q. We then analyze the scaling behavior and the space-time structure of belief-aligned clusters in 4, before we summarize our results in the last section.

2 Model

The definition of the model is equivalent to the polycontextural networks model we recently proposed elsewhere [17], and is repeated here for the sake of completeness. The polycontextural network is a stochastic model, where N agents interact in a network. Each agent A_n with $n \in \{1 \dots N\}$ is endowed with a certain characteristic $c_n \in C$, where C contains all alternative states the characteristic may have. For all agents, the set of expressions is the same. Thus, the characteristic of each agent is given as a standard basis vector $\boldsymbol{e}_i$ of length C with 1 in the ith position and 0 in every other position.

Additionally, each agent has an individual dictionary (its belief system) that bijectively maps the 'objective outside world' of the agent to its personal cognition. Formally, this dictionary is a bijective function $\sigma : C \to C$ and can be written as a $C \times C$ permutation matrix T_n. If one agent A_n observes the characteristic c_m of another agent, the observing agent perceives $T_n c_m$ instead of the 'true' (objective) c_m. The result of such an observation can be interpreted as the agent's belief about the other agent's actual belief [26]. In the following, we will assume that the characteristics c_n are colors. Due to this definition, our model does not have objective truth values – a predefined understanding of color – but $C!$ different and equally correct belief systems (here:

color mappings). In contrast to most other models of belief dynamics we hence only have perceived (subjective) and no 'true' (objective) beliefs.

Each agent is equipped with two internal counters: $\#O_n$ and $\#K_n$, and every update step proceeds as follows:

- One agent A_n is randomly selected, and one of its neighbours A_m is chosen.
- Agent A_n subjectively observes the characteristic of A_m, denoted as c_m but subjectively perceived as $T_n c_m$.
- If A_n already has the same characteristic as the one perceived from A_m, only its internal counter $\#O_n$ is increased by 1.
- Otherwise, A_n changes its own characteristic to the subjectively observed one and increments both its internal counters $\#O_n$ and $\#K_n$ by one.
- If the fraction of times, represented by $\#K_n/\#O_n$, that A_n has not changed its characteristic exceeds the parameter q, the agent will randomly select a new belief system from the $C!$ available ones and reset both counters to zero. Note that $\#O_n$ is always incremented at least once before its final step, so the fraction $\#K_n/\#O_n$ is always defined.

In what follows, we are mainly interested in the dynamics and the organization of the belief system (the "color translation tables"). In this context, the characteristics of the agents are just signals to transmit information about their own belief system and will not be analyzed further.

In the initial publication, we showed that – depending on the value of q – the polycontextural networks show a transition that mediates between an active phase where the belief systems of the agents change frequently, and an inactive (frozen) phase, where almost no belief system changes occur. In both phases, the belief systems are not correlated over long scales. In contrast, close to the transition point we observe long-range correlations as clusters of mutually compatible belief systems. One should note that compatible does not necessarily mean that the belief systems of two nodes n and m are equal, but only that their color mappings mutually agree in all colors, which means $T_n \times T_m = \mathbb{I}$.

As an example, Fig. 1 shows the largest cluster of compatible belief systems (C_{max}) vs. the threshold parameter q for triangular and square lattices with different numbers of nodes and after $t = 20,000$ simulated steps. As outlined above, we observe two phases $q \ll q_c$ and $q \gg q_c$, where no clusters of compatible belief systems emerge. Only for an intermediate value $q \approx q_c$ clusters of compatible beliefs can build up.

