Charged pion vortices in rotating systems

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Possibilities for formation of the charged pion field vortices in a rotating empty vessel (in vacuum) and in the rotating pion gas with a dynamically fixed particle number at zero temperature are studied within the $\lambda|\phi|^4$ model. It is shown that in the former case at a rapid rotation a supervortex of a charged pion field can be formed. Important role played by the electric field is demonstrated. Field configurations in presence and absence of the pion self-interaction are found. Conditions for formation of the vortex lattice at the rotation of the charged pion gas at zero temperature are studied. Observational effects are discussed.

1. Introduction

In heavy-ion collisions at some collision stage a hadron fireball is formed. At LHC and top RHIC collision energies number of produced pions exceeds the baryon/antibaryon number by an order of magnitude [1,2]. At the fireball expansion stage from chemical to thermal freeze out the pion number is approximately (dynamically) conserved. If the state formed at the chemical freeze-out was overpopulated by pions, then during the cooling they may form the Bose-Einstein condensate characterized by the dynamically fixed pion number, as was suggested in [3] and studied then in a number of works, cf. [4–7]. The ALICE Collaboration observed a significant suppression of three and four pion Bose–Einstein correlations in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV at the LHC [8]. Analysis [6] indicated that about 5% of pions could stem from the Bose-Einstein condensate.

Estimates show on angular momenta $L \sim \sqrt{s}Ab/2 \lesssim 10^6\hbar$ in peripheral heavy-ion collisions of Au + Au at $\sqrt{s} = 200$ GeV, for the impact parameter $b = 10$ fm, where A is the nucleon number of the ion [9]. The global polarization of $\Lambda(1116)$ hyperon observed by the STAR Collaboration in non-central Au-Au collisions [2] indicated existence of a vorticity with rotation frequency $\Omega \simeq (9 \pm 1) \cdot 10^{21}$ Hz $\simeq 0.05 m_\pi$, $m_\pi \simeq 140\,\mathrm{MeV}$ is the pion mass.

Formation of vortices in resting quantum liquids is energetically unfavorable. Vortex structures in the rotating liquid helium and cold Bose gases of nonrelativistic bosons have been extensively studied, cf. [10, 11].

Besides a rotation, also strong magnetic fields are expected to occur at heavy-ion collisions and in compact stars. Estimates [12] predicted values of the magnetic field up to $\sim (10^{17} - 10^{18})$G for peripheral heavy-ion collisions at the energy $\sim$ GeV per nucleon. Also, fields $H \lesssim (10^{15} - 10^{16})$G should exist at surfaces of magnetars, and m.b. still stronger fields in interiors.

Question about condensation of the noninterating charged pions in vacuum at a simultaneous action of the rotation and a strong magnetic field was studied in [13]. Work [14] included the pion self-interaction within the $\lambda|\phi|^4$ model and suggested appearance of a giant pion vortex (supervortex). Limit $H \to 0$ was not allowed and effects of the electric field were disregarded. Reference [15] studied possibilities of the appearance of the pion-σ supervortex in the rapidly rotating empty vessel and in rotating nuclear systems, as well as formation of the vortex lattice in the rotating pion gas at zero temperature, $T = 0$, within the σ model with taking into account the charge effects. In the given paper these problems will be considered on example of the $\lambda|\phi|^4$ model. Units $\hbar = c = 1$ will be used.

2. Charged pion field in rotation frame

Let us study behavior of the charged pion vacuum and a pion gas at $T = 0$ with a dynamically fixed particle number, in the rigidly rotating cylindrical system at the constant rotation frequency $\vec{\Omega} \parallel z$ in the cylindrical coordinates (r, θ, z), $\nabla = (\partial_r, \partial_\theta/r, \partial_z)$, $r = \sqrt{x^2 + y^2}$. Interval in rotation frame is [16], $(ds)^2 = (1 - \Omega^2 r^2)(dt)^2 + 2\Omega y dx dt - 2\Omega x dy dt - (dr_3)^2$, $r_3 = \sqrt{r^2 + z^2}$, with the tetrad $e^t_0 = e^x_1 = e^y_2 = e^z_3 = 1$, $e^x_0 = y\Omega$, $e^y_0 = -x\Omega$. Other elements are zero; $e_\alpha = e^\beta_\alpha \partial_\beta$, and thereby $e_0 = \partial_t + y\Omega\partial_x - x\Omega\partial_y$, $e_i = \partial_i$. Lattin index $i = 1, 2, 3$, Greek index $\alpha, \beta = 0, 1, 2, 3$. Region $r > 1/\Omega$ is beyond light cone.

In presence of the electromagnetic field $A^\alpha_{\rm lab}$ in the laboratory frame, the pion term in the Lagrangian density in the rotation frame renders [13,14,16]:

$$\mathcal{L}_\pi = |(D_t + y\Omega D_x - x\Omega D_y)\phi|^2 - |D_i\phi|^2 - m^2_\pi|\phi|^2 - \tfrac{\lambda|\phi|^4}{2}\,, \qquad (1)$$

where $D_\alpha = \partial_\alpha + ieA_\alpha$, $eA_\alpha = eA^{\rm lab}_\beta e^\beta_\alpha$, e is the charge of the electron, λ is a positive constant. Let us focus on the case $V_{\rm lab} = V_{\rm lab}(r)$ produced by the external charge density $n^{\rm lab}_p(\vec{r})$, $\vec{A}_{\rm lab} = 0$ at $\Omega = 0$. In the rotation frame we seek solution of the equation of motion in the form of the individual vortex with the center at $r = 0$, cf. [10], $\phi = \phi_0\chi(r)e^{i\xi(\theta)-i\mu t+ip_z z}$, with $\phi_0 = const$, $p_z = const$. Being interested in description of ground state we put $p_z = 0$. Circulation of the $\xi(\theta)$-field, $\oint d\vec{l}\,\nabla\xi = 2\pi\nu$, yields integer values of the winding number $\nu = 0, \pm1, ...$ and $\nabla\xi = \nu/r$, $\xi = \nu\theta$, thereby. The quantity μ has the sense of the energy of the ground state level of the π^-. In case of the pion gas at $T = 0$ it coincides with the chemical potential.

