Calculable neutrino Dirac mass matrix and one-loop $\bar{\theta}$ in the minimal left-right symmetric model

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We revisit the contribution to the strong CP parameter $\bar{\theta}$ from leptonic CP violation at one-loop level in the minimal left-right symmetric model in the case of parity as the left-right symmetry. The Hermitian neutrino Dirac mass matrix M_D can be calculated using the light and heavy neutrino masses and mixings. We propose a parameterization of the right-handed neutrino mixing matrix V_R and construct the heavy neutrino mass that maintains the Hermiticity of M_D. We further apply it to evaluate the one-loop $\bar{\theta}$, denoted as $\bar{\theta}_{loop}$, as a function of the sterile neutrino masses for explicit examples of V_R. By requiring the magnitude of $\bar{\theta}_{loop} \lesssim 10^{-10}$, we derive the upper limits on the sterile neutrino masses, which are within reach of direct searches at the Large Hadron Collider and neutrinoless double beta decay experiments. Furthermore, our parameterization is applicable to other phenomenological studies.

arXiv:2404.16740v1 [hep-ph] 25 Apr 2024

I. INTRODUCTION

The standard model (SM) of particle physics has achieved great success. However, the origin of neutrino masses and the strong CP problem remains unsolved, serving as compelling motivations for physics beyond the SM (BSM). These two problems might have intrinsic connections, even though they appear in the weak and strong sectors at low energies.

If neutrinos are Majorana fermions, they could acquire Majorana masses in the seesaw mechanism [1–5], making them naturally small. In the type-I [1–5] and type-II [6–11] seesaw mechanisms, right-handed neutrinos and scalar triplet are introduced, respectively. In the minimal left-right symmetric model (MLRSM) [5, 9, 12–15], the neutrino masses can receive contributions from both type-I and type-II seesaw mechanisms $M_\nu = M_L - M_D^T M_N^{-1} M_D$ (cf. Eq. (11)). In case of parity or charge conjugation as the left-right symmetry [16], dubbed case $\mathcal{P}$ or $\mathcal{C}$, respectively, the MLRSM is highly predictive, which has been extensively studied [17–25]. Moreover, it was found that in the MLRSM, one can calculate the neutrino Dirac mass matrix M_D in terms of the light and heavy neutrino masses and mixings [26–29]. As a contrast, the expression of neutrino Dirac mass matrix M_D in Casas-Ibarra parameterization [30] in type-I seesaw models is still dependent on an arbitrary complex orthogonal matrix.

The strong CP problem is about the extremely small parameter $\bar{\theta} \lesssim 10^{-10}$ [31–33] that violates CP in the strong sector of the SM. The most popular solution to the strong CP problem is the Peccei-Quinn mechanism [34, 35], which leads to the existence of the axion [36, 37] and has thus drawn a lot of theoretical attention [38–44] as well as experimental interest [45]. Additionally, the strong CP problem can also be addressed by imposing discrete symmetries [46–51]. Parity solutions to the strong CP problem in the left-right symmetric models were considered in Refs. [46–48], and have been further studied recently [52–55]. In both SM and BSM scenarios, we can separate $\bar{\theta} = \theta + \arg\det(M_u M_d)$, where θ is the coefficient of $G\tilde{G}$ term in the Lagrangian, and $\arg\det(M_u M_d)$ is included since the up-type and down-type quark mass matrices M_u and M_d are in general non-Hermitian [56]. In the MLRSM of case $\mathcal{P}$, θ vanishes at tree level and $\bar{\theta}$ is equal to $\arg\det(M_u M_d)$. It has been shown that $\bar{\theta} \simeq \sin\alpha \tan(2\beta) m_t/(2m_b)$ [21, 52], where α and β are defined in Eq. (5), m_t and m_b denote the masses of top and bottom quarks, respectively. Thus in order to satisfy the constraint from measurements of neutron electric dipole moments [31–33] on $\bar{\theta} \lesssim 10^{-10}$, $\sin\alpha\tan(2\beta) \to 0$ is required.