An analysis of the cluster sizes of the triangular network close to the critical point ($q \approx q_c$) revealed that during the build-up of the clusters,

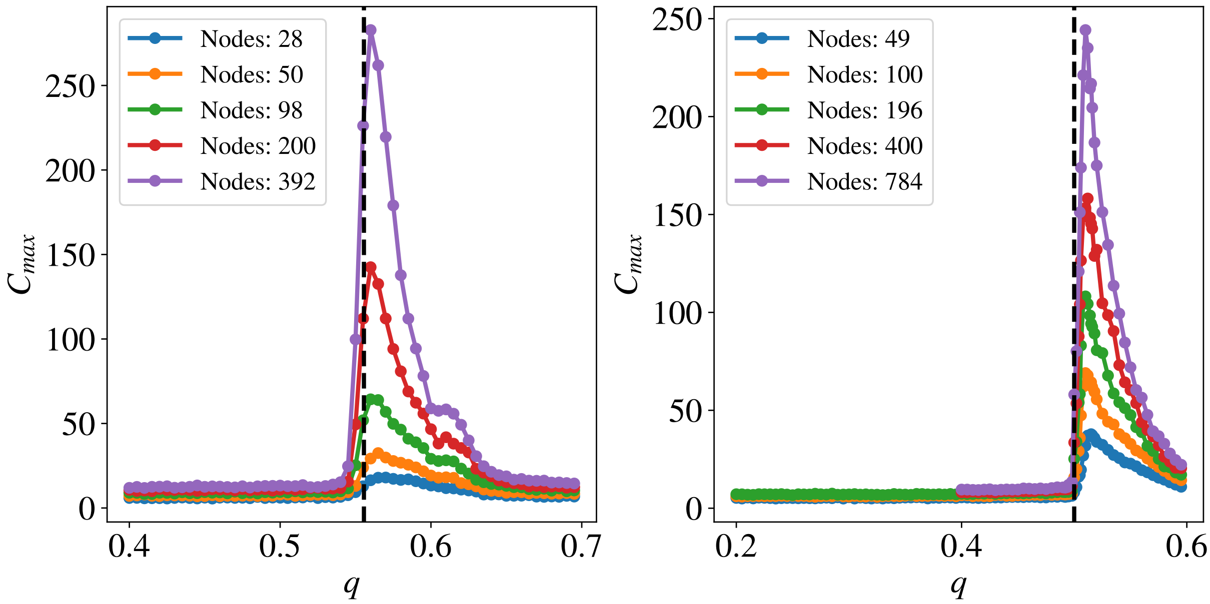

Fig. 1 Size of the largest cluster (C_{max}) vs. the threshold parameter q for (left) triangular and (right) square lattices with different numbers of nodes. Simulated steps: $t = 20,000$, averaged over 100 runs.

the distribution follows a power-law with an exponent of $\alpha = -2.3$ (Fig. 2 (left)). Additionally, it was observed that for both network types the size of the largest cluster C_{max} scales with the linear system size L according to

$$C_{\max}(L) \sim L^{d_f}, \tag{1}$$

where d_f is the (possibly) fractal dimension of the cluster [28, 29] (Fig. 2(right)).

In the following, we will provide a detailed analysis of the transition point, as well as the scaling properties and the space-time structure of the clusters.

3 Separation of three different phases

As suggested by Fig. 1, depending on the value of the parameter q our model shows three different dynamical phases. In Ref. [17] we provided a simple mechanistic explanation for these three phases. Here, we extend this argument and analytically derive approximate values for the boundaries between these domains.

For our approximation, we assume random-d-regular networks. We consider two connected actors: A and B. Let us assume that there is no correlation between the two actors, which means that at each time step B randomly changes its color to one of the C available. If A observes the