Lagrangian density with account of the rotation can be presented as

$$\mathcal{L}_{\pi,V} = \mathcal{L}_\pi + \mathcal{L}_V\,, \quad \mathcal{L}_\pi = |\widetilde{\mu}\phi|^2 - |\partial_i\phi|^2 - m^2_\pi|\phi|^2 - \tfrac{\lambda|\phi|^4}{2}\,, \qquad (2)$$
$$\mathcal{L}_V = \tfrac{(\nabla V)^2}{8\pi e^2} + n_p V\,, \quad \widetilde{\mu} = \mu + \Omega\nu - V(r)\,.$$

The rotation term acts as a constant contribution to the electric potential for $r < 1/\Omega$. Equation of motion for the vortex field in the rotation frame is

$$[\widetilde{\mu}^2 + \Delta_r - \nu^2/r^2 - m^2_\pi]\chi(r) - \lambda|\phi_0|^2\chi^3(r) = 0\,, \qquad (3)$$

$\Delta_r = \partial^2_r + \partial_r/r$, and the equation for the electric field is

$$\Delta V = 4\pi e^2(n_p - n_\pi)\,, \quad n_\pi = \frac{\partial\mathcal{L}_{\pi,V}}{\partial\mu} = 2\widetilde{\mu}|\phi|^2\,, \qquad (4)$$

n_π is the charged pion field (particle) density. Simplifying, we will assume that n_p is produced by very heavy particles (e.g., by protons being 7 times heavier than pions) and thereby we put $n_p \simeq n_p^{\rm lab}$ and we take $n_p = n_p^0\theta(R-r)$, and either $n_p^0 = const > 0$ or zero, and $\theta(x)$ is the step function, $R < 1/\Omega$.

The angular momentum associated with the charged pion field ϕ is

$$\vec{L}_\pi = \int d^3X[\vec{r}_3 \times \vec{P}_\pi] \ , \quad P^i_\pi = T^{0i}_\pi = -\tfrac{\partial \mathcal{L}_\pi}{\partial \partial_t \phi}\nabla\phi - \tfrac{\partial \mathcal{L}_\pi}{\partial \partial_t \phi^*}\nabla\phi^*, \tag{5}$$

T^{0i}_π is the $(0i)$ component of the energy-momentum. The energy density is

$$E_{\pi,V} = E_\pi + E_V = \mu n_\pi - \mathcal{L}_{\pi,V} \ . \tag{6}$$

Question arises what is distribution of vortices if they appear: a supervortex with the winding number $\nu \gg 1$ or the lattice of vortices with $\nu = 1$ each?

3. Rotation and laboratory frames

The question how to treat the rotating reference frame and the response of the Bose field vacuum and the Bose gas at $T = 0$ (i.e. the Bose-Einstein condensate) on the rotation in this frame is rather subtle due to necessity to fulfill the causality condition $r < 1/\Omega$. Thereby, we will associate the rotation frame with a rotating rigid body of a finite transversal size. For instance, we may consider either vacuum or the pion gas inside a long empty cylindric vessel of a large internal transversal radius R, external radius $R_>$, hight $d_z \gg R$, a large mass M and constant mass-density ρ_M, rotating in the z direction with constant cyclic frequency Ω at $\Omega < \Omega_{\rm caus} = 1/R_>$, as requirement of causality. Otherwise solid vessel will be destroyed by rotation.

In case of the vortex placed in the center of the cylindric coordinate system $P_\theta = 2\widetilde{\mu}|\phi|^2\nu/r = n_\pi\nu/r$ and using (5) we have $L^\pi_z = 2\pi d_z \int_0^R r dr \nu n_\pi = \nu N_\pi$.

There are two possibilities: (1) the charged pion system responses on the rotation creating the vortex field in the rotation frame, and (2) it does not rotate, cf. [10, 11]. Further, there are two possibilities: (i) conserving rotation frequency $\vec{\Omega}_{fin} = \vec{\Omega}_{in}$ of the rotating rigid body representing the rotation frame, and (ii) conserving angular momentum $\vec{L}_{fin} = \vec{L}_{in}$.

In case (i) the kinetic energy of the vessel measured in the laboratory (resting) reference frame, $\mathcal{E}_{in} = \pi\rho_M\Omega^2_{in}d_z(R^4_> - R^4)/4$, given for simplicity for the nonrelativistic motion, does not change with time, i.e. $\mathcal{E}_{in} = const$. The loss of the energy due to a radiation is recovered from an external source. We will consider situation when in the laboratory frame the pion vortex field does not appear from the vacuum. It is so if external fields (the electric field V in our case) are not too strong. However there is still a possibility of the formation of the pion field from the vacuum in the rotation reference frame. In presence of Bose excitations the final energy of the system is given by

$$\mathcal{E}_{fin} = \mathcal{E}_{in} + \mathcal{E}_\pi[\Omega_{in}] \, ,$$

$\mathcal{E}_\pi[\Omega_{in}]$ is the rotation part of the energy associated with a boson field in the rotating system. Note that in a deep electric potential well in absence of the material walls, instability for the creation of $\pi^\pm$ pairs occurs, if the pion ground energy level, μ, reaches $-m_\pi$. In the rotating piece $r < R$ of the vacuum inside the vessel the pions can be produced via reactions on the walls of the vessel already when the lowest energy level, μ, reaches zero. Further, studying vacuum in empty rotating vessel we put $\mu = 0$ whereas in a formal treatment of rotation frame one would put $\mu = -m_\pi$. The condition for the formation of the condensate in these cases is

$$\mathcal{E}_\pi[\Omega_{in}] < 0 \,. \tag{7}$$

The same condition holds, if we deal with the pion gas with a dynamically fixed particle number, with the difference that for the gas the chemical potential, $\mu > 0$, is determined from condition of the fixed particle number.