However, even if the quark mass matrices are (nearly) Hermitian, leptonic CP violation would induce $\bar{\theta}$ at one-loop level, which might exceed the aforementioned bound as pointed out in Ref. [57]. Instead of being a problem, Senjanovic et al. [58] demonstrated that the one-loop $\bar{\theta}$ in the MLRSM implies an upper bound on the masses of sterile neutrinos, which is complementary to the direct searches at the Large Hadron Collider [59]. As obtained in Ref. [58], $\bar{\theta}_{loop}$ is proportional to $\mathrm{Im}\,\mathrm{Tr}\left(M_N^\dagger M_N [M_D, M_\ell]\right)$, where M_ℓ denotes the charged lepton mass matrix, and the neutrino Dirac mass matrix M_D is determined by the light and heavy neutrino masses and mixings. However, it was shown that [25] $\bar{\theta}_{loop}$ might vanish in the type-I seesaw dominance scenario for specific benchmark choices of the right-handed neutrino mixing matrix V_R, which hindered the attempt to search for sterile neutrinos contributing to $\bar{\theta}$ with neutrinoless double beta ($0\nu\beta\beta$) decay [25].

In this work, we propose a parameterization of right-handed neutrino mixing V_R in the MLRSM of case $\mathcal{P}$ and construct the heavy neutrino mass matrix M_N, for which the Hermiticity of the neutrino Dirac mass matrix M_D is maintained. We then evaluate the one-loop $\bar{\theta}$ for

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the general seesaw relation for explicit examples of V_R, and obtain non-vanishing $\bar{\theta}_{loop}$ as a function of the sterile neutrino masses. By using the bound $|\bar{\theta}_{loop}| \lesssim 10^{-10}$, we can then obtain the upper limits of the sterile neutrino masses.

The remainder of the paper is organized as follows. In the next section, we provide a brief introduction of the MLRSM of case $\mathcal{P}$. Sec. III delves into the calculation of neutrino Dirac mass matrix M_D in the Senjanovic-Tello method, and the parameterization of V_R and M_N. In Sec. IV, we evaluate the one-loop $\bar{\theta}$ for explicit examples of V_R. We conclude in Sec. V.

II. MINIMAL LEFT-RIGHT SYMMETRIC MODEL

The MLRSM is based on the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, which was proposed to explain the origin of neutrino masses [5, 9]. Three right-handed neutrinos ν_R and scalar triplets $\Delta_{L,R}$ are introduced

$$\ell_{L,R} = \begin{pmatrix} \nu \\ e \end{pmatrix}_{L,R} , \quad \Delta_{L,R} = \begin{pmatrix} \delta^+_{L,R}/\sqrt{2} & \delta^{++}_{L,R} \\ \delta^0_{L,R} & -\delta^+_{L,R}/\sqrt{2} \end{pmatrix} , \tag{1}$$

where the flavor indices of leptons are omitted. Besides, the scalar bi-doublet Φ exists, which is written as

$$\Phi = [\phi_1, i\sigma_2\phi_2^*] , \quad \phi_i = \begin{pmatrix} \phi_i^0 \\ \phi_i^- \end{pmatrix} , \quad i = 1, 2 , \tag{2}$$

where σ_2 is the second Pauli matrix. If parity is taken as the left-right symmetry, i.e., case $\mathcal{P}$, we have

$$\Delta_L \leftrightarrow \Delta_R , \quad \Phi \leftrightarrow \Phi^\dagger . \tag{3}$$

The leptonic Yukawa interactions are

$$\begin{aligned} \mathcal{L} = &-\bar{\ell}_L \left(Y_1\Phi - Y_2\sigma_2\Phi^*\sigma_2\right)\ell_R \\ &- \frac{1}{2}\left(\ell_L^T C Y_L i\sigma_2\Delta_L\ell_L + \ell_L^T C Y_R i\sigma_2\Delta_R\ell_R\right) \\ &+ \text{h.c.} , \end{aligned} \tag{4}$$

where $C = i\gamma^0\gamma^2$ is the charge conjugation matrix, h.c. denotes the Hermitian conjugate terms. The left-right symmetry is spontaneously broken once the right-handed triplet Δ_R develops a vacuum expectation value (vev), $v_R = \langle\delta_R^0\rangle$. After the electroweak symmetry breaking, Φ develops vevs