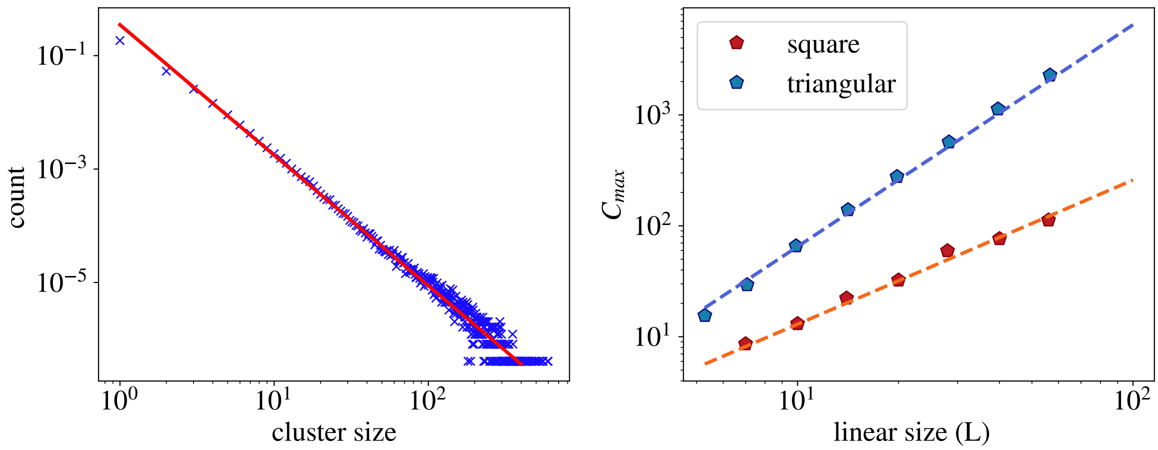

Fig. 2 Left: Cluster size distribution of the triangular network for $q = 0.57$ at $t = 5000$ (N=2450 nodes, averaged over 1000 runs). The red line indicates a power law with exponent $\alpha = -2.3$. Right: Size of the largest cluster C_{max} over the linear size $L = N^{0.5}$ of the system. Both systems were simulated at $q \approx q_c$ for $t = 20,000$ time steps. The dashed lines indicate the respective power law fits with (green) $d_f = 1.3$ and (red) $d_f = 2$. (Figure adapted from [17])

color of B, it perceives $T_A c_B$. Since the translation tables are bijective mappings, A's observed color will be similar to its own color in $1/C$ cases. Neglecting stochastic fluctuations we hence know that A_n already has the same characteristic as the observed one with a probability of $1/C$. Now, if q is larger than $1 - 1/C$, this small fraction of correct observations is enough for A to not change its translation table. The same holds for all other agents. Above the threshold of $q_+ = 1 - 1/C$ we hence have a *frozen dynamic*. Due to the limited number of colors, it is possible that small clusters exist just by accident, but the individual actors do not actively change their belief systems in order to align with neighbors.

We now turn to the lower threshold. We consider an actor A that has only neighbors with the same belief system as A. We call the randomly selected neighbor that A observes B. We further assume that since A's last update step also B has performed one update, one observation. During B's last update, it observed A's color with a probability of $1/d$ (d is the degree of the random-regular graph). Since A and B share the same belief system, in this case, B changed its color such that A now observes the color that matches its own. With a probability of $1 - 1/d$, B did not observe A but another of its neighbors. Since we assumed an uncorrelated environment, with a probability of $1/C$ this color nonetheless matches the one, which A wants to observe. Hence, given that A and B share the same belief system, the probability that A observes the correct color is given by:

$$p_- = \frac{1}{d} + \frac{d-1}{d} \cdot \frac{1}{C}. \qquad (2)$$

In the above derivation, we assumed that since A's last update step also B has performed one update. It might also happen that this is wrong and instead, B has not performed any update. In this case, however, one can apply the same arguments to A instead of B. The result does not change.

If we again neglect any stochastic fluctuations, we can state that nodes requiring more than p_- correct observations cannot stabilize, even if they share the same belief system with their neighborhood. Below the threshold of $q_- = 1 - p_-$ we hence have random dynamics where no clusters of shared tables can emerge.

We have shown that below p_- no cluster can build up, since the actors require more agreement than possible even in a system with equal belief systems. Likewise, we have shown that no cluster can emerge above p_- because the actors require less agreement than randomly observed. Within the boundaries of p_- and p_+ the growth of clusters is possible and they are statistically stable. There is, however, a drastic difference between both limits: If q is smaller, but close to p_+, small clusters consisting of only a few nodes can already form stable structures. In contrast, if q is only slightly larger than p_- the only stable cluster is the giant cluster that covers the full system.

In Fig. 3 we show the size of the largest cluster vs. q for random regular networks with different degrees. Additionally, we added two gray lines that denote the lower (dashed) and upper (solid) threshold for the build-up of clusters. Even though our approximation is only of the first order and neglects the possibility of structural effects (e.g. triangles in the network), we observe a strong agreement between the analytical results and the simulations.