In case (ii) the vessel is rotated owing to the initially applied angular momentum $\vec{L}_{in} = \int d^3X[\vec{r}_3 \times \vec{P}_{in}]$, $\vec{P}_{in} = \rho_M[\vec{\Omega}_{in}, \vec{r}_3]$. The value $\vec{L}_{in}$ is conserved, provided one ignores a weak radiation, but it can be redistributed between the massive vessel (stiff subsystem) and the pion field (a softer subsystem),

$$\vec{L}_{in} = \vec{L}_{M,fin} + \vec{L}_\pi^{\rm lab} \,, \quad \mathcal{E}_{fin} = \pi\rho_M\Omega_{fin}^2 d_z(R_>^4 - R^4)/4 + \mathcal{E}_\pi^{\rm lab} \,, \tag{8}$$

and for the gas with fixed particle number we have $\mathcal{E}_\pi^{\rm lab} = \mathcal{E}_\pi[\nu, \Omega = 0]$. Employing (8) and neglecting $O(1/M)$ term we obtain

$$\delta\mathcal{E} = \mathcal{E}_{fin} - \mathcal{E}_{in} \simeq -L_\pi^{\rm lab}\Omega_{in} + \mathcal{E}_\pi^{\rm lab}[\nu, \Omega = 0] \,. \tag{9}$$

The vortex-condensate field appears provided $\delta\mathcal{E} < 0$. For $V_0 < m$, $\mathcal{E}_\pi[\nu, \Omega = 0] > 0$. Conditions (7) and (9) should coincide, see below.

4. Charged pion vortex field in absence of self-interaction

Equation of motion, boundary conditions, energy in rotation frame. Let us consider the case $\lambda = 0$. From Eq. (3) we have

$$\left[\partial_r^2 + \partial_r/r - \nu^2/r^2 + (\widetilde{\mu}^2 - m^2)\right] \chi(r) = 0 \,. \tag{10}$$

Eq. (10) describes a spinless relativistic particle of energy $\epsilon_{n,\nu} = \mu$, mass m and z-projection of the angular momentum ν, placed in the potential well $U(r) = -\Omega\nu + V(r)$ for $r < R$. Behavior at $r > R$ depends on the boundary condition put at $r = R$. We are interested in the description of the ground state, then $\mu = \min\{\epsilon_{n,\nu}\}$ plays a role of the chemical potential. The term $-2\Omega\nu(\mu - V)|\phi|^2$ in the energy density is associated with the Coriolis force and the term $-\Omega^2\nu^2|\phi|^2$ is an attractive relativistic $\propto 1/c^2$ contribution to the centrifugal force term $(\nu^2/r^2)|\phi|^2$.

The Schrödinger equation for a nonrelativistic spinless particle follows from the Klein-Gordon equation (10) after replacement $\mu \to m + \mu_{\rm n.r.}$ and subsequent dropping of small quadratic terms $(\mu_{\rm n.r.} + \Omega\nu - V)^2$. Then we get

$$\left[-\Delta_r/(2m) - \Omega\nu + V(r) + \nu^2/(2mr^2)\right]\chi = E_{\rm n.r.}\chi, \tag{11}$$

$U_{ef} = -\Omega\nu + V(r) + \nu^2/(2mr^2)$, $E_{\rm n.r.} = \mu_{\rm n.r.}$. So, rotation in the rotating frame acts similarly to a constant electric potential acting on a nonrelativistic particle with the projection of the angular momentum ν.

In the Schrödinger equation the rotation is ordinary introduced employing the local Galilei transformation, with the speed given by $\vec{W} = [\vec{\Omega} \times \vec{r}_3]$. It results in the replacement in the Schrödinger equation, cf. [17],

$$-\Delta/(2m) \to -(\nabla - im\vec{W})^2/(2m) - m\vec{W}^2/2 \to -\Delta/(2m) - \Omega\nu\,, \tag{12}$$

that yields the same Eq. (11), at which we arrived considering the problem in the rotation frame. Thus we see that uniform rotation acts in nonrelativistic case similarly to a uniform rather weak magnetic field described by the vector-potential $\vec{A} = \frac{1}{2}[\vec{H}, \vec{r}_3]$. In relativistic case shift of variables $\partial_t \to \partial_t - \Omega\partial_\theta$ in the Klein-Godron equation in the rotation frame is not equivalent to the shift $\nabla \to \nabla - im[\vec{\Omega}, \vec{r}_3]$ in the Hamiltonian and subtraction of the $\frac{m\vec{W}^2}{2}$ term associated with the motion of the system as a whole, cf. [16].

Further let us for simplicity consider the case $V \simeq -V_0 = const$ for $r < R$. V_0 can be treated as a contribution to the chemical potential. For example we may assume that an ideal rotating vessel is placed inside the cylindrical co-axial charged capacitor or itself it represents the capacitor. Appearance of the field $\phi \neq 0$ produces a dependence of V on r. However, if ϕ is a rather small, we can continue to consider $V = -V_0 \simeq const$. Employing dimensionless variable $x = r/r_0$, with

$$r_0 = 1/\sqrt{\bar{\mu}^2 - m^2}\,, \quad \bar{\mu} = \mu + \Omega\nu + V_0\,, \tag{13}$$

for $\bar{\mu} > m$, $V_0 = const$, from Eq. (10) we obtain equation

$$(\partial_x^2 + x^{-1}\partial_x - \nu^2/x^2)\chi + \chi = 0\,. \tag{14}$$

Simplest appropriate boundary conditions are

$$\chi(0) = 0\,, \quad \chi(R/r_0) = 0\,. \tag{15}$$

Further to be specific let us consider $\Omega, \nu > 0$. Appropriate solution of Eq. (14) is the Bessel function $\chi(r) = J_\nu(r/r_0)$ for $\nu > 0$, cf. [18]. For $x \to 0$ we have $J_\nu \sim x^\nu$. The energy of the $n = 1$ level is determined by the first zero of the function $J_\nu(R/r_0) = 0$, $j_{n=1,\nu=0} \simeq 2.403$. The $n = 1, \nu = 1$ zero yields $j_{1,1} = R/r_0 \simeq 3.832$, $j_{1,\nu}$ increases with increase of integer values of ν. For $\nu \gg 1$, $j^{\rm as}_{1,\nu} \to \nu + 1.85575\nu^{1/3}$, e.g., $j_{1,100} \simeq 108.84$.