$$\langle\Phi\rangle = v\,\text{diag}(c_\beta, -s_\beta e^{-i\alpha}) \tag{5}$$

with $c_\beta \equiv \cos\beta$, $s_\beta \equiv \sin\beta$ and $v \simeq 174$ GeV. Then the left-handed triplet Δ_L would get the vev v_L, which is generally complex [60] and proportional to v^2/v_R [9, 61]. Defining $N_L = \nu_R^c$, one obtains the neutrino mass terms

$$\mathcal{L}_\nu = -\frac{1}{2}(\bar{\nu}_L^c, \bar{N}_L^c) M_n \begin{pmatrix} \nu_L \\ N_L \end{pmatrix} + \text{h.c.} , \tag{6}$$

where the full neutrino mass matrix is defined as

$$M_n \equiv \begin{pmatrix} M_L & M_D^T \\ M_D & M_N \end{pmatrix} . \tag{7}$$

The neutrino Majorana and Dirac neutrino mass matrices are

$$M_L = \frac{v_L}{v_R} U_e^T M_N^* U_e^* , \tag{8}$$

$$M_N = Y_R^* v_R , \tag{9}$$

$$M_D = -v\left(Y_1 c_\beta + Y_2 s_\beta e^{-i\alpha}\right) . \tag{10}$$

After block diagonalizing the neutrino mass matrix, we can obtain the light neutrino masses

$$M_\nu = M_L - M_D^T \frac{1}{M_N} M_D , \tag{11}$$

which is a general seesaw relation including contributions from both type-I and type-I mechanisms. If v_L is negligibly small, it is reduced to the type-I seesaw dominance scenario.

As shown in Ref. [28], in the MLRSM of case $\mathcal{P}$ we have

$$M_D - U_e M_D^\dagger U_e \propto s_\alpha t_{2\beta} , \tag{12}$$

where $s_\alpha \equiv \sin\alpha$, $t_{2\beta} \equiv \tan(2\beta)$, U_e is the matrix that diagonalizes the charged lepton mass matrix. Thus, in the limit $s_\alpha t_{2\beta} \to 0$, M_D is Hermitian and $U_e = \pm\mathbb{1}$.

III. CALCULABLE NEUTRINO DIRAC MASS MATRIX

A. Senjanovic-Tello method

It has been shown by Senjanovic and Tello [27, 28], the neutrino Dirac mass matrix M_D can be determined with the light and heavy neutrino masses and mixings in the limit $s_\alpha t_{2\beta} \to 0$. In the following, we will briefly introduce the general method they proposed in Ref. [28].

From $M_D = M_D^\dagger$, Eq. (11) can be expressed as

$$HH^T = \frac{v_L}{v_R}\mathbb{1} - \frac{1}{\sqrt{M_N}} M_\nu^* \frac{1}{\sqrt{M_N}} , \tag{13}$$

where the Hermitian matrix H is defined as

$$H = \frac{1}{\sqrt{M_N}} M_D \frac{1}{\sqrt{M_N}} . \tag{14}$$

One can then decompose HH^T as

$$HH^T = OsO^T , \tag{15}$$

using the fact that HH^T is symmetric. In the above, O is a complex orthogonal matrix and s is the symmetric normal form. The matrices O and s are obtained from Eqs. (13) (15). The matrix H itself can be expressed as

$$H = O\sqrt{s}EO^\dagger . \tag{16}$$

with E being determined by the Hermitian condition $H = H^\dagger$.

$$\sqrt{s}E = E\sqrt{s^*} \ , \quad E^T = E^* = E^{-1} \ . \tag{17}$$

Comparing Eq. (16) with Eq. (14), one readily get

$$M_D = \sqrt{M_N} O \sqrt{s} E O^\dagger \sqrt{M_N^*} \ . \tag{18}$$

Notice that O, s and E depend on M_ν and M_N, the neutrino Dirac mass matrix, we can calculate M_D once the light and heavy neutrino masses and mixings are known.