The described behavior is characteristic of a nucleation transition [30–32], where for given parameters a nucleus starts to grow to form a large cluster that can eventually reach the size of the full system. Phase transitions like the observed nucleation transition are defined by an abrupt change in the characteristic of a system upon a small change of a macroscopic parameter, often called a control parameter. In the polycontextural networks, the control parameter is given by q which governs how responsive the agents are to changing their belief system. The specific characteristic that reacts to a change of q is then the number of table changes per time step. Based on this motivation, we introduce the quantity P_T, which measures the probability that the system has reached a frozen state (no color changes in the last time step) and serves as an order parameter of the system, i.e., as an indicator of the collective state. In Fig. 4 we show the sudden jump of the order parameter P_T for random regular networks with different degrees and at different times $t = \{500, 1.000, 5.000, 20.000\}$. The dashed line denotes the transition point as predicted

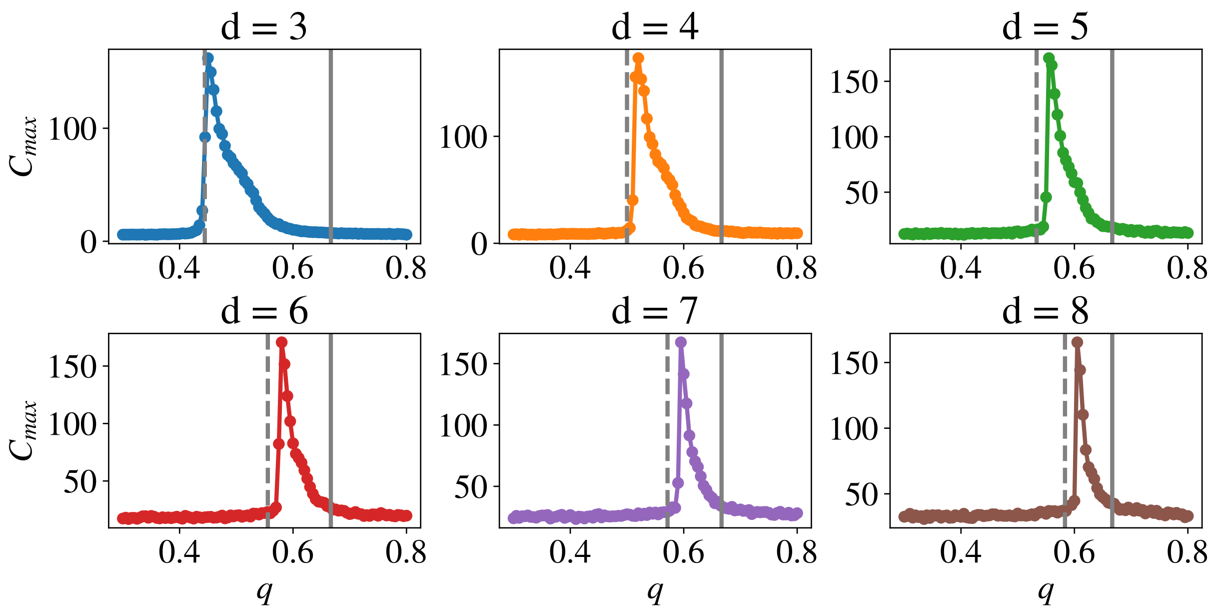

Fig. 3 Size of the largest cluster (C_{max}) vs. the threshold parameter q for random-regular networks with different degrees d. Simulated steps: $t = 80,000$, averaged over 100 runs