Employing the boundary condition $\chi(x = R/r_0 = j_{n,\nu}) = 0$ we find

$$\epsilon_{n,\nu} = \mu = -\Omega\nu - V_0 + m\sqrt{1 + j^2_{n,\nu}/(R^2m^2)}\,, \tag{16}$$

$\bar{\mu} = \sqrt{m^2 + j_{n,\nu}^2/R^2}$. Notice that with increasing quantity $\Omega\nu + V_0$ the $n, \nu \neq 0$ levels become more bound than the level $n = 1, \nu = 0$. From (16) we also find that the roots $\epsilon_{n,\nu}$ for $V_0 = 0$ do not reach zero for $\Omega R < 1$. Thus for $V_0, \lambda = 0$ the field ϕ would not appear at the rotation of empty vessel.

One could employ the boundary condition $\chi'(r = R) = 0$ instead of the condition $\chi(r = R) = 0$. In both cases there is no current through the surface $r = R$. Such a change of the boundary condition would not affect our conclusion that at $V_0 = 0$ the energy level does not cross zero. Note that in cases of the vacuum and the Bose-Einstein condensate in the vessel, usage of one of mentioned boundary conditions is motivated provided the typical frequency of atomic transitions in the solid wall, $\omega_{\rm at}$, is larger than difference between energies of the first excited energy levels and the ground state level, $\sim 1/R$ for $\nu \gg mR$ and $\sim \nu/(R^2 m)$ in oppisite case. Otherwise one should use exact matching conditions for $\chi(r = R)$ and $\chi'(r = R)$.

Storm of charged pion field in rotation frame. The value of the rotation frequency, at which Eq. (16) could be fulfilled for $\mu = \epsilon_{1,\nu} \le 0$, $V_0 \neq 0$ is

$$\Omega \ge \Omega_c = \Omega(\epsilon_{1,\nu} = 0) = (\sqrt{m^2 + j_{1,\nu}^2/R^2} - V_0)/\nu\,, \tag{17}$$

where $\sqrt{m^2 + j_{1,\nu}^2/R^2} \to \sqrt{m^2 + \nu^2/R^2}$ for $\nu \gg 1$.

For $1 \le \nu = c_1 mR \ll mR$, i.e. at $c_1 \ll 1$, $mR \gg 1$, from Eq. (16) we get

$$\epsilon_{1,\nu} \simeq -V_0 - \Omega\nu + m + ... \tag{18}$$

The levels $\epsilon_{1,\nu}$ reach zero for $V_0 > m(1 - c_1)$ at $\Omega = 1/R$. The critical rotation frequency is then given by

$$\Omega_c = \Omega(\epsilon_{1,\nu} = 0, c_1 \ll 1) \simeq (m - V_0)/\nu > 0. \tag{19}$$

For $V_0 < m$, Ω_c decreases with increasing ν. For $V_0 > m$, $\Omega_c \sim O(1/(mR^2))$. The latter case is similar to the case of the gas with fixed particle number.

For $\nu = c_1 mR \gg mR \gg 1$ (at $c_1 \gg 1$) from (16) we have

$$\epsilon_{1,\nu} \simeq -V_0 + (-\Omega R + 1)\nu/R + Rm^2/(2\nu) + 1.86\nu^{1/3}/R + ... \tag{20}$$

Setting in (20) the limiting value $\Omega^{\rm caus} = 1/R$ we see that $\epsilon_{1,\nu} \to -V_0 + m/(2c_1) + ...$ for $1 \ll c_1 \ll \sqrt{mR}$. Thus the level $\epsilon_{1,\nu}$ may reach zero for $\Omega R < 1$ at $V_0 > V_{0c} = m/(2c_1)$. With increasing ν, V_{0c} decreases. Formation of the supervortex becomes energetically favorable for $\sqrt{mR} \gg c_1 \gg 1$ at

$$\Omega > \Omega_c = \Omega(\epsilon_{1,\nu} = 0, c_1 \gg 1) \simeq \frac{1}{R} - \frac{V_0 - m/(2c_1)}{c_1 mR}\,, \quad V_0 > V_{0c} = \frac{m}{2c_1}\,. \tag{21}$$

In case (i) the minimal critical value $V_{0c} \sim \sqrt{m/R}$ for $c_1 \sim \sqrt{mR} \gg 1$. For $V_0 = V_{0c}$, we have $\Omega_c \to 1/R$. For $c_1 \gg \sqrt{mR}$, $V_{0c} \simeq 1.86(m/R^2)^{1/3} \ll m$. The amplitude of the arising vortex field is limited by redistribution of the charge, which we did not take into account assuming that $V_0 \simeq const$. In case (ii) c_1 is also limited by conservation of the initial angular momentum.

In the rotation frame, in case (i), using solution (16) and (6), (14) we find

$$\mathcal{E}_\pi(\Omega) = \epsilon_{n,\nu} N_\pi = \epsilon_{n,\nu} d_z 4\pi \int_0^R r dr \sqrt{m_\pi^2 + \frac{j_{n,\nu}^2}{R^2}} \phi_0^2 \chi^2(r). \tag{22}$$

For $n_p \neq 0$, $\mathcal{E}_{\pi,V} \simeq \mathcal{E}_\pi(\Omega) + V_0 Z$, where $Z = n_p \pi R^2 d_z$.

In case (ii) the angular momentum needed for formation of the vortex is taken from the bucket walls. Presenting Eq. (16) as $\epsilon_{n,\nu} = \epsilon_{n,\nu}[\Omega = 0] - \Omega\nu$ and comparing Eqs. (9) and (22) we get $\delta\mathcal{E} = \mathcal{E}_\pi(\Omega) = \epsilon_{n,\nu} N_\pi$.

Notice that in case of the vacuum placed in a strong static electric field in absence of the rotation, the charged bosons are produced nonlocally via tunneling from the lower continuum to the upper continuum. The typical time of such processes is exponentially large $\tau \sim e^{m^2/|eE|}/m$ for the strength of the electric field $|eE| \ll m$. In case of the rotating empty vessel the charged pion field can be produced in more rapid processes, in reactions of particles of the rotating wall of the vessel. Also, as one of possibilities to create the vortex field, one may inject inside the vessel an admixture of protons. Accelerated protons will then produce radiation of the charged pion pairs and the latter can then form the vortex field.