Although the above method is applied to the general seesaw relation in the Hermitian case (cf. Eq. (11)), no general M_D could be obtained since M_N is arbitrary [28]. In terms of the physical masses and neutrino mixing matrices,

$$M_\nu = V_L^* m_\nu V_L^\dagger \ , \quad M_N = V_R m_N V_R^T \ , \tag{19}$$

thus we should have a priori knowledge of V_R and m_N besides the inputs of m_ν and V_L from the measurements of neutrino oscillation [62].

If $V_R = V_L$ is assumed,we could obtain [27, 28]

$$M_D = V_L m_N \sqrt{\frac{v_L}{v_R} - \frac{m_\nu}{m_N}} V_L^\dagger \ . \tag{20}$$

While it is straightforward to calculate M_D for a different V_R, the following condition

$$\mathrm{ImTr}\left[\frac{v_L}{v_R} - \frac{1}{M_N} M_\nu^*\right]^n = 0 \ , \quad n = 1, 2, 3 \tag{21}$$

makes it more complicated, which results from the Hermiticity of H. The above relation implies that the phases of light and heavy neutrino mass matrices are not independent [27].

That is to say, for any V_R being assumed, it is necessary to verify the condition in Eq. (21) with the resulting heavy neutrino mass matrix M_N. Therefore, an appropriate choice of V_R is crucial and non-trivial.

B. Parameterization of V_R and M_N

Notice that if v_L is real, the condition in Eq. (21) is reduced to $\mathrm{ImTr}\left[M_N^{-1} M_\nu^*\right]^n = 0$. This enables us to obtain possible forms of M_N and V_R, the details of which are given in Appendix A.

We find that in the MLRSM of case $\mathcal{P}$ for Hermitian M_D and real v_L, the right-handed neutrino mixing matrix V_R can be parameterized as

$$V_R = P V_L \sqrt{m_N m_\nu}^{-1} \ , \tag{22}$$

where P is a Hermitian or anti-Hermitian matrix,

$$P = \pm P^\dagger \ . \tag{23}$$

For convenience, we can further write V_R as

$$V_R = \hat{P} V_L \ , \quad \hat{P} \equiv P V_L \sqrt{m_N m_\nu}^{-1} V_L^\dagger \ . \tag{24}$$

Note that P has the mass dimension one, while $\hat{P}$ is dimensionless. As V_L and V_R are unitary [28], it follows that $\hat{P}$ must also be a unitary matrix, thereby imposing constraint on P. If $V_R = V_L$, $\hat{P} = \mathbb{1}$, we readily get the Hermitian matrix $P = V_L \sqrt{m_N m_\nu} V_L^\dagger$.

From Eq. (22), one can construct the heavy neutrino mass matrix

$$M_N = P M_\nu^{-1} P^T \ , \tag{25}$$

which satisfies the condition in Eq.(21).

If $V_R = V_L$, and $m_N = v_R/v_L m_\nu$, using Eq. (25), we can readily get $M_N = v_R/v_L M_\nu^*$. Thus the above parameterization of M_N is compatible with the type-II seesaw dominance scenario.

IV. ONE-LOOP $\bar{\theta}$

As pointed out in Ref. [57], $\bar{\theta}$ can be generated from the leptonic CP violation, which contributes to the Higgs potential at one-loop level:

$$V \supset \left[\alpha_2 \,\mathrm{Tr}\left(\Delta_R^\dagger \Delta_R\right) + \mathrm{h.c.}\right] \mathrm{Tr}\left(\tilde{\Phi}\Phi\right) \ , \tag{26}$$

where the coupling α_2 is complex and $\tilde{\Phi} \equiv \sigma_2 \Phi^* \sigma_2$. It is shown that [57, 58]

$$\bar{\theta}_{loop} \simeq \frac{1}{16\pi^2} \frac{m_t}{m_b} \,\mathrm{Im}\,\mathrm{Tr}\left(Y_R^\dagger Y_R [Y_1, Y_2]\right) \ln \frac{M_{Pl}}{v_R} \ , \tag{27}$$