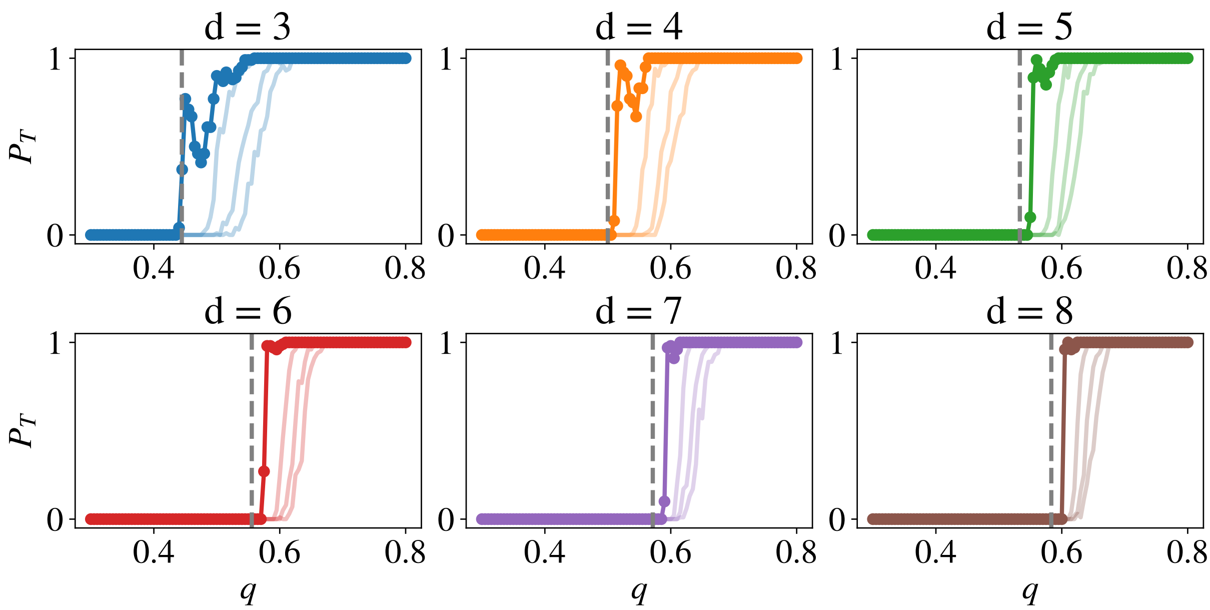

Fig. 4 Order parameter P_T vs. the threshold parameter q for random-regular networks with different degrees d at different times $t = \{500, 1.000, 5.000, 20.000\}$ (averaged over 100 runs). The dashed line denotes the respective lower critical value.

by Eq. 2. The remaining fluctuations especially apparent for networks with small degrees are due to competition between two or more large clusters.

An important quantity when analyzing phase transitions is the susceptibility [33]. The susceptibility indicates how sensitive the order parameter reacts to changes in the control parameter. Therefore, it peaks at the critical point where the order parameter P_T has high fluctuations [34]. We have already discussed that in our system clusters cannot grow if there are either no table changes $q > q_+$ or if there are too many table changes $q < q_-$. This corresponds to the two phases where either $P_T = 1$ or $P_T = 0$. We can therefore directly link the ability of clusters to form to the magnitude of the susceptibility. The observed size of the clusters is hence an indication of the susceptibility of the system.

For the presented approximation, we assumed random d-regular graphs. In what follows, we present a scaling analysis of the polycontextural network model. To make our results comparable to other simple and well-studied models that show phase transitions, we will focus on planar network topologies like triangular and square lattices with periodic boundary conditions. Since these networks have a constant degree, we can use our approximation to predict the transition point of these networks. In Fig. 5 (e) and (f) we show the plots for the order parameter P_T for the triangular and square lattices. In our approximation, we neglected any structural effects. However, in Ref [17] we showed that especially triangular structures have a stabilizing effect on clusters. This explains why the transition point of the triangular network is shifted to the left compared to our approximation.

4 Scaling

Scaling analyses are mostly performed when analysing second-order (continuous) phase transitions. However, it is also known that grain growth and particle coarsening can create statistical self-similarity, according to which the system shows self-similarity and hence scale invariance in a statistical sense [35]. Given that our polycontextural networks serve as a minimal model to study the build-up of believe-aligned clusters it is useful to clarify what scaling behavior these clusters exhibit.