Ideal pion gas with dynamically fixed particle number in rotating system. In case of the ideal pion gas at $T = 0$ characterized by the dynamically fixed particle number N_π, being put in a resting vessel on the ground state level, the value $\phi_0^2[\Omega = 0]$ is found from the normalization condition $N_\pi = \mu\phi_0^2[\Omega = 0]\pi R^2 d_z$. In the rotation frame $\epsilon_{1,\nu} = \mu > 0$, the constant $\phi_0^2[\Omega]$ is found from the condition (22) yielding $N_\pi \simeq 2\bar{\mu}\phi_0^2[\Omega]\pi R^2 d_z J_{\nu+1}^2(R/r_0[\bar{\mu}])$, with r_0 and $\bar{\mu}$ given in Eq. (13), i.e. the relation between the (dynamically) fixed value N_π and constant $\bar{\mu}$. Value $\epsilon_{n,\nu} = \mu$ depends on Ω through the relation (13).

To understand will the gas be at rest or rotating with the angular velocity Ω we should compare $\mathcal{E}_\pi[\Omega] = N_\pi \epsilon_{1,\nu}$ and $\mathcal{E}_\pi[\Omega = 0]$. The minimal value of $\mathcal{E}_\pi[\Omega = 0]$ corresponds to $\nu = 0$ and for $m_\pi R \gg 1$ is given by

$$\mathcal{E}_\pi[\Omega = 0] \simeq N_\pi[-V_0 + m_\pi + j_{1,0}^2/(2R^2 m_\pi)] \,. \tag{23}$$

For $\nu \ll R m_\pi$ and any V_0 we find that for

$$\Omega > \Omega_{c1}^{\mathrm{id}}(\nu) \simeq (j_{1,\nu}^2 - j_{1,0}^2)/(2\nu R^2 m_\pi) \,, \tag{24}$$

$$\mathcal{E}_\pi[\Omega] - \mathcal{E}_\pi[\Omega = 0] \simeq N_\pi[-\Omega\nu + (j_{1,\nu}^2 - j_{1,0}^2)/(2R^2 m_\pi)] < 0 \,, \tag{25}$$

$\Omega_{c1}^{\mathrm{id}}(\nu)$ is minimal for $|\nu| = 1$. Comparing (24) and (17), (19) we see that $\Omega_{c1}^{\mathrm{id}} \ll \Omega_c(\lambda = 0)$ at least for small values V_0, i.e. in presence of a pion gas vortices appear already at much smaller rotation frequencies than in case of the rotating vacuum. For the former case at $m_\pi R \gg 1$ we deal with a slow rotation for $\Omega \sim \Omega_{c1}^{\mathrm{id}}(1)$. For a single vortex with $\nu = 1$ we have $\delta\mathcal{E}^{(1)} \simeq -[\Omega - \Omega_{c1}^{\mathrm{id}}(1)]N_\pi$. For ν_{tot} single vortices, each with $\nu = 1$, the energy gain is $\delta\mathcal{E} = \nu_{\mathrm{tot}}\delta\mathcal{E}^{(1)}$. For $\Omega > \Omega_{c1}^{\mathrm{id}}$ at increasing rotation frequency individual vortices may form lattice. For $\Omega < \Omega_{c1}$, production of vortices is energetically not profitable. However, if a vortex appeared by a reason, it would survive due to conservation of ν.

5. Self-interacting complex scalar field in rotating system

Equation of motion, boundary conditions, energy. For $\lambda \neq 0$, $V_0 \simeq const$, using Eq. (3) in the dimensionless variable $x = r/r_0$, we arrive at equation:

$$(\partial_x^2 + x^{-1}\partial_x - \nu^2/x^2)\chi + \chi - \lambda\phi_0^2 r_0^2 \chi^3 = 0\,, \tag{26}$$

where r_0 is given by Eq. (14). Choosing $\lambda\phi_0^2 r_0^2 = 1$ we have

$$\phi_0 = \sqrt{(\bar{\mu}^2 - m_\pi^2)/\lambda} \times \theta((\bar{\mu}^2 - m_\pi^2)/\lambda), \tag{27}$$

being the solution of Eq. (26) at $x \to \infty$ satisfying condition $\chi(x \to \infty) \to 1$.

For case of the rotating vacuum, for $\Omega - \Omega_c^\pi \sim \Omega_c^\pi$, $V_0 \ll m_\pi$, cf. Eq. (30) below, we have $r_0 \ll R$ for $m_\pi R \gg 1$. For the pion gas at a low density and small Ω and V_0, from Eq. (4) it follows that $n_\pi \simeq 2\bar{\mu}(\bar{\mu}^2 - m_\pi^2)/\lambda$, and we have $\bar{\mu} \simeq m_\pi + n_\pi\lambda/(4m_\pi^2)$ for $n_\pi \ll m_\pi^3/\lambda$ and thus $r_0 \simeq \sqrt{2m_\pi/(n_\pi\lambda)}$ in this case and $r_0 \ll R$ for $m_\pi R \gg 1$ and $r_0 \ll R/\nu$ for $c_1 \ll 1$.

There exist asymptotic solution of Eq. (26): $\chi \propto x^{|\nu|}$ for $x \to 0$ and $\chi = 1 - \nu^2/(2x^2)$ for $x \gg \nu$. Then the field ϕ is expelled from the vortex core and the equilibrium value (27) is recovered at $r \gg \nu r_0$. The solution is modified for $R - r \sim r_0$ to fulfill condition $\chi(R/r_0) = 0$.