where the Dirac Yukawa couplings Y_R and $Y_{1,2}$ are defined in Eq. (4), and $M_{Pl} = 1.22 \times 10^{19}$ GeV denotes the Planck scale. In terms of the mass matrices, we have [58]

$$\bar{\theta}_{loop} \simeq \frac{1}{16\pi^2} \frac{m_t}{m_b} \frac{1}{v_R^2 v^2} \times \mathrm{Im}\,\mathrm{Tr}\left(M_N^T M_N^* [M_D, M_\ell]\right) \ln \frac{M_{Pl}}{v_R} \ . \tag{28}$$

where the charged lepton mass matrix M_ℓ is diagonal due to $U_e = \pm\mathbb{1}$. If $V_R = V_L$, by using the expressions of M_N and M_D given in Eqs. (19) (20), we can easily verify that $\bar{\theta}_{loop}$ is exactly zero. This also applies when $V_R = \mathbb{1}$ [25].

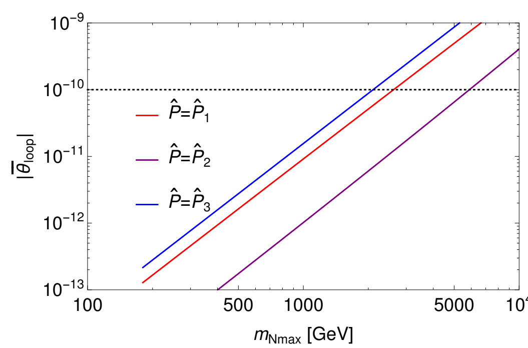

Figure 1: The magnitude of $\bar{\theta}_{loop}$ as a function of the heaviest sterile neutrino mass $m_{N\max}$, which is assumed to be m_4.

In order to evaluate $\bar{\theta}_{loop}$ for other choices of V_R, we use the parameterization in Sec. III B, and consider $V_R = \hat{P}V_L$ with the following textures of $\hat{P}$:

$$\hat{P}_1 = i\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}\ ,\quad \hat{P}_2 = i\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}\ ,$$
$$\hat{P}_2 = i\begin{pmatrix}\frac{1}{2}&\frac{1}{2}&-\frac{\sqrt{2}}{2}\\\frac{1}{2}&\frac{1}{2}&\frac{\sqrt{2}}{2}\\-\frac{\sqrt{2}}{2}&\frac{\sqrt{2}}{2}&0\end{pmatrix}\ , \tag{29}$$

where we have included the factor of i to maintain the Hermiticity of the neutrino Dirac mass matrix M_D [1]. One can directly verify that for these cases the matrix $P = \hat{P}V_L\sqrt{m_N m_\nu}V_L^\dagger$ is anti-Hermitian.

We assume that the active neutrino masses $m_\nu \equiv \mathrm{diag}(m_1, m_2, m_3)$ are in the normal hierarchy, and that the sterile neutrino masses $m_N \equiv \mathrm{diag}(m_4, m_5, m_6)$ are correlated with m_ν:

$$m_1 m_4 = m_2 m_5 = m_3 m_6\ , \tag{30}$$

and choose the parameters as

$$m_1 = 10^{-3}\ \mathrm{eV}\ ,\quad v_L = 1\ \mathrm{eV}\ ,\quad v_R = 15\ \mathrm{TeV}\ . \tag{31}$$

For the cases in Eq. (29), we obtain the magnitude of non-vanishing $\bar{\theta}_{loop}$ as a function of the heaviest sterile neutrino $m_{\mathrm{Nmax}} = m_4$ in Fig. 1. Since $\bar{\theta}_{loop}$ approximately increases with the sterile neutrino masses $(m_{\mathrm{Nmax}})^{5/2}$, by requiring $|\bar{\theta}_{loop}| \lesssim 10^{-10}$ [2], we obtain the upper bound on the sterile neutrino masses, which was highlighted in Ref. [58]. For $\hat{P} = \hat{P}_1$, $\hat{P}_2$ and $\hat{P}_3$, we obtain $m_{\mathrm{Nmax}} \lesssim 2.5$ TeV, 6 TeV and 2 TeV, respectively.