The appearance of self-similarity and scale invariance close to q_c (Fig. 2) implies that the system (in principle) does not have a characteristic length scale $\xi_\perp$. In our finite system, the characteristic length is restricted by the size of the system ($L \approx \xi_\perp$) which means L determines the relevant length scale. Contrary, for large $L/\xi_\perp$ the system shows an equilibrium behavior and the dynamics are almost independent of L as we have already observed in Fig. 1. As known from the theory of finite-size scaling, the spatial ($\perp$) correlation lengths follows $\xi_\perp \sim (q - q_c)^{-\nu_\perp}$, which gives rise to the scaling law [29]:

$$P_{max}(t, L, q) = L^{-\beta/\nu_\perp} f((q - q_c)L^{1/\nu_\perp}) \quad (3)$$

where $f(x)$ is a scaling function. In Fig. 5 (a) and (b) we apply the scaling relation to the curves for C_{max}, where, for better comparability between the two networks, we set $L = N$. The collapse of the curves indicates similarity in the spatial domain.

Besides the scaling with regard to the system size, our system also shows self-similarity during its time evolution. This gives rise to dynamical scaling (Family-Vicsek scaling) as it is known from cluster growth by diffusion-limited aggregation [36]:

$$n(t) \sim t^{-w} f((q - q_c)t^{-z}). \tag{4}$$

In Fig. 5 (c) and (d) we show the dynamic scaling for both the triangular and the square lattice. As expected, in both systems the curves collapse, indicating self-similarity at different times.

5 Conclusion

In this paper, we studied polycontextural networks for different system sizes as well as for different simulation length. We derived analytical approximations for the lower and upper bound of the observed phase transitions and showed their correspondence with numerical simulations. Additionally, we analyzed the scaling behavior of the belief clusters in polycontextural networks.

The analysis of belief systems is a current research topic in social science [37, 38]. In particular, the question of how monological belief systems, closed-off clusters of mutually supportive beliefs, stabilize, is of interest [39]. Here, the observed dynamics are the result of a complicated interplay between one's own beliefs and the influence of others. It has recently been argued that the mutual influence of different people cannot be explained from an objective perspective. Rather, the perceived beliefs of agent A depend strongly on A's belief system [26]. Each observation needs hence to be modeled as a subjective process. This concept of subject-based cognition has a long tradition in philosophy: Hegel's dialectic and later Gotthard Günther's theory of polycontexturality are based on this premise [40, 41]. With our polycontextural networks, we developed a simple system that incorporates these philosophical concepts into a numerical model.

The phenomenon that specific views are strengthened rather than undermined by confrontation with counter-arguments is often observed in social epistemic structures like echo chambers [42]. Here, members often hold a belief system such that the existence of opposite beliefs reinforces their original belief system [43]. Our model represents the same effect by assuming that the same facts are perceived and communicated differently due to different belief systems.

The in-depth analysis we presented here can help to understand the emergence of belief-aligned clusters. In follow-up studies, it might then be interesting to compare the characteristics of our