To clarify how to fulfill the boundary condition at $r = R$ we can solve Eq. (26) employing the variable $y = (r - R)/r_0$, $x = y + R/r_0$, now at boundary conditions $\chi(y \to -\infty) = 1$ and $\chi(y = 0) = 0$. At $\nu r_0 \ll R$ for typical dimensionless distances $y \sim 1$, the angular momentum term, $\sim \nu^2 r_0^2/R^2 \ll 1$, and the curvature term, $\sim r_0/R \ll 1$, can be dropped, which means that geometry can be considered as effectively one-dimensional one, cf. a similar argumentation employed in [19]. Then appropriate solution gets the form

$$\phi = -\phi_0 e^{i\nu\theta} \mathrm{th}[(r - R)/(\sqrt{2}r_0)],\ \ \nu r_0 \ll R\,, r < R\,. \tag{28}$$

For $V_0 \simeq const$ and for $(\bar{\mu}^2 - m_\pi^2)/\lambda > 0$ using (6), (2), (26) we find

$$\mathcal{E}_{\pi,V} \simeq 2\pi d_z \int_0^R r dr \left[-\tfrac{(\bar{\mu}^2 - m_\pi^2)^2 \chi^4}{2\lambda} + \tfrac{2\mu\tilde{\mu}(\bar{\mu}^2 - m_\pi^2)\chi^2}{\lambda} + n_p V_0 \right]. \tag{29}$$

Rotating vacuum. In this case $\mu = \bar{\mu} - \Omega\nu - V_0 = 0$. For $n_p = 0$ only first term in square brackets (29) remains. So, production of the vortex field becomes energetically favorable, $\mathcal{E}_{\pi,V}(\Omega) < 0$, for $\Omega\nu + V_0 > m_\pi$, i.e. for

$$\Omega > \Omega_c^\pi = (m_\pi - V_0)/\nu\,, \quad c_1 > (m_\pi - V_0)/m_\pi > 0\,. \tag{30}$$

Note that Ω_c^π approximately coincides with Ω_c given above by Eq. (19) derived for $\lambda = 0$ and $c_1 \ll 1$, but differs from (21). On the other hand the asymptotic solution $\chi = 1 - \nu^2/(2x^2)$ works only for $R \gg \nu r_0$, being valid for $\Omega - \Omega_c^\pi \sim \Omega_c^\pi$, $c_1 \ll 1$, as well as for Ω near $1/R$ and $V_0 \gg V_c$.

Non-ideal gas with fixed particle number in rotating system. Now let us consider non-ideal pion gas at $T = 0$ with dynamically fixed particle number at the condition $\mu \simeq m + O(n_\pi/m^2) \gg \Omega\nu + V_0$. This case is similar to that occurs for cold atomic gases, and He-II when $\mu \simeq m_{\rm He}$ and $\Omega\nu + V_0 \ll m_{\rm He}$.

In presence of the rotation, in the rotation frame, using the asymptotic solution $\chi = 1 - \nu^2/(2x^2)$ of the equation of motion for $x \gg \nu$ we find that the energy balance is controlled by the kinetic energy of the vortex, $\mathcal{E}^{(1)}_{\rm kin}$, and the rotation contribution $L\Omega$ extracted from the first two terms in squared brackets (29). The same consideration can be performed in the laboratory frame employing Eq. (9). In the latter case the kinetic energy associated with the single vortex line with the logarithmic accuracy is given by

$$\mathcal{E}^{(1)}_{\rm kin} \simeq \int d^3X|\nabla\phi|^2 = \ln(\widetilde{R}/r_0)d_z\pi\nu^2 n_\pi/\bar{\mu}. \tag{31}$$

At large distances r we cut integration at $r \sim \widetilde{R} \gg r_0$, $\widetilde{R}$ being the transversal size of the vessel R in case of the single vortex line with the center at $r = 0$, and the distance R_L between vortices in case of the lattice of vortices. At small distances integration is naturally cut at $r \sim r_0$.

Let us consider the system at approximately constant density n_π. Then from the condition $\mathcal{E}^{(1)}_{\rm kin} - \vec{L}\vec{\Omega} < 0$ we find that the first vortex filament appears (together with the anti-vortex) for

$$\Omega > \Omega^\lambda_{c1}(\nu) = \nu\ln(R/r_0)/(R^2\bar{\mu}), \tag{32}$$

and for a low density and slowly rotating gas $\bar{\mu} \simeq \mu \simeq m + O(n_\pi/m^2)$. Note that Eq. (32) differs only by a logarithmic pre-factor from a similar expression Eq. (24) valid for the ideal gas. We should put $\nu = 1$ and take $\widetilde{R} \sim R$, getting the minimum value of Ω^λ_{c1}. In case $R \gg r_0$, following (32) we have $\Omega^\lambda_{c1}R \ll 1$ for a slow rotation, $c_1 \ll 1$.

Under action of fluctuations the vortex lines may form spirals and rings, cf. [11, 15]. Vortices may also form a lattice and then the system mimics rotation of the rigid body characterized by the linear velocity $v_{\rm rig} = \Omega R < 1$. In case of a vortex lattice we get [10, 15]:

$$N^{\rm rig}_v\kappa = n_v\pi R^2\kappa = 2\pi R\cdot\Omega R, \quad \kappa = 2\pi\nu/\bar{\mu}. \tag{33}$$

Here $N^{\rm rig}_v = R^2/R^2_L$ is the total number of vortices inside the vessel of the internal radius R, which should be formed at given Ω in order the interior of the vessel would rotate as a rigid body together with the walls, and $n_v = 1/(\pi R^2_L)$ is the corresponding number of vortices per unit area. Thus distance between vortices $R_L = \sqrt{\nu}/\sqrt{\bar{\mu}\Omega}$, decreases with increasing Ω.

The energy gain due to the rigid-body rotation of the lattice of vortices mimicking the rotation of the vessel is given by [10],

$$\delta\mathcal{E} \simeq N^{\rm rig}_v[\mathcal{E}^{(1)}_{\rm kin}(R_L) - L^v(R_L,\nu)\Omega]. \tag{34}$$

This result is obtained within a simplifying assumption of a uniform distribution of vortices [20]. A more accurate result computed for the triangular lattice [17, 21], differs only by a factor $\frac{\pi}{2\sqrt{3}} \simeq 0.91$ from that found for the uniform approximation. Also, following simplifying consideration of Ref. [10] we disregarded a small difference of the rotation angular velocity of the vortex lattice, ω, from that of the vessel Ω. For $Rm \gg 1$ this difference proves to be a tiny quantity, cf. [15]. Minimization of (34) yields

$$N_v^{\rm rig} L_v(R_L, \nu) = N_v^{\rm rig} \pi n_\pi \nu^2 d_z / (2\bar{\mu}\Omega) \,. \tag{35}$$

Setting (35) to (34) and using (33) we obtain the equilibrium energy

$$\delta\mathcal{E} \simeq n_\pi \nu \Omega \pi R^2 d_z [\ln(R_L/r_0) - 1/2] \,. \tag{36}$$

Minimum of $\delta\mathcal{E}$ corresponds to $\nu = 1$.