As a benchmark, we take $\hat{P} = \hat{P}_1$ and assume $m_4 = 2.86$ TeV, $m_5 = 3.32$ GeV, and $m_6 = 57.2$ MeV. The heavy neutrino mass matrix is given by

$$M_N = \begin{pmatrix} -1.95\times10^{12} - 7.05\times10^{5}i & -1.16\times10^{12} + 5.60\times10^{10}i & 6.38\times10^{11} + 6.47\times10^{10}i \\ -1.16\times10^{12} + 5.60\times10^{10}i & -6.96\times10^{11} + 6.69\times10^{10}i & 3.85\times10^{11} + 2.03\times10^{10}i \\ 6.38\times10^{11} + 6.47\times10^{10}i & 3.85\times10^{11} + 2.03\times10^{10}i & -2.09\times10^{11} - 4.24\times10^{10}i \end{pmatrix}\ \mathrm{eV}\ .$$

Using the Senjanovic-Tello method, we obtain the matrices in Eq.(16)

$$O = \begin{pmatrix} -0.1344 + 0.04691i & -0.4861 - 0.006028i & 0.8648 + 0.003902i \\ 0.6396 - 0.0002750i & 0.6240 + 0.01683i & 0.4499 - 0.02296i \\ 0.7584 + 0.008545i & -0.6125 + 0.02193i & -0.2263 - 0.03073i \end{pmatrix}\ ,$$
$$E = \mathbb{1}\ ,\ s = \begin{pmatrix} 8.576\times10^{-7} & 0 & 0 \\ 0 & 4.602\times10^{-10} & 0 \\ 0 & 0 & 6.867\times10^{-13} \end{pmatrix}\ ,$$

and the neutrino Dirac mass matrix

$$M_D = \begin{pmatrix} 545380. & 343623. + 13636.i & -204348. + 16763.i \\ 343623. - 13636.i & 272102. & -116364. + 17374.i \\ -204348. - 16763.i & -116364. - 17374.i & 109404. \end{pmatrix}\ \mathrm{eV}\ .$$

The resulting value of $\bar{\theta}_{loop}$ is -1.241335×10^{-10}. While the heaviest sterile neutrino is within reach of direct searches at the Large Hadron Collider [59], the lighter sterile neutrinos could give significant contributions to neutrinoless double beta decay [25].

It is worth noting that due to $s_\alpha t_{2\beta} \propto \mathrm{Im}\,\alpha_2$, the neutrino Dirac mass matrix M_D is not exactly Hermitian. A delicate examination of non-Hermitian M_D was recently conducted in Ref. [29]. Nevertheless, the correlation between M_D and $\bar{\theta}$ poses a challenge for recursive evaluation. Unless there is accidental cancellation, assuming that M_D is exactly Hermitian, as we have done, is adequate for estimating the upper limit on the sterile neutrino masses.

V. CONCLUSION

In this work, we have proposed a parameterization of right-handed neutrino mixing $V_R = PV_L\sqrt{m_N m_\nu}^{-1}$ with P being a Hermitian or anti-Hermitian matrix in the MLRSM of case $\mathcal{P}$, and constructed heavy neutrino mass matrix as $M_N = PM_\nu^{-1}P^T$. In this parameterization, the Hermiticity of the neutrino Dirac mass matrix M_D is maintained. We then evaluate the one-loop $\bar{\theta}$ generated from leptonic CP violation for the general seesaw relation with explicit examples of V_R and obtain non-vanishing $\bar{\theta}_{loop}$ as a function of the sterile neutrino masses. By requiring $|\bar{\theta}_{loop}| \lesssim 10^{-10}$, we obtain the up-

[1] In the type-I seesaw dominance scenario, where v_L is negligibly small, M_D in Eq. (20) is anti-Hermitian if we assume $V_R = V_L$, a fact that appears to have been overlooked. We find that for $V_R = iV_L$, $\bar{\theta}_{loop}$ also vanishes, which can be verified analytically.

[2] We obtain that $\bar{\theta}_{loop}$ has definite sign for $\hat{P}$ being given in Eq. (29).

per bound on the sterile neutrino masses.