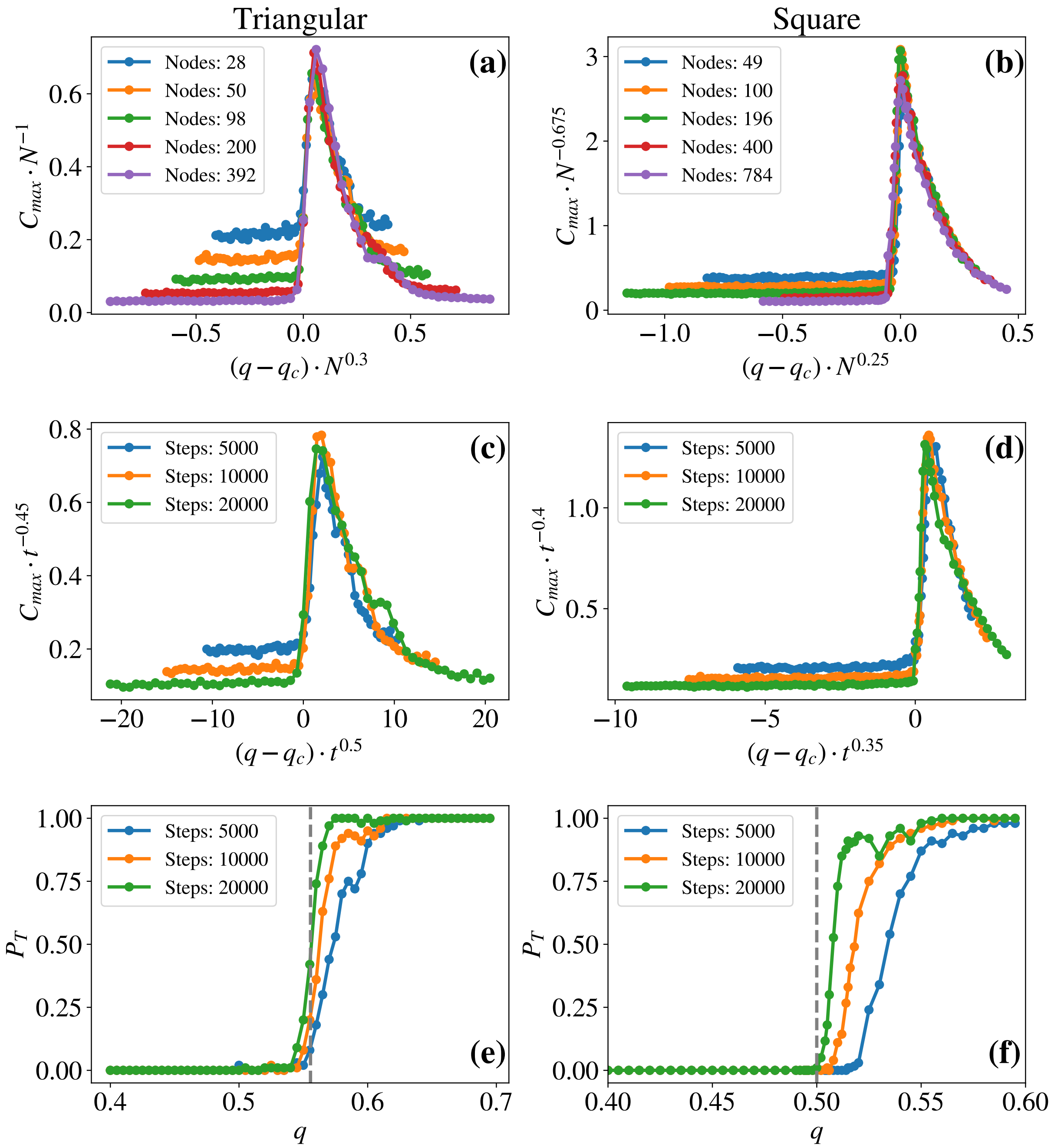

Fig. 5 **(a)** $C_{max} \cdot N^{-1}$ vs $(q - q_c) \cdot N^{0.3}$ for five different sizes of a triangular lattice with a simulation time of $t = 20,000$. The collapse of the curves indicates the validity of the finite-size scaling. **(b)** $C_{max} \cdot N^{-0.675}$ vs $(q - q_c) \cdot N^{0.25}$ for five different sizes of a square lattice with a simulation-time of $t = 20,000$. The collapse of the curves indicates the validity of the finite-size scaling. **(c)** Dynamic scaling for the largest cluster in a triangular lattice. The size of the system is $N = 98$. **(d)** Dynamic scaling for the largest cluster in a square lattice. The size of the system is $N = 100$. **(e)** The probability P_T that the system has reached a frozen state with no colour change for a triangular lattice at different simulation times. **(f)** The probability P_T that the system has reached a frozen state with no colour change for a square lattice at different simulation times. The dashed line indicates the transition point as indicated by our approximation from Sec. 3

theoretical model with clusters of election outcomes [44] or the structures that can e.g. be observed in interaction networks [45]. A deeper awareness of the mechanisms can help to counter disinformation campaigns and opinion polarization.

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Statements and Declarations

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