Thus, we demonstrated that at the rotation frequency $\Omega > \Omega_{c1}^{\lambda}$ in the rotating vessel filled by a pion gas (in our case at $T = 0$) there may appear charged pion vortices, which at subsequent increase of Ω (for $\Omega > 2\Omega_{c1}^{\lambda}$) may form the lattice mimicking the rigid-body rotation.

With a further increase of the rotation frequency the lattice can be destroyed. The minimal distance $R_L \sim r_0$ at a dense packing of vortices corresponds to the number of vortices per unit area $n_v \sim 1/(\pi r_0^2)$ in Eq. (33) and the maximum rotation frequency is $\Omega \simeq \Omega_{c2} \sim 1/(r_0^2 m)$. For $\Omega > \Omega_{c2}$, the ϕ vortex-state should disappear completely. Note that for an extended rotating system the value $\Omega_{c2} R \gg 1$, however now it may not contradict to causality since the system may consist of independently rotating vortices. Note also that in cold atomic gases breakup of lattice occurs for $\Omega > \Omega_h \sim \Omega_{c2} r_0/R \ll \Omega_{c2}$ when in the center of vessel arises a hole, cf. [17].

6. Some consequences

Rotating supercharged nuclei and nuclearites. Let us consider a supercharged nucleus or a piece of nuclear matter (nuclearite) of a large atomic number $A \simeq 2Z$ for $n_p = n_0/2$, Z is the proton charge of the nucleus and n_0 is the nuclear saturation density, $n_0 \simeq m_\pi^3/2$. For $V_0 \sim Ze^2/R > m_\pi$ the ground state π^- energy level reaches zero and in reactions, e.g. $n \to p + \pi^-$, there may appear the charged π^- condensate. For $Z|e^3| \gg 1$ the charge of protons is screened by π^- condensate at least to the value of a surface charge $Z_s \lesssim Z/(Z|e^3|)^{1/3}$, cf. [19]. The surface energy term is small and the total energy is given by $\mathcal{E} \simeq \mathcal{E}_{\pi,V} + \mathcal{E}_A$, $\mathcal{E}_A \simeq -32Z$ MeV. Let for simplicity $\lambda \to 0$ and $m^*(n_0)$ is the effective mass of the pion in muclear matter at $n = n_0$. Then in case of a resting nuclearite one obtains $\mathcal{E} \simeq (m^* - 32\,\mathrm{MeV})\mathrm{Z}$. Thus, if $m^*(n_0)$ were $< 32\,\mathrm{MeV}$ or if there existed spinless charged bosons of such a mass, there would exist nuclearites and nuclei-stars bounded by nuclear and electromagnetic forces, cf. [22, 23]. However, in spite of the attractive pion-nucleon interaction, pions have a larger effective mass at $n \sim n_0$, cf. [23, 24].

In case of a rotating supercharged nucleus, using (20), (22), for $\epsilon_{1,\nu} = \mu = 0$, and for $\sqrt{m_\pi R} \gg c_1 \gg 1$ we have $V_0 \simeq m_\pi(1 - \Omega R)c_1$ and

$$\mathcal{E} - \mathcal{E}_{in} \simeq [m_\pi(1 - \Omega R)c_1 - 32\mathrm{MeV}]Z. \tag{37}$$

Vortex condensate state appears for $\mathcal{E} - \mathcal{E}_{in} < 0$. This estimate is changed only a little in a realistic case, $\lambda \sim 1$ within the $\lambda|\phi|^4$ model. Thus a charged pion supervortex should be formed at least for Ω close to $1/R$. Additionally, we should notice that the rotating charged nucleus forms a magnetic field, which presence still improves conditions for formation of the supervortex [15].

Thus in case of a rotating cold piece of the nuclear matter it might be profitable to form a charged pion vortex field, which will for a while stabilize the system. The kinetic energy of such a rotating nuclear system is then lost on a long time scale via a surface electromagnetic radiation. For very large number of baryons, A, such a radiation is strongly suppressed, e.g. pulsars radiate their energy during $\gtrsim 10^6$yr.

Pion vortices in heavy-ion collisions. In heavy-ion collisions at LHC, RHIC conditions typical parameters of the pion fireball estimated in the resonance gas model [25] are: the temperature $T \simeq 155$ MeV, the volume is 5300fm^3, the $\pi^\pm$ density is $n_\pi \simeq m_\pi^3$ and we estimate the electric potential as $V_0 \sim Ze^2/R \sim 0.2m_\pi$ for central collisions, where Z is now the charge of the fireball. For peripheral collisions typical values of V_0 can be larger.

Estimates performed for peripheral heavy-ion collisions at $\sqrt{s} = 200$ GeV give for the rotation angular momentum values $L_z \lesssim 10^6$ that yields $\nu \lesssim 10^3$. Measured global polarization gave for vorticity $\Omega_{\rm exp}(200{\rm GeV}) \simeq 0.05{\rm m}_\pi$, cf. [2, 9]. Taking for size of the overlapping region of colliding nuclei $R = 10$ fm we get $\Omega < \Omega_{\rm caus} = 1/R \simeq 0.14m_\pi$. After the chemical freeze out up to the thermal freeze out, at $T_{\rm th} < T(t) \lesssim m_\pi$, the pion number can be considered dynamically fixed. Estimates show that $T_{\rm BEC} > T_{\rm th}$, where $T_{\rm BEC}$ is the critical temperature of the pion Bose-Einstein condensation [3, 5, 7]. Thus we may expect $\Omega\nu \gg m_\pi$. A rough estimate yields that $\Omega_{c1} < \Omega_{\rm exp}(200{\rm GeV})$, whereas $\Omega_{c2} \sim m_\pi^2$. Thus the fireball can be stabilized for a while by the formed pion supervortex or the lattice of vortices. Also at these conditions one may think about occurrence of the charged kaon vortices.

It is believed that the spin polarization of particles emitted in heavy-ion collisions is induced by the coupling of the angular momentum produced by colliding nuclei and the spin of particles distributed in the matter. Nucleons participate in production of strange particles, e.g., Λ hyperons. The polarization of the Λ is measured, cf. [26]. Being formed, pion vortices absorb a part of the angular momentum. At the freeze out they return part of the angular momentum back to baryons affecting Λ polarization. Also, pion and baryon momentum distributions for particles involved in the vortex structures should be different from the ordinary thermal distributions.