Our parameterization of V_R and M_N is applicable to other phenomenological studies. In particular, it enables us to study the interplay of $0\nu\beta\beta$ decay and one-loop $\bar{\theta}$ mediated by the sterile neutrinos in the MLRSM of case $\mathcal{P}$.

Appendix A: Hermitcity of H

In Sec. III B, we have provided the parameterization of the right-handed neutrino mixing V_R and heavy neutrino mass matrix M_N, which guarantees the Hermiticity of the matrix H hence neutrino Dirac mass matrix M_D. In this appendix, we will give more details.

From Eq. (21), we expand

$$\begin{aligned}&\left[\frac{v_L}{v_R} - \frac{1}{M_N}M_\nu^*\right]^n \\ =&C_1 + C_2\frac{1}{M_N}M_\nu^* + ... + C_n(\frac{1}{M_N}M_\nu^*)^n ,\end{aligned} \tag{A1}$$

where all the C_n for $n \in \mathbb{N}$ are real numbers. So all we need to check is $\left(M_N^{-1}M_\nu^*\right)^n$.

First, we assume

$$M_N = PM_\nu^{-1}Q , \tag{A2}$$

where Q is a matrix with dimension one.

We observe that if $Q = \pm P^*$,

$$\begin{aligned}&\mathrm{ImTr}\left[\left(\frac{1}{M_N}M_\nu^*\right)^n\right] \\ =&\pm \mathrm{ImTr}\left[(P^{-1*}M_\nu P^{-1}M_{\nu^*})^n\right] .\end{aligned} \tag{A3}$$

Defining $A = {P^{-1}}^* M_\nu$, we have

$$\begin{aligned}\mathrm{Tr}[(AA^*)^n] &= \mathrm{Tr}\left[A^*(AA^*)^{n-1}A\right] \\ &= \mathrm{Tr}[(A^*A)^n] ,\end{aligned} \tag{A4}$$

so that

$$\mathrm{ImTr}\left[\left(\frac{1}{M_N}M_\nu^*\right)^n\right] = \mathrm{ImTr}\left[\left(\frac{1}{M_N}M_\nu^*\right)^{n*}\right] , \tag{A5}$$

which implies that

$$\mathrm{ImTr}\left[\left(\frac{1}{M_N}M_\nu^*\right)^n\right] = 0 . \tag{A6}$$

Therefore, the condition in Eq. (21) is satisfied.

From Eq. (A2), we have

$$\begin{aligned}M_N =&PV_L m_\nu^{-1}V_L^T Q \\ =&V_R m_N V_R^T .\end{aligned} \tag{A7}$$

To find a possible form of V_R, we define

$$F = \sqrt{m_\nu}X\sqrt{m_N} , \tag{A8}$$

where X is an orthogonal matrix, $XX^T = \mathbb{1}$, and obtain

$$m_\nu = F m_N^{-1} F^T . \tag{A9}$$

Then Eq. (A7) becomes

$$PV_L(F^T)^{-1}m_N F^{-1}V_L^T Q = V_R m_N V_R^T . \tag{A10}$$

If

$$V_R = PV_L(F^T)^{-1} , \quad V_R^T = F^{-1}V_L^T Q , \tag{A11}$$

the relation in Eq. (A10) must be satisfied. Hence, $Q = P^T = \pm P^*$ with P being a Hermitian or anti-Hermitian matrix. Without loss of generality, we assume $X = \mathbb{1}$, and then $F = \sqrt{m_\nu m_N}$. Other choices of X can also yield appropriate V_R and P, which may also be of interest.

ACKNOWLEDGEMENTS

GL would like to thank Jordy de Vries and Juan Carlos Vasquez for fruitful discussions, and Ravi Kuchimanchi for helpful correspondence regarding Ref. [57]. DYL is grateful to Ken Kiers for generously sharing the codes of Ref. [29] and for providing valuable suggestions and help. This work is partly supported by the National Natural Science Foundation of China under Grant No. 12347105, Fundamental Research Funds for the Central Universities in Sun Yat-sen University (23qnpy62), and SYSU startup funding.

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