Superfluidity of baryon Cooper pairs and boson condensates in neutron stars. Nucleons in pulsars form neutron-neutron and proton-proton Cooper pairs, playing a role of boson excitations, and $\bar{\mu} \simeq 2m_N$, where m_N is the nucleon mass. Vortices in nucleon superfluids form the lattice, which mimics the rigid-body rotation of the matter. Besides the nucleons, the hyperons may appear in the interiors of neutron stars with the mass $M \gtrsim 1.4M_\odot$, forming Cooper pairs. Taking $\nu = 1$ and using Eq. (33) one gets estimation $n_v \simeq 6.3 \cdot 10^3 (P/{\rm sec.})^{-1}$ vortices/cm^2 provided rotation period P is measured in seconds, cf. [27]. Then for the Vela pulsar, $P \simeq 0.083$ sec, the distance between vortices is $4 \cdot 10^{-3}$ cm. The charged pion condensate superfluid also can be formed in neutron star interiors [12]. Employing $\bar{\mu} \simeq m_\pi$ we estimate the density of pion vortices as $n_v \simeq 5 \cdot 10^2 (P/\text{sec.})^{-1}$ vortices/cm^2. Similarly, the kaon [28] and $\rho^\pm$ condensates [29] may form lattice structures.

Rotating vessel. Let $n_p = 0$. In case when an ideal rotating vessel is placed inside a cylindrical co-axial charged capacitor (with plates placed at $r = R_{ex}$ and $r = R_{in}$ for $R_{ex} > R_{in} > R_>$) when Z_{in} is the charge placed on the internal surface of the capacitor and $-Z_{ex}$ is the charge of the external surface, the electric field between plates is $E(r > R_{in}) = -2Z_{ex}/(rd_z)$ and $V(r = R_{in}) = R_{in}E(R_{in})\ln(R_{ex}/R_{in})$. For $\rho_\pi d_z \pi R^2 \ll Z_{ex}$ we have $V(r < R) \simeq V(r = R_{in})$. For a surface charge density $\sigma \sim 10^4$v/cm and $R_{in} \sim 10$ m, $R_{ex} \gtrsim (2{-}3)R_{in}$, one gets $V_0 > m_\pi$ and the supervortex arises even for $c_1 \ll 1$.

In case of the pion gas, with the help of Eq. (33) we may estimate number of vortices in the lattice at the rigid-body rotation $N_v = \bar{\mu}\Omega R^2/\nu < \bar{\mu}(R/R_>)^2 R_>/\nu$, for $\Omega < 1/R_>$, $1/R_> = 3 \cdot 10^6$Hz for $R_> = 10$m. Then, with $R = 1$cm and $\bar{\mu} \sim m_\pi$ we estimate $N_v < 10^{10}/\nu$, $R_L \gtrsim \sqrt{\nu/\Omega[\text{Hz}]}10^{-2}$cm.

Formation of vortices in magnetic field. In [13] rotation of the vacuum of non-interacting charged pions was considered in presence of a strong external uniform constant magnetic field H. The number of permitted states is given by $N = |e|HS/(2\pi) = |e|HR^2/2$ and for $|e|H < 2/R^2$ we have $N < 1$. Thus results [13] do not describe the case $H \to 0$, which we have studied above.

Uniform magnetic field inside the rotating ideal vessel can be generated, if the vessel is put inside a solenoid or if we deal with the charged rotating cylindrical capacitor. In the latter case simple estimate shows that for $R = 1$ cm it is sufficient to switch on a tiny external field $|e|H > 10^{-8}$G in order to get $N > 1$ and thereby to overcome the problem with absence of the solution $\mu = 0$ of Eq. (16) at $V_0 = 0$. Now, at $H \neq 0$, $N > 1$, dispersion equation renders $\epsilon_{1,\nu} = -\Omega\nu - V_0 + \sqrt{m^2 + |e|H}$ and instead of Eq. (17) we obtain

$$\nu\Omega_c^H = \nu\Omega(\epsilon_{1,\nu} = 0) = -V_0 + \sqrt{m^2 + |e|H}\,, \tag{38}$$

and $\nu\Omega_c^H \simeq -V_0 + m$ for $|e|H \ll m_\pi^2$, compare with Eq. (19), the latter is valid only for $c_1 \ll 1$. The dependence on R disappeared, cf. [13]. The degeneracy factor $0 < \nu \leq N$. Fields $H \lesssim (10^5 - 10^7)$G can be generated at the terrestrial laboratory conditions. For $|e|H \sim 10^6$ G at $R = 1$ cm we have $N \sim 10^{14}$ and $\nu \lesssim N$ for $c_1 \lesssim 10$ and $\Omega_c^H \lesssim 10^9$ Hz.

Injection of the proton gas in rotating vessel. In absence of the capacitor, in case of the rotation of the charge neutral empty vessel, in which an amount of heavy positively charged particles is injected (e.g. protons), the positive charge density n_p can be compensated by the produced negatively charged pion vortex field, i.e. $|n_\pi| = \bar{\mu}|\phi|^2 \simeq n_p$. Maximum value of $|\phi|^2$ at $m_\pi^2 \gg eH \sim Ze^2\Omega/R \gg 1/R^2$ corresponds to $\bar{\mu} \simeq m_\pi$, and thus the minimum of the pion supervortex energy in the rotation frame is given by $\mathcal{E}_\pi = (m_\pi - \Omega\nu)Z$ and it becomes negative for $\Omega > m_\pi/\nu$, for $c_1 > 1$.

7. Concluding remarks

In this work within the $\lambda|\phi|^4$ model we studied possibilities of the formation of the charged $\pi^{\pm}$ vortex fields in various systems: in the rotating cylindrical empty vessel; in the vessel filled by the charged pion gas at the temperature $T = 0$ (Bose-Einstein condensate) with a (dynamically) fixed particle number; and in case of rotating nuclear systems. In case of the vessel filled by a pion gas at $T = 0$, an analogy was elaborated with cold Bose gases and the condensed ^{4}He. Various applications of the results were discussed.